



CMSC 170: Games

Min-Max Algorithm

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LEARNING OUTCOMES



At the end of the session, the students should be able to:

- understand the Min-Max Algorithm;
- understand the use of Alpha and Beta Pruning ; and
- implement min-max algorithm and alpha beta pruning in a 2-player deterministic game.



Games

Every game has different sets of properties.



Games

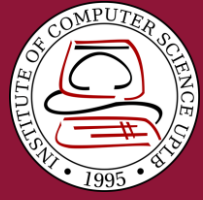
We will discuss 2-player deterministic games and how to make an AI agent.



States

We can define it as S as the set of states.

Ex. $S = \{S_0\}$ //start of the game



Player

There are also P as the of players.

Ex. $P = \{P_1, P_2\}$



Result

There are also results in games. $\text{Result}(s,a) \rightarrow s'$



Results(s,a) $\rightarrow s'$

s' are the new states made when action a is done to a state s .



Actions(s,p)

provides the set of possible actions/moves for a player.



Terminal

It returns true when the state is terminal or there is an end result or false otherwise.



Utility(s,p)

Returns the current value of the state s to a player p .



Utility

It expresses the value of the current game state to the player. It is usually expressed as +/- numbers or 0's or 1's.



Zero-Sum Games

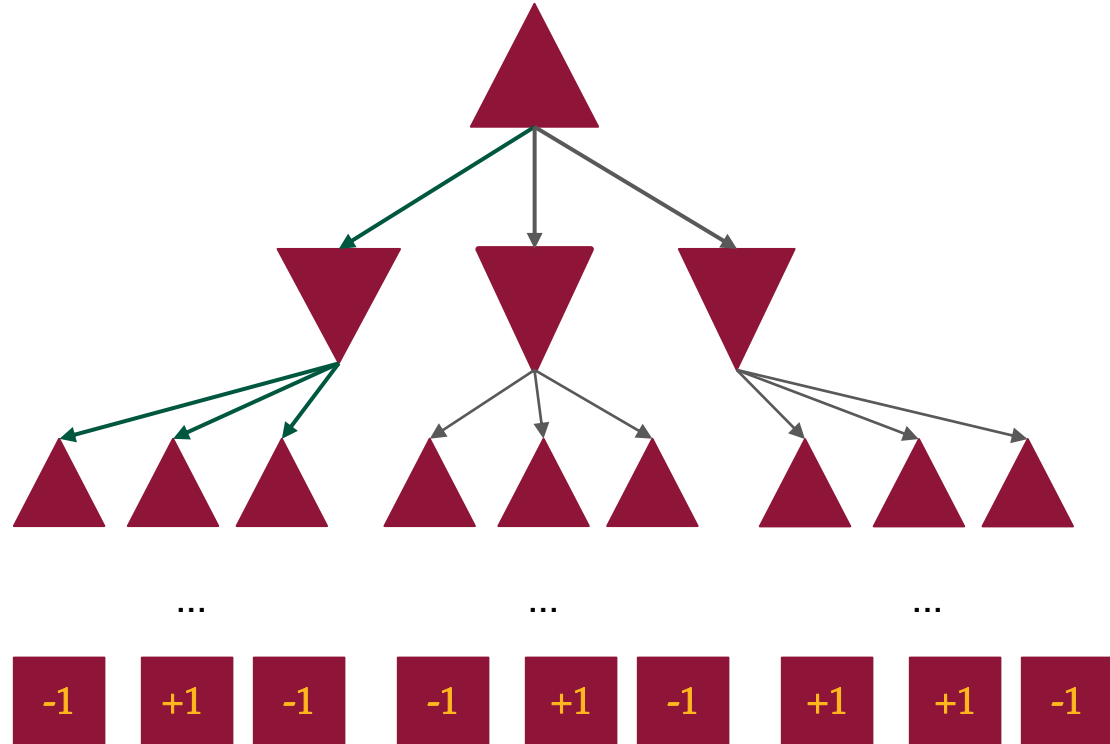
These are games where the overall sum of the utilities for both the players is 0.



Zero-Sum Games

Ex. The utility for a win is 1 and for the loss is -1.

