

# Time-critical reactive systems (verification)

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# Traces

## Definition

A **timed trace** over a **temporal LTS** is a (finite or infinite) sequence  $\langle t_1, a_1 \rangle, \langle t_2, a_2 \rangle, \dots$  in  $\mathcal{R}^+ \times Act$  such that there exists a path

$$\langle l_0, \eta_0 \rangle \xrightarrow{d_1} \langle l_0, \eta_1 \rangle \xrightarrow{a_1} \langle l_1, \eta_2 \rangle \xrightarrow{d_2} \langle l_1, \eta_3 \rangle \xrightarrow{a_2} \dots$$

such that

$$t_i = t_{i-1} + d_i$$

with  $t_0 = 0$  and, for all clock  $x$ ,  $\eta_0 x = 0$ .

Intuitively, each  $t_i$  is an absolute time value acting as a **time-stamp**.

## Warning

All results from now on are given over an arbitrary **temporal LTS**; they naturally apply to  $\mathcal{T}(ta)$  for any timed automata  $ta$ .

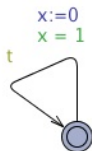
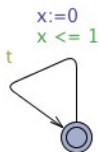
# Traces

Given a **timed trace**  $tc$ , the corresponding **untimed trace** is  $(\pi_2)^\omega tc$ .

## Definition

- two states  $s_1$  and  $s_2$  of a timed LTS are **timed-language equivalent** if the **set of finite timed traces** of  $s_1$  and  $s_2$  coincide;
- ... similar definition for **untimed-language equivalent** ...

## Example



are not **timed-language equivalent**

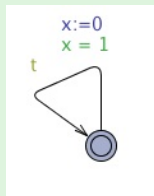
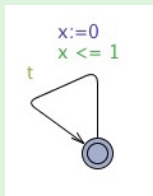
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## Example



are not **timed-language equivalent**

$\langle (0, t) \rangle$  is not a trace of the TLTS generated by the second system.

# Bisimulation

## Timed bisimulation (between temporal LTS)

A relation  $R$  is a **timed simulation** iff whenever  $s_1 R s_2$ , for any action  $a$  and delay  $d$ ,

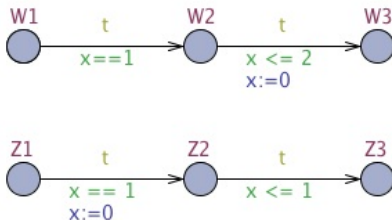
$$s_1 \xrightarrow{a} s'_1 \Rightarrow \text{there is a transition } s_2 \xrightarrow{a} s'_2 \wedge s'_1 R s'_2$$

$$s_1 \xrightarrow{d} s'_1 \Rightarrow \text{there is a transition } s_2 \xrightarrow{d} s'_2 \wedge s'_1 R s'_2$$

And a **timed bisimulation** if its converse is also a bisimulation.

# Bisimulation

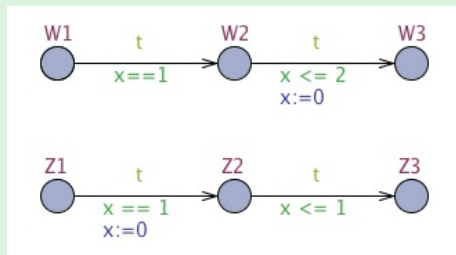
## Example



W1 bisimilar to Z1?

# Bisimulation

## Example



W1 bisimilar to Z1?

$$\langle\langle W1, \{x \mapsto 0\}\rangle, \langle Z1, \{x \mapsto 0\}\rangle\rangle \in R$$

where

$$R = \begin{array}{ll} \{\langle\langle W1, \{x \mapsto d\}\rangle, \langle Z1, \{x \mapsto d\}\rangle\rangle & | d \in \mathcal{R}_0^+\} \cup \\ \{\langle\langle W2, \{x \mapsto d+1\}\rangle, \langle Z2, \{x \mapsto d\}\rangle\rangle & | d \in \mathcal{R}_0^+\} \cup \\ \{\langle\langle W3, \{x \mapsto d\}\rangle, \langle Z3, \{x \mapsto e\}\rangle\rangle & | d, e \in \mathcal{R}_0^+\} \end{array}$$

