Projeto - Grupo 12

June 6, 2021

[1]:
```
# importing Qiskit
from qiskit import Aer, IBMQ
from qiskit import QuantumCircuit, ClassicalRegister, QuantumRegister
from qiskit import execute, transpile

from qiskit.tools.visualization import plot_histogram, plot_state_city,␣
 ↪plot_state_hinton
from qiskit.visualization import plot_bloch_multivector

provider = IBMQ.load_account()

# Backend overview
import qiskit.tools.jupyter

from qiskit.tools.monitor import backend_overview, backend_monitor

# Import measurement calibration functions
from qiskit.ignis.mitigation.measurement import (complete_meas_cal,␣
 ↪tensored_meas_cal,
                                                 CompleteMeasFitter,␣
 ↪TensoredMeasFitter)

import matplotlib.pyplot as plt
%matplotlib inline

import math as m
import numpy

backend_statevector = Aer.get_backend('statevector_simulator')
backend_qasm_sim = Aer.get_backend("qasm_simulator")
```

1 Projeto - Grupo 12

- Anabela Pereira, a87990
- André Gonçalves, a87942

Neste trabalho realizado no âmbito da unidade curricular de Interação e Concorrência foi nos proposto encontrar um número numa lista de oito elementos. No nosso caso, grupo número 12 modulo 8 é 4.

Para ser mais simples e não estarmos a repetir código frequentemente , definimos logo no início uma função para desenhar as esferas, uma função para fazer as matrizes, uma função para invocar as funções de cima e por último uma função para desenhar os histogramas, respetivamente.

[2]:
```
def get_psi(circuit):

    result = execute(circuit, backend_statevector).result()
    psi = result.get_statevector(circuit)
    display(plot_bloch_multivector(psi))

def print_unitary(circuit):

    backend = Aer.get_backend('unitary_simulator')
    unit=execute(circuit, backend).result().get_unitary(circuit)
    #print(unit)
    print(unit.real)

def display_circuit(circuit,disp,psi,unitary):
    if disp:
        display(circuit.draw(output="mpl"))
    if psi:
        get_psi(circuit)
    if unitary:
        print_unitary(circuit)

def display_results(counts_sim):
    return plot_histogram(counts_sim)

def display_counts(circuit, shots):

    result = execute(circuit, backend_qasm_sim, shots=shots).result()
    counts_sim = result.get_counts(circuit)
    return counts_sim
```

Tomemos então N=4 numa lista não ordenada. Para tal usamos o algoritmo de Grover.

1.0.1 Algoritmo de Grover

O algoritmo de Grover permite encontrar um número N numa lista não ordenada de n elementos.

Os passos do algoritmo são: 1. Inicializar o circuito com todos os estados possíveis com a mesma amplitude.

$$|s\rangle = \frac{1}{\sqrt{n}} \sum_{x=0}^{n-1} |x\rangle$$

2. Repetir os seguintes passos $\sqrt{n}$ vezes:
    1. Aplicar o oráculo U_w
    2. Aplicar o difusor U_D
3. Medir os estados quânticos resultantes

Oráculo U_w O óráculo vai inverter o sinal do número que estámos á procura. Se estamos á procura de w então

$$U_w|w\rangle = -|w\rangle U_w|\psi\rangle = |\psi\rangle, \psi \neq w$$

Difusor U_D O difusor vai inverter de novo o sinal do número que estámos á procura e aumentar a sua amplitude.

$$U_D = 2|a\rangle\langle a| - I|a\rangle = H^{\oplus n}|0\rangle^{\oplus n}$$

1.0.2 Implementação

Queremos procurar $N = 4$ numa lista não ordenada de $2^3 = 8$ elementos então o nosso circuito terá 3 qubits.

Como $4 = 100$ queremos um oráculo U_4 tal que

$$U_4|100\rangle = -|100\rangle U_4|\psi\rangle = |\psi\rangle, \psi \neq 100$$

U_4 corresponde à matriz

$$\begin{bmatrix} 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & -1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \end{bmatrix}$$

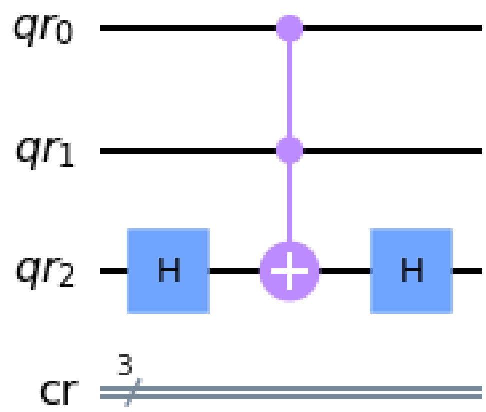
Como sabemos que o seguinte circuito corresponde á matriz

$$\begin{bmatrix} 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & -1 \end{bmatrix}$$

conseguimos chegar ao oráculo que queremos.

[3]:
```
qr=QuantumRegister(3,'qr')
cr=ClassicalRegister(3,'cr')
```

```
qc= QuantumCircuit(qr, cr)

qc.h(qr[2])
qc.ccx(qr[0],qr[1],qr[2])
qc.h(qr[2])

display_circuit(qc,True,False,True)
```

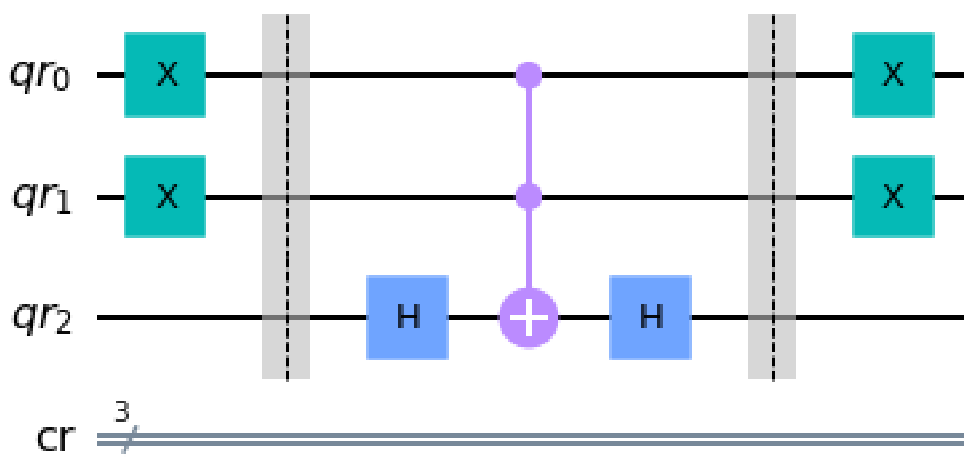



```
[[ 1.00000000e+00  0.00000000e+00  0.00000000e+00  0.00000000e+00
   1.01465364e-17  0.00000000e+00  0.00000000e+00  0.00000000e+00]
 [ 0.00000000e+00  1.00000000e+00  0.00000000e+00  0.00000000e+00
   0.00000000e+00  1.01465364e-17  0.00000000e+00  0.00000000e+00]
 [ 0.00000000e+00  0.00000000e+00  1.00000000e+00  0.00000000e+00
   0.00000000e+00  0.00000000e+00  1.01465364e-17  0.00000000e+00]
 [ 0.00000000e+00  0.00000000e+00  0.00000000e+00  1.00000000e+00
   0.00000000e+00  0.00000000e+00  0.00000000e+00 -1.99673462e-16]
 [ 1.01465364e-17  0.00000000e+00  0.00000000e+00  0.00000000e+00
   1.00000000e+00  0.00000000e+00  0.00000000e+00  0.00000000e+00]
 [ 0.00000000e+00  1.01465364e-17  0.00000000e+00  0.00000000e+00
   0.00000000e+00  1.00000000e+00  0.00000000e+00  0.00000000e+00]
 [ 0.00000000e+00  0.00000000e+00  1.01465364e-17  0.00000000e+00
   0.00000000e+00  0.00000000e+00  1.00000000e+00  0.00000000e+00]
 [ 0.00000000e+00  0.00000000e+00  0.00000000e+00 -1.79380389e-16
   0.00000000e+00  0.00000000e+00  0.00000000e+00 -1.00000000e+00]]
```