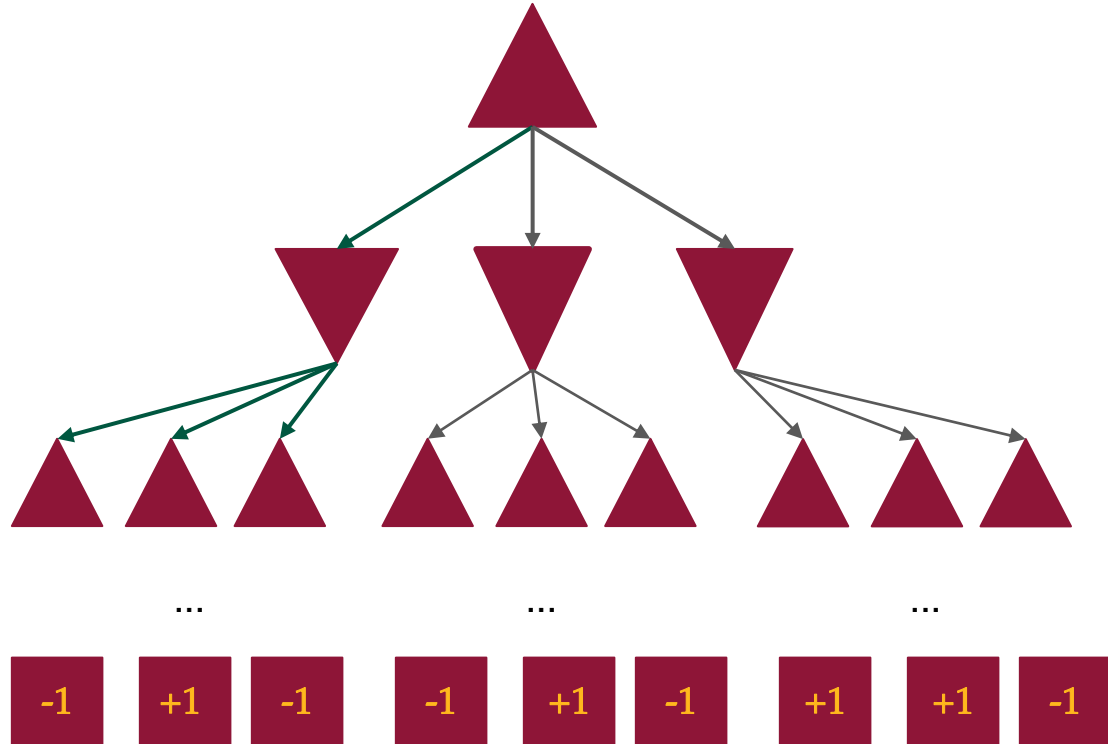
MINMAX ALGORITHM



MINMAX ALGORITHM



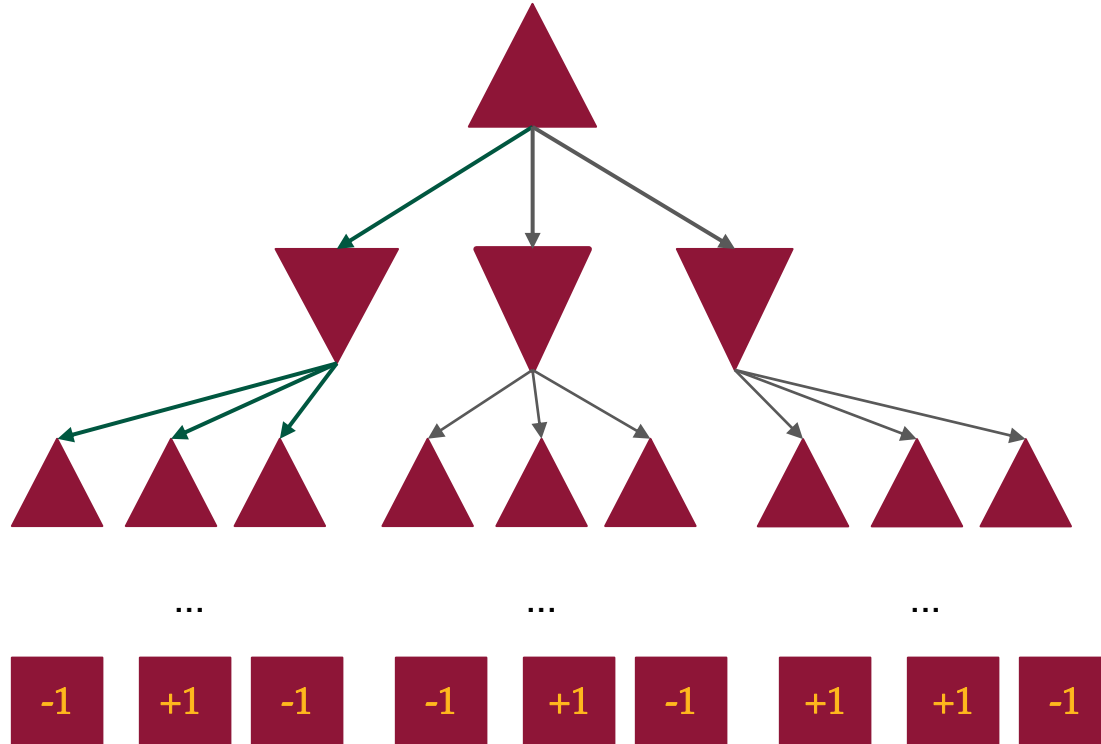
Game Tree



MINMAX ALGORITHM



