Computer Graphics

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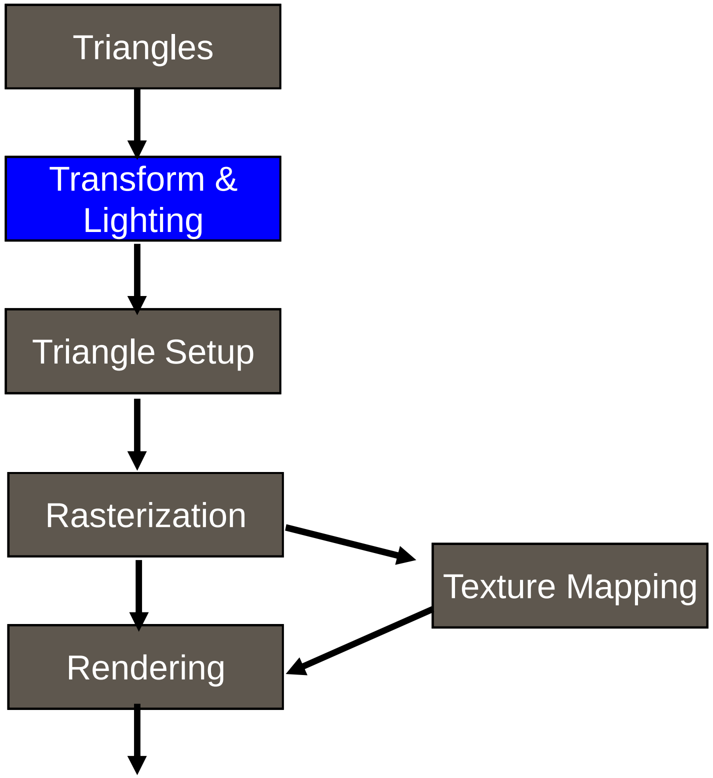
Wrap up from last course

- Geometrical Transformation
- Viewing Transformation
- Projection Transformation
- Viewport Transformation

Part I:
Conventional 3D Graphics Pipeline



Conventional 3D Graphics Pipeline

Lighting

Color Model

Illumination Model

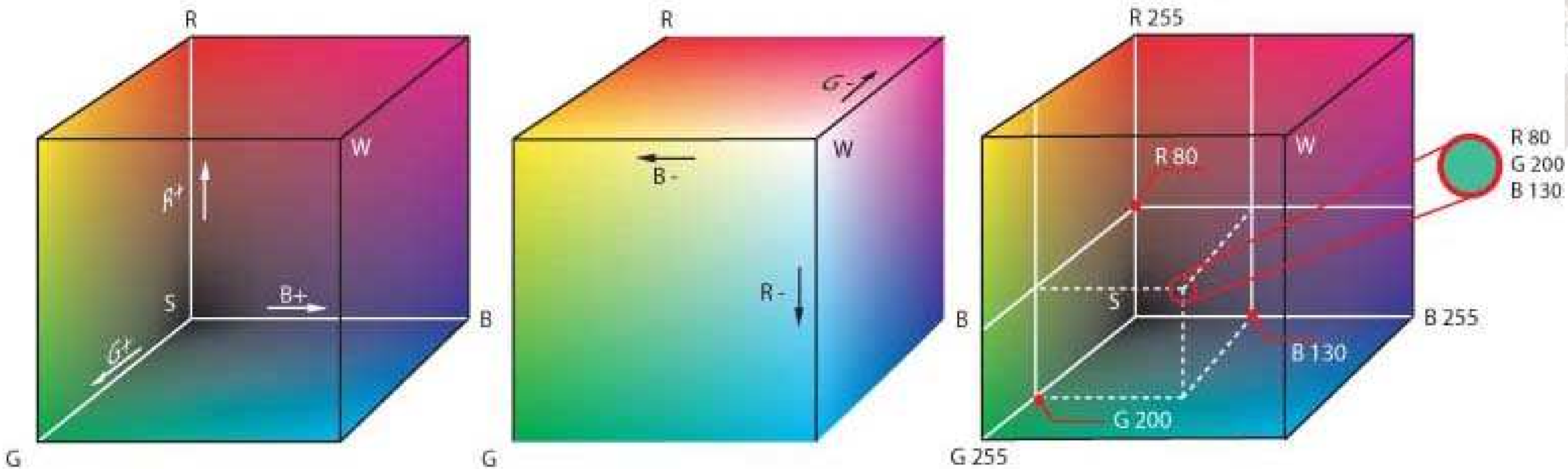
Polygon Shading

Color Model

- A color model is an abstract mathematical model describing the way color can be represented.
- Commonly, three or four values (components) are used to represent a specific color, such as RGB or CMYK

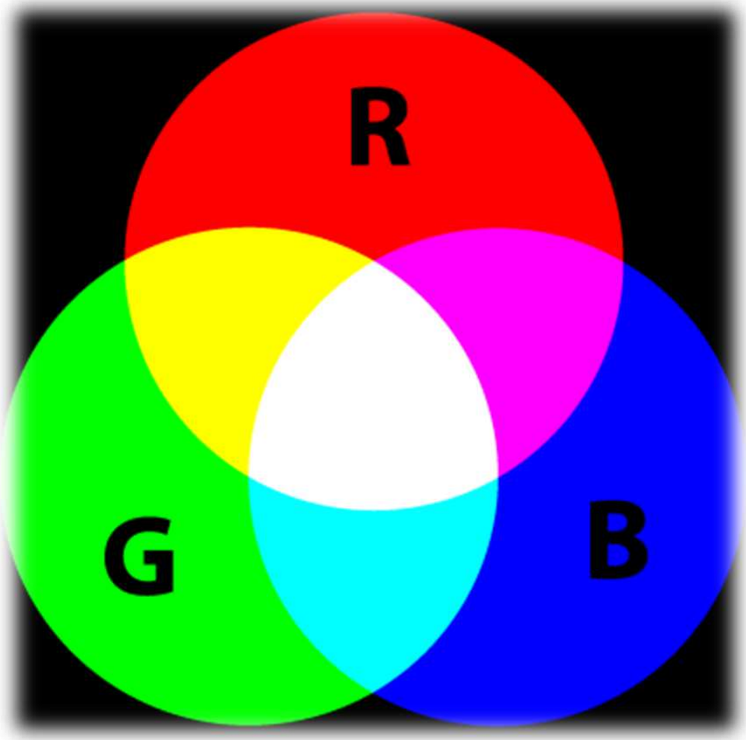
Color Space

- A color space is defined by a mapping function, e.g., $f(r, g, b)$, and the color model



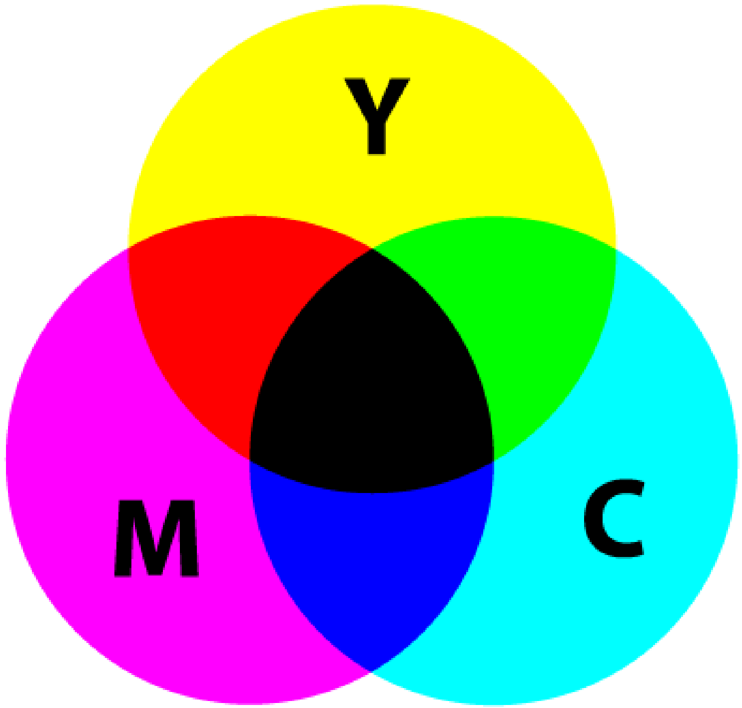
RGB Color Model

- Additive color model
- Used in sensing, representation, and display of images in electronic systems, such as TV and computer



CMYK Color Model

- Subtractive color model
- Used in color printing



Shading

- **Shading is a light-material interaction in determining the color for each point of objects or scenes**

Flat Shading

Smooth Shading

Advanced Shading

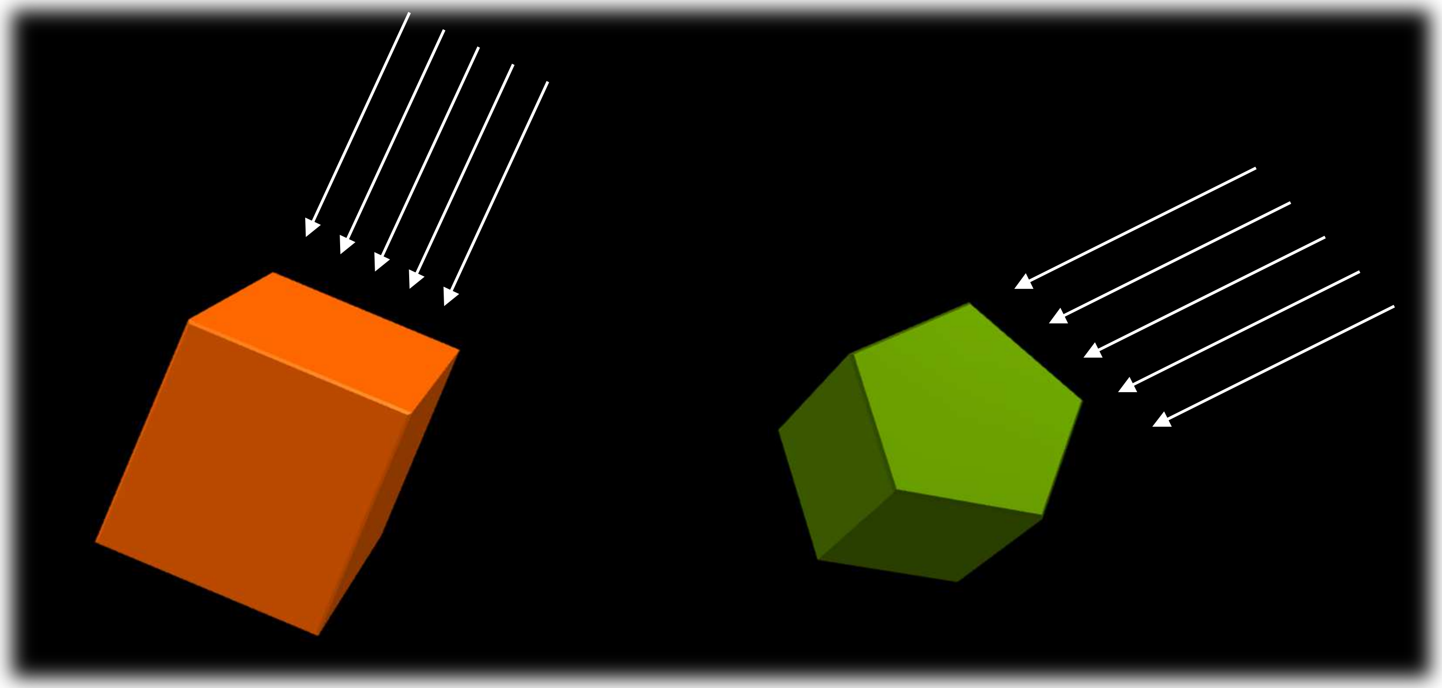
- Global illumination

Factors that affect Shading

- Light sources
  - Ambient light, directional light, positional light, spot light…
- Material properties
  - Ambient reflection, diffuse reflection, specular reflection…
- Location of the viewer
  - Position of perceiving specular highlight
- Surface orientation
  - Surface normal, vertex normal