Este circuito segue o seguinte esquema: $|111\rangle \mapsto -|111\rangle$ e $|\psi\rangle \mapsto |\psi\rangle$, $\psi \neq 111$.

Para chegar ao oráculo que precissámos colocamos gates X nos qubits 0 e 1 antes assim o novo circuito faz $|100\rangle \mapsto -|111\rangle$ e $|\psi\rangle \mapsto |\psi\rangle$, $\psi \neq 100$ então colocamos gates X nos qubits 0 e 1 também depois e assim obtemos $|100\rangle \mapsto -|100\rangle$ e $|\psi\rangle \mapsto |\psi\rangle$, $\psi \neq 100$, que é o que queremos.

[4]:
```
qr=QuantumRegister(3,'qr')
cr=ClassicalRegister(3,'cr')
qc= QuantumCircuit(qr, cr)

qc.x(qr[:2])
qc.barrier()

qc.h(qr[2])
qc.ccx(qr[0],qr[1],qr[2])
qc.h(qr[2])

qc.barrier()
qc.x(qr[:2])

display_circuit(qc,True,False,True)
```

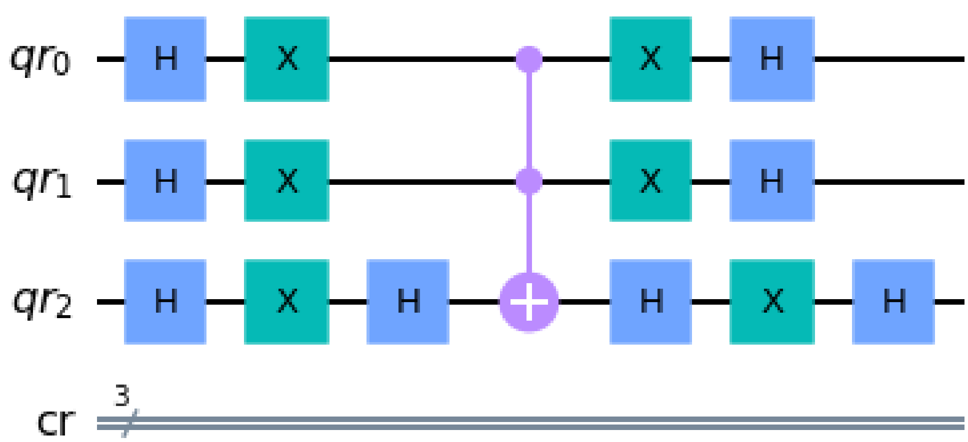



```
[[ 1.00000000e+00  0.00000000e+00  0.00000000e+00  0.00000000e+00
  -1.99673462e-16  0.00000000e+00  0.00000000e+00  0.00000000e+00]
 [ 0.00000000e+00  1.00000000e+00  0.00000000e+00  0.00000000e+00
   0.00000000e+00  1.01465364e-17  0.00000000e+00  0.00000000e+00]
 [ 0.00000000e+00  0.00000000e+00  1.00000000e+00  0.00000000e+00
   0.00000000e+00  0.00000000e+00  1.01465364e-17  0.00000000e+00]
 [ 0.00000000e+00  0.00000000e+00  0.00000000e+00  1.00000000e+00
   0.00000000e+00  0.00000000e+00  0.00000000e+00  1.01465364e-17]
 [-1.79380389e-16  0.00000000e+00  0.00000000e+00  0.00000000e+00
  -1.00000000e+00  0.00000000e+00  0.00000000e+00  0.00000000e+00]
 [ 0.00000000e+00  1.01465364e-17  0.00000000e+00  0.00000000e+00
```

```
  0.00000000e+00  1.00000000e+00  0.00000000e+00  0.00000000e+00]
[ 0.00000000e+00  0.00000000e+00  1.01465364e-17  0.00000000e+00
  0.00000000e+00  0.00000000e+00  1.00000000e+00  0.00000000e+00]
[ 0.00000000e+00  0.00000000e+00  0.00000000e+00  1.01465364e-17
  0.00000000e+00  0.00000000e+00  0.00000000e+00  1.00000000e+00]]
```

O nosso difusor será $U_D = H^{\oplus 3}(-2|000\rangle\langle 000| + I)H^{\oplus 3} = -H^{\oplus 3}(2|000\rangle\langle 000| - I)H^{\oplus 3}$ que equivale ao seguinte circuito:

[5]:
```
qr=QuantumRegister(3,'qr')
cr=ClassicalRegister(3,'cr')
qc= QuantumCircuit(qr, cr)

qc.h(qr)
qc.x(qr)
qc.h(qr[2])
qc.ccx(qr[0],qr[1],qr[2])
qc.h(qr[2])
qc.x(qr)
qc.h(qr)

display_circuit(qc,True,False,True)
```




```
[[ 0.75 -0.25 -0.25 -0.25 -0.25 -0.25 -0.25 -0.25]
 [-0.25  0.75 -0.25 -0.25 -0.25 -0.25 -0.25 -0.25]
 [-0.25 -0.25  0.75 -0.25 -0.25 -0.25 -0.25 -0.25]
 [-0.25 -0.25 -0.25  0.75 -0.25 -0.25 -0.25 -0.25]
 [-0.25 -0.25 -0.25 -0.25  0.75 -0.25 -0.25 -0.25]
 [-0.25 -0.25 -0.25 -0.25 -0.25  0.75 -0.25 -0.25]
```

```
 [-0.25 -0.25 -0.25 -0.25 -0.25 -0.25  0.75 -0.25]
 [-0.25 -0.25 -0.25 -0.25 -0.25 -0.25 -0.25  0.75]]
```

1.0.3 Implementação no Qiskit

Somos o grupo 12, portanto, o nosso N é $4 = 12 \mod 8$, que em binário é '100'.

Começamos por criar uma função que cria um circuito com n qubits:

[6]:
```
def create_circuit(n):
    qr=QuantumRegister(n,'qr')
    cr=ClassicalRegister(n,'cr')
    qc= QuantumCircuit(qr, cr)
    return qr,cr,qc
```

De seguida, uma função que inizializa os qubits na superposition aplicando Haddamard gates a cada qubit.

