Mathematics for Computer Science

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(slides from Luis Soares Barbosa)

Orders & Induction

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Orders

- preorder
- partial order
- linear or total order

Special elements in $(A, \sqsubseteq)$

- least (and greatest) element:

$$\forall_{x \in A} \,.\, m \sqsubseteq x$$

- minimal (and maximal) element:

$$\forall_{x \in A} \,.\, x \sqsubseteq m \Rightarrow x = m$$

- lower (and upper) bound of a subset $S \subseteq A$: is an element $z \in A$ such that

$$\forall_{s \in S} \,.\, z \sqsubseteq m$$

Lattices

- Lattices are partial orders in which every element has a *lub* and *glb*
- *lub* - least upper bound (supremo)
- *glb* - greatest lower bound (infimo)

Well-founded orders and induction

- A *well founded order* is a partial order in which every non empty subset has a minimal element
- ... this is the structure required for induction to be defined
- Example: induction over natural numbers
- Example: induction over sets
- Example: induction over sequences