

BANKSY

A paraconsistent inference engine to support research in age-related macular degeneration

COMPETE2030-FEDER-00892000 - Project 15253

**Project Presentation
LMF meeting, 1 October 2025**



Luís S. Barbosa (www.di.uminho.pt/~lsb/)

Vagueness and contradiction

Modelling contexts in which bivalent reasoning, even if of a probabilistic nature, is not enough, entailing the need to capture

- lack of information (vagueness or uncertainty)
- excess of information (potential inconsistency)

Vagueness is addressed in fuzzy logics,

but potentially contradictory information arises in a number of scenarios.
(e.g. knowledge representation, data integration, etc.)

Vagueness and contradiction

or simply from the need to **articulate different perspectives** on complex information:

local consistency

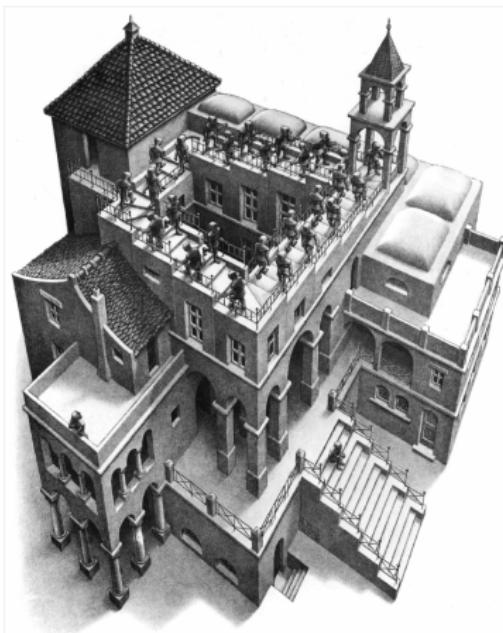


global inconsistency



Why we should care?

Information (reality?) is not what we thought it was ...



M. C. Escher, *Ascending and descending*

Indeed, data is a mined field

- Not only the **values and structure** of data changes but also the **logic** under which this information needs to be understood changes as well
- Informational states may exhibit **potentially inconsistent** (or partially consistent) data, reflecting the **diversity of judgements** (e.g., from different domain experts).
- Moreover, they may be linked **positively** (witnessing e.g. the existence of a relationship) and **negatively** (recording whatever prevents such a relationship)
- Finally, the weights of such transitions are, in most cases, **non-complementary**, opening an inference arena encompassing both classical, vague, and even (controlled forms of) inconsistent reasoning.

Paraconsistent Logic

Originally developed in Latin America in 50's, mainly by F. Asenjo and Newton da Costa, accommodates inconsistency **in a controlled way**, treating **inconsistent information as potentially informative**.

Paraconsistency is the study of logical systems in which the presence of a contradiction does not imply triviality, therefore avoiding the **principle of explosion** according to which any statement can be proven from a contradiction.

Separating

contradiction from **deductive triviality**
inconsistency from **contradiction**
consistency from **absence of contradiction**

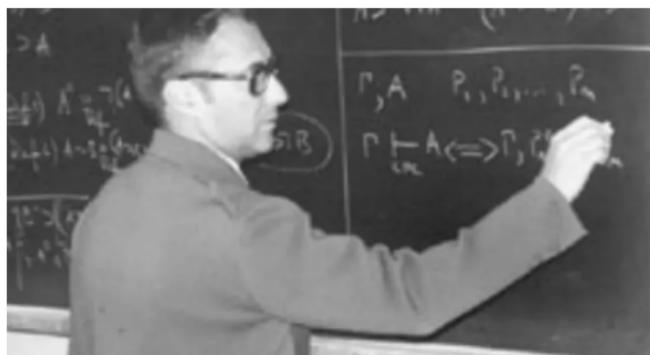
A brief history of an idea

- 1910 [J. Łukasiewicz](#) and [N. Vasiliev](#): denying the law of noncontradiction would lead to still meaningful, although non-Aristotelian logics
- [S. Jaśkowski](#)'s study of empirical theories including contradictory assumptions
- 1958: [Newton da Costa](#) (1929 - 2024) seminal paper

Nota sobre o conceito de contradição.
Anuário da Sociedade Paranaense de Matemática, 2 (1), 1958.

paved the way to the remarkable influence of the [Brazilian School](#)

A brief history of an idea

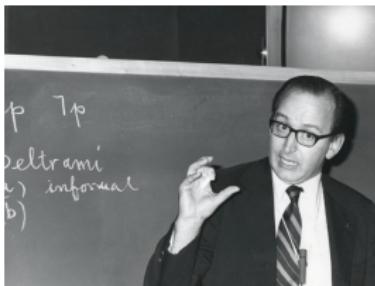


"I decided to do it the other way round: mathematics with contradictions. Existence in mathematics means anything but the absence of contradiction. Contradictions begin to appear at the edges of mathematics. There are always problems."