[7]:
```
def init():
    qc.h(qr)
    qc.barrier()
```

Vamos agora criar um oráculo de mudança de fase.

[8]:
```
def oraculo():

    qc.x(qr[:2])

    qc.h(qr[2])
    qc.ccx(qr[0],qr[1],qr[2])
    qc.h(qr[2])

    qc.x(qr[:2])

    qc.barrier()
```

Após a criação do oráculo é necessário fazer a amplificação, por isso criamos o diffusor.

[9]:
```
def diffusor():

    qc.h(qr)
    qc.x(qr)
    qc.h(qr[2])
    qc.ccx(qr[0],qr[1],qr[2])
    qc.h(qr[2])
    qc.x(qr)
    qc.h(qr)
```

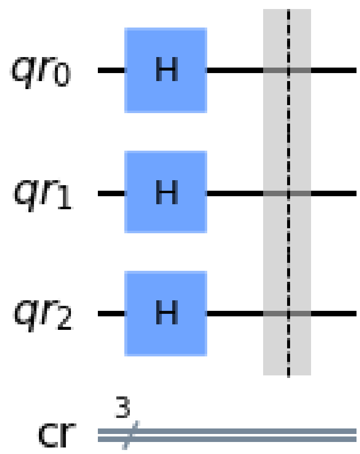
```
    qc.barrier()
```

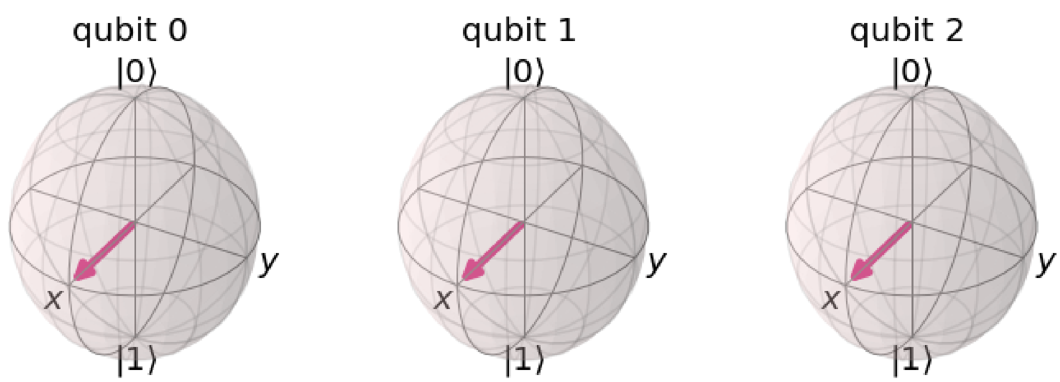
Começemos agora por aplicar o algoritmo. Criamos um circuito com 3 qubits.

[10]:
```
qr,cr,qc = create_circuit(3)
```

Após a criação do circuito iniciamos o mesmo na superposição.

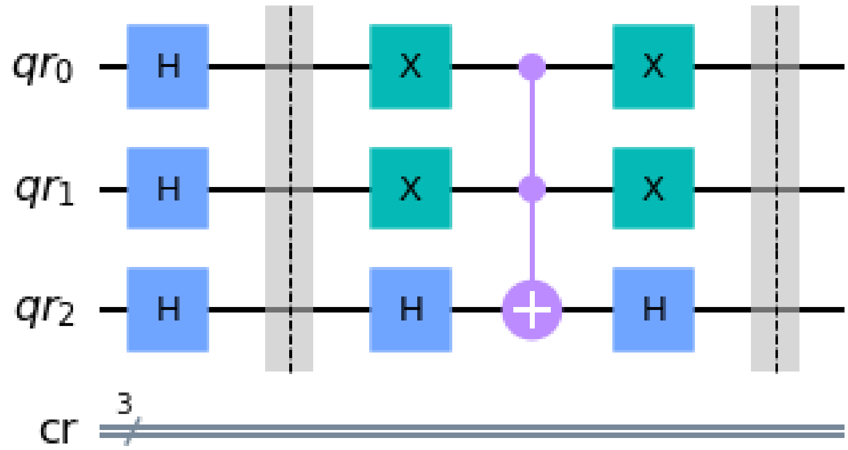
[11]:
```
init()
display_circuit(qc,True,True,False)
```

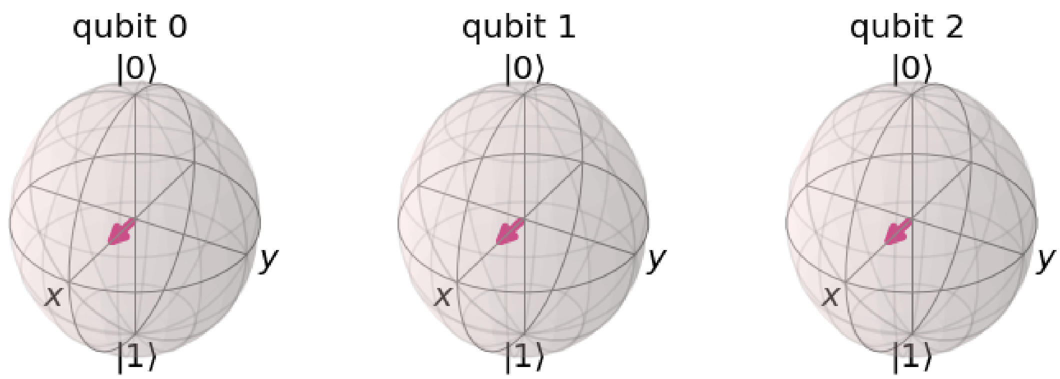




De seguida aplicamos o oráculo.

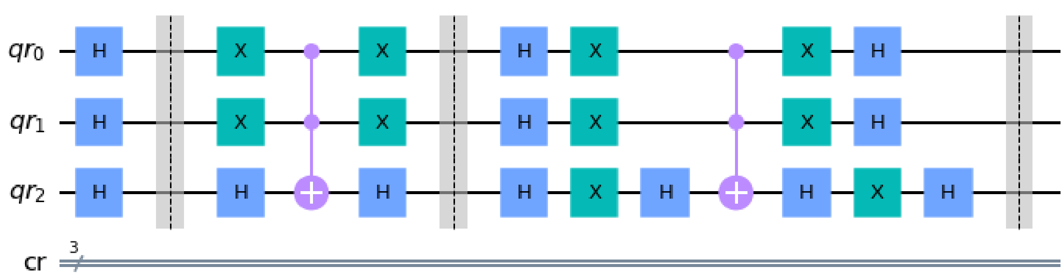
[12]:
```
oraculo()
display_circuit(qc,True,True,False)
```

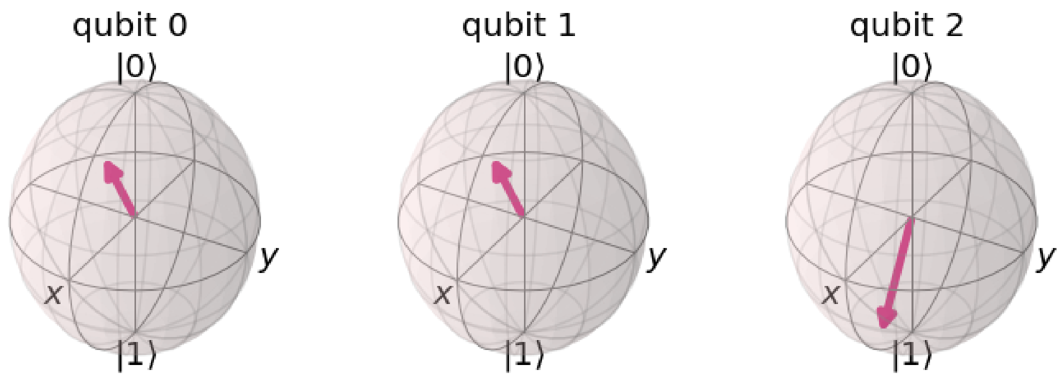




Depois aplicamos o difusor.

