Grover

x numa base de dados c/ N elementos

clássico $O(N)$ Quântico $O(\sqrt{N})$

Ingredientes: 1) sobreposições

2) Oráculo: função como no Deutsch.

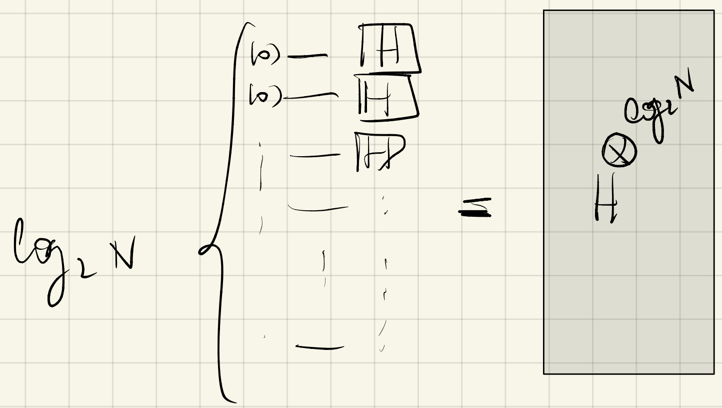
3) Difusor: Amplitude Amplification.

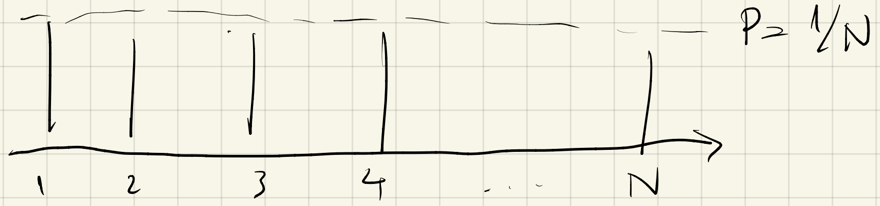
→ Amplificar a amplitude do elemento q queremos encontrar.

1) sobreposições

n qubits $\longrightarrow$ 2^n elementos.

N elementos $\rightarrow$ $\sim \log_2 N$ qubits.





$$|DB\rangle = H|0\rangle \otimes H|0\rangle \otimes H|0\rangle \cdots$$

$$= \frac{1}{\sqrt{2}}(|0\rangle+|1\rangle) \otimes \frac{1}{\sqrt{2}}(|0\rangle+|1\rangle) \otimes \cdots$$

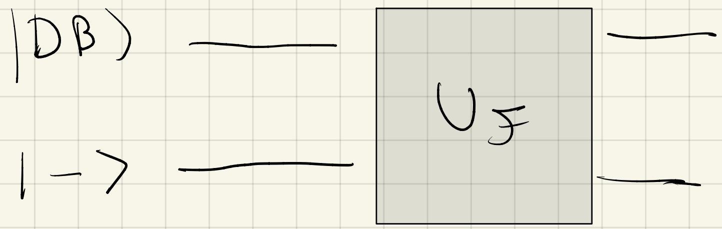
$$= \bigotimes_{i=1}^{\log_2 N} \frac{1}{\sqrt{2}}(|0\rangle_i+|1\rangle_i)$$

$$= \frac{1}{\sqrt{N}}\left[|0\cdots0\rangle + |0\cdots1\rangle + \cdots + |11111\cdots1\rangle \right]$$

$$= \frac{1}{\sqrt{2^n}}$$

2) <u>ORÁCULO</u>: função q marca elemento

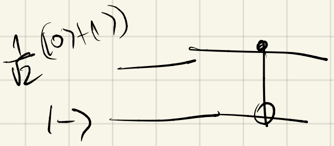
"Diz-nos o q está nesta posição de memória".