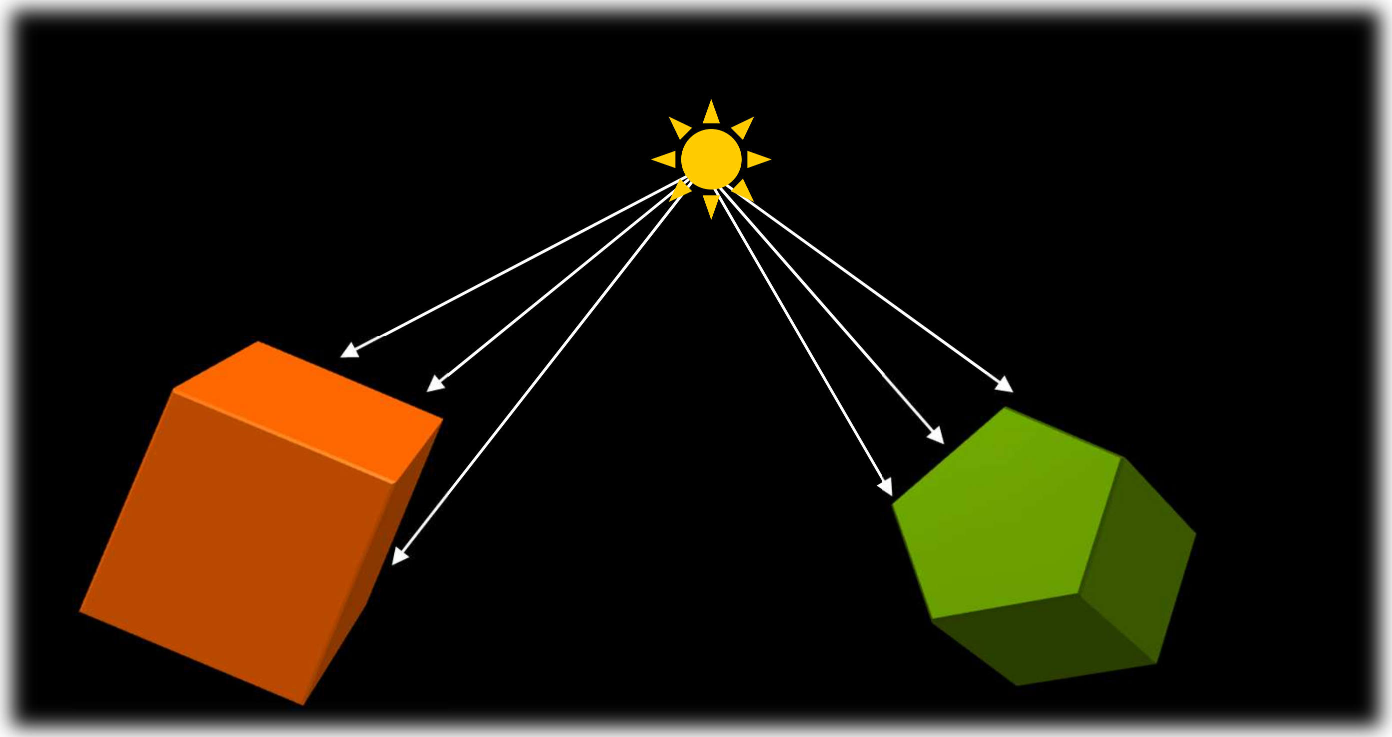
Light Sources

- Directional Light
  - Light source located at infinite far away. Such as sun



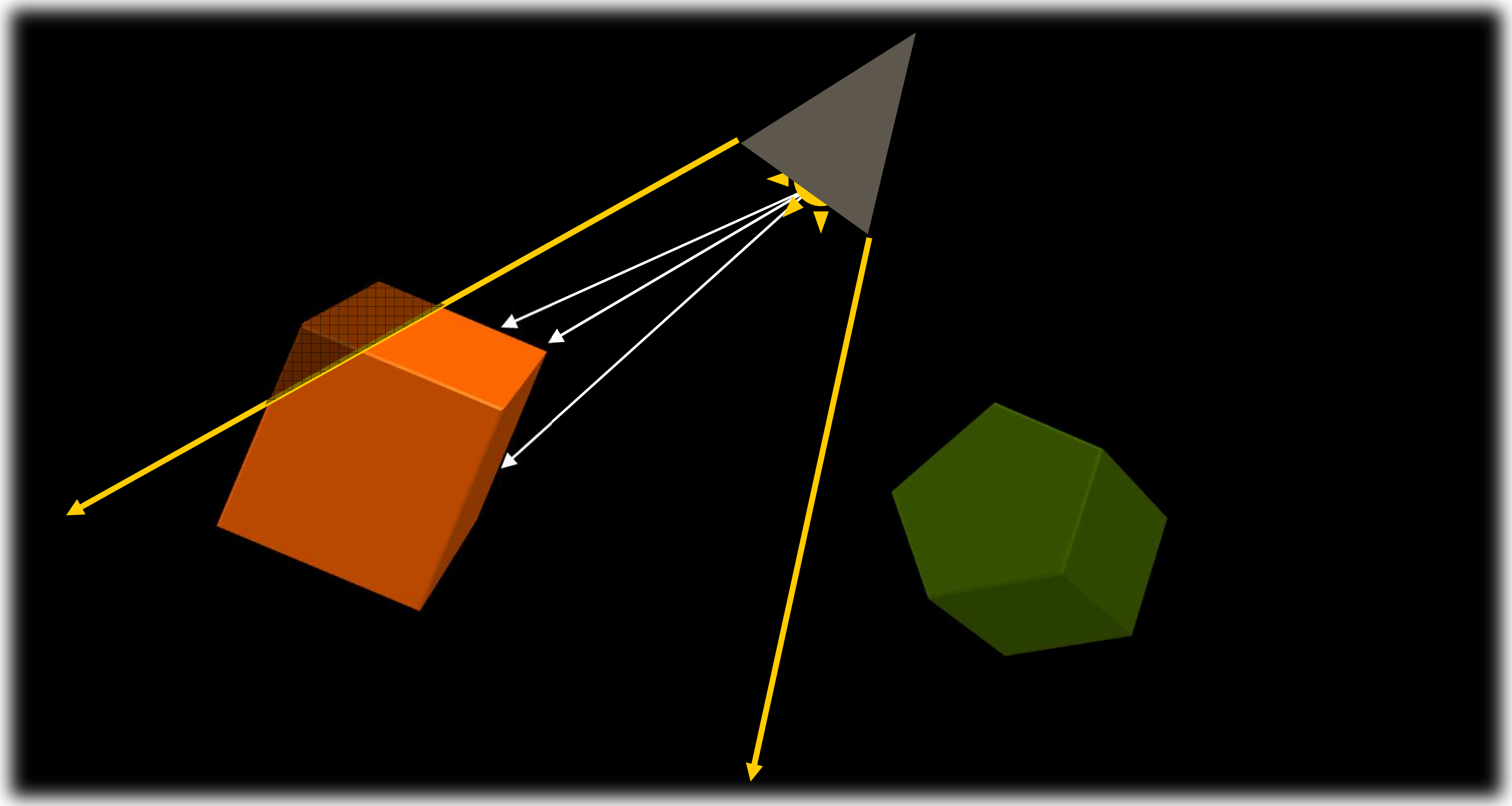
Light Sources

- **Positional Light (Point Light)**
  - **Light source located at a specific position**



Light Sources

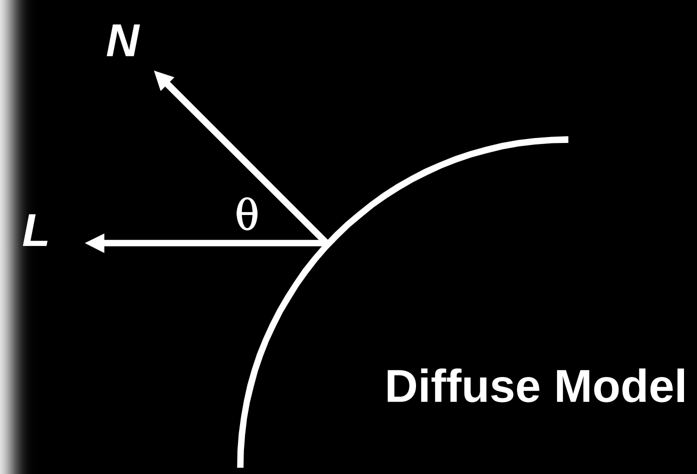
- Spot Light
  - Light source located at a specific position with certain cutoff range

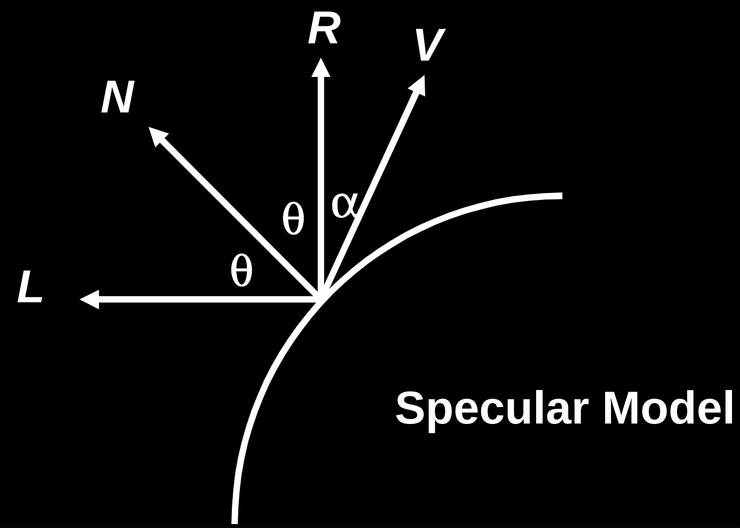


Lighting Equation

- Intensity = Ambient + Diffuse + Specular

$$I = I_a k_a + \sum_{p=1}^{m} f_p I_p (k_d (N \cdot L_p) + k_s (R_p \cdot V)^n)$$