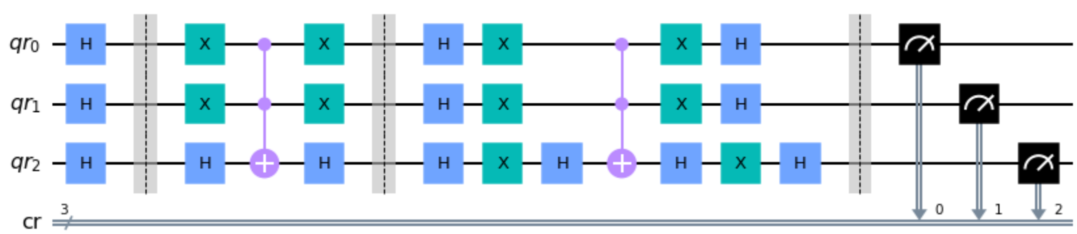
[13]:
```
diffusor()
display_circuit(qc,True,True,False)
```



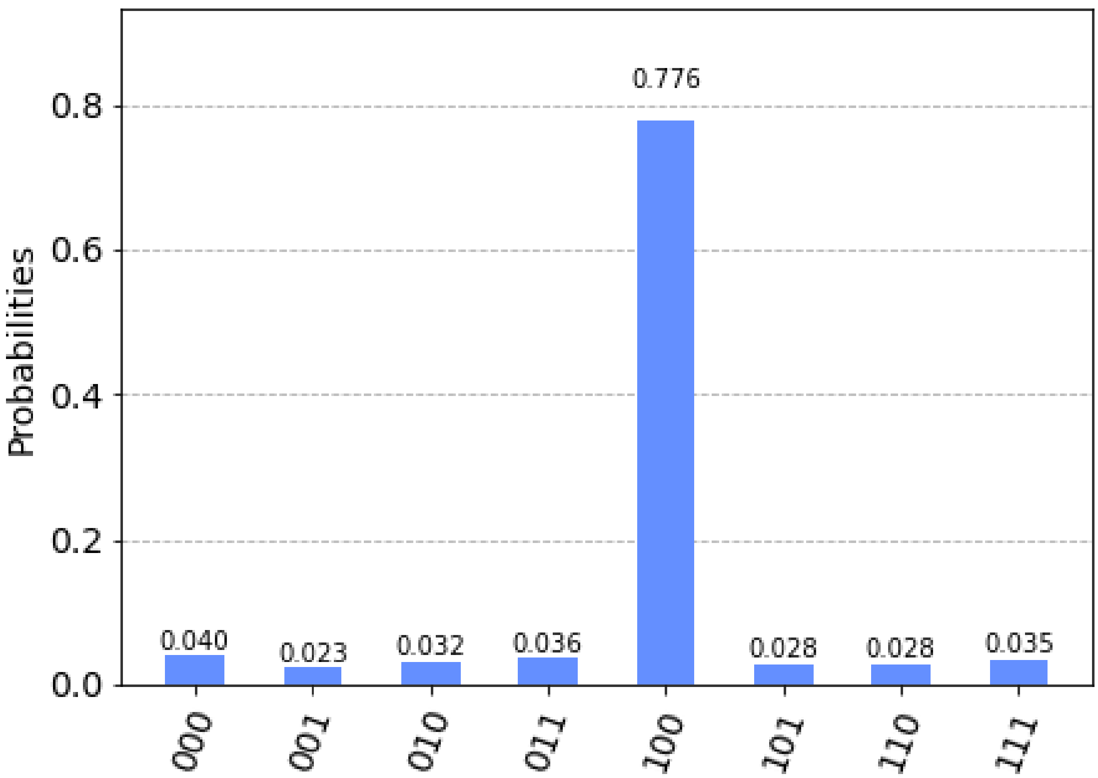


Como já fizemos uma iteração e vamos medir para ver o resultado em bits.

[14]:
```
qc.measure(qr, cr)
display_circuit(qc,True,False,False)
counts_sim = display_counts(qc, 1024)
display_results(counts_sim)
```

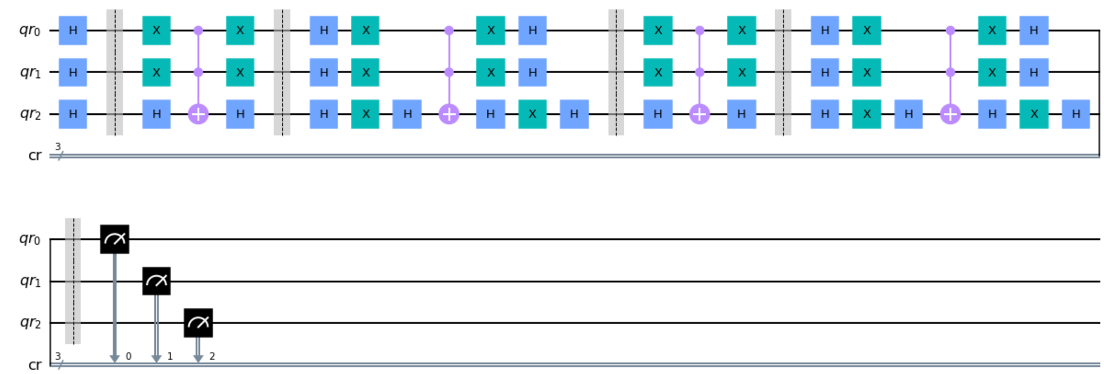


[14]:

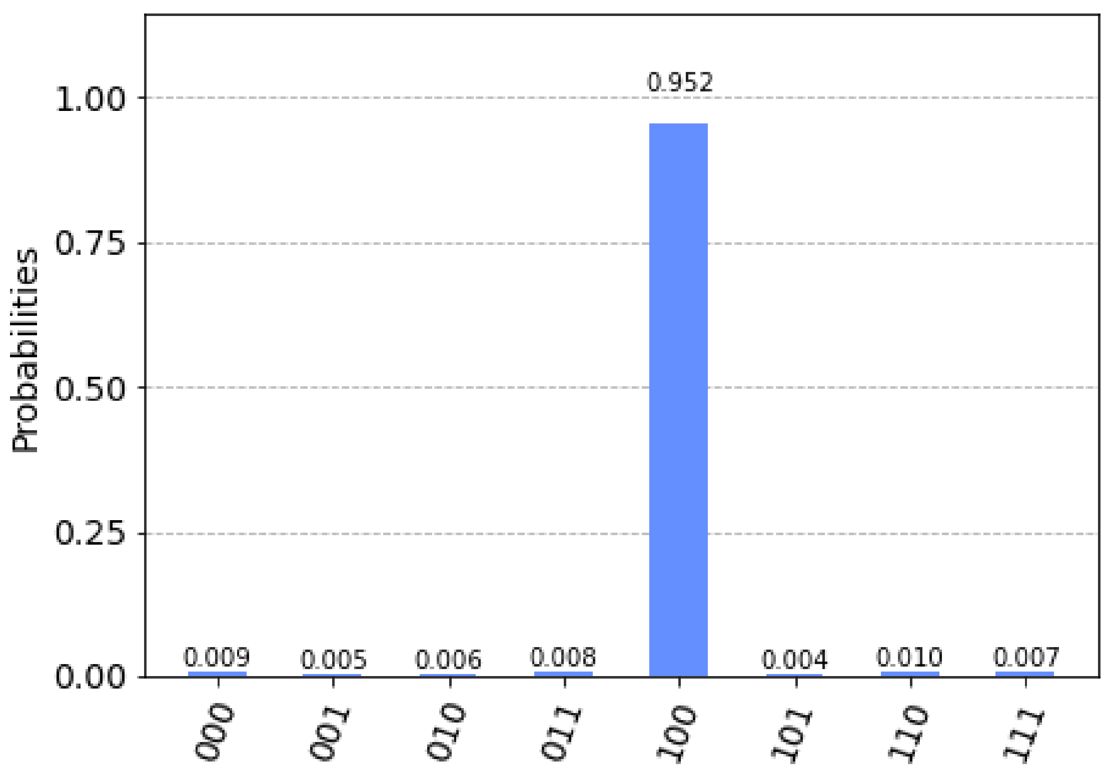


Observámos que obtivemos o resultado esperado com apenas 1 iteração. Como sabemos devemos fazer $\sqrt{n}$ iterações, vamos fazer agora 2 iterações.

[15]:
```
qr,cr,qc = create_circuit(3)

init()
for i in range(2):
    oraculo()
    diffusor()

qc.measure(qr,cr)
display_circuit(qc,True,False,False)
counts_sim = display_counts(qc, 1024)
display_results(counts_sim)
```



[15]:



1.0.4 Simulação com ruído

[16]:
```
from qiskit.providers.aer.noise import NoiseModel
import qiskit.providers.aer.noise
```

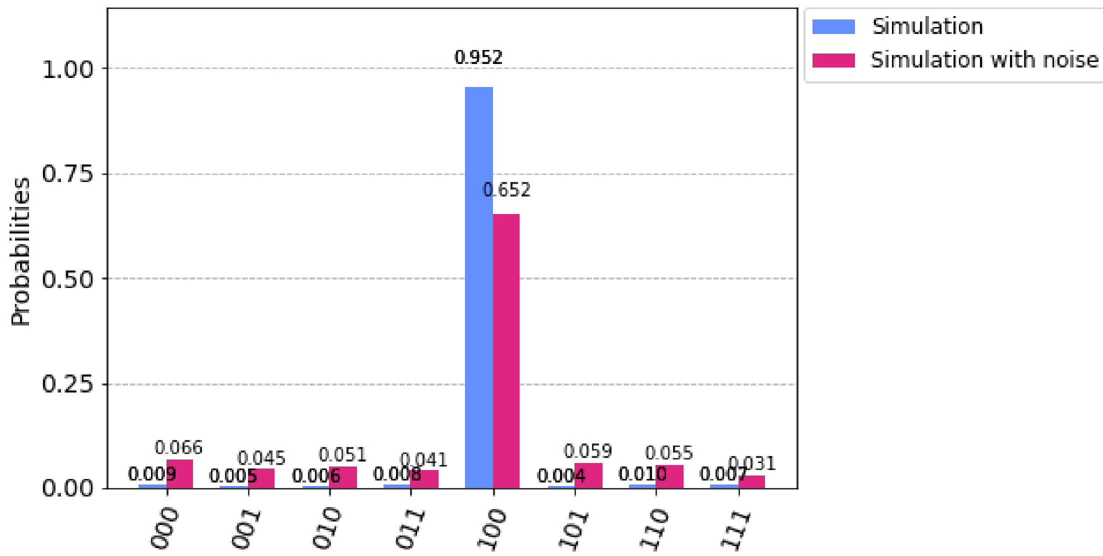
[17]:
```
provider.backends()
```

[17]:
```
[<IBMQSimulator('ibmq_qasm_simulator') from IBMQ(hub='ibm-q', group='open',
project='main')>,
 <IBMQBackend('ibmqx2') from IBMQ(hub='ibm-q', group='open', project='main')>,
 <IBMQBackend('ibmq_16_melbourne') from IBMQ(hub='ibm-q', group='open',
project='main')>,
 <IBMQBackend('ibmq_armonk') from IBMQ(hub='ibm-q', group='open',
project='main')>,
 <IBMQBackend('ibmq_athens') from IBMQ(hub='ibm-q', group='open',
project='main')>,
 <IBMQBackend('ibmq_santiago') from IBMQ(hub='ibm-q', group='open',
project='main')>,
 <IBMQBackend('ibmq_lima') from IBMQ(hub='ibm-q', group='open',
project='main')>,
 <IBMQBackend('ibmq_belem') from IBMQ(hub='ibm-q', group='open',
project='main')>,
 <IBMQBackend('ibmq_quito') from IBMQ(hub='ibm-q', group='open',
project='main')>,
 <IBMQSimulator('simulator_statevector') from IBMQ(hub='ibm-q', group='open',
project='main')>,
 <IBMQSimulator('simulator_mps') from IBMQ(hub='ibm-q', group='open',
project='main')>,
 <IBMQSimulator('simulator_extended_stabilizer') from IBMQ(hub='ibm-q',
group='open', project='main')>,
 <IBMQSimulator('simulator_stabilizer') from IBMQ(hub='ibm-q', group='open',
project='main')>,
 <IBMQBackend('ibmq_manila') from IBMQ(hub='ibm-q', group='open',
project='main')>]
```

[18]:
```
backend_device = provider.get_backend('ibmq_santiago')
coupling_map = backend_device.configuration().coupling_map
# Construct the noise model from backend properties
noise_model = NoiseModel.from_backend(backend_device)
basis_gates = noise_model.basis_gates
```

[19]:
```
result_noise = execute(qc, backend_qasm_sim,shots = 1024,
                       noise_model=noise_model,
                       coupling_map=coupling_map,
                       basis_gates=basis_gates).result()
counts_noise = result_noise.get_counts(qc)

plot_histogram([counts_sim, counts_noise], legend=[ "Simulation", "Simulation␣
↪with noise" ])
```

[19]:



[20]:
```
print(counts_sim)
```

```
{'101': 4, '001': 5, '110': 10, '100': 975, '010': 6, '000': 9, '011': 8, '111':
7}
```

[21]:
```
print(counts_noise)
```

```
{'101': 60, '010': 52, '110': 56, '100': 668, '111': 32, '000': 68, '011': 42,
'001': 46}
```