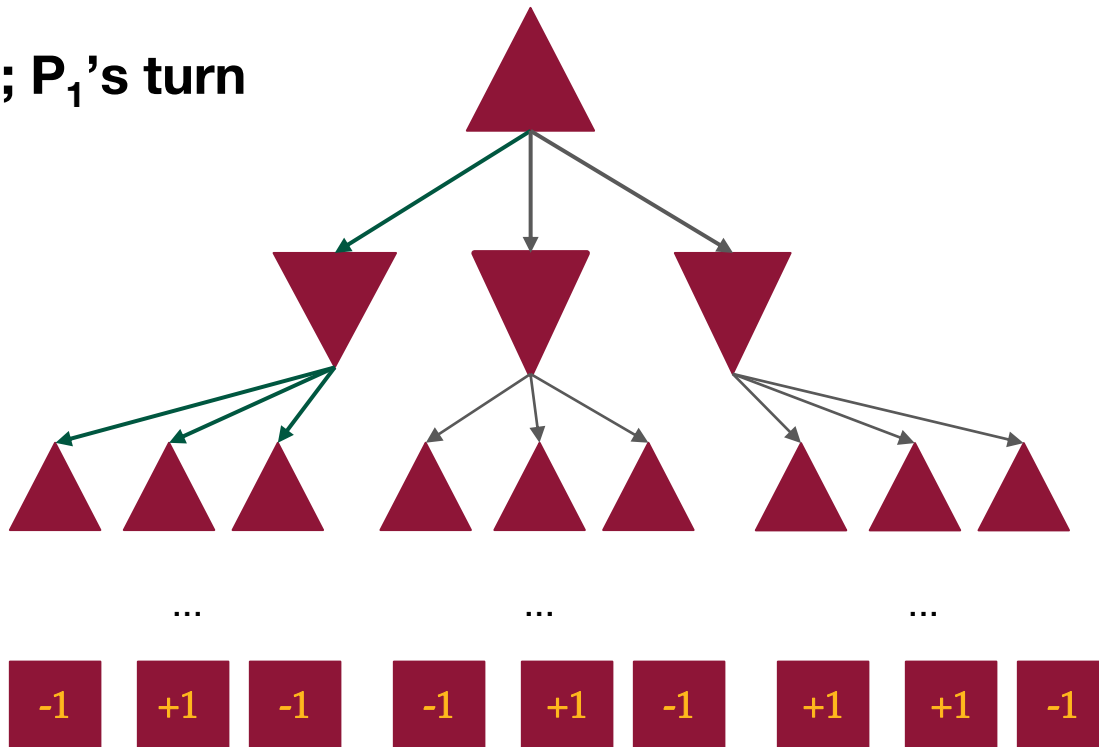
Game Tree



MINMAX ALGORITHM



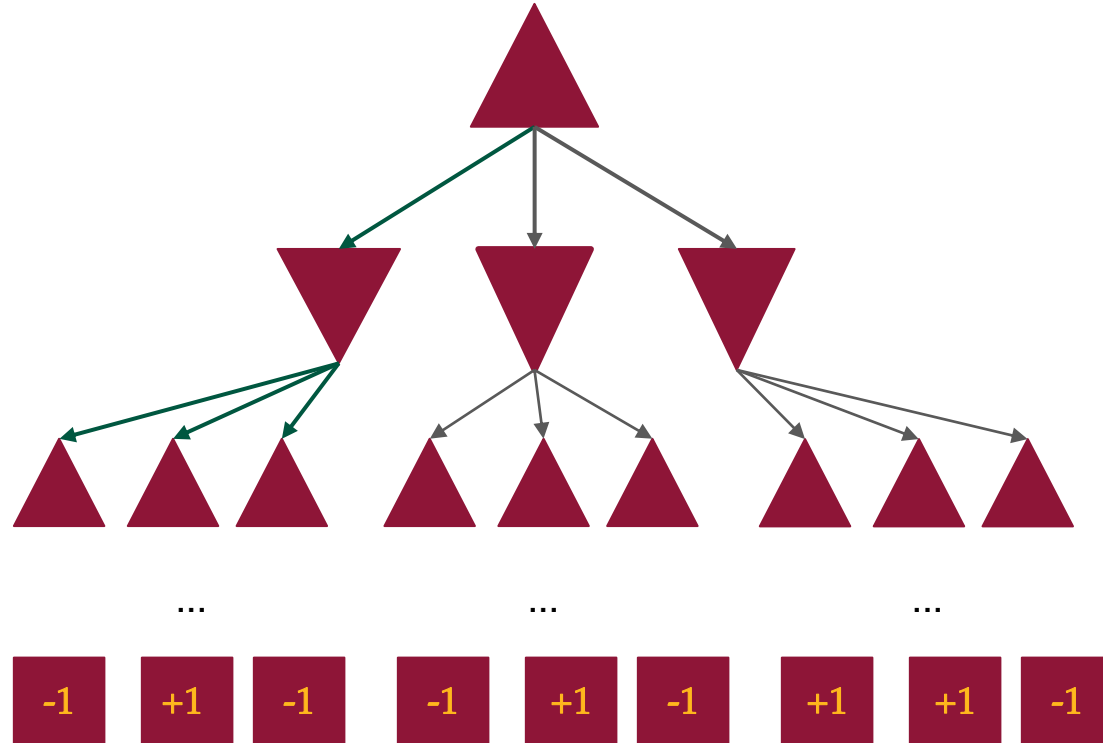
Start of game; P_1 's turn



MINMAX ALGORITHM