[Newton da Costa, available from youtube, 2019]

A brief history of an idea

Another pioneer was the Argentinian mathematician and music composer, with a main role in the development of relevance logic,



Florencio Gonzalez Asenjo (1926-2013)

and his seminal paper

A calculus of antinomies
Notre Dame Journal of Formal Logic, Vol. 7, 1966.

A brief history of an idea

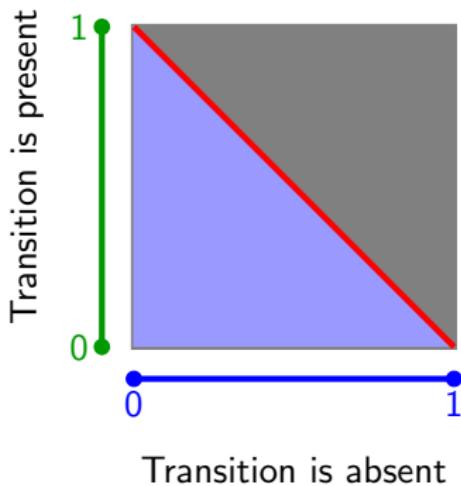
- A main development in the Brazilian School led to [the logic of formal inconsistency](#) (LFI) in which consistency of a sentence is made explicit by a unary operator (W. Carnielli, M. E. Coniglio, J. Marcos)
- 1990: *Mathematical Reviews* added a new entry [03B53](#) entitled [Paraconsistent Logic](#) later expanded to [Logics admitting inconsistency](#)
- ... and [applications](#) pop out: from Philosophy of Science to Mathematics, from Economics to Quantum Mechanics.
- ... and in Computer Science: AI, databases, semantics of concurrency, quantum computation ...

The CIDMA & INESC TEC approach

The approach

- Transition systems with both **positive** and **negative** accessibility relations, with **non complementary weights**:
 - one weighting the possibility of a transition to be present (e.g. the state remaining coherent),
 - the other weighting the possibility of being absent (i.e. becoming unstable)
- used in **software modelling** and as Kripke frames for a **modal logic**

The CIDMA & INESC TEC approach

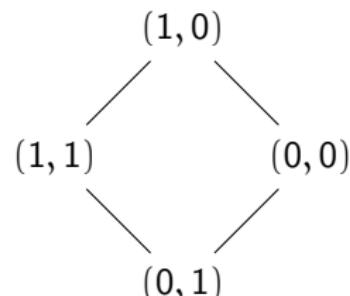


Current research: starting in 2022

- Semantic models and their composition
- Suitable notions of modality and logics
- Equivalence, bisimulation, and approximation
- Structured specification and development
- The algebraic counterpart (generalizing Kleene algebras)
- Paraconsistent automata theory
- Reactivity in paraconsistent transition structures
- ...

Paraconsistent Structures and their Logics

- Generalising Belnap-Dunn *FOUR*
- to PLTS parametric on a iMTL-algebra, and corresponding twisted structure,
- a modal logic with positive and negative modalities, and
- combinators and a multimodal logic



- Cruz, Madeira & Barbosa: [A logic for paraconsistent transition systems](#) *Non-Classical Logics: Theory and Applications*, 2022.
- Cruz, Madeira & Barbosa: [Paraconsistent transition systems](#) LSFA'22 (Logical and Semantic Frameworks with Applications), 2022.
- Barbosa & Madeira: [Capturing qubit decoherence through paraconsistent transition systems](#). Engineering of Quantum Programming , IEEE 2023
- **** Cunha, Madeira & Barbosa: [Paraconsistent transition structures: compositional principles and a modal logic](#)**. *Math. Struc. Comp. Sci.*, Elsevier (in print) 2025

