# Verification of Timed Systems

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# The Satisfaction Problem

Given a system $S$ and a property $\varphi$ show that

$$\underset{\text{semantics of } S}{\underbrace{[\![S]\!]}} \qquad \underset{\models :: \text{Models} \times \text{Formulae} \to \{\mathtt{tt}, \mathtt{ff}\}}{\underbrace{\models}} \qquad \varphi$$

The choice of which logical language to use for writing $\varphi$ depends on the underlying computational paradigm

# A Logical Language for Timed Systems

Variant of Computation Tree Logic with two types of formulae

$\boxed{\text{description of state and path properties}}$

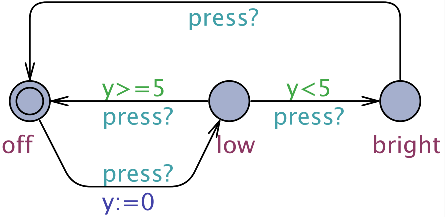
# State formulae

**Grammar**

$\Psi ::= \ell \mid c \mid \texttt{deadlock} \mid \texttt{not}\ \Psi \mid \Psi\ \texttt{or}\ \Psi \mid \Psi\ \texttt{and}\ \Psi$

We can thus express current locations $\ell$, clock constraints $c \in \mathcal{C}(C)$, and the presence of deadlocks

# Back to the lamp




**Exercise**

Write formulae for the following statements

1. The lamp is `low`
2. Not `off` and $y > 25$
3. If it is `low` or `bright` then $y \leq 3600$

# Path Formulae

**Grammar**

$$\Pi \;::=\; A\Box\,\Psi \mid A\Diamond\,\Psi \mid E\Box\,\Psi \mid E\Diamond\,\Psi \mid \Phi \rightsquigarrow \Psi$$

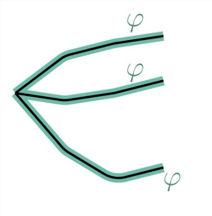
where

- $A$, $E$ quantify (universally and existentially, resp.) over paths
- $\Box$, $\Diamond$ quantify (universally and existentially, resp.) over states in a path

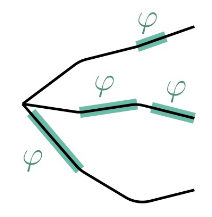
Paths can be seen as possible executions

# Path Formulae pt. II

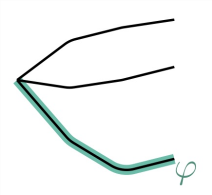
$A\Box\,\varphi$



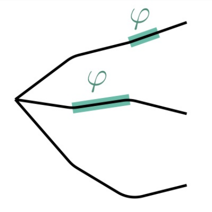

$A\Diamond\,\varphi$




$E\Box\,\varphi$




$E\Diamond\,\varphi$



# Safety Properties

**$A\Box\varphi$ and $E\Box\varphi$**

Something bad will never happen

Examples:

- A nuclear reactor's temperature will never exceed a certain threshold
- We will never reach deadlock
- There is at least one execution in which we never reach deadlock

# Reachability Properties

$E\lozenge\, \varphi$

Something good can happen

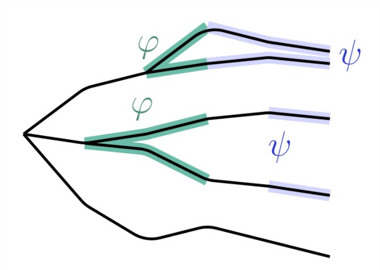
Examples:

- All adventurers reach the other side.
- All adventurers reach the other side in $\leq 17$ minutes.
- ...

# Path Formulae pt. III

For all paths if $\varphi$ holds at some point then $\psi$ will also hold later on

$$\varphi \rightsquigarrow \psi$$



# Liveness Properties

**$A \Diamond \phi$ and $\phi \rightsquigarrow \psi$**

Something good will eventually happen or
if something happens then something good will eventually happen

Examples:

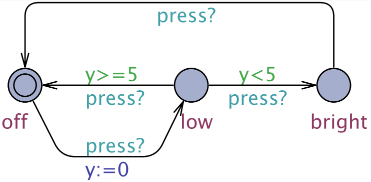
- Always when pressing the on button the television will eventually turn on
- If the philosopher requests a fork she will eventually get it
- If the plane asks to land it will eventually land

# Exercises

Write the sentences below in CTL

1. The system never enters in deadlock
2. The location $\ell$ is reachable
3. In all executions we reach location $\ell$
4. If we reach location $\ell$ we will inevitably reach location $s$
5. There exists at least one execution where variable `i` is always below or equal 10
6. The two philosophers never eat at the same time

# Back to the lamp




## Exercise

1. The lamp can become bright;
2. The lamp will eventually become bright;
3. The lamp can never be on for more than 3600s;
4. It is possible to never turn on the lamp;
5. Whenever the light is bright, the clock $y$ is non-zero;
6. Whenever the light is bright it will eventually become off.

# Table of Contents

Chapter's Conclusion

# Keypoints

Communicating systems as timed automata

Syntax

Semantics for rigorous analysis and verification

UPPAAL as an important tool of the cyber-physical engineer

# Keypoints

Time as the main physical process . . .

. . . others will appear in the next lectures

Barely scratched the surface . . .

. . . more about the theory of timed automata in the website

# Open Challenges

Quantum communicating systems and computational tools

Reasoning precisely about imprecisions

The thorny Reachability Problem

. . .