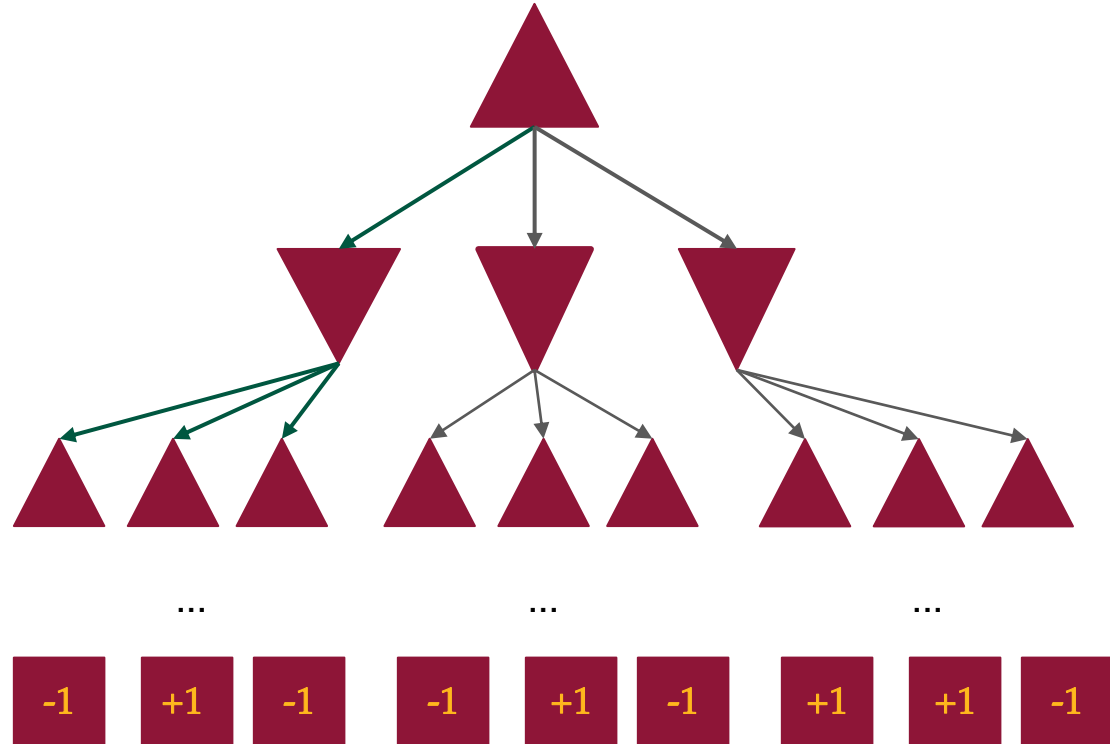
P_2 's turn



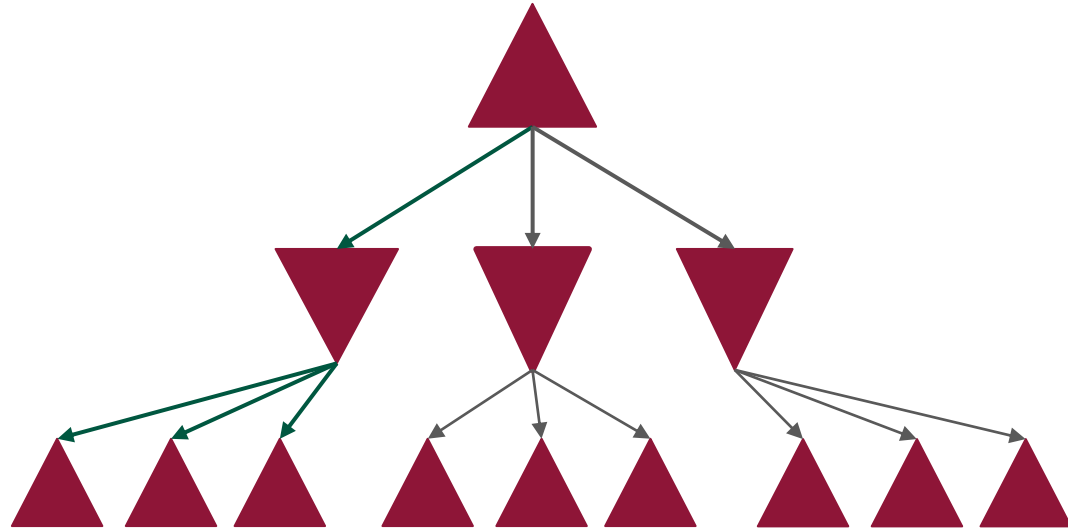
MINMAX ALGORITHM



P_1 's turn again



MINMAX ALGORITHM