# Bisimulation

## Untimed bisimulation

A relation  $R$  is an **untimed simulation** iff whenever  $s_1 R s_2$ , for any action  $a$  and delay  $t$ ,

$$s_1 \xrightarrow{a} s'_1 \Rightarrow \text{there is a transition } s_2 \xrightarrow{a} s'_2 \wedge s'_1 R s'_2$$

$$s_1 \xrightarrow{d} s'_1 \Rightarrow \text{there is a transition } s_2 \xrightarrow{d'} s'_2 \wedge s'_1 R s'_2$$

And it is an **untimed bisimulation** if its converse is also a untimed bisimulation.

Alternatively, it can be defined over a modified LTS in which all delays are abstracted on a unique, special transition labelled by  $\epsilon$ .



# Properties: expression and satisfaction

## The satisfaction problem

Given a **timed automata**,  $ta$ , and a **property**,  $\phi$ , show that

$$\mathcal{T}(ta) \models \phi$$

- in which logic language shall  $\phi$  be specified?
- how is  $\models$  defined?

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# Expressing properties: Uppaal

## Uppaal variant of Ctl

- **state formulae**: describes individual states in  $\mathcal{T}(ta)$
- **path formulae**: describes properties of paths in  $\mathcal{T}(ta)$

# Expressing properties: Uppaal

## State formulae

Any expression which can be evaluated to a boolean value for a state (typically involving the **clock constraints** used for guards and invariants and similar constraints over integer variables):

$$x \geq 8, i == 8 \text{ and } x < 2, \dots$$

Additionally,

- $ta.\ell$  which tests **current location**:  $(\ell, \eta) \models ta.\ell$  provided  $(\ell, \eta)$  is a state in  $\mathcal{T}(ta)$
- **deadlock**:  $(\ell, \eta) \models \forall_{d \in \mathcal{R}_0^+}. \text{there is no transition from } \langle \ell, \eta + d \rangle$

# Expressing properties: Uppaal

## Path formulae

$$\Pi ::= A \Box \Psi \mid A \Diamond \Psi \mid E \Box \Psi \mid E \Diamond \Psi \mid \Phi \rightsquigarrow \Psi$$

$$\Psi ::= ta.\ell \mid g_c \mid g_d \mid \text{not } \Psi \mid \Psi \text{ or } \Psi \mid \Psi \text{ and } \Psi \mid \Psi \text{ imply } \Psi$$

where

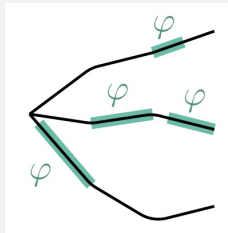
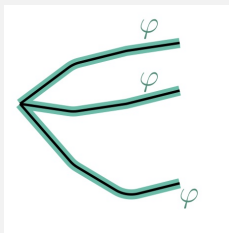
- $A, E$  quantify (universally and existentially, resp.) over **paths**
- $\Box, \Diamond$  quantify (universally and existentially, resp.) over **states in a path**

also notice that

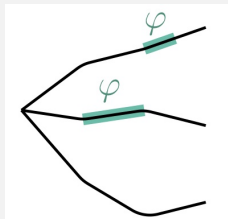
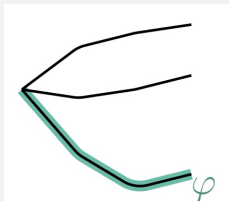
$$\Phi \rightsquigarrow \Psi \stackrel{\text{abv}}{=} A \Box (\Phi \Rightarrow E \Diamond \Psi)$$

# Expressing properties: Uppaal

$A\Box\varphi$  and  $A\Diamond\varphi$

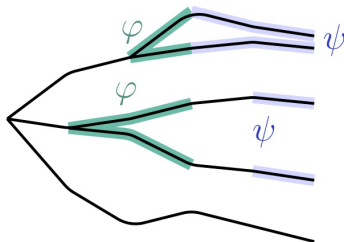


$E\Box\varphi$  and  $E\Diamond\varphi$



# Expressing properties: Uppaal

$\varphi \rightsquigarrow \psi$



# Reachability properties

$E \Diamond \phi$

Is there a path starting at the initial state, such that a state formula  $\phi$  is eventually satisfied?