[22]:
```
def resume(counts_raw):
    s0=s1=0
    k=counts_raw.keys()
    lk=list(k)
    for c in lk:
        if c[0]=='0':
            s0 = s0 + counts_raw.get(c)
        else:
            s1 = s1 + counts_raw.get(c)
    return({'0':s0, '1':s1})
```

[23]:
```
cn = resume(counts_noise)
print(cn)
```

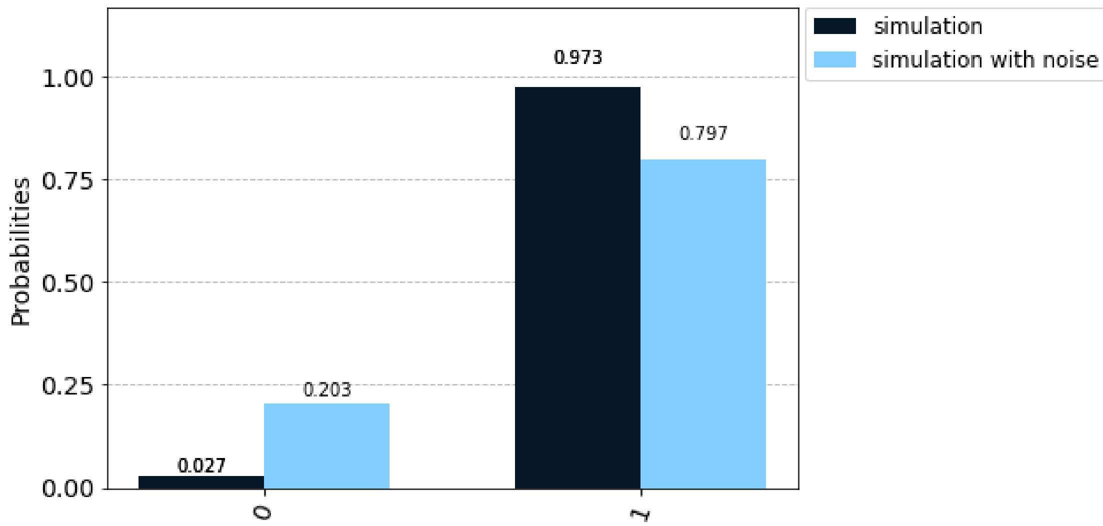
```
{'0': 208, '1': 816}
```

[24]:
```
c = resume(counts_sim)
print(c)
```

```
{'0': 28, '1': 996}
```

[25]:
```
plot_histogram([c,cn], legend= ['simulation','simulation with noise'],␣
↪color=['#061727','#82cfff'])
```

[25]:



1.0.5 Num computador quântico

[26]:
```
%qiskit_backend_overview
```

```
VBox(children=(HTML(value="<h2 style ='color:#ffffff; background-color:#000000;padding-top: 1%
```

Vamos agora correr o circuito num computador quântico. Vamos utilizar ibmq_belem.

[28]:
```
backend_device = provider.get_backend('ibmq_belem')
print("Running on:", backend_device)
```

```
Running on: ibmq_belem
```

[29]:
```
backend_device
```

```
VBox(children=(HTML(value="<h1 style='color:#ffffff;background-color:#000000;padding-top: 1%;pa
```

[29]:
```
<IBMQBackend('ibmq_belem') from IBMQ(hub='ibm-q', group='open', project='main')>
```

[30]:
```
job_r = execute(qc, backend_device, shots=1024)

jobID_r = job_r.job_id()
```

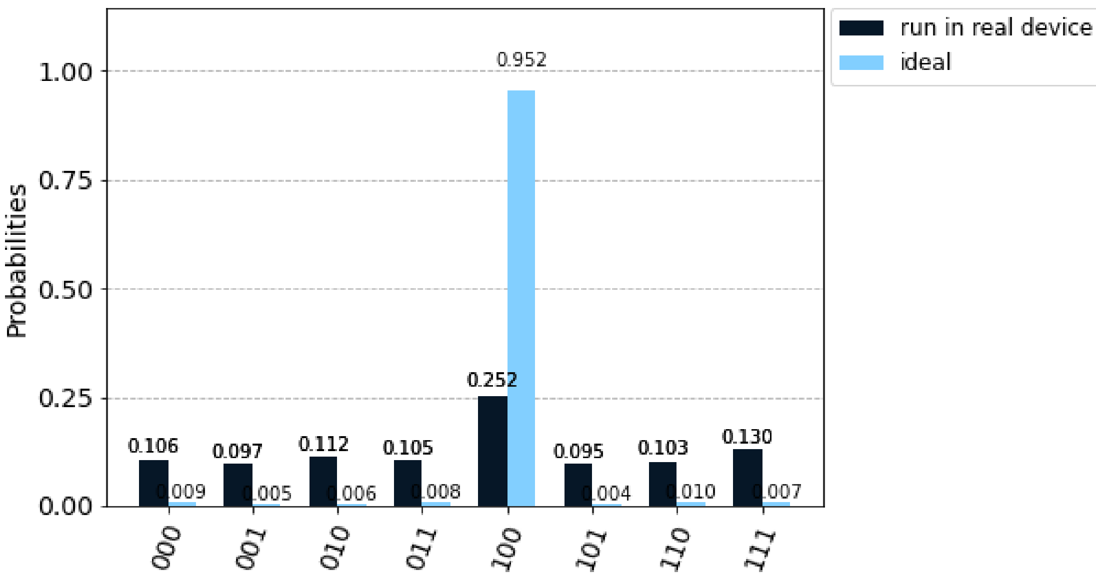
```
print('JOB ID: {}'.format(jobID_r))
```

```
JOB ID: 60bd1e4625cc6e024d65d716
```

[31]:
```
job_get=backend_device.retrieve_job("60bd1e4625cc6e024d65d716")

result_r = job_get.result()
counts_run = result_r.get_counts(qc)
```