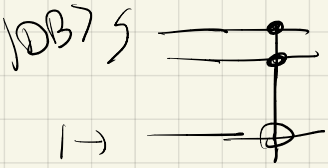


<u>Ex</u>: $|DB\rangle = \frac{1}{2}\left[|00\rangle + |01\rangle + |10\rangle + |11\rangle \right]$

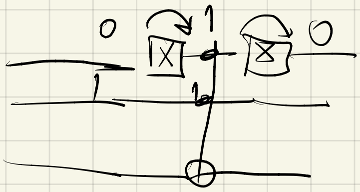
ancilla $\rightarrow |-\rangle = \frac{1}{\sqrt{2}}(|0\rangle - |1\rangle)$

queremos marcar o estado $|11\rangle$





Se quiséssemos $|01\rangle$

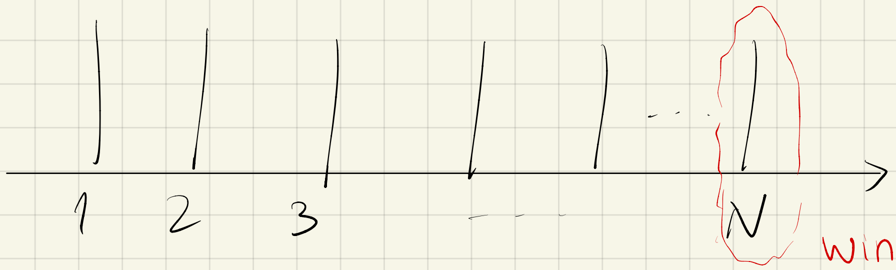


$$X = \sigma_x = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}$$

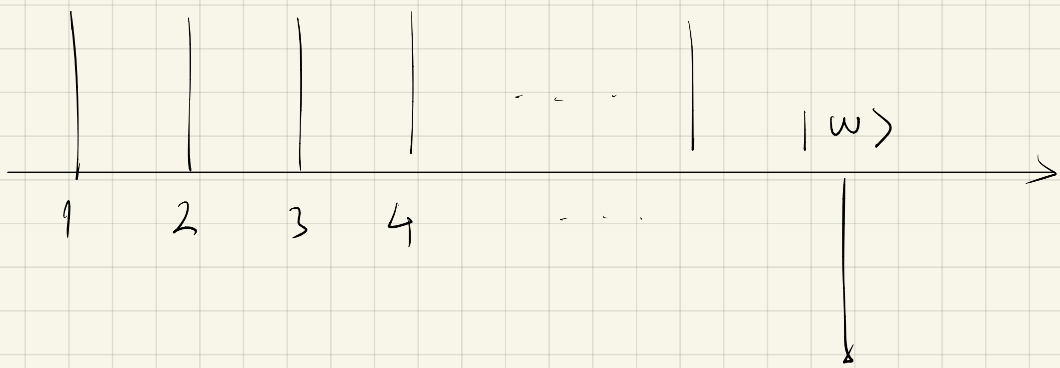
$$\text{bitflip em } |-\rangle = \frac{1}{\sqrt{2}}\left(|0\rangle - |1\rangle\right) = \frac{1}{\sqrt{2}}\left(|1\rangle - |0\rangle\right)$$

