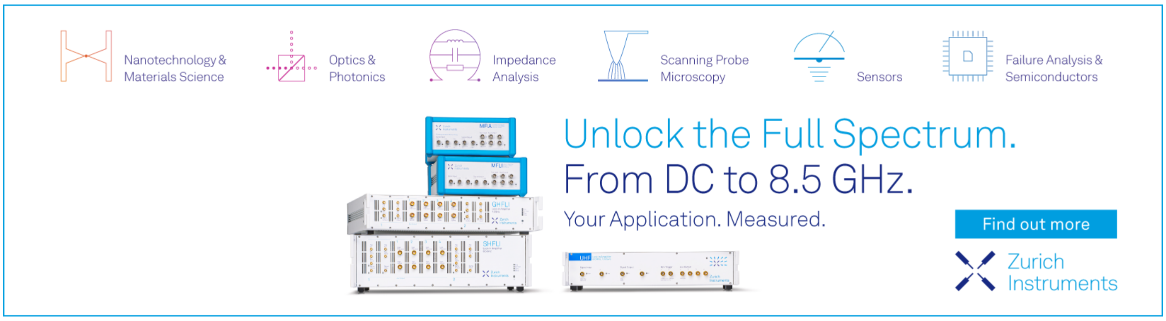
Journal of Applied Physics

RESEARCH ARTICLE | APRIL 15 1991

Preisach modeling of magnetostrictive hysteresis

A. A. Adly; I. D. Mayergoyz; A. Bergqvist

J. Appl. Phys. 69, 5777–5779 (1991)
https://doi.org/10.1063/1.347873



Preisach modeling of magnetostrictive hysteresis

A. A. Adly and I. D. Mayergoyz
Electrical Engineering Department and Institute for Advanced Computer Studies, University of Maryland, College Park, Maryland 20742

A. Bergqvist
Electric Power Research Centre, The Royal Institute of Technology, 10044 Stockholm, Sweden

Preisach-type models are suggested for the description of strain hysteresis of magnetostrictive materials. It is pointed out that the strain hysteresis has some peculiar properties in comparison with magnetic hysteresis. These peculiarities of strain hysteresis are discussed in detail along with the identification problem for these models.

The essence of magnetostrictive phenomenon is the dependence of strain on magnetization. Since the magnetization can be varied by applied magnetic fields, this opens the opportunity to control the strain of magnetostrictive rods by magnetic fields. For this reason, magnetostrictive materials (especially Terfenol-type materials with gigantic magnetostriction) are very attractive as actuator materials for many applications such as robotics, active vibration damping, micromotors, etc. However, magnetostrictive materials exhibit hysteresis, which represents a problem for fine-positioning applications. If hysteretic effects of magnetostrictive materials could be predicted, then actuator controllers could be designed to correct for these effects. This would result in high-precision actuators powered by magnetostrictive materials. It is clear that mathematical models for magnetostrictive hysteresis could facilitate the design of the above controllers.

Recently, the classical Preisach model has been applied to the description of *magnetic* hysteresis of magnetostrictive (Terfenol-type) materials[1]; namely, the hysteretic behavior of magnetization with respect to variations of magnetic field has been modeled. However, it is known that magnetostrictive materials exhibit hysteretic behavior with respect to variations of two variables: magnetic field and stress. Thus, the problem of developing Preisach-type models with two inputs presents itself. These models are discussed in this paper. It is quite possible that they may find some applications beyond the area of magnetostrictive hysteresis.

For the sake of generality, we consider a hysteresis nonlinearity which can be characterized by two inputs $u(t)$ and $v(t)$ and an output $f(t)$. In magnetostriction applications, $u(t)$ is the magnetic field, $v(t)$ is the stress, while $f(t)$ is the strain. We shall discuss the following Preisach-type model with two inputs:

$$f(t)=\int_{\alpha\geqslant\beta}\int \mu(\alpha,\beta,v(t))\,\hat{\gamma}_{\alpha\beta}u(t)\,d\alpha\,d\beta + \int_{\alpha\geqslant\beta}\int \nu(\alpha,\beta,u(t))\,\hat{\gamma}_{\alpha\beta}v(t)\,d\alpha\,d\beta. \qquad (1)$$

In the above model, the dependence of the functions μ and ν on $v(t)$ and $u(t)$, respectively, reflects the cross coupling between two inputs.