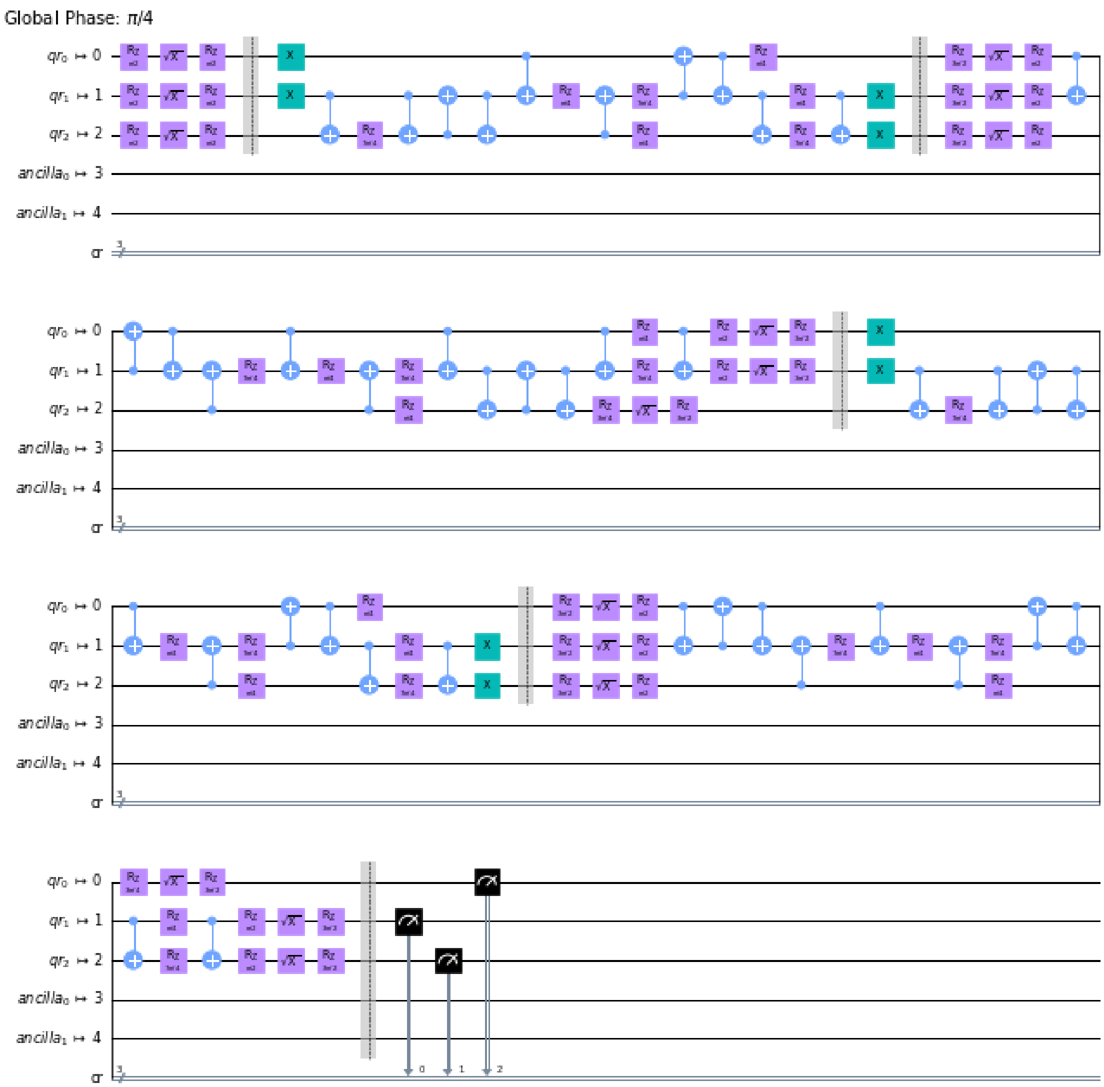
[32]:
```
#result = execute(qc, Aer.get_backend("qasm_simulator"), shots=1024).result()
#counts_sim = result.get_counts(qc)

plot_histogram([counts_run, counts_sim ], legend=[ 'run in real device',␣
↪'ideal'], color=['#061727','#82cfff'])
```

[32]:



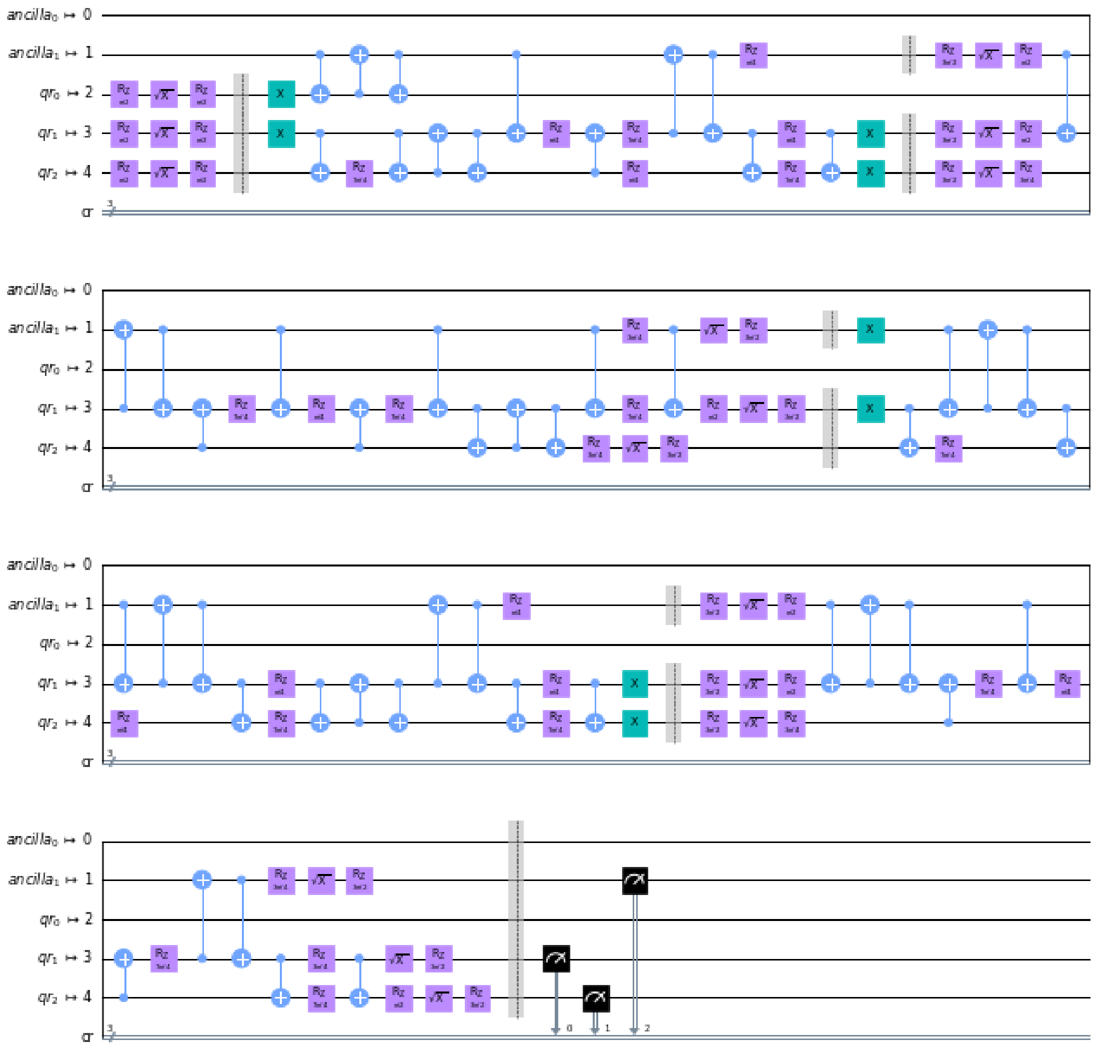
[33]:
```
qc_real = transpile(qc, backend=backend_device)
qc_real.draw(output='mpl', scale=0.5)
```

[33]:



[34]:
```
qc_otimizado = transpile(qc, backend=backend_device, optimization_level=2)
qc_otimizado.draw(output='mpl', scale=0.5)
```

[34]:



[35]:
```
qc_real.depth()
```

[35]: 78

[36]:
```
qc_otimizado.depth()
```

[36]: 82

[37]:
```
job_exp = execute(qc_otimizado, backend_device, shots = 1024)

# job_id allows you to retrive old jobs
jobID = job_exp.job_id()
```

```
print('JOB ID: {}'.format(jobID))

job_exp.result().get_counts(qc_otimizado)
```

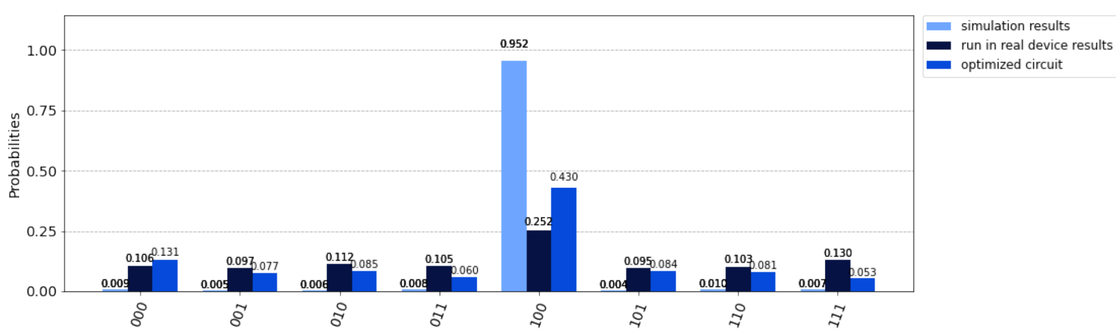
```
JOB ID: 60bd1e6043987e233ff11556
```

[37]:
```
{'000': 134,
 '001': 79,
 '010': 87,
 '011': 61,
 '100': 440,
 '101': 86,
 '110': 83,
 '111': 54}
```

[38]:
```
#with optimization 2
job_get_o=backend_device.retrieve_job("60bd1e6043987e233ff11556")

result_real_o = job_get_o.result(timeout=3600, wait=5)

counts_opt = result_real_o.get_counts(qc_otimizado)
```

[39]:
```
legend = [ 'simulation results','run in real device results', 'optimized␣
 ↪circuit']
color = ['#6ea6ff','#051243','#054ada']

plot_histogram([counts_sim, counts_run, counts_opt], legend = legend,␣
 ↪color=color, figsize=(15, 5))
```

[39]:



1.0.6 Mitigação de erros com IGNIS

[40]:
```
meas_calibs, state_labels = complete_meas_cal(qubit_list=range(3), qr=qr,␣
↪circlabel='mcal')
```

[41]:
```
job_ignis = execute(meas_calibs, backend=backend_device, shots=1024)

jobID_run_ignis = job_ignis.job_id()

print('JOB ID: {}'.format(jobID_run_ignis))
```

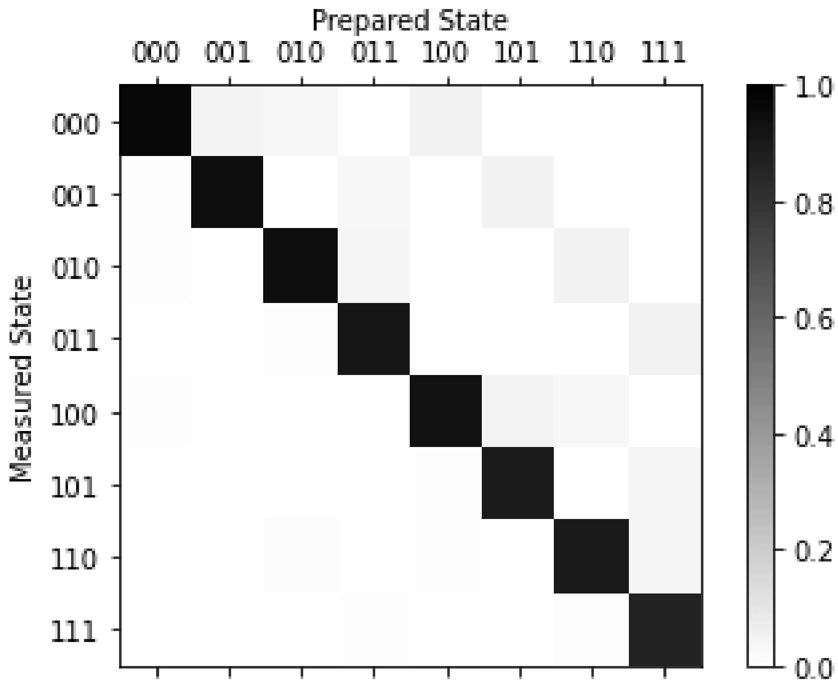
```
JOB ID: 60bd1e8325cc6e17cd65d71b
```

[42]:
```
%qiskit_job_watcher
```

```
Accordion(children=(VBox(layout=Layout(max_width='710px', min_width='710px')),), layout=Layout

<IPython.core.display.Javascript object>
```

[43]:
```
job_get=backend_device.retrieve_job("60bd1e8325cc6e17cd65d71b")

cal_results = job_get.result()
```

[44]:
```
meas_fitter = CompleteMeasFitter(cal_results, state_labels, circlabel='mcal')

# Plot the calibration matrix
meas_fitter.plot_calibration()
```



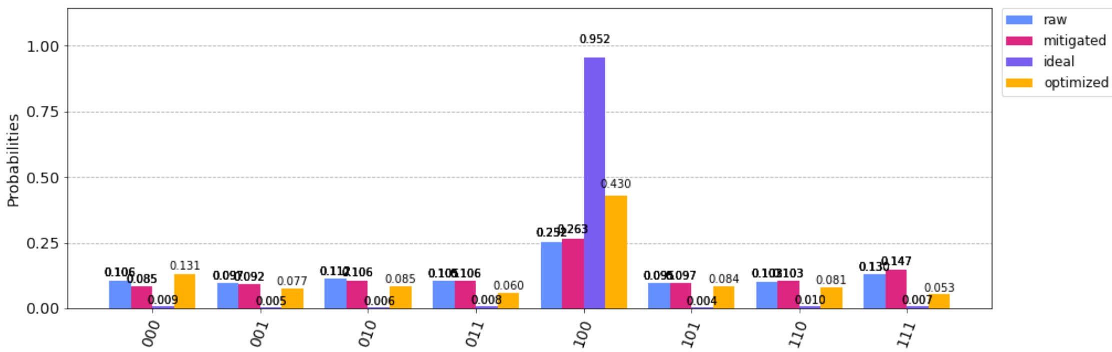
[45]:
```
print("Average Measurement Fidelity: %f" % meas_fitter.readout_fidelity())
```

```
Average Measurement Fidelity: 0.917603
```

Aplicamos um filtro baseado na matriz de calibração.

[46]:
```
# Get the filter object
meas_filter = meas_fitter.filter

# Results with mitigation
mitigated_results = meas_filter.apply(result_r)
mitigated_counts = mitigated_results.get_counts()

plot_histogram([counts_run, mitigated_counts, counts_sim, counts_opt],␣
 ↪legend=['raw', 'mitigated', 'ideal', 'optimized'], figsize=(15,5))
```

[46]:



[47]:
```
%qiskit_version_table
```

<IPython.core.display.HTML object>