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Are exchange rate changes normally distributed?

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Abstract

Although assumed to be normal, daily returns in reality are leptokurtic. Monthly returns, however, are shown to be more normally distributed. Evidence was found of dependence on consecutive daily price changes, which may be an explanation for the leptokurtosis.

JEL classification: C12

1. Introduction

The distribution of price movements is important in economic modeling. If distribution is normal, the variance is an appropriate measure of dispersion. If not, there may be problems with, for instance, option pricing models. Some adjustments in the models may then be necessary.

When examining the daily price changes in stocks and currencies, we can see that there are more observations near the mean and in the tails than is customary with normal distributions. We calculated the kurtosis of selected series of daily price changes, including the Standard and Poor's stock index (SP500) and exchange rates involving the US dollar (USD), the Japanese yen (JPY), the German mark (DEM), and the British pound (GBP). The series cover 12.5 years, from 1980 to 1992. The kurtosis ranges from 4.8 for the yen/pound exchange rate to 6.2 for the Standard and Poor's. In all cases, they deviate significantly from 3. The leptokurtosis was established before by, for example, Fama (1963), Westerfield (1977), So (1987) and Wolff and Reyes (1988).

The reason for daily price changes not being normally distributed may be that traders are not able to interpret new information immediately, and that interpretation takes a number of days. This may indicate dependence of price changes on consecutive days. Another explana-

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tion might be that daily price changes are drawn from several normal distributions (see, for example, Fama and Roll, 1971).

In both cases, price changes that cover more than one day would tend to follow a normal distribution more closely. Whether this is true is examined by using the expression for symmetric stable Paretian distributions.

2. Symmetric stable Paretian distributions

Stable Paretian distributions may be used to describe leptokurtic distributions. They may be defined by the log characteristic function:[1]

$$\log \phi(t) = \mathrm{i}\delta t - \gamma |t|^{\alpha} , \tag{1}$$

where $\delta, \gamma \geq 0$ and $0 < \alpha \leq 2$ are real numbers and i is the imaginary unit $(\mathrm{i}^2 = -1)$. Symmetric stable Paretian distributions are described completely by the parameters δ, γ and α.

- δ is the location parameter. In the series examined δ is the average price change and does not seem to deviate significantly from 0.
- γ is equal to c^{α}, where c is the scale parameter (comparable with the σ of normal distributions).
- α is the characteristic exponent. This parameter determines the shape of the distribution. When $\alpha = 2$, distribution is normal. When α is smaller than 2, the number of observations near the mean and at both extremes of the distribution will be higher than is usual with normal distributions.

The fact that α is characteristic for the distribution can be shown as follows. Suppose that from a series of independent variables (x_n), distributed in a stable Paretian way, N adjacent values are summed to form a new series of independent variables $(x_1' = x_1 + \cdots + x_N;$ $x_2' = x_{N+1} + \cdots + x_{2N}$, etc.). The original log characteristic function changes into

$$\mathrm{i}N\delta t - N\gamma |t|^{\alpha} . \tag{2}$$

Only the location and scale parameters change. They become N times the old ones. The characteristic exponent α, however, remains unchanged.

This property of stable Paretian distributions is used to determine whether the found leptokurtic distributions remain leptokurtic when changes cover more than one day. First, the α of the daily changes is determined. It is expected to be smaller than 2. From the series of daily changes, five new series are formed by summing 2, 4, 8, 16 or 30 adjacent price changes, respectively. When the new series are leptokurtic, α will also be smaller than 2. If they tend to be more normal, α will increase to 2.

[1] In the case of non-symmetric distributions a third term should be added, but this does not affect the following analysis; see Stuart and Ord (1987, p.147) and Mandelbrot (1963).

3. Determination of α

The procedure developed by Fama and Roll (1968, 1971) is followed to determine α. The principal idea is that for every distribution, there is a unique α, which can be determined from close observation of the tails of the distribution. After all, the thickness of the tails is indicative of the leptokurtosis. Fama and Roll (1968) proved that the estimation of α is robust and reliable when looking at the observations in the 95th fractile.