Ambient Light

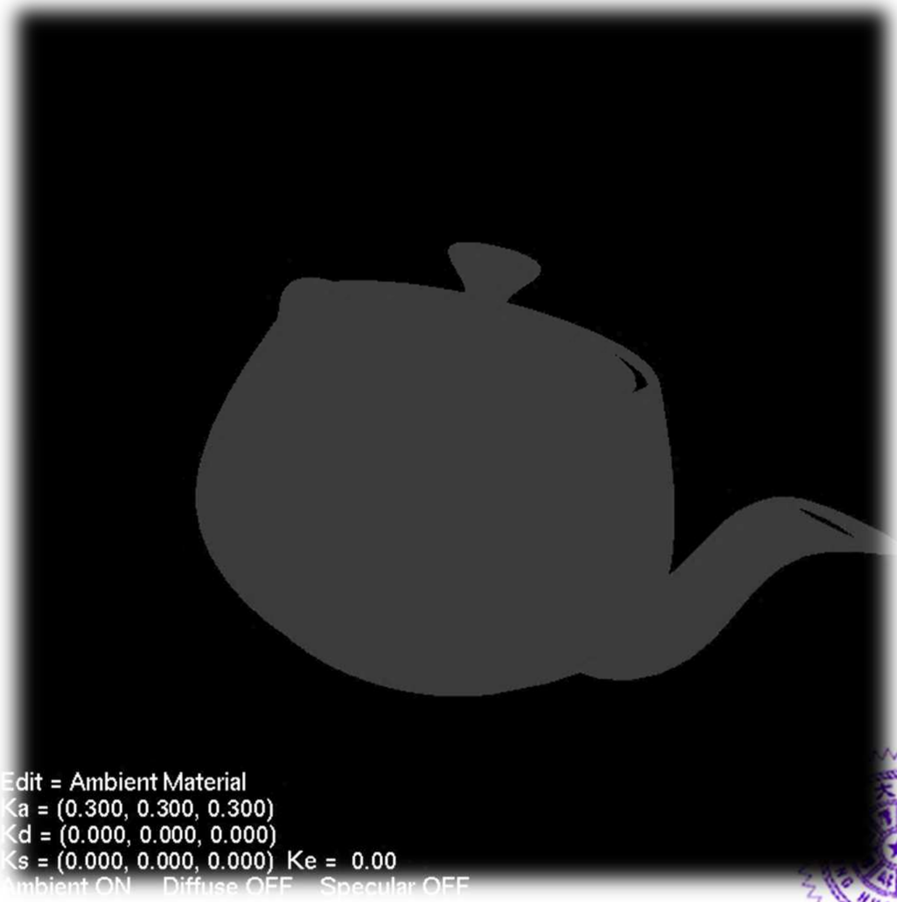
- **Illumination surrounding a scene without providing any specific light source**

$$I = I_a k_a$$

I: resulting intensity

I_a: ambient light intensity

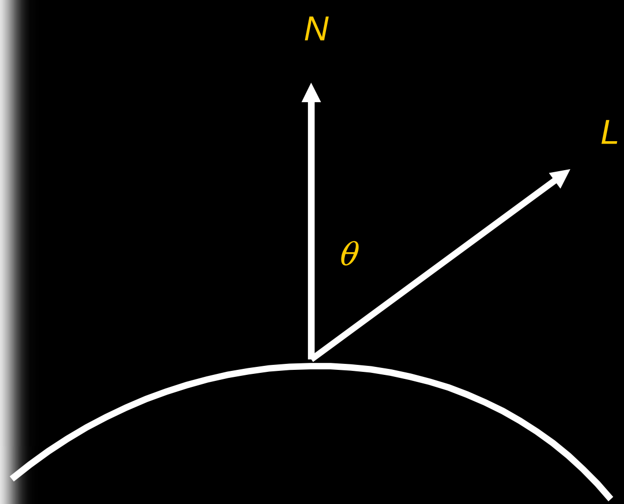
k_a : ambient reflection coefficient



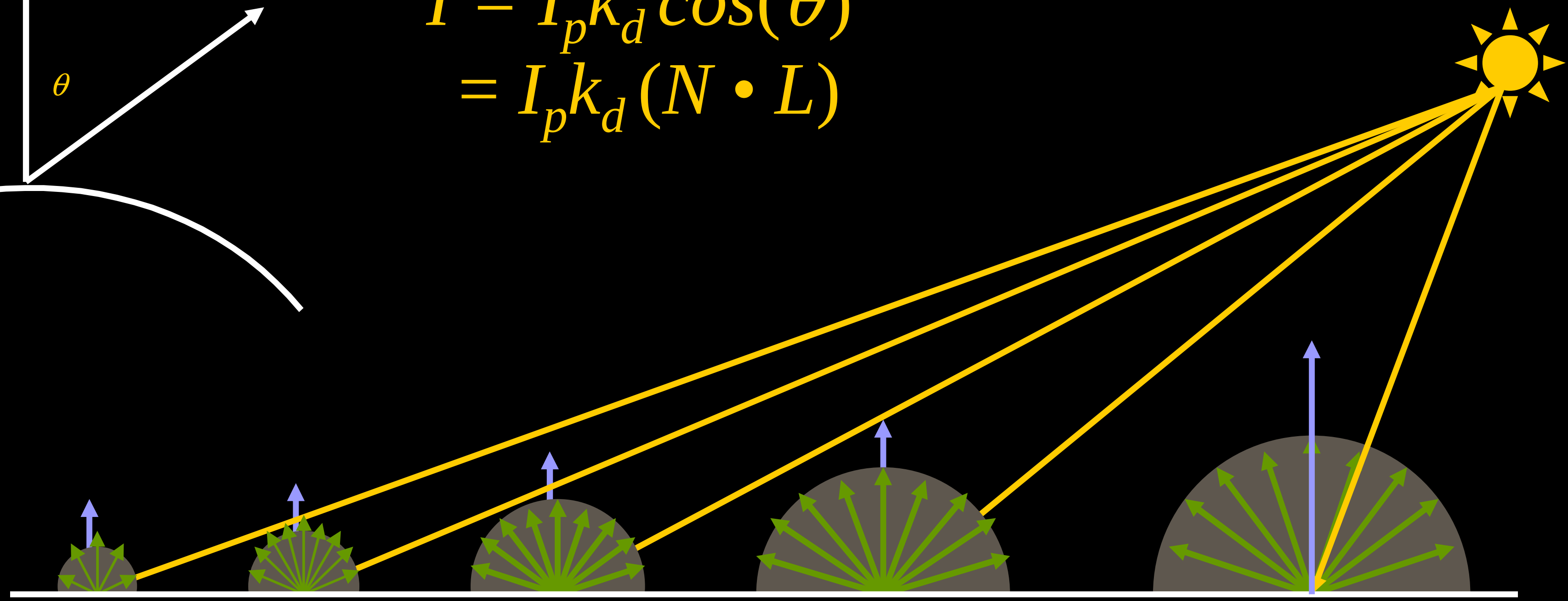
Diffuse Reflection

- **Lambert's Cosine Law**

I_p: point light source intensity
k_d: diffuse reflection coefficient
N: normalized normal vector
L: normalized light direction vector

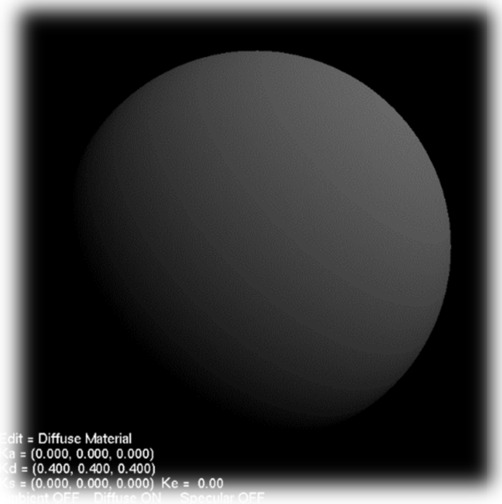


$$I = I_p k_d \cos(\theta) = I_p k_d (N \bullet L)$$

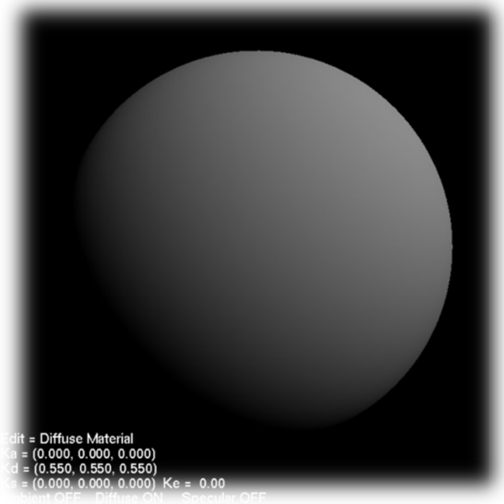


Diffuse Reflection

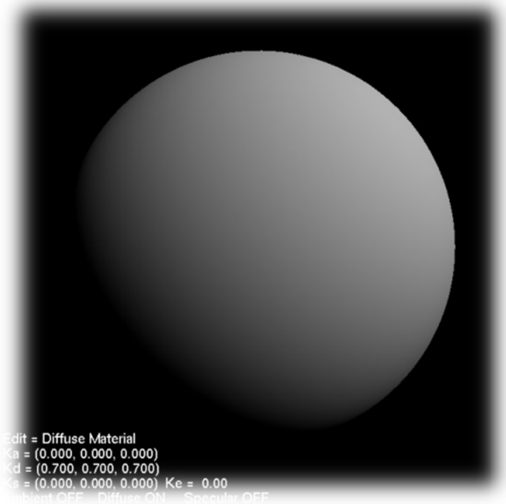
- **Example**
  - **Fixed point light source at (1.0, 1.0, 1.0)**