The investigation of the model (1) is greatly facilitated by its equivalent representation as a model with moving supports for $\mu(\alpha,\beta,v(t))$ and $\nu(\alpha,\beta,u(t))$. To arrive at this representation, we introduce the sets $R_{u(t)}$, and $R_{v(t)}$, which are defined as $(\alpha,\beta)\in R_{u(t)}$ if $\beta_0\leqslant\beta\leqslant u(t)\leqslant\alpha\leqslant\alpha_0$, $(\alpha,\beta)\in R_{v(t)}$ if $\beta_0\leqslant\beta\leqslant v(t)\leqslant\alpha\leqslant\alpha_0$.

Consider first a state of negative saturation and a subsequent monotonic increase of inputs until they reach some values $u(t)$ and $v(t)$. Let $f^{+}_{u(t)v(t)}$ be the resulting output value.

Similarly, starting from the state of positive saturation, we can define $f^{-}_{u(t)v(t)}$.

Now, by using simple transformations we can derive the following representation for the model (1):

$$f(t)=\int_{R_{u(t)}}\int \mu(\alpha,\beta,v(t))\,\hat{\gamma}_{\alpha\beta}u(t)\,d\alpha\,d\beta + \int_{R_{v(t)}}\int \nu(\alpha,\beta,u(t))\,\hat{\gamma}_{\alpha\beta}v(t)\,d\alpha\,d\beta + \tfrac{1}{2}\left(f^{+}_{u(t)v(t)}+f^{-}_{u(t)v(t)}\right). \qquad (2)$$

The last expression can be regarded as a generalization of the formulas derived for the generalized Preisach model.[2]

By using the same line of reasoning as in Refs. 2 and 3 it can be established that the model (2) has the following two characteristic properties.

Wiping-out property. Only the alternating series of past dominant extrema of $u(t)$ and $v(t)$ are stored by the model (2), while all other past extrema of $u(t)$ and $v(t)$ are wiped out.

Property of equal vertical chords. All minor hysteretic loops corresponding to the same consecutive extremum values of $u(t)$ for the same fixed value of v have equal vertical chords regardless of the past history of variations of $u(t)$ and $v(t)$. The same is true for minor hysteretic loops formed as a result of back-and-forth variations of $v(t)$ for any fixed value of u.

It turns out that the model (2) has also the following distinct property.

Path independence property. Consider two points (u_1,v_1) and (u_2,v_2) on the u-v plane and a set of paths connecting these points and corresponding to *monotonic* variations of *both* $u(t)$ and $v(t)$ (see Fig. 1). Then, the output increment predicted by the model (2) does not depend on a particular monotonic path between the points (u_1,v_1) and (u_2,v_2).

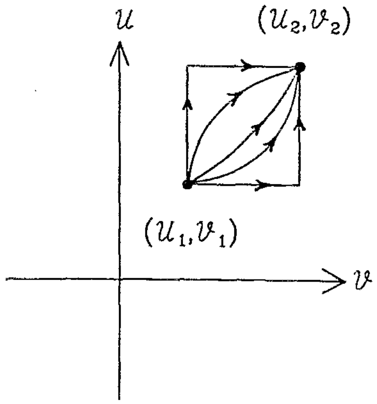


FIG. 1. Multiple paths connecting two arbitrary points on the u-v plane.

We next proceed to the discussion of the identification problem for the model (2). In order to determine the functions $\mu(\alpha,\beta,v)$ and $\nu(\alpha,\beta,u)$, we shall use the first-order transition curves $f_{\alpha\beta v}$ and $f_{u\alpha\beta}$. These curves are measured for piecewise monotonic input variations started from the state of negative saturation. Namely, $f_{\alpha\beta v}$ is the output resulting from monotonic increases of two inputs to the values α and v, respectively, and subsequent monotonic decrease of $u(t)$ to the value β. The output value $f_{u\alpha\beta}$ can be determined in a similar way.