- Often used to perform sanity checks on a model:
  - is it possible for a sender to send a message?
  - can a message possibly be received?
  - ...
- Do not by themselves guarantee the correctness of the protocol (i.e. **that any message is eventually delivered**), but they validate the basic behavior of the model.



# Safety properties

$A \Box \phi$  and  $E \Box \phi$

Something bad will never happen  
or something bad will possibly never happen

## Examples

- In a nuclear power plant the temperature of the core is always (invariantly) under a certain threshold.
- In a game a safe state is one in which we can still win, ie, will possibly not lose.

In Uppaal these properties are formulated positively: something good is invariantly true.

# Liveness properties

$A \Diamond \phi$  and  $\phi \rightsquigarrow \psi$

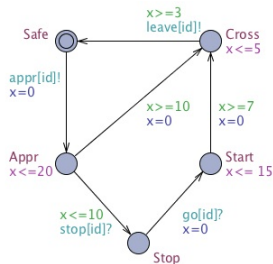
Something good will eventually happen

or if something good happen, then something else will eventually happen!

## Examples

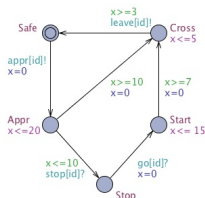
- When pressing the on button, then eventually the television should turn on.
- In a communication protocol, any message that has been sent should eventually be received.

# The train gate example



- $E \leftrightarrow \text{Train}(0).\text{Cross}$   
(Train 0 can reach the cross)
- $E \leftrightarrow \text{Train}(0).\text{Cross}$  and  $\text{Train}(1).\text{Stop}$   
(Train 0 can be crossing bridge while Train 1 is waiting to cross)
- $E \leftrightarrow \text{Train}(0).\text{Cross}$  and  $(\text{forall } (i:\text{id}-t) \ i \neq 0 \text{ imply } \text{Train}(i).\text{Stop})$   
(Train 0 can cross bridge while the other trains are waiting to cross)

# The train gate example



- `A[] Gate.list[N] == 0`  
There can never be  $N$  elements in the queue
- `A[] forall (i:id-t) forall (j:id-t) Train(i).Cross && Train(j).Cross imply i == j`  
There is never more than one train crossing the bridge
- `Train(1).Appr -> Train(1).Cross`  
Whenever a train approaches the bridge, it will eventually cross
- `A[] not deadlock`  
The system is deadlock-free

# Mutual exclusion

## Properties

- **mutual exclusion**: no two processes are in their critical sections at the same time
- **deadlock freedom**: if some process is trying to access its critical section, then eventually some process (not necessarily the same) will be in its critical section; similarly for exiting the critical section

# Mutual exclusion

## The Problem

- Dijkstra's original asynchronous algorithm (1965) requires, for  $n$  processes to be controlled,  $\mathcal{O}(n)$  read-write registers and  $\mathcal{O}(n)$  operations.
- This result is a theoretical limit (proved by Lynch and Shavit in 1992) which compromises scalability.

but it can be overcome by introducing specific **timing constraints**

## Two *timed* algorithms:

- **Fisher's protocol** (included in the Uppaal distribution)
- **Lamport's protocol**

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## Two *timed* algorithms:

- [Fisher's protocol](#) (included in the Uppaal distribution)
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# Fisher's algorithm

## The algorithm

```
repeat
  repeat
    await  $id = 0$ 
     $id := i$ 
    delay( $k$ )
  until  $id = i$ 
  (critical section)
   $id := 0$ 
forever
```