Terminal states



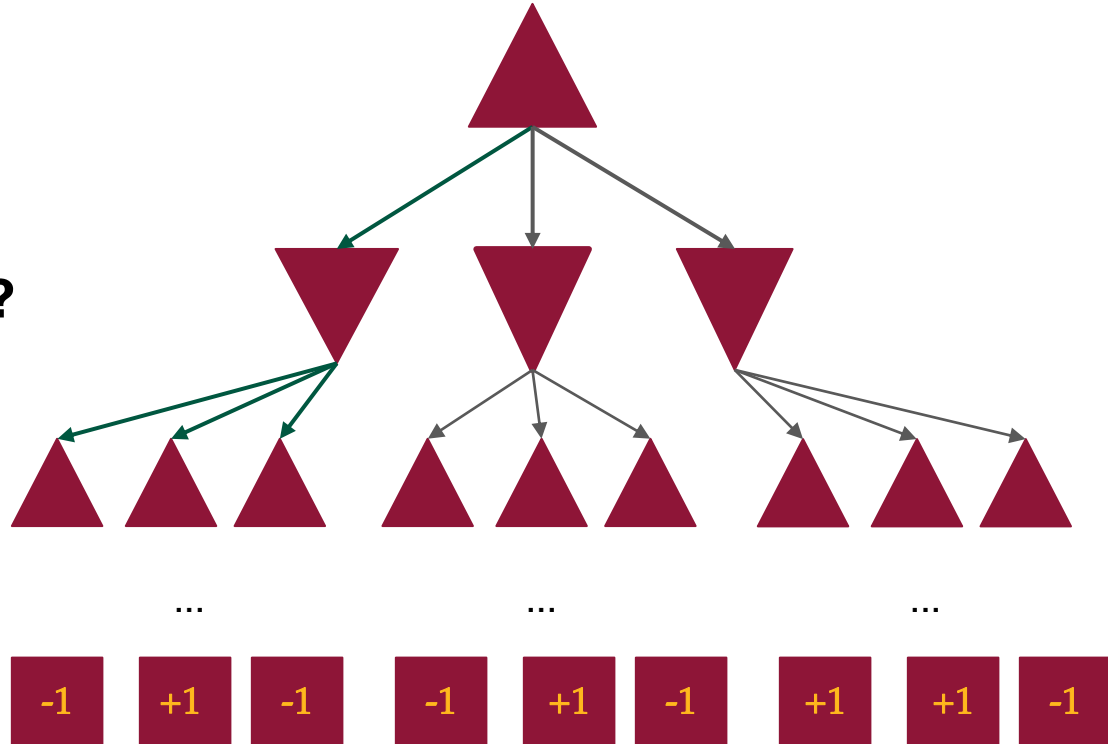


Each node has a utility value. P_1 attempts to **maximize** the values; P_2 attempts to **minimize** the