$$= (-)\frac{1}{\sqrt{2}}\left(|0\rangle - |1\rangle\right)$$

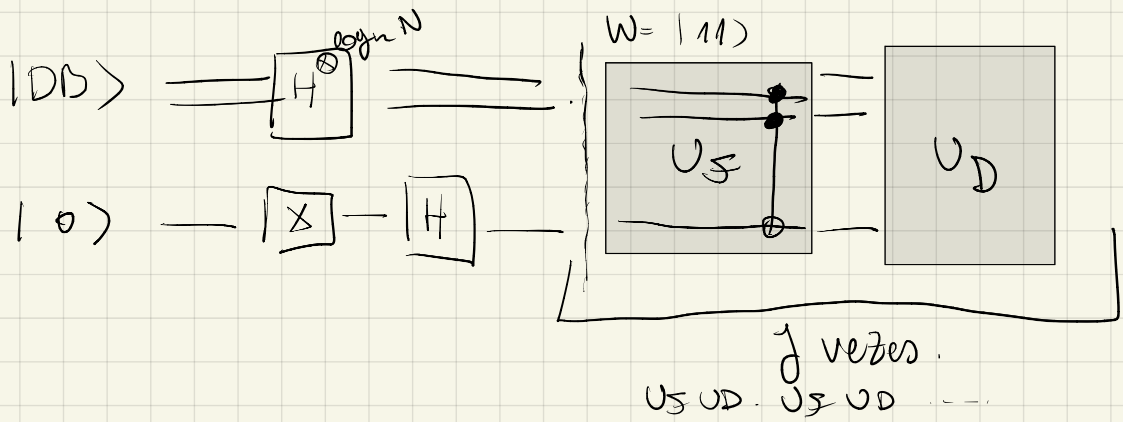
$$= -|-\rangle$$



↓ Atuamos c/ oráculo para marcar $|11\rangle$ ou N



Circuito até ao momento



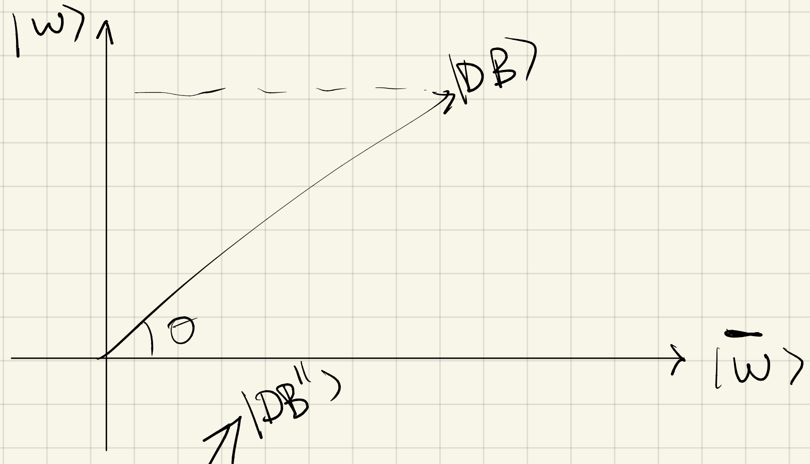
$$|DB\rangle = \frac{1}{N}\left[|00\ldots0\rangle + |00\ldots1\rangle + \ldots + |111\ldots1\rangle \right]$$

$$\text{2 qubits} = \frac{1}{2}\left[|00\rangle + |01\rangle + |10\rangle + |11\rangle \right]$$

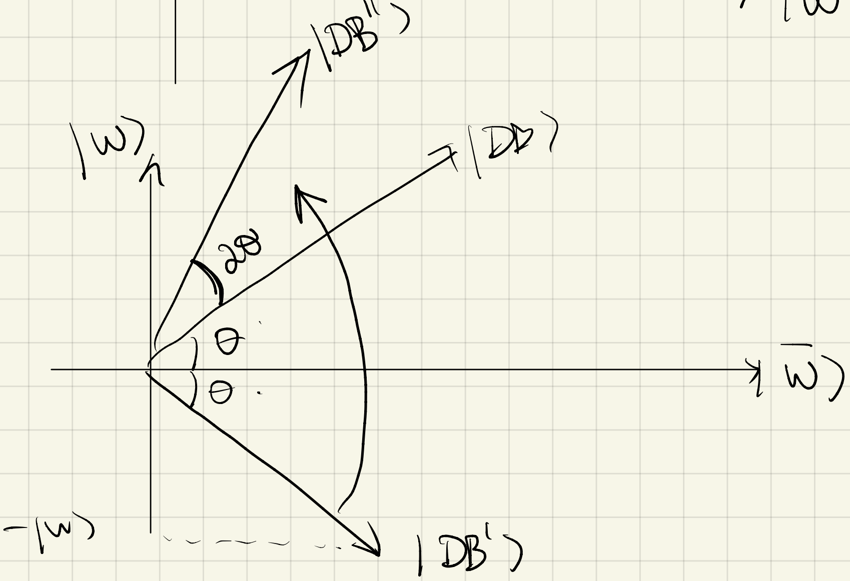
winner $|w\rangle$

$$= \operatorname{sen}(\theta)\,|w\rangle + \cos(\theta)\,|\bar{w}\rangle$$

$$\operatorname{sen}(\theta) = \frac{1}{2}$$



oráculo :



$$U_D = 2\,\underbrace{|DB\rangle\langle DB|} - \mathbb{I}$$

$$U_D\,|DB'\rangle$$

$$= H^{\otimes \log_2 n}\left(2\,|0\rangle\langle 0| - \mathbb{I}\right) H^{\otimes \log_2 n}$$

$$= H^{\otimes \log_2 n}\left(2\left(\begin{pmatrix}1\\0\\0\\0\\\vdots\\0\end{pmatrix}\begin{pmatrix}1 & 0 & 0 & \cdots & 0\end{pmatrix}\right) - \mathbb{I}\right) H^{\otimes \log_2 n}$$