We shall define the functions

$$F(\alpha,\beta,v)=\tfrac{1}{2}(f_{\alpha\beta v}-f^{+}_{\beta v}),$$
$$G(\alpha,\beta,u)=\tfrac{1}{2}(f_{u\alpha\beta}-f^{+}_{u\beta}). \tag{3}$$

By using (2) and the α-β diagram technique, it is easy to show[3,4] that

$$F(\alpha,\beta,v)=\int_{R(\alpha,\beta)}\int \mu(\alpha',\beta',v)d\alpha'\,d\beta'$$
$$=\int_{\beta}^{\alpha}\left(\int_{\beta_0}^{\beta}\mu(\alpha',\beta',v)d\beta'\right)d\alpha', \tag{4}$$

$$G(\alpha,\beta,u)=\int_{R(\alpha,\beta)}\int \nu(\alpha',\beta',u)d\alpha'\,d\beta'$$
$$=\int_{\beta}^{\alpha}\left(\int_{\beta_0}^{\beta}\nu(\alpha',\beta',u)d\beta'\right)d\alpha', \tag{5}$$

where $R(\alpha,\beta)$ is the rectangle shown in Fig. 2.

From (4) and (5), and by using similar arguments as those in Refs. 3 and 4, we derive

$$\mu(\alpha,\beta,v)=\frac{\partial^2 F(\alpha,\beta,v)}{\partial\alpha\,\partial\beta},$$
$$\nu(\alpha,\beta,u)=\frac{\partial^2 G(\alpha,\beta,u)}{\partial\alpha\,\partial\beta}. \tag{6}$$

As far as the numerical implementation of the model (2) is concerned, it can be accomplished without the formulas (6), but by employing explicit expressions for the integrals in (2) in terms of the functions $F(\alpha,\beta,v)$ and $G(\alpha,\beta,u)$. The derivation of these expressions proceeds as follows.

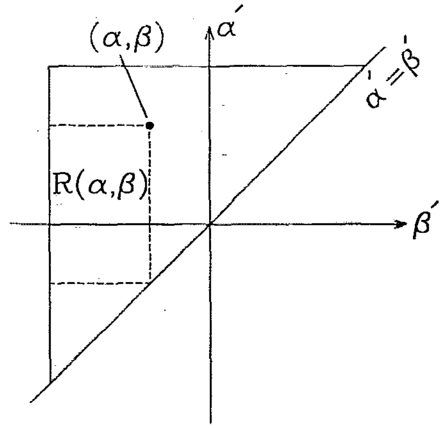


FIG. 2. Geometric representation of the rectangle $R(\alpha,\beta)$.

It can be shown that the model (2) can be represented in the following equivalent form

$$f(t)=2\left(\int_{S^+(t)}\int \mu(\alpha,\beta,v(t))d\alpha\,d\beta + \int_{\Omega^+(t)}\int \nu(\alpha,\beta,u(t))d\alpha\,d\beta\right)+f^{+}_{u(t)v(t)}. \tag{7}$$

The set $\Omega^+(t)$ has a similar geometry.

From this figure follows

$$\int_{S^+(t)}\int \mu(\alpha,\beta,v(t))d\alpha\,d\beta = \sum_{k=1}^{n(t)}\int_{R_k}\int \mu(\alpha,\beta,v(t))d\alpha\,d\beta. \tag{8}$$

By using (4), we find

$$\int_{R_k}\int \mu(\alpha,\beta,v(t))d\alpha\,d\beta = F(M_k^{(u)},m_k^{(u)},v(t)) - F(M_{k+1}^{(u)},m_k^{(u)},v(t)), \tag{9}$$

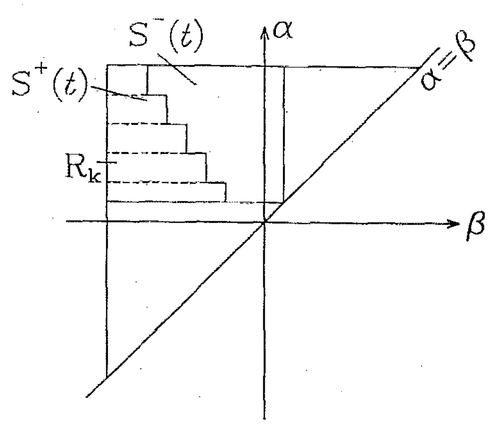


FIG. 3. Geometric representation of the set $S^+(t)$.