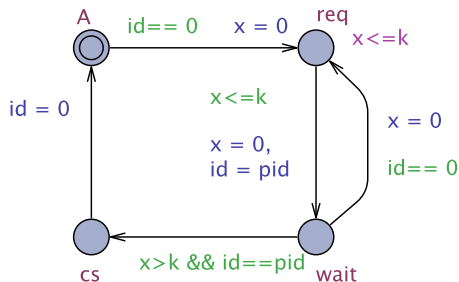


# Fisher's algorithm

## Comments

- One shared read/write register (the variable  $id$ )
- Behaviour depends crucially on the value for  $k$  — the **time delay**
- Constant  $k$  should be **larger than the longest time that a process may take to perform a step while trying to get access to its critical section**
- This choice guarantees that whenever process  $i$  finds  $id = i$  on testing the loop guard it can enter safely its critical section: **all** other processes are out of the loop or with their index in  $id$  overwritten by  $i$ .

# Fisher's algorithm in Uppaal



- Each process uses a local clock  $x$  to guarantee that the upper bound between its successive steps, while trying to access the critical section, is  $k$  (cf. **invariant** in state *req*).
- **Invariant** in state *req* establishes  $k$  as such an upper bound
- **Guard** in transition from *wait* to *cs* ensures the correct delay before entering the critical section

# Fisher's algorithm in Uppaal

## Properties

```
% P(1) requests access => it will eventually wait
P(1).req → P(1).wait
% the algorithm is deadlock-free
A[] not deadlock
% mutual exclusion invariant
A[] forall (i:int[1,6]) forall (j:int[1,6])
    P(i).cs && P(j).cs imply i == j
```

- The algorithm is **deadlock-free**
- It ensures mutual exclusion if the correct timing constraints.
- ... but it is critically sensible to small violations of such constraints: for example, replacing  $x > k$  by  $x \geq k$  in the transition leading to cs compromises both **mutual exclusion** and **liveness**.

# Lamport's algorithm

## The algorithm

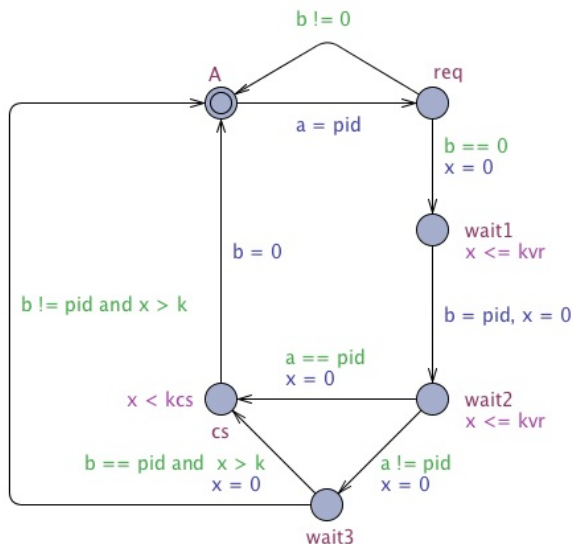
```
start :  $a := i$   
       if  $b \neq 0$  then goto start  
        $b := i$   
       if  $a \neq i$  then delay( $k$ )  
           else if  $b \neq i$  then goto start  
       (critical section)  
        $b := 0$ 
```

# Lamport's algorithm

## Comments

- Two shared read/write registers (variables  $a$  and  $b$ )
- Avoids **forced waiting** when no other processes are requiring access to their critical sections

# Lamport's algorithm in Uppaal



# Lamport's algorithm

## Model time constants:

$k$  — time delay

$kvr$  — max bound for register access

$kcs$  — max bound for permanence in critical section

Typically

$$k \geq kvr + kcs$$

## Experiments

	$k$	$kvr$	$kcs$	verified?
Mutual Exclusion	4	1	1	Yes
Mutual Exclusion	2	1	1	Yes
Mutual Exclusion	1	1	1	No
No deadlock	4	1	1	Yes
No deadlock	2	1	1	Yes
No deadlock	1	1	1	Yes