$$= \;\;''\;\; 2\begin{pmatrix}1 & 0 & 0 & 0 & \cdots & 0\\ 0 & 0 & & & & 0\\ 0 & & 0 & & & \\ 0 & & & 0 & & \\ \vdots & 0 & & & \ddots & \\ 0 & & & & & 0\end{pmatrix} - \mathbb{I} \;\;''$$

$$= H^{\otimes \log_2 N}\begin{pmatrix}1 & & & & 0\\ & -1 & & & \\ & & -1 & & \\ & & & -1 & \\ 0 & & & & -1\end{pmatrix}\begin{array}{l}\rightarrow |00\cdots0\rangle\\ \rightarrow |00\cdots1\rangle\\ \vdots\\ \\ |111\cdots1\rangle\end{array} H^{\otimes \log_2 N}$$

$$|1\rangle \;—\boxed{Z}—\; = -|1\rangle$$

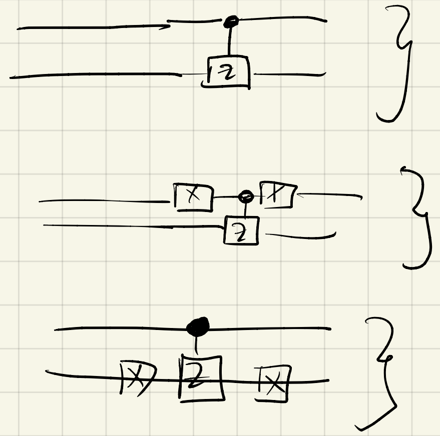
$$Z = \begin{pmatrix}1 & 0\\ 0 & -1\end{pmatrix}$$

$$|0\rangle \;—\boxed{Z}—\; |0\rangle$$

$|11\rangle$ $\quad\quad -|11\rangle$

$|01\rangle$ $\quad\quad -|01\rangle$

$|10\rangle$ $\quad\quad -|10\rangle$

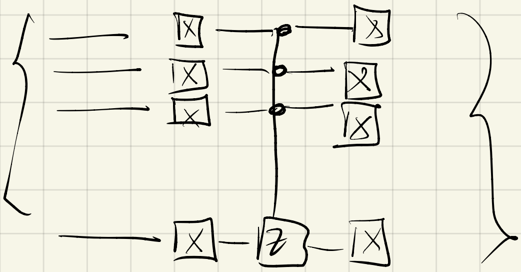


Truque: em vez de invertermos todos os $2^n - 1$ elementos usamos propriedades da fase global (− em evidência)

$$U_D = H^{\otimes \log_2 n} \begin{pmatrix} 1 & & & & \\ & -1 & & & 0 \\ & & -1 & & \\ & 0 & & -1 & \\ & & & & \ddots \\ & & & & & -1 \end{pmatrix} H^{\otimes \log_2 N}$$

$$= - H^{\otimes \log_2 N} \begin{pmatrix} -1 & & & \\ & 1 & & 0 \\ 0 & & \ddots & \\ & & & 1 \end{pmatrix} H^{\otimes \log_2 N}$$