value thus resulting to an adversarial environment.

MINMAX ALGORITHM



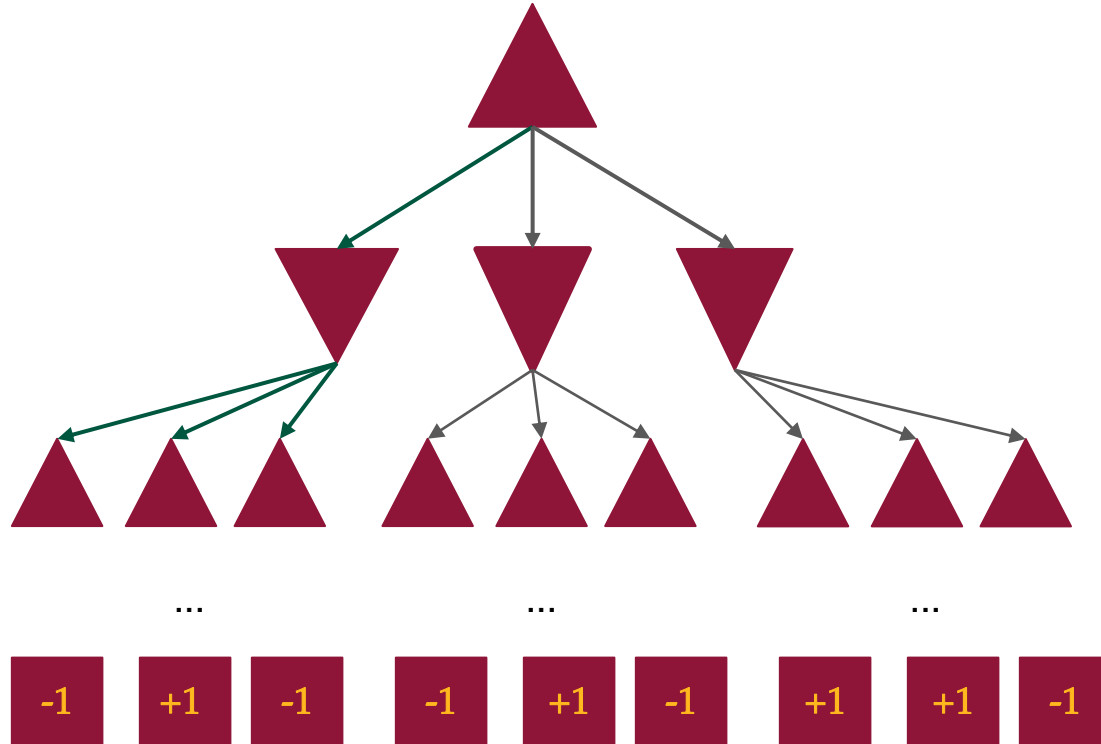
How to get utility
of the inner nodes?