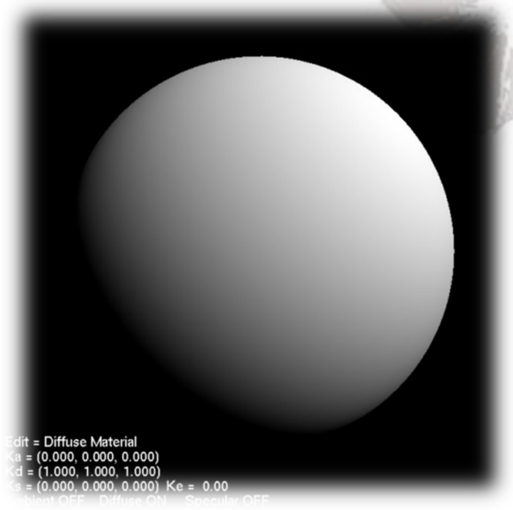
k_d = 0.4



k_d = 0.55



k_d = 0.7

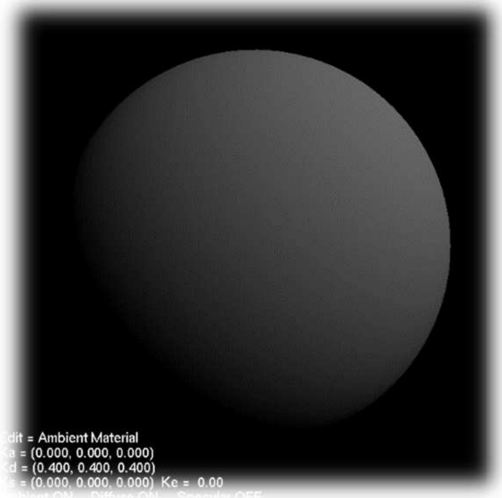


k_d = 1.0

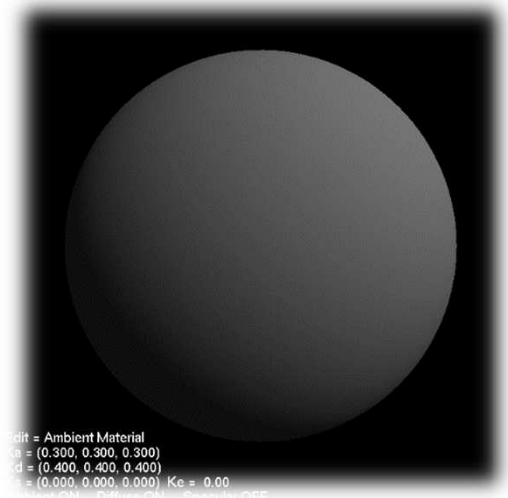
Ambient + Diffuse Reflection

- **Example**
  - **k_d = 0.4**

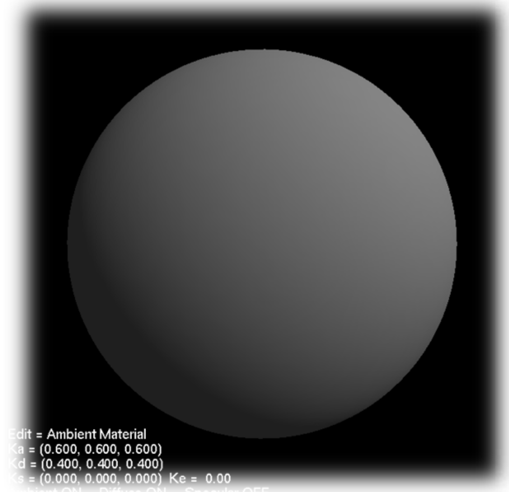
$$I = I_a k_a + I_p k_d (N \bullet L)$$



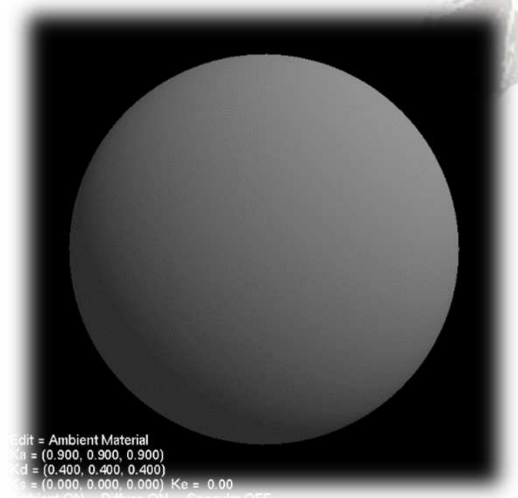
k_a = 0.0



k_a = 0.3



k_a = 0.6



k_a = 0.9

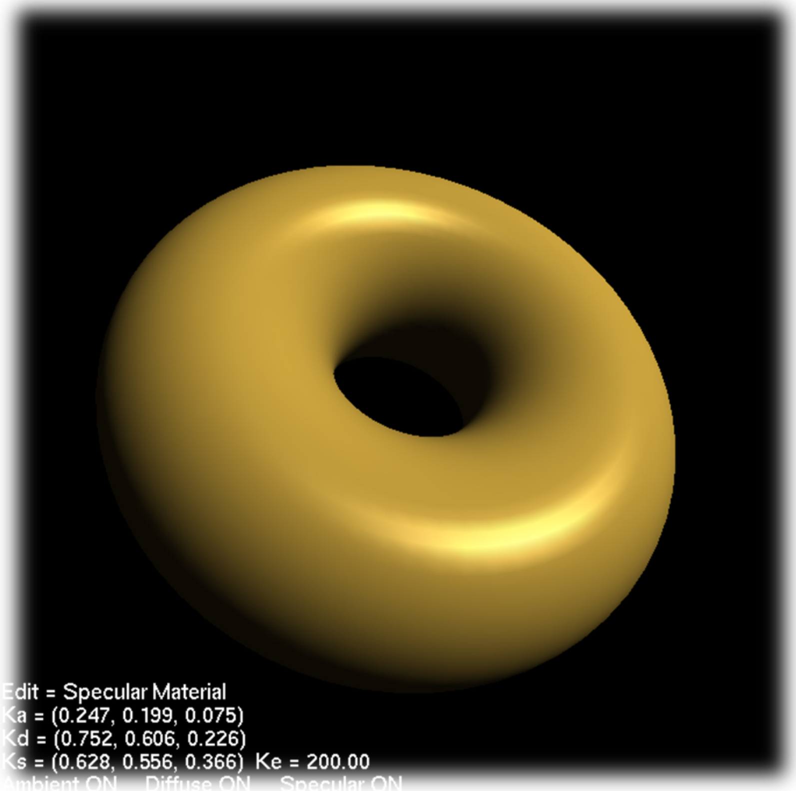
Light Source Attenuation

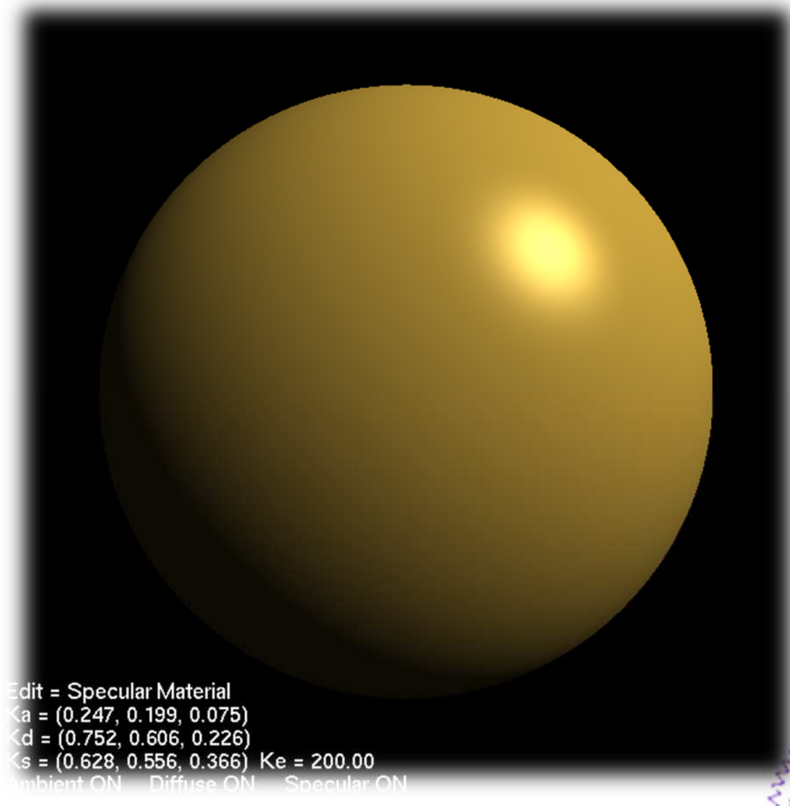
- **Light source intensity will attenuate with respect to the distance between the light source and the object**

$$I = I_a k_a + f_{att} I_p k_d (N \bullet L) \qquad f_{att} = \min\left(\frac{1}{c_1 + c_2 d_L + c_3 d_L^2}, 1\right)$$

Specular Highlight

- Specular highlight is the bright spot of light on object being illuminated

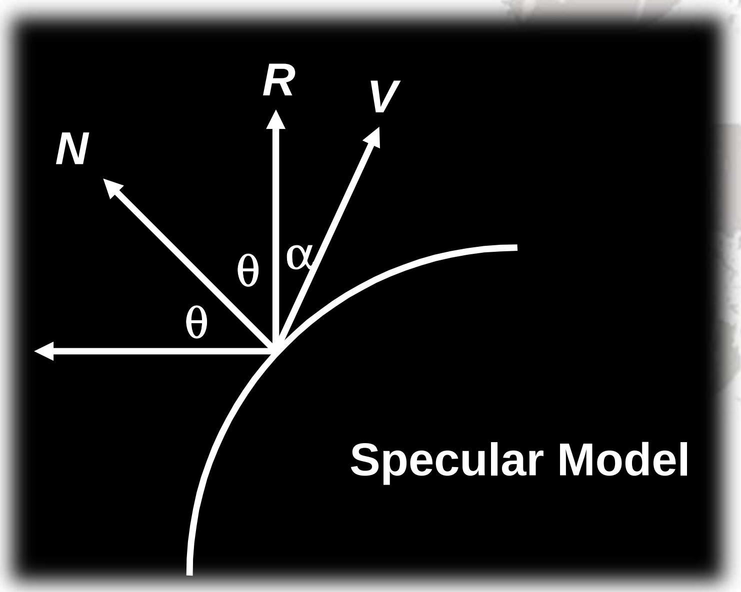




Specular Highlight

$$I_s = I_p k_s \cos^n \alpha = I_p k_s (R_p \cdot V)^n$$

$$I = I_a k_a + f_p I_p (k_d (N \cdot L_p) + k_s (R_p \cdot V)^n)$$



I: Intensity of final illumination

I_a: Intensity of ambient light

k_a: Ambient reflection coefficient

f_p: Attenuation function of point light source *p*

k_d: Diffuse reflection coefficient

N: Normalized normal vector

L_p: Normalized light source direction of point light source *p*

k_s: Specular reflection coefficient

R_p: Normalized light source reflection vector of point light source *p*

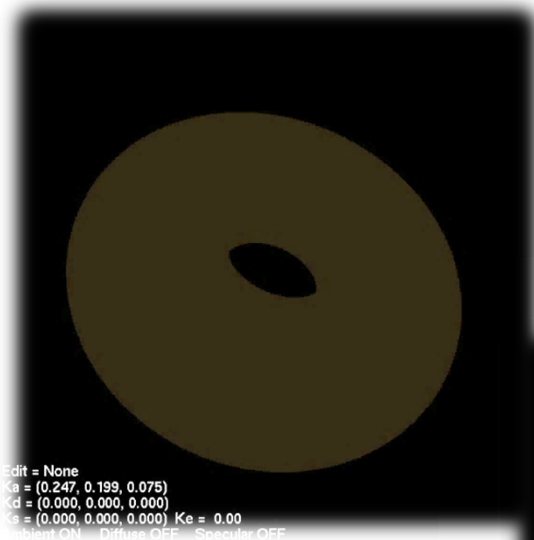
V: Normalized viewpoint direction vector

n: Material's specular reflection exponent

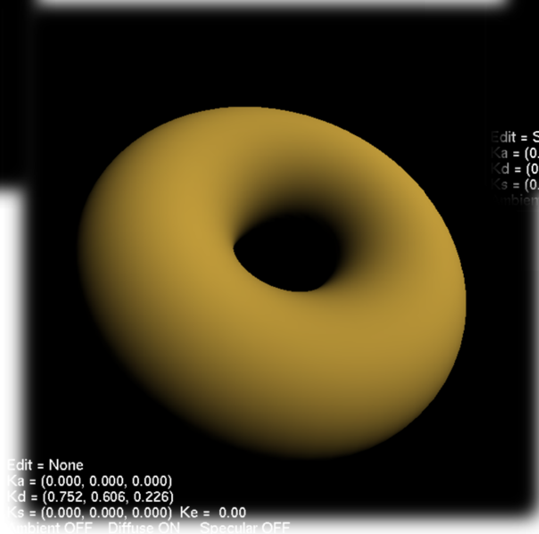
Specular Highlight

◆ **Phong Reflection Model**

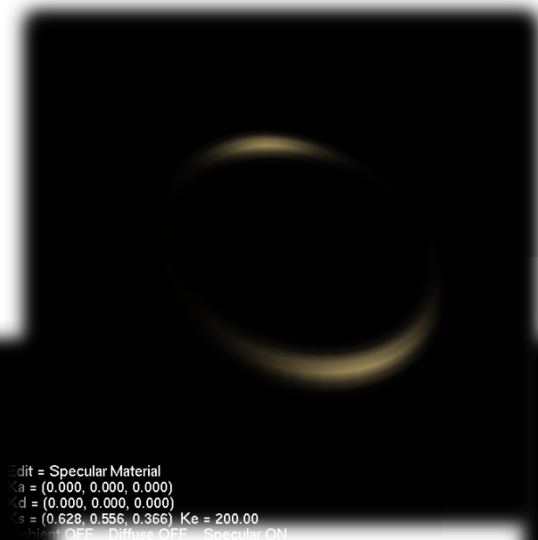
$$I = I_a k_a + f_p I_p (k_d (N \cdot L_p) + k_s (R_p \cdot V)^n)$$



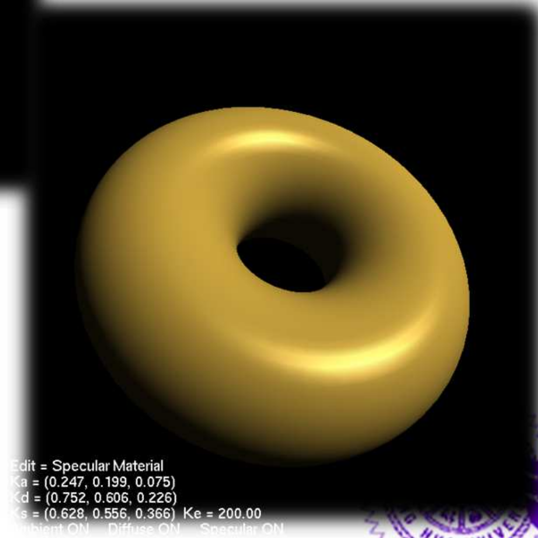
Ambient



Diffuse



Specular



Final illumination

Modified Phong Reflection Model

- Also called Blinn-Phong Reflection Model
  - Original Phong model requires to calculate the reflection vector and view vector for each point
  - Blinn suggested to use an approximated way to calculated the specular reflection term by introducing the halfway vector