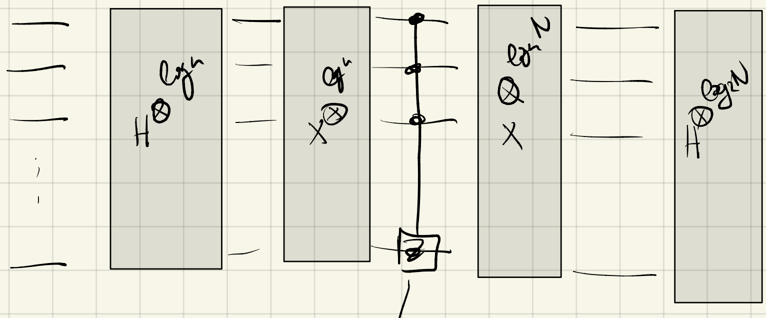
$= \log_2 N$



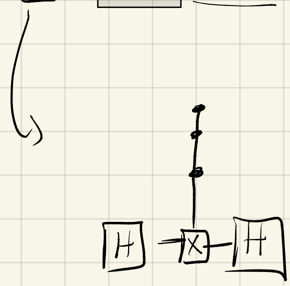
$-|000\cdots0\rangle$

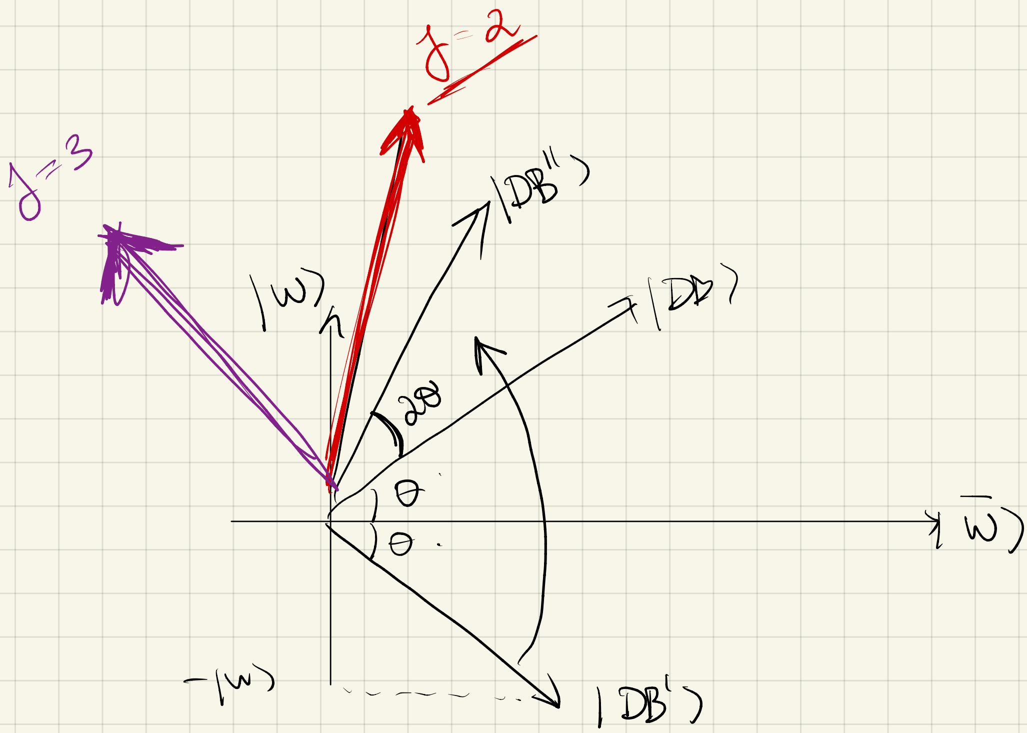
$|0000\cdots0\rangle$

U_D circuito final:



$$Z = H \cdot X \cdot H$$





$$|DB\rangle = \operatorname{sen}(\theta)\,|w\rangle + \cos(\theta)\,|\bar{w}\rangle$$

$$\operatorname{sen}(\theta) = \frac{1}{N}$$

$$\operatorname{sen}^2(\theta) \approx 1$$

$$\operatorname{sen}^2\big((2J+1)\theta\big) \approx 1$$

$$\downarrow$$

$$(2J+1)\theta \approx \operatorname{arcsen}(1)$$

$$2J+1 \approx \frac{\operatorname{arcsen}(1)}{\theta}$$

$$2J \approx \frac{\operatorname{arcsen}(1)}{\theta} - 1$$

$$J \approx \frac{\operatorname{arcsen}(1)}{2\theta} - \frac{1}{2}$$

$$\operatorname{sen}(\theta) = \frac{1}{\sqrt{N}} \qquad \approx \frac{\operatorname{arcsen}(1)}{2\operatorname{arcsen}\left(\frac{1}{\sqrt{N}}\right)} - \frac{1}{2}$$

$$\theta = \operatorname{arcsen} \frac{1}{\sqrt{N}}$$

$$= \operatorname{arcsen} \frac{1}{\sqrt{2^n}} \qquad \approx \frac{\pi}{4}$$

$$\operatorname{arcsen}\left(\frac{1}{\sqrt{N}}\right) \approx \frac{1}{\sqrt{N}}$$