MINMAX ALGORITHM



Use the terminal
states to
Return the values
by following the
min-max behavior
of P_1 and P_2





value (s):

- if s \square is : utility(s)
- if s \triangle is : maxValue(s)
- If s ∇ is : minValue(s)



maxValue(s):

$m = -\infty$

for a, s' in $\text{successors}(s)$:

$v = \text{value}(s')$

$m = \max(m, v)$

return m



minValue(s):

$m = +\infty$

for a, s' in successors(s):

$v = \text{value}(s')$

$m = \min(m, v)$

return m



It has a time complexity of
 $O(B^m)$



BRANCHING FACTOR

approximate number of choice
(successors) a player has at each node.



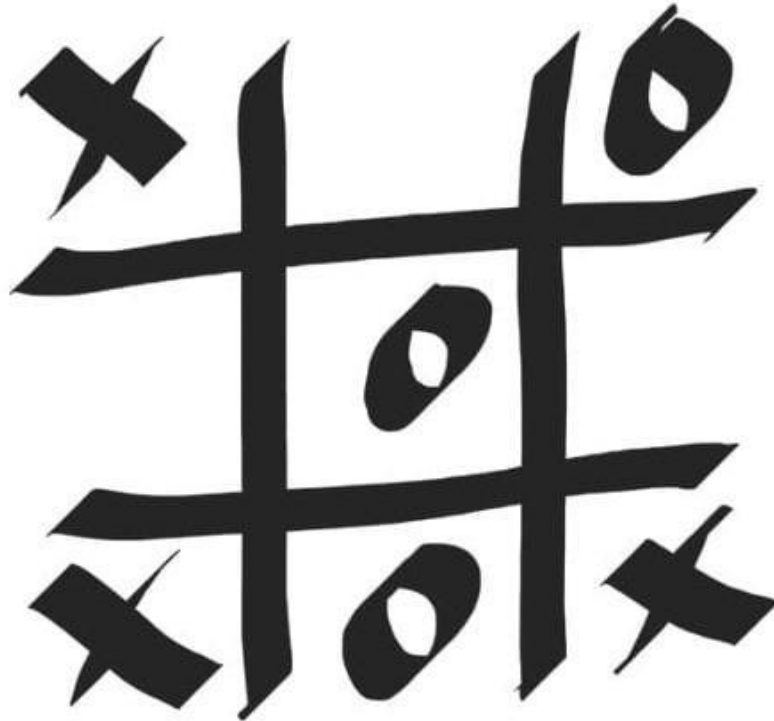
DEPTH

total number of turns for P_1 and P_2 to reach a terminal state

TICTACTOE GAME



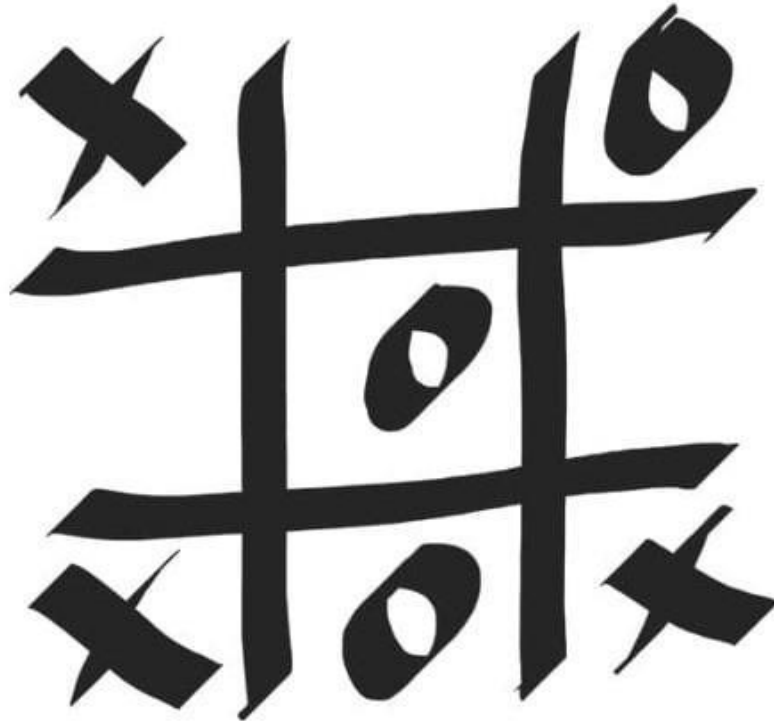
This is our
current state s .



TICTACTOE GAME



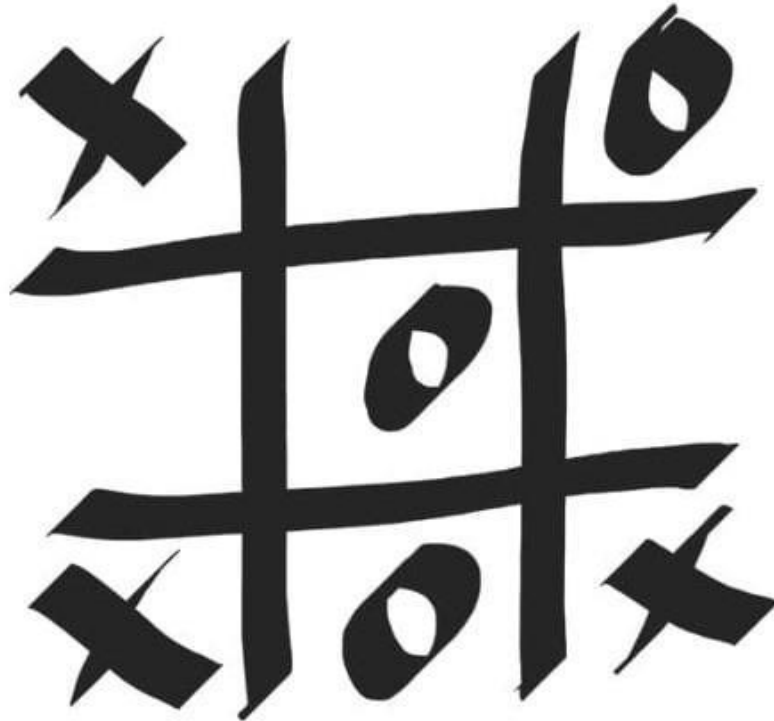
We set X as the maximizer and O as the minimizer.