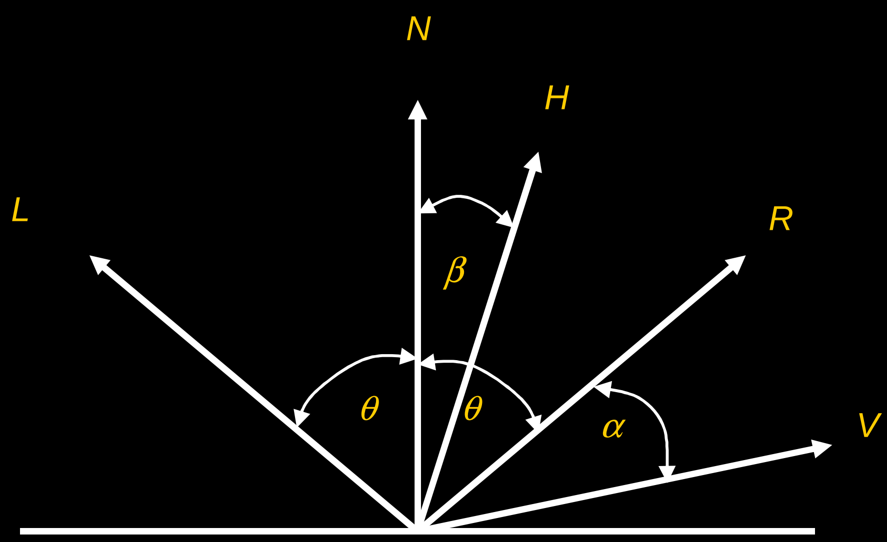
The Halfway Vector

- The halfway vector is the normalized vector halfway between the viewpoint and the light vector

$$H = \frac{L+V}{|L+V|}$$

$$\theta + \beta = \theta - \beta + \alpha$$

$$\beta = \frac{1}{2}\alpha$$



Multiple Light Sources

$$I = I_a k_a + \sum_{p=1}^{m} f_p I_p (k_d (N \cdot L_p) + k_s (N \cdot H_p)^{n'})$$

I: Intensity of final illumination

I_a: Intensity of ambient light

k_a: Ambient reflection coefficient

f_p: Attenuation function of point light source *p*

k_d: Diffuse reflection coefficient

N: Normalized normal vector

L_p: Normalized light source direction of point light source *p*

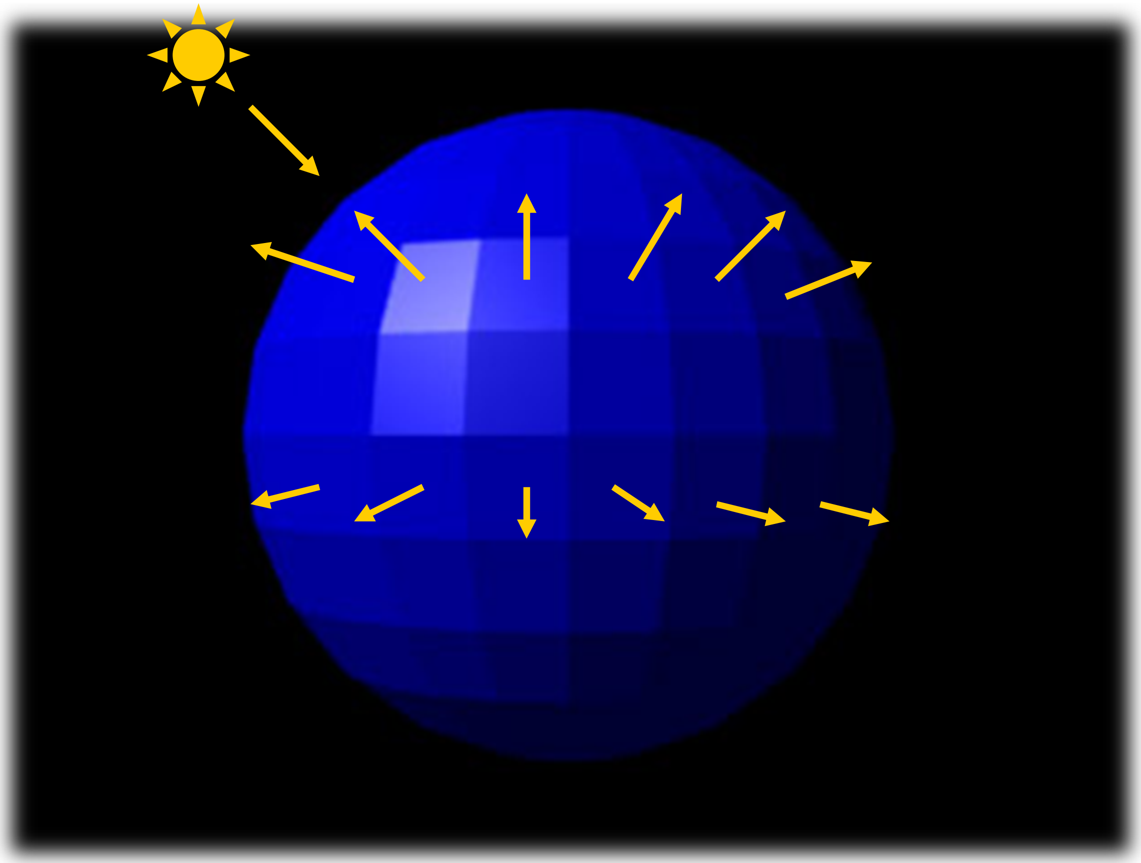
k_s: Specular reflection coefficient

H_p: Normalized half vector between viewpoint and the point light source *p*

n': Material's specular reflection exponent (Blinn-Phong Reflection Model)

Flat Shading

- Using face normal to derive polygon color

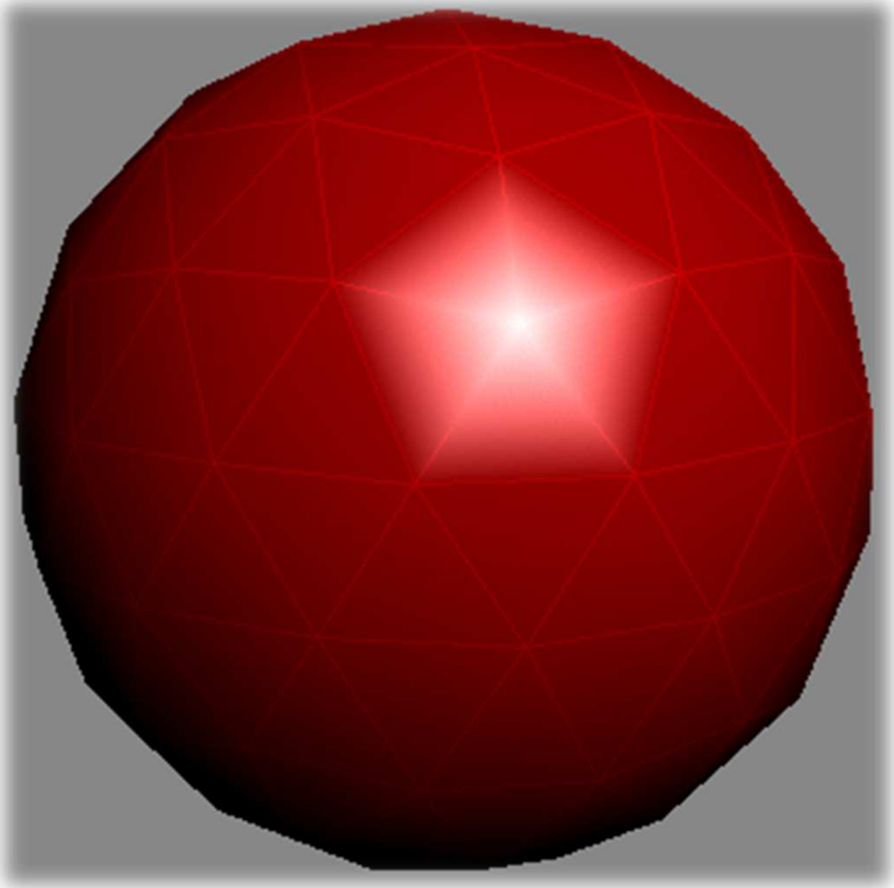


Smooth Shading

- **The colors for the interior of the polygon are interpolated between vertex colors**

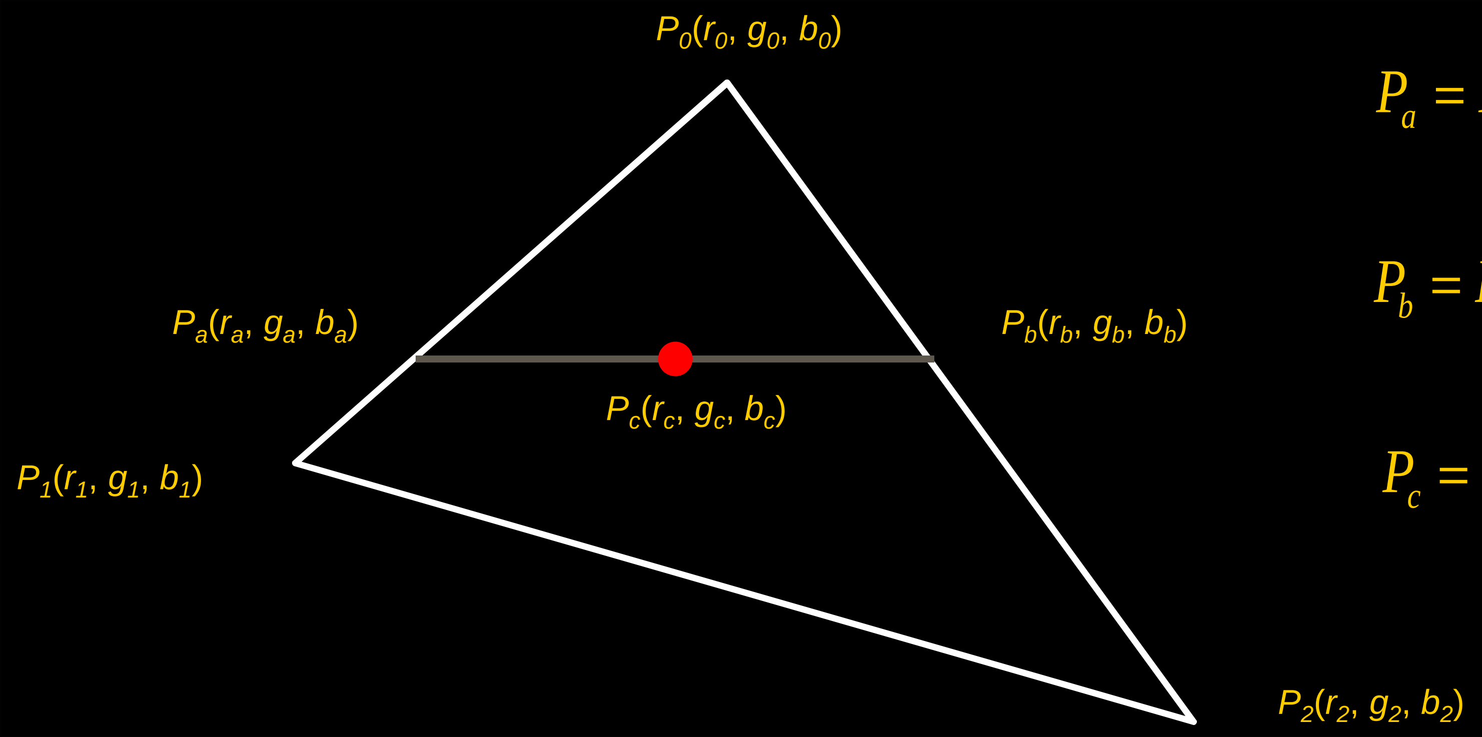
Smooth Shading

- Gouraud Shading



Smooth Shading

- Gouraud Shading
  - Compute colors for each vertices respectively
  - Interpolate pixel colors through vertex colors