$$\boxed{\operatorname{arcsen}(x) \approx x \ \text{ para } \ |x| \ll 1}$$

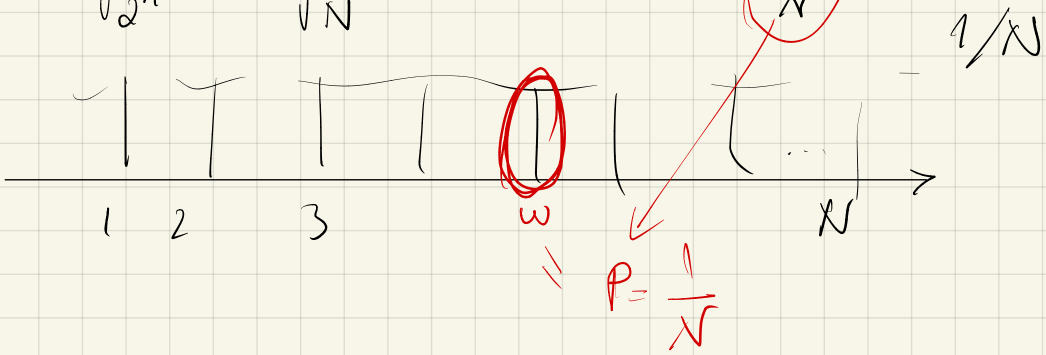
N é grande

$$\approx \frac{\pi}{4}\sqrt{N} - \frac{1}{2}$$

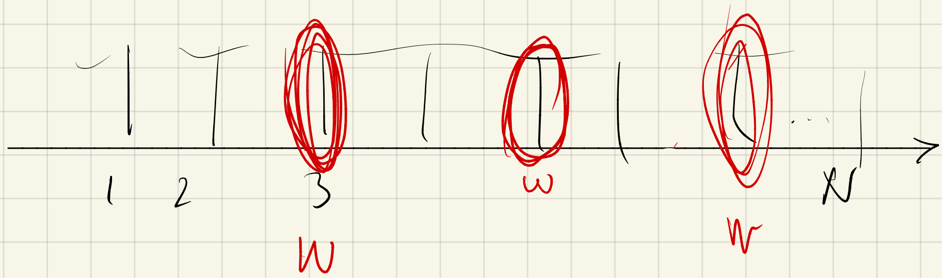
$$j = \left\lfloor \frac{\pi}{4}\sqrt{N} \right\rfloor$$

$$|DB\rangle = \sin(\theta)\,|w\rangle + \cos(\theta)\,|\bar{w}\rangle$$

$$\sin(\theta) = \frac{1}{\sqrt{2^n}} = \frac{1}{\sqrt{N}} \qquad \sin^2(\theta) = \frac{1}{N}$$



Para M soluções:



$$\sin(\theta) \neq \frac{1}{\sqrt{N}} \longrightarrow \sin(\theta) = \sqrt{\frac{M}{N}}$$

$$\operatorname{sen}^2\left((2J+1)\theta\right) \approx 1$$

$$\operatorname{sen}(\theta) = \sqrt{\frac{M}{N}} \iff$$

$$\theta = \operatorname{arcsen}\left(\sqrt{\frac{M}{N}}\right)$$

isolar # iterações J:

$$(2J+1)\theta \approx \operatorname{arcsen}(1)$$

$$2J+1 \approx \frac{\operatorname{arcsen}(1)}{\operatorname{arcsen}\left(\sqrt{\frac{M}{N}}\right)}$$

$$J \approx \frac{\operatorname{arcsen}(1)}{2\operatorname{arcsen}\left(\sqrt{\frac{M}{N}}\right)} - \frac{1}{2}$$

$$\approx \frac{\pi}{4}\,\frac{1}{\operatorname{arcsen}\left(\sqrt{\frac{M}{N}}\right)} - \frac{1}{2}$$

$\operatorname{arcsen}(x) = x$, $|x| \ll 1$ ou seja N é grande

$$\approx \frac{\pi}{4}\sqrt{\frac{N}{M}} - \frac{1}{2}$$

$$\approx \left\lfloor \frac{\pi}{4}\sqrt{\frac{N}{M}} \right\rfloor \rightarrow$$

Para M soluções

$$O\left(\sqrt{\frac{N}{M}}\right)$$