where $M_k^{(u)}$ and $m_k^{(u)}$ form the alternating series of past dominant extrema of $u(t)$. From (8) and (9), we conclude

$$\int_{S^+(t)}\int \mu(\alpha,\beta,v(t))d\alpha\, d\beta = \sum_{k=1}^{n(t)} [F(M_k^{(u)},m_k^{(u)},v(t)) - F(M_{k+1}^{(u)},m_k^{(u)},v(t))]. \tag{10}$$

By using the same line of reasoning as before, we can derive the expression

$$\int_{\Omega^+(t)}\int \nu(\alpha,\beta,u(t))d\alpha\, d\beta = \sum_{k=1}^{l(t)} [G(M_k^{(v)},m_k^{(v)},u(t)) - G(M_{k+1}^{(v)},m_k^{(v)},u(t))]. \tag{11}$$

By substituting (11) and (10) into (9), we obtain

$$f(t)=f^+_{u(t)v(t)} + \sum_{k=1}^{n(t)} [F(M_k^{(u)},m_k^{(u)},v(t)) - F(M_{k+1}^{(u)},m_k^{(u)},v(t))] + \sum_{k=1}^{l(t)} [G(M_k^{(v)},m_k^{(v)},u(t)) - G(M_{k+1}^{(v)},m_k^{(v)},u(t))]. \tag{12}$$

This is the final expression for the output in terms of experimentally measured functions F and G defined by (3).

We next formulate the following result.

Representation theorem. The wiping-out property, the property of equal vertical chords, and the path independence property constitute necessary and sufficient conditions for the representation of actual two input hysteresis nonlinearities by the model (2) on the set of piecewise monotonic inputs.

In conclusion, we consider one peculiar property of strain hysteresis in comparison with magnetic hysteresis. This peculiarity stems from even symmetry of strain hysteresis which exhibits butterfly-shaped major hysteretic loops with respect to variations of magnetic field (see Fig. 4). It is clear that first-order transition curves $f_{\alpha\beta v}$ will also exhibit even symmetry. This can be expressed mathematically as

$$f_{\beta'\alpha' v}=f_{\alpha\beta v} \quad \text{and} \quad f^-_{\alpha' v}=f^+_{\beta v}, \tag{13}$$

if $\beta' = -\alpha$ and $\alpha' = -\beta$. By using (13), it can be easily proven that

$$\mu(\alpha,\beta,v) = -\mu(-\beta,-\alpha,v). \tag{14}$$

Thus, the function $\mu(\alpha,\beta,v)$ is of odd symmetry with respect to the line $\alpha = -\beta$. In the case of magnetic hysteresis, the μ function has an even symmetry with respect to the same line.

It follows from (14) that the function $\mu(\alpha,\beta,v)$ cannot be positive everywhere on the α-β plane. Actually, this function changes its sign across the line $\alpha = -\beta$. As a result, first- and higher-order transition curves are not enclosed by major hysteretic loops. This property has been observed, and it also distinguishes the strain hysteresis from magnetic hysteresis.

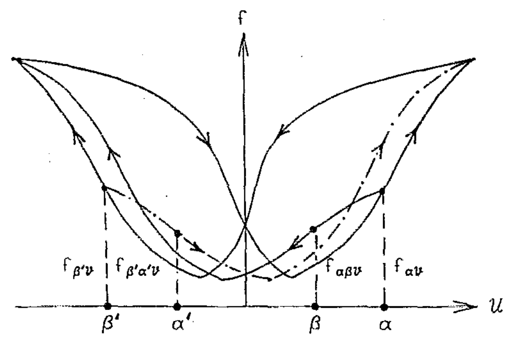


FIG. 4. Even symmetry of strain hysteresis.

ACKNOWLEDGMENT

The reported research has been supported by the U.S. Department of Energy, Engineering Research Program.

[1] J. B. Restorff, H. T. Savage, A. E. Clark, and M. Wun-Fogle, J. Appl. Phys. **67**, 5016 (1990).
[2] I. D. Mayergoyz and G. Friedman, IEEE Trans. Magn. **MAG-24**, 212 (1988).
[3] I. D. Mayergoyz, Phys. Rev. Lett. **56**, 1518 (1986).
[4] I. D. Mayergoyz, J. Appl. Phys. **57**, 3803 (1985).