First one has to arrange the observed daily price changes from the lowest to the highest value. Then the 95th fractile observation should be standardized:

$$\hat{z} = \frac{x(\alpha, 0.95) - \delta}{c} . \tag{3}$$

where $x(\alpha, 0.95)$ is the 95th fractile observation from the arranged series. Both values, Δ and c, have to be estimated. As mentioned, δ does not deviate significantly from 0. The c can be estimated by using an adjusted average of the 0.72 and 0.28 fractiles. Fama and Roll (1968, p. 822) conclude that the error with this procedure is less than 0.4%:

$$\hat{c} = \frac{1}{2}\frac{1}{0.827}[x(\alpha, 0.72) - x(\alpha, 0.28)] . \tag{4}$$

Now the calculated standardized value, $\hat{z}$, can be compared with the z-values from known standard Paretian distributions. For this, Fama and Roll (1968) provide a table of standard z-values for 12 distributions with different α-values. The α-value of the matching standard distribution is also the α-value of the observed distribution.

4. Results

The procedure described in Section 3 is followed for the six exchange rate combinations and the Standard & Poor's stock index. The results are summarized in Table 1. As expected, the α-values for the original series of daily changes were smaller than 2. However, the α-value increases as the number of summed observations, N, increases from 2 to 30. The conclusion

Table 1
Estimated values for the characteristic exponent α for six exchange rate combinations and the Standard & Poor's stock index, for the original series of daily returns (1), the two-daily returns (2), etc. In the case of a normal distribution, $\alpha = 2$. In this test, adjacent observations were summed

	1	2	4	8	16	30
USD/JPY	1.50	1.58	1.53	1.59	1.89	2
USD/DEM	1.56	1.66	1.78	1.69	2	2
USD/GBP	1.56	1.68	1.73	1.82	2	2
JPY/DEM	1.56	1.57	1.68	1.73	1.79	2
JPY/GBP	1.60	1.58	1.66	1.75	1.62	1.89
DEM/GBP	1.53	1.49	1.51	1.49	1.44	1.51
SP500	1.52	1.74	1.78	1.89	1.55	2

Table 2
Estimated values for the characteristic exponent α for six exchange rate combinations and the Standard & Poor's stock index, for daily returns (1), two-daily returns (2), etc. In the case of a normal distribution, $\alpha = 2$. In this test, observations were randomly drawn from the original series before summation

	1	2	4	8	16	30
USD/JPY	1.50	1.76	1.77	1.98	2	2
USD/DEM	1.56	1.72	1.98	2	2	2
USD/GBP	1.56	1.78	1.78	2	2	2
JPY/DEM	1.56	1.65	1.87	2	2	2
JPY/GBP	1.60	1.71	1.88	2	2	2
DEM/GBP	1.52	1.70	1.82	2	2	2
SP500	1.52	1.75	2	1.92	1.86	2

should be that the distributions of monthly changes are more normal than the original distribution of daily changes. The only clear exception is the DEM/GBP exchange rate, where α remains at the original level. For the JPY/GBP the α-value increases at a slow rate, and does not reach 2, even when the number of summed observations is 30.

The exceptional behavior of the DEM/GBP exchange rate might be attributed to the Exchange Rate Mechanism of the European Monetary System. This mechanism limited the price changes for the DEM/GBP exchange rate during a considerable part of the observed period. This may have caused some dependence between daily price changes. In order to test the lack of independence of daily changes, a modified procedure was followed.[2] The modification relates to the summing of the observed daily changes. In the previous procedure adjacent values were summed to form new series of changes over 2, 4, 8, 16 and 30 days, respectively. In the modified procedure values were drawn randomly from the original series before summing. If the price changes of consecutive days are independent, the modified procedure will have no effect on the results. If they are not independent, the α-values of the randomized series will increase more rapidly to 2. The results are shown in Table 2.

With the randomized experiment all series show a more marked increase in α-values. Even the DEM/GBP exchange rate now seems to be more normally distributed. Apparently the daily price changes are not completely independent.

5. Conclusion

For both stock prices and exchange rates, daily price changes are not distributed normally. There are more observations around the mean and in the tails than is customary with normal distributions. In this paper it has been shown that changes covering more than one day are more normally distributed than daily changes. With the exception of the mark–pound exchange rate, monthly price changes seem to be drawn from a normal distribution.

An explanation for the deviating mark–pound exchange rate was found in the lack of independence of consecutive price changes. A reason may be that during a considerable

[2] This procedure is also followed by Hall et al. (1989) for futures price changes.

period both currencies participated in the European Exchange Rate Mechanism. Some degree of dependence in consecutive price changes was also found with the other exchange rates and the Standard & Poor's stock index.

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