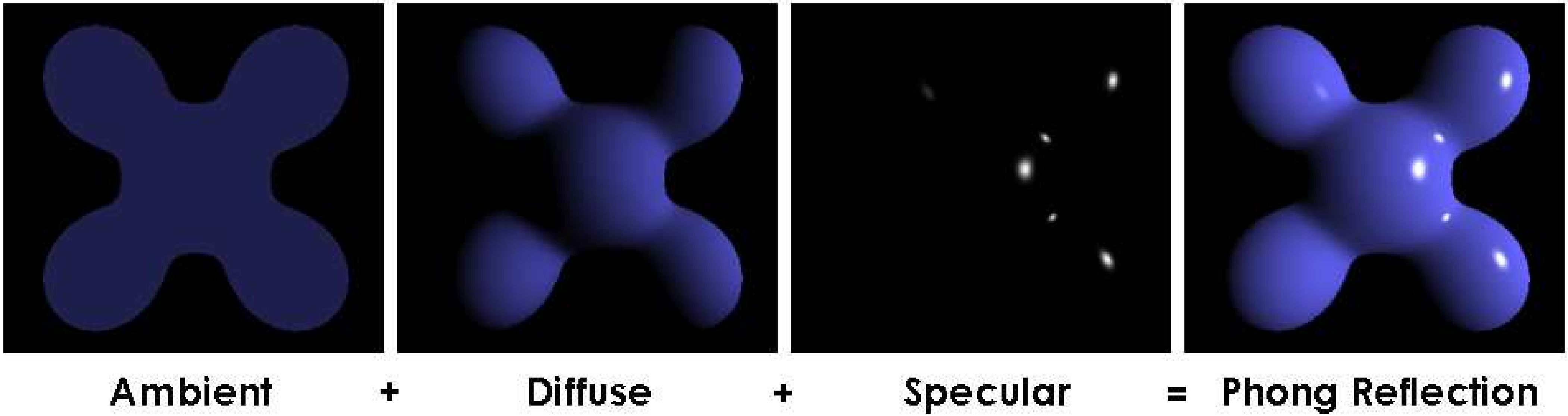
$$P_a = P_0\left(1-\frac{y_a-y_0}{y_1-y_0}\right)+P_1\frac{y_a-y_0}{y_1-y_0}$$

$$P_b = P_0\left(1-\frac{y_b-y_0}{y_2-y_0}\right)+P_2\frac{y_b-y_0}{y_2-y_0}$$

$$P_c = P_a\left(1-\frac{x_c-x_a}{x_b-x_a}\right)+P_b\frac{x_c-x_a}{x_b-x_a}$$

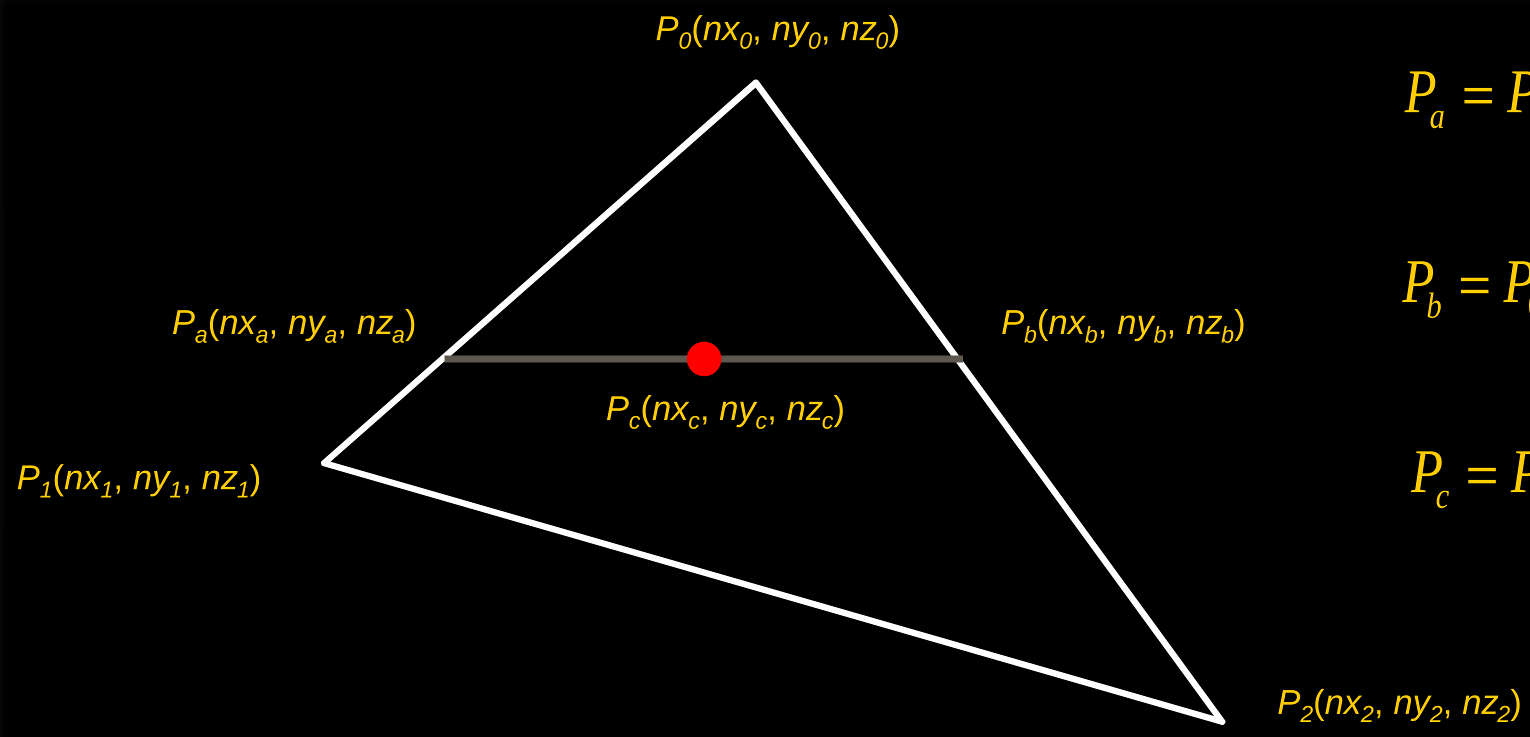
Smooth Shading

- Phong Shading



Smooth Shading

- **Phong Shading**
  - **Interpolate pixel normal through vertex normals**
  - **Compute pixel color with derived pixel normal**



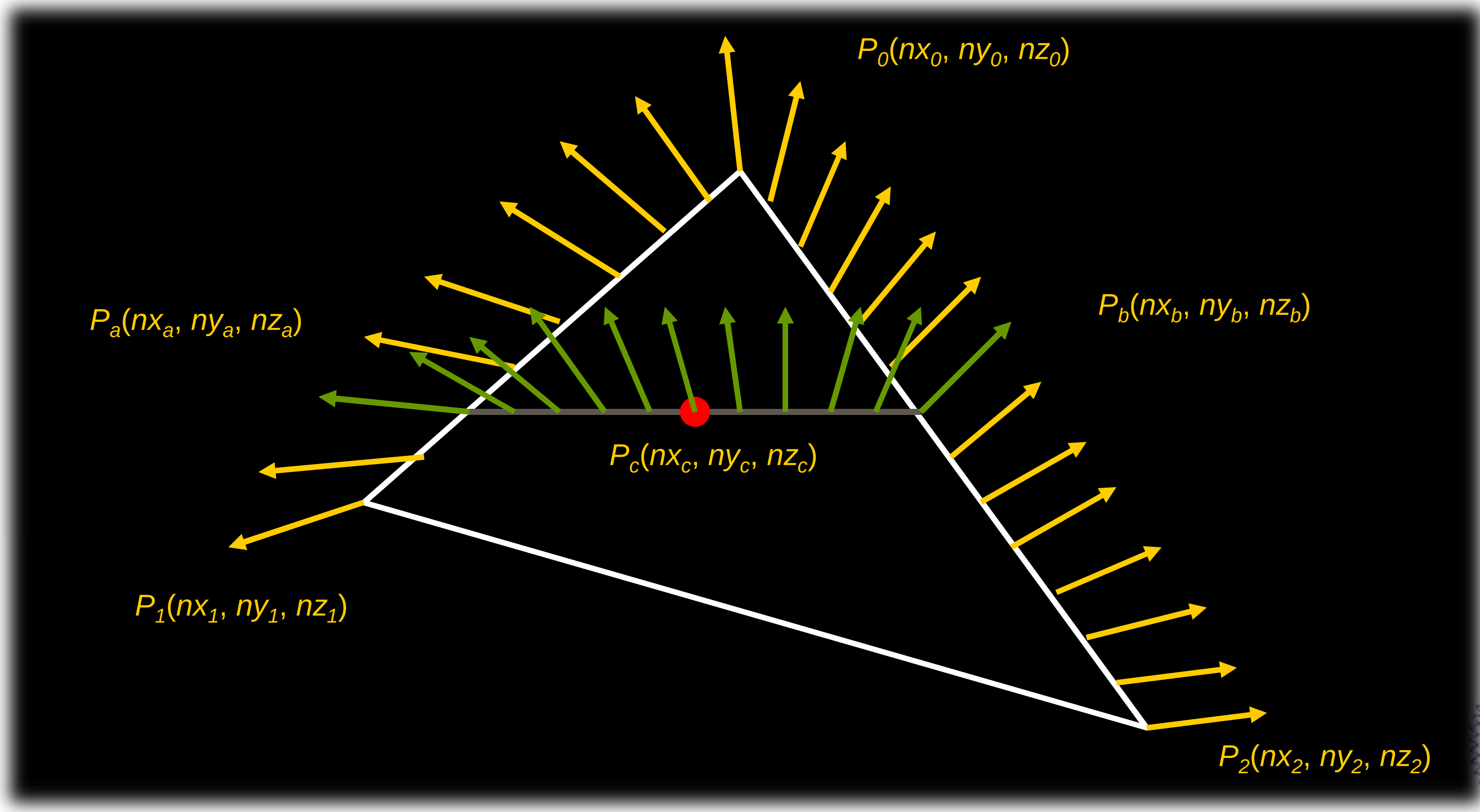
$$P_a = P_0\left(1-\frac{y_a-y_0}{y_1-y_0}\right)+P_1\frac{y_a-y_0}{y_1-y_0}$$

