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# Robustness Of Deep Neural Networks Using Trainable Activation Functions

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*Dedicated to  
Donald Knuth*



## Abstract

This document is an example which shows the main features of the  $\text{\LaTeX} 2_{\epsilon}$  class `sapthesis.cls` developed by with the help of GuIT (Gruppo Utilizzatori Italiani di  $\text{\TeX}$ ).



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# Chapter 1

## Introduction

### 1.1 Intriguing Properties of Neural Networks

Here we informally state the problem of adversarial attacks in ML models especially wrt to Neural Networks. Why is it of fundamental importance for the progress of the field from both practical (nns cant yet be deployable in critical scenarios for such reasons ) and theoretical (Madry arguments around interperatability and robustness) perspectives

### 1.2 Smooth Activation Functions and Robustness

Recently a link has been proposed between activation functions and the robustness of Neural Networks (Smooth Adversarial Training). In particular, authors showed how they managed to improve the robustness by replacing the traditional Rectified Linear Units activation functions with smoother alternatives such as ELUs, SWISH, PReLU

Building up from this result we thought we could find benefits by leveraging recently proposed smooth trainable activation functions called Kernel Based Activation Functions (Scardapane et al.), which already showed great results in standard tasks, in the context of adversarial attacks.

### 1.3 Structure of the Thesis

Description of the remaining chapters



## Chapter 2

# Fundamentals

### 2.1 Deep Neural Networks

Broadly speaking, the field of Machine Learning is the summa of any algorithmic methodology whose aim is to automatically find meaningful patterns inside data without being explicitly programmed on how to do it. Well known examples are: Search Trees (ref.), Support Vector Machines (ref. ), Clustering (ref.) and, more recently, Neural Networks (ref. ). During the last two decades Neural Networks gained a lot of attention for their outstanding performances in different tasks like image classification (ref. ImageNet, over human level), speech and audio processing(ref ).

#### 2.1.1 Definition

Neural Networks (NNs) are often used in the context of Supervised Learning where the objective is to model a parametric function  $f_\theta: \mathcal{X} \rightarrow \mathcal{Y}$  given  $n$  input-output pairs  $S = \{(\mathbf{x}_i, y_i)_{i=1}^n\}$  with  $\mathbf{x}_i \in \mathcal{X}$  and  $y_i \in \mathcal{Y}$  such that

$$f_\theta \sim f$$

where  $f$  is assumed to be the real input-output distribution that we want to learn. In plain words, this means that we want to find the best set of parameters  $\theta^*$  for the model such that, for any unseen input  $x_{new}$  we have that  $f_{\theta^*}(x_{new})$  is as close as possible to  $f(x_{new})$

For the sake of explanation, assume the input, which in practice can be very complex and unstructured e.g. made of: graphs, text, sounds, ecc, to be embedded in an input space  $\mathcal{X} = \mathbb{R}^d$ . The simplest form of a neural network is then given by

$$f_W(x) = \sigma(Wx + b)$$

where the parameters of the network are the elements of a  $u \times d$  matrix  $W$  and a  $u$ -dimensional vector called bias. The last element in this picture is the  $\sigma$  function applied at the end which consists of a non-linear function and is the key component to introduce non linearity in NNs allowing them to model highly non-linear functions. We call it *activation function*

### **2.1.2 Training**

### **2.1.3 CNNs: Convolutional Neural Networks**

### **2.1.4 From Neural Networks to Deep Neural Networks**

## **2.2 Adversarial Examples Theory**

Formal definition of what is an adversarial attacks plus currently well known attacks

## **2.3 Defenses**

Review of the literature on defenses to improve robustness: provable robustness, adversarial training

## **2.4 Kernel Based Activation Functions**

## Chapter 3

# Related Works

### 3.1 K-Winners Take All

### 3.2 Smooth Adversarial Training





## Chapter 4

# Solution Approach

Comparing the activations's distributions for different activation functions (ReLU, KWTa, Kafs) seem to suggest Kafs might be good candidates to improve model robustness

### 4.1 Lipschitz Constant Approach

On the limitations of current Lipschitz-Constant based approaches especially when involving Kafs

### 4.2 Fast is Better than Free Adversarial Training

Adversarial training (Madry et al.) and current methods to improve the efficiency (Fast is better than free)



## Chapter 5

# Evaluation

### 5.1 VGG Inspired Architectures Results

### 5.2 Explofing Gradients with KafNets

The exploding gradients problem with KafResNet, why is it happening? (still to clarify)

### 5.3 ResNet20 Inspired Architectures Results



## Chapter 6

# Future Works

Different Kernels, resolve the exploding gradient problem and scale to ImageNet  
Perform more adaptive attacks to assess the robustness of kafresnets as is the current standard (Carlini et al.)



## Chapter 7

# Conclusions

This thesis tries to add to the bag of evidences in literature that smoother architectures might benefit improvements in adversarial resiliency

