Imaginary Numbers - Exercise 1

1. i^{25}

Solution:

1. There is a pattern in imaginary numbers.
   $i^0 = 1 \quad i^1 = i \quad i^2 = -1 \quad i^3 = -i$
   $i^4 = 1 \quad i^5 = i \quad i^6 = -1 \quad i^7 = -i$
   ...
   Then we apply modular arithmetic: $25 \mod 4 = 1$

$$i^{25} = i^1 = i$$

complex Numbers - Exercise 2

Gate S is a phase gate.

$$\begin{bmatrix} 1 & 0 \\ 0 & i \end{bmatrix}$$

It does nothing to state $|0\rangle$. When the initial state is $|1\rangle$ the gate applies a rotation giving by the complex number i.

What is the phase of gate S?

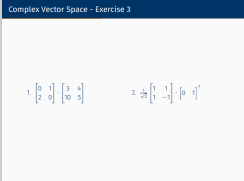
Solution:

$\rho = \sqrt{((-1)^2 + 0)} = \sqrt{1} = 1$

$\theta = \tan^{-1}(1/0) = \tan^{-1}(+\infty)$ or ANGLE whose tangent equals infinity.

$\tan(\theta) = \frac{\sin(\theta)}{\cos(\theta)}$, so $\tan(\theta) = +\infty$ implies $\cos(\theta) = 0$.

$\theta = \frac{\pi}{2}$



Solution:

1. $$\begin{bmatrix} 0 & 1 \\ 2 & 0 \end{bmatrix} \cdot \begin{bmatrix} 3 & 4 \\ 10 & 5 \end{bmatrix} = \begin{bmatrix} 0 \times 3 + 1 \times 10 & 0 \times 4 + 1 \times 5 \\ 2 \times 3 + 0 \times 10 & 2 \times 4 + 0 \times 5 \end{bmatrix} = \begin{bmatrix} 0 + 10 & 0 + 5 \\ 6 + 0 & 8 + 0 \end{bmatrix} = \begin{bmatrix} 10 & 5 \\ 6 & 8 \end{bmatrix}$$
2. $$\frac{1}{\sqrt{2}} \begin{bmatrix} 1 & 1 \\ 1 & -1 \end{bmatrix} \cdot \begin{bmatrix} 0 & 1 \end{bmatrix}^T = \frac{1}{\sqrt{2}} \begin{bmatrix} 1 & 1 \\ 1 & -1 \end{bmatrix} \cdot \begin{bmatrix} 0 \\ 1 \end{bmatrix} = \frac{1}{\sqrt{2}} \begin{bmatrix} 0 + 1 \\ 0 - 1 \end{bmatrix} = \frac{1}{\sqrt{2}} \begin{bmatrix} 1 \\ -1 \end{bmatrix}$$

Complex Vector Spaces - Exercise 4

In **quantum computation**, the most used bases are $|0\rangle$ and $|1\rangle$.

Exercise

Write a qubit state as linear combination of these basis.

Solution:

$$\frac{|0\rangle + |1\rangle}{\sqrt{2}}$$

1. Proof that the quantum state $|0\rangle$ is orthogonal to $|1\rangle$.
2. Proof that the quantum state $|+\rangle$ is orthogonal to $|-\rangle$.

Solution:

1. $\langle 0|1\rangle$ needs to be 0.

   Recall $|0\rangle = \begin{bmatrix} 1 \\ 0 \end{bmatrix}$ and $|1\rangle = \begin{bmatrix} 0 \\ 1 \end{bmatrix}$

   $\langle 0|1\rangle = \begin{bmatrix} 1 & 0 \end{bmatrix} \begin{bmatrix} 0 \\ 1 \end{bmatrix} =$
   $(1 \times 0) + (0 \times 1) = 0$

2. $\langle +|-\rangle$ needs to be 0.

   Recall $|+\rangle = \frac{|0\rangle+|1\rangle}{\sqrt{2}} = \frac{1}{\sqrt{2}} \begin{bmatrix} 1 \\ 1 \end{bmatrix}$

   and $|-\rangle = \frac{|0\rangle-|1\rangle}{\sqrt{2}} = \frac{1}{\sqrt{2}} \begin{bmatrix} 1 \\ -1 \end{bmatrix}$

   $\langle +|-\rangle =$
   $(\frac{1}{\sqrt{2}} \begin{bmatrix} 1 & 1 \end{bmatrix}) \cdot (\frac{1}{\sqrt{2}} \begin{bmatrix} 1 \\ -1 \end{bmatrix}) =$
   $\frac{1}{2}((1 \times 1) + (1 \times (-1))) = 0$