$$P_b = P_0\left(1-\frac{y_b-y_0}{y_2-y_0}\right)+P_2\frac{y_b-y_0}{y_2-y_0}$$

$$P_c = P_a\left(1-\frac{x_c-x_a}{x_b-x_a}\right)+P_b\frac{x_c-x_a}{x_b-x_a}$$

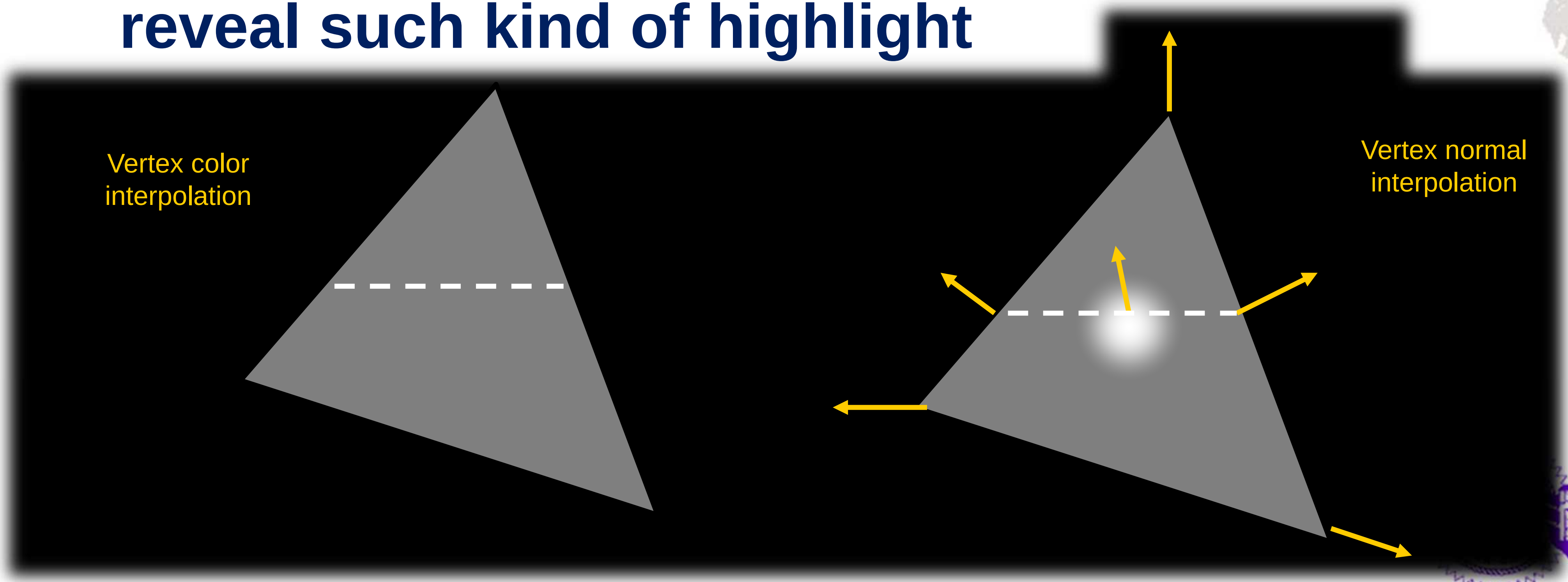
Smooth Shading

- Phong Shading
  - Interpolate pixel normal through vertex normals



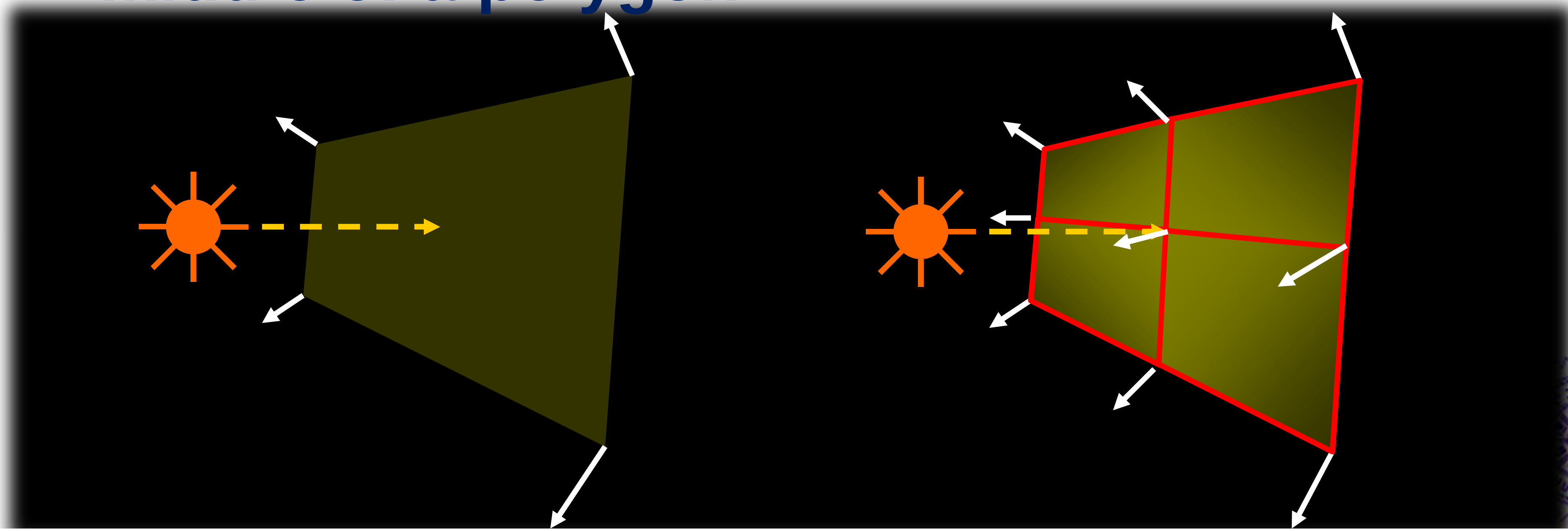
Smooth Shading

- **Difference between Gouraud shading and Phong shading**
  - **If specular highlight is located inside the triangle only, then Gouraud shading cannot reveal such kind of highlight**



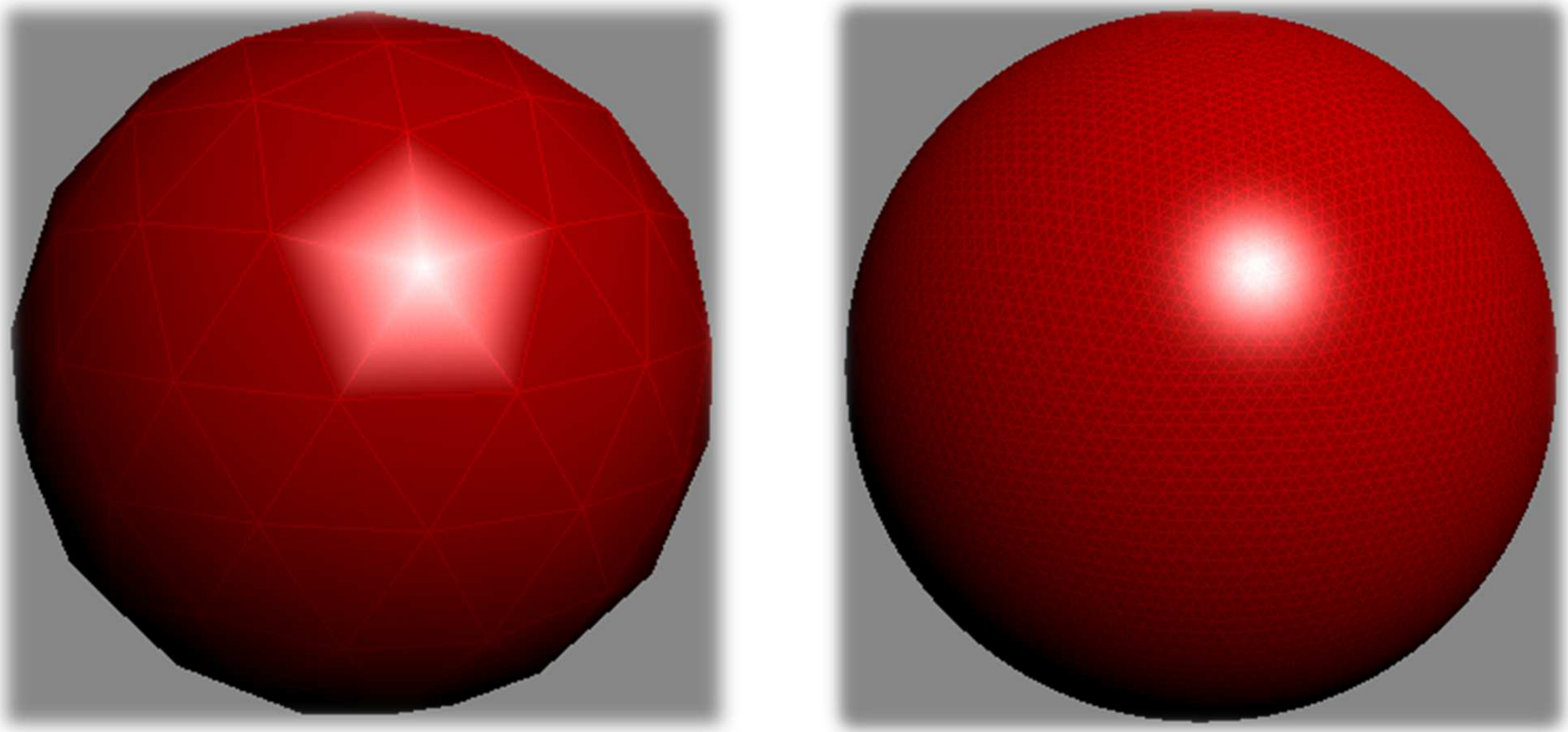
Polygon Size Mattered

- Large polygon size might degrade the lighting quality
  - Darker if the light source is closer to the polygon
  - No specular highlight can be perceived in the middle of a polygon



Polygon Size Mattered

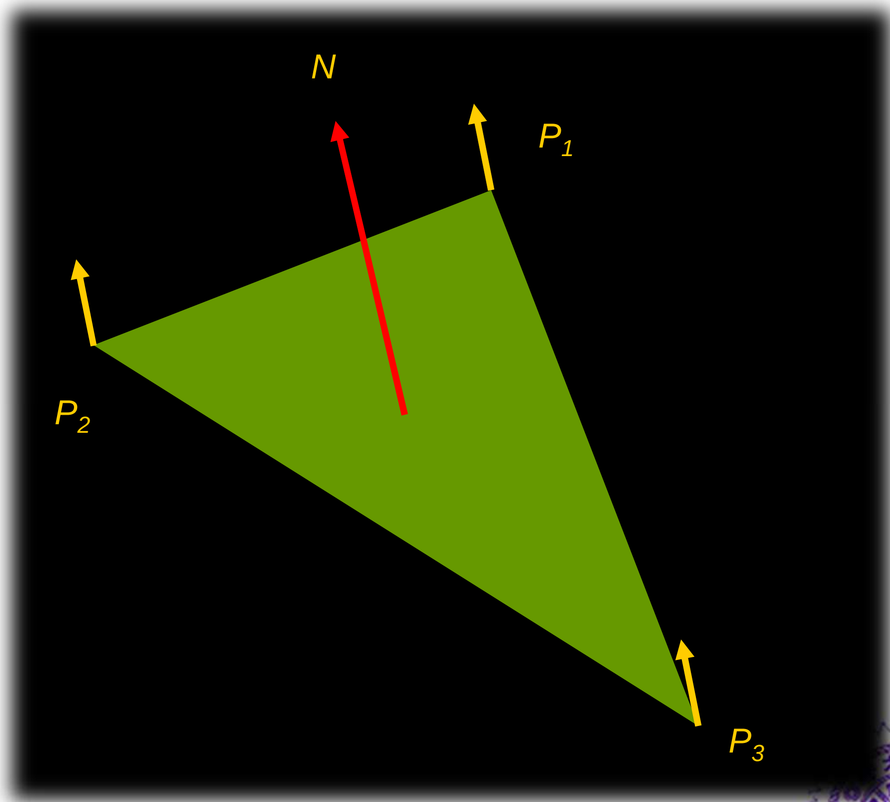
- Subdivide large polygon into smaller polygons to gain better shading result