Structured Specification and Development

- Instantiation of Sannella and Tarlecki's stepwise implementation for the structured specification of PLTS from their abstract design down to the concrete implementation stage.
 - Structured specification logic à la CASL
 - Paraconsistent institution parametric on a twisted structure enriched with regular modalities
-
- Cunha, Madeira & Barbosa: Stepwise Development of Paraconsistent Processes TASE'23 (Theoretical Aspects of Software Engineering), 2023.
 - Cunha, Madeira & Barbosa: Structured specification of paraconsistent transition systems FSEN'23 (Fundamentals of Software Engineering), 2023.
 - ** Cunha, Madeira & Barbosa: Specification of paraconsistent transition systems, revisited. *Sci Comp Programming*, 240, Elsevier, 2024

PKAT: The algebraic counterpart

- Study **paraconsistent KAT** to reason about uncertain or inconsistent computations in a (quasi)-equational way
 - made concrete in an algebra of paraconsistent relations
-
- Cunha, Madeira & Barbosa: [Paraconsistent relations as a variant of Kleene algebras](#)
LSFA'24, 2024.

The project

- Aims to develop the **theory** and application of paraconsistent reasoning
- ... driven by a **concrete case-study**: **AMD diagnosis**
- ... and oriented toward the construction and validation of an **inference tool**

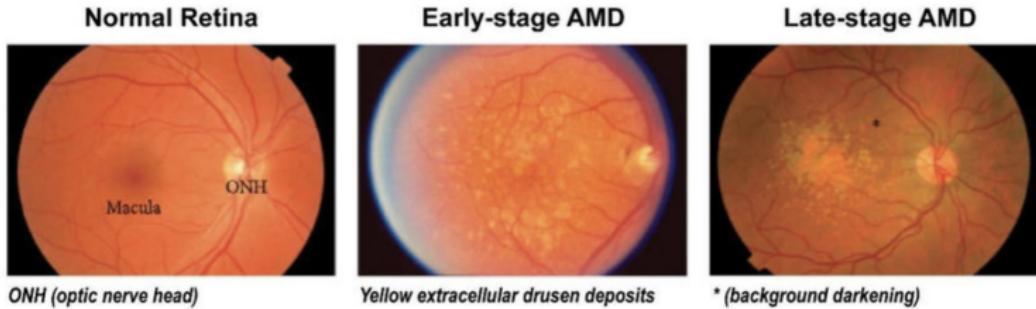
The project success (as well as the possibility of more ambitious follow-ups) **depends crucially** on the tool to be developed and the way the case-study is addressed:

can paraconsistency really help?

Age-related macular degeneration (AMD) diagnosis

A disease of the macula, leading cause of vision loss in people over 55.

- multifactorial disease, with a complex pathophysiology, for which the onset and progression are different, with different risk factors, environmental and genetic, contributing.
- progression not well understood (e.g. different rates/patterns for similar patients and even between the two eyes of the same patient).



Age-related macular degeneration (AMD) diagnosis

- Highly **heterogeneous** data collected in large epidemiological studies over long time spans
- Crucial role played by **expert assessment**: the evidence level assigned to each data factor (e.g. an image) as an enabler for a specific future development of the disease may vary from an expert to another, leading to complex data consolidation processes.
- Thus, **potentially contradicting** medical judgements cannot be swept under the carpet.

Such is the concrete research context for BANKSY: we intend to develop an **inference framework** able to **reason about contradictory, or even inconsistent data in a sound and effective way**.

The project

- Three partners (need for team integration and fostering synergies)
- Three years (1 Sep 2025 to 31 August 2028)
- A complementary team (including, from the outset, a PhD student)
- Some funding for human resources, missions, and dissemination provided under the **simplified cost model**, 249.5K

Tasks

- T1 - **Paraconsistent Inference: Foundations and Methods**
Sep 2025 to Feb 2026 (6m) | 12,5
- T2 - **Paraconsistent Engineering in the Medical Domain**
Sep 2025 to August 2027 (24m) | 67,4 K
- T3 - **Tool Development**
Sep 2026 to August 2028 (24m) | 86,8K
- T4 - **Tracing and Understanding AMD**
Mar 2027 to August 2028 (18m) | 80,3K
- T5 - **Project Management**
Sep 2025 to August 2028 | 2.5K