TICTACTOE GAME



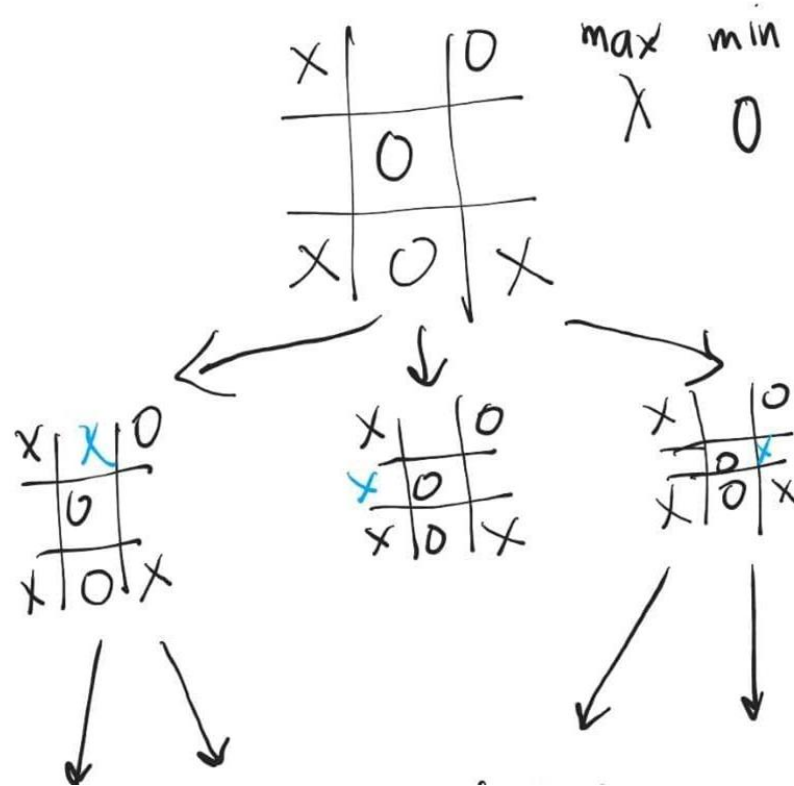
Its X's Turn



TICTACTOE GAME



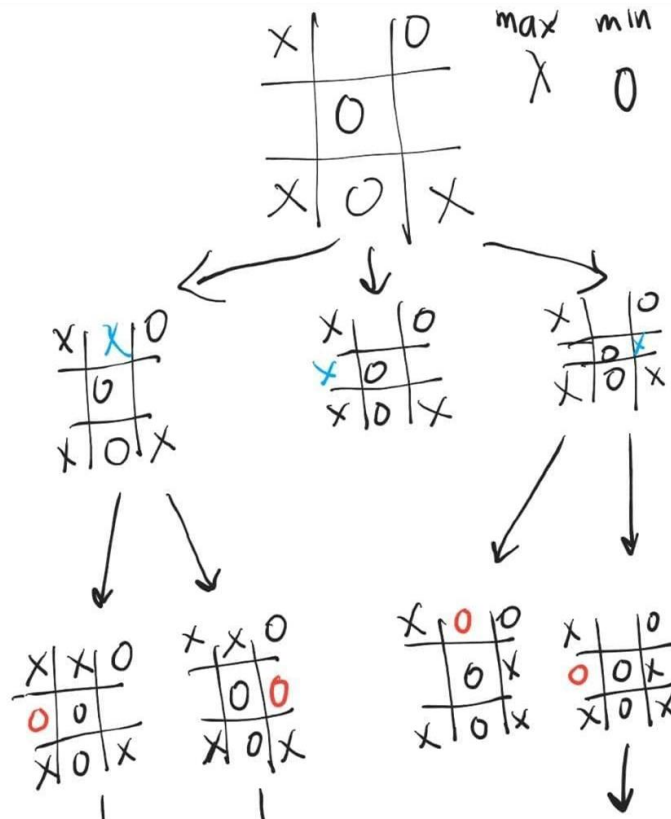
Its X's Turn



TICTACTOE GAME



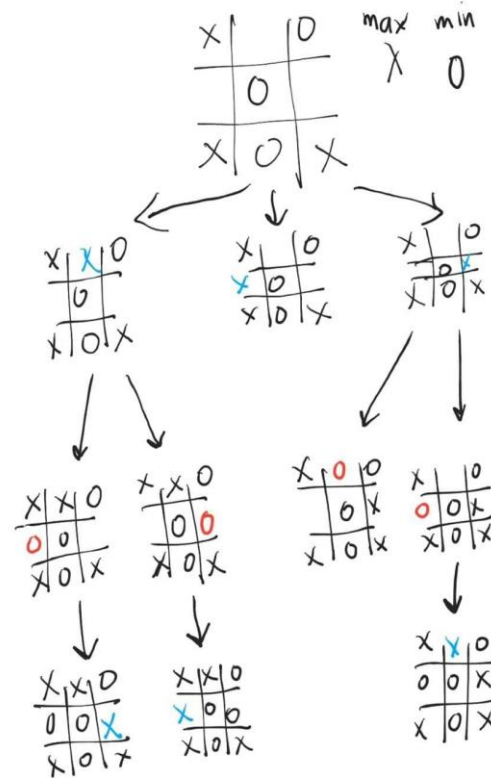
Its 0's Turn



TICTACTOE GAME



Its X's Turn





ALPHA-BETA PRUNING

The algorithm used to avoid searching subtrees of moves that won't be considered.



ALPHA(α)

The best solution (lower bound) for the maximizer on the path to the root.



BETA(β)

The best solution (upper bound) for the minimizer on the path to the root.



value (s, α , β):

- if s \square is : utility(s)
- if s \triangle is : maxValue(s, α , β)
- If s ∇ is : minValue(s, α , β)



maxValue(s, α , β):

$m = -\infty$

for a, s' in successors(s):

$v = \text{value}(s', \alpha, \beta)$

$m = \max(m, v)$

if $v \geq \beta$:

return m

$\alpha = \max(\alpha, m)$

return m



minValue(s, α , β):

$m = +\infty$

for a, s' in successors(s):

$v = \text{value}(s', \alpha, \beta)$

$m = \min(m, v)$

if $v \leq \alpha$:

return m

$\beta = \min(\beta, m)$

return m



For questions and inquiries, you can email
me at

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EXERCISE on MinMax Algorithm



Create a tictactoe program with a MinMax AI agent. It must have a user interface. It must ask whether the user or the AI would go first.

SOME REMINDERS:

- Naming convention for exercise: `surname_minmax`.
- Python or Java can only be used for the exercise.
- Do not forget to put a journal in your ReadMe file in Github.
- Lastly, **Honor and Excellence**.