Vertex Normal Derivation

- Flat shading
  - Vertex normal is equal to polygon face normal

$$N = (P_2 - P_1) \times (P_3 - P_1)$$

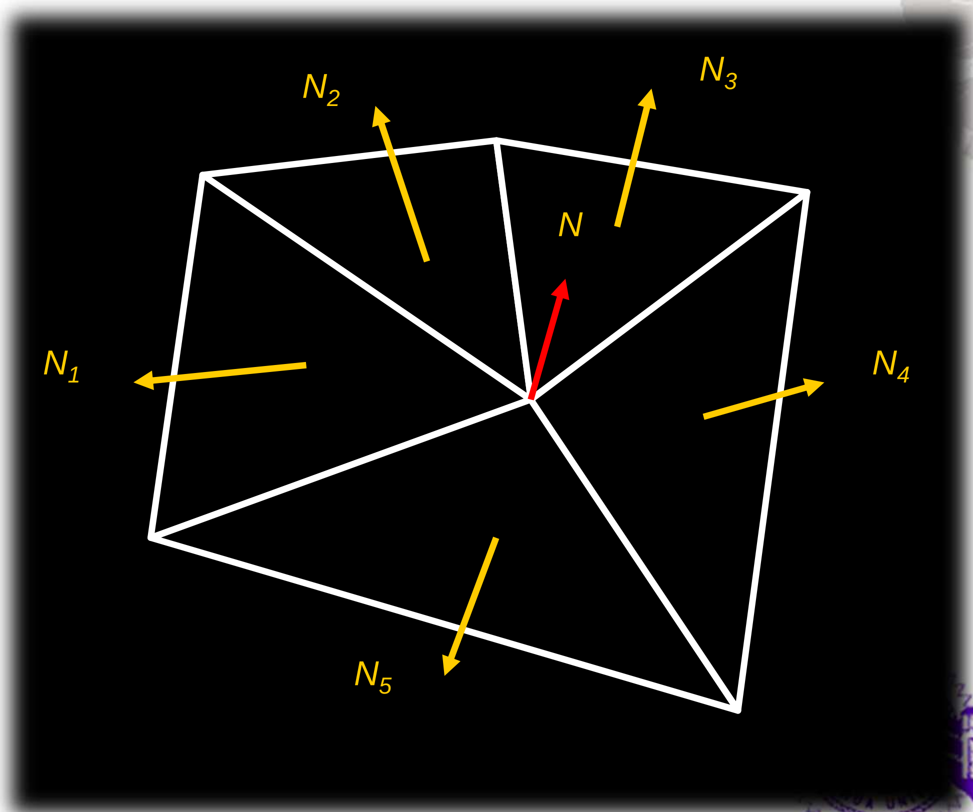


Vertex Normal Derivation

- Smooth shading
  - Vertex normal is equal to the sum of polygon face normals of adjacent polygons

$$N_{sum} = (N_1 + N_2 + \ldots + N_m)$$

$$N = N_{sum} / |N_{sum}|$$

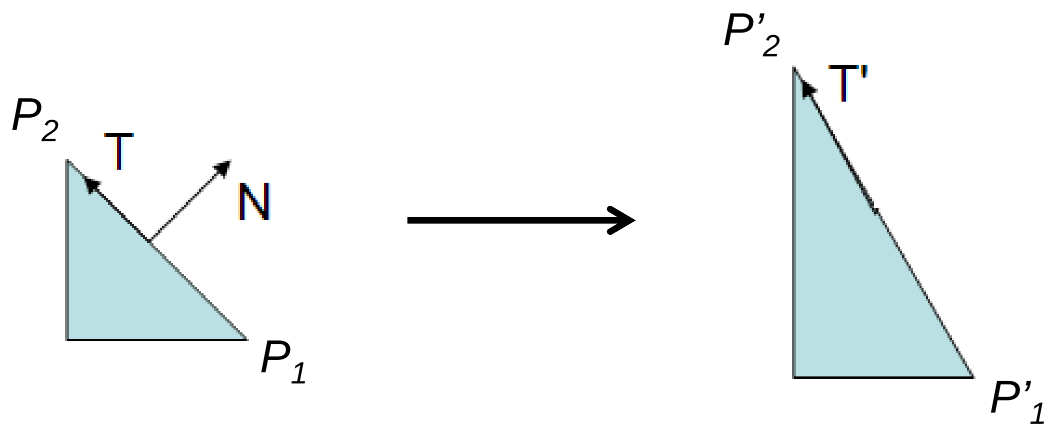


Normal Transformation

- Model transform and viewing transform together can transform a vertex from object space to eye space
- Normal is related to lighting process and the lighting calculation is performed in eye space
- So, normal has to transform to eye space as well
- But, what will happen if we transform normal using the same model-view matrix?

Normal Transformation

- **If we transform normal using model-view matrix M, then…**



$T = P_2 - P_1$
$M \cdot T = M \cdot (P_2 - P_1)$
$T' = M \cdot P_2 - M \cdot P_1 = P'_2 - P'_1$

$N = Q_2 - Q_1$
$M \cdot N = M \cdot (Q_2 - Q_1)$
$N' = M \cdot Q_2 - M \cdot Q_1 = Q'_2 - Q'_1$

$N \cdot T = 0$
But, after normal transformed by model-view matrix M
$N' \cdot T' \neq 0$

Normal Transformation

- **Normal transformation should be taking care if you have done any model and viewing transformations to the geometric data**

A vector in homogeneous coordinate is represented as
$T = (x, y, z, 0) = (x_2, y_2, z_2, 1) – (x_1, y_1, z_1, 1)$

A normal in homogeneous coordinate is represented as $N = (n_x, n_y, n_z, 0)$

Since T and N are orthogonal, thus $N \cdot T = 0$
We also know that after model-view transformation, T' and N' should remain orthogonal.
That is, $N' \cdot T' = 0$.

Normal Transformation

- **Represent the dot product by matrix multiplication and let *M* be the model-view matrix, we have**

$$N \cdot T = \begin{pmatrix} n_x & n_y & n_z & 0 \end{pmatrix} \begin{pmatrix} x \\ y \\ z \\ 0 \end{pmatrix} = 0$$

Eye space normal

Object space normal

Normal Transformation

$$N' \cdot T' = \boxed{\begin{pmatrix} n_x & n_y & n_z & 0 \end{pmatrix} M^{-1}} \boxed{M \begin{pmatrix} x \\ y \\ z \\ 0 \end{pmatrix}} = 0 \Rightarrow \begin{pmatrix} n'_x \\ n'_y \\ n'_z \\ 0 \end{pmatrix} = (M^{-1})^T \begin{pmatrix} n_x \\ n_y \\ n_z \\ 0 \end{pmatrix}$$

Model-view Transformation

Lighting Procedure

- Define the vertex normals
  - Lighting is the interaction between vertex normals and the light source
- Define light sources
  - Light source properties
- Select lighting model
  - Determine which lighting equation is used
- Define material properties
  - Define the percentage of reflectance to the light source

Complete OpenGL Lighting Formula

Object can emit light itself

Global ambient light

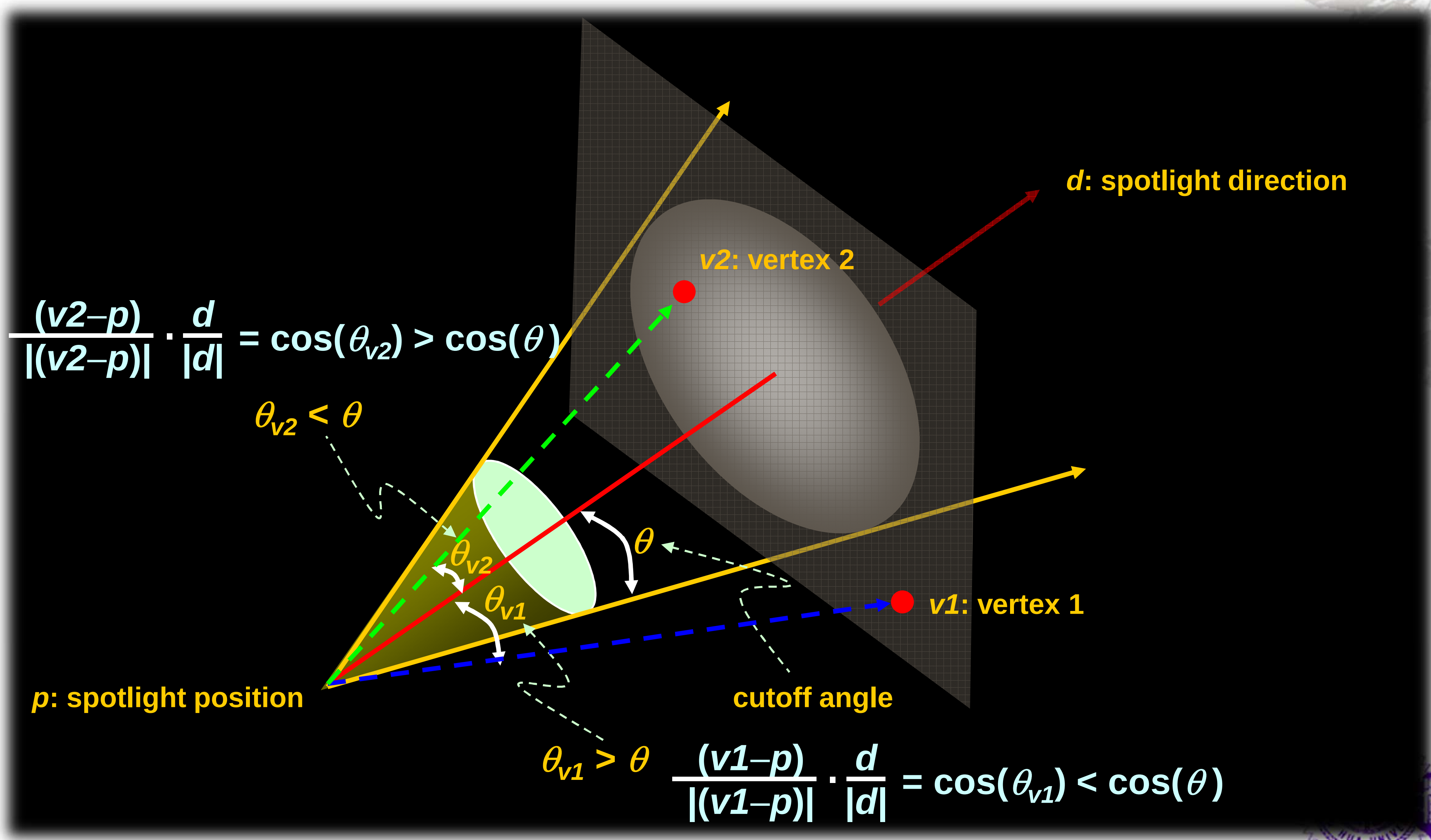
Light source contribution

$$\text{vertex color} = \boxed{\text{emission}_{\text{material}}} +$$

$$\boxed{\text{ambient}_{\text{light model}} * \text{ambient}_{\text{material}}} +$$

$$\boxed{\begin{aligned}&\sum_{i=0}^{n-1}\left(\frac{1}{k_c + k_l d + k_q d^2}\right)_i * (\text{spotlight effect})_i * \\ &[\text{ambient}_{\text{light}} * \text{ambient}_{\text{material}} + \\ &(\max\{\mathbf{L}\cdot\mathbf{n}, 0\}) * \text{diffuse}_{\text{light}} * \text{diffuse}_{\text{material}} + \\ &(\max\{\mathbf{s}\cdot\mathbf{n}, 0\})^{\text{shininess}} * \text{specular}_{\text{light}} * \text{specular}_{\text{material}}]_i\end{aligned}}$$

Spotlight Effect



Spotlight Effect

- Spotlight Effect =
  - 1, if the light source is not a spotlight
  - 0, if the light source is a spotlight but the vertex lies outside the cone of illumination produced by the spotlight
  - Otherwise, spotlight effect = $(\max\{v \cdot d, 0\})^{spot\_exp}$
    - v is the unit vector from the spotlight to the vertex
    - d is the spotlight direction

Q&A