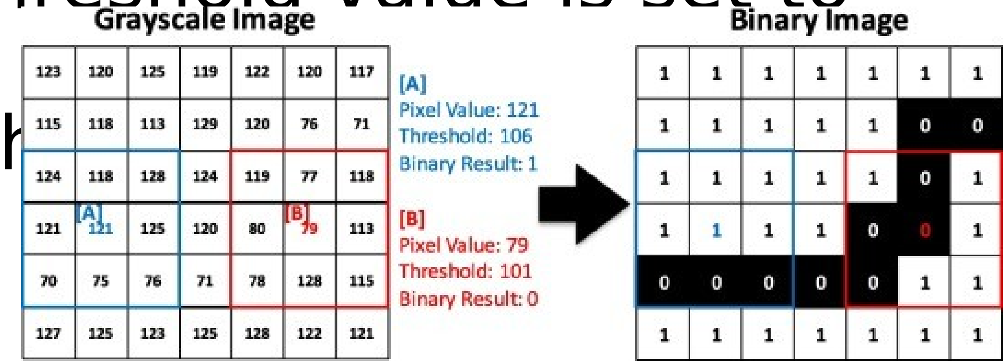
Character Segmentation

For Optical Character Recognition

Binarization

- The first step in character segmentation is grey scaling the image, taking all colors and transforming the pixel values into only a value between 0 and 1 depicting how black the image is at that point.
- Then, the image is binarized, this consists of turning all pixels into either black or white pixels, foreground text and background.
- a threshold value is set to
  xel
  d th



Binarization Methods and their accuracy

- Binarization methods can use a local threshold depending which are different in different regions, or a global threshold which is used in the entire document.
- F-Measure is the harmonic mean of Precision and Recall. Its value should be high for better results.
- PSNR is used to check the similarity between two images. It is used for images having noise
- NRM is calculated using the unmatched pixels between the binarized image and ground truth image.
- MPM is used to evaluate how the binarized image constitutes the contour of ground truth image
- This paper explains all the methods:

https://ieeexplore.ieee.org/stamp/stamp.jsp?arnumber=7784945

TABLE I: RESULTS ON DIBCO 2009[10][11]

Method	F-Measure	PSNR	NRM	MPM
Otsu	78.72	15.34	5.77	13.3
Niblack	55.82	9.89	16.4	61.5
Sauvola	85.41	16.39	6.94	3.2
Bernsen	52.48	8.89	14.29	113.8
LMM	91.06	18.5	7	0.3
AC	93.5	19.65	3.74	0.43
Global-Local	83.66	15.60	4.80	32.9

TABLE II: RESULTS ON H-DIBCO 2010[10][12]

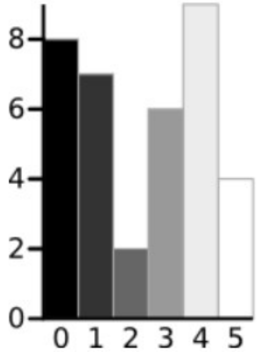
Method	F-Measure	PSNR	NRM	MPM
Otsu	85.27	17.51	9.77	1.35
Niblack	74.1	15.73	19.06	1.06
Sauvola	75.3	15.96	16.31	1.96
Bernsen	41.3	8.57	21.18	115.98
LMM	85.49	17.83	11.46	0.37
AC	92.03	20.12	6.14	0.25
MCI	90.67	19.15	4.7	0.6

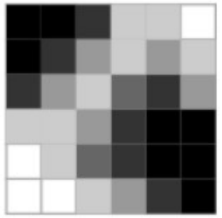
Otsu’s Method

- Put simply, Otsu’s method finds the global threshold where the different pixel values in each class (background and foreground) have the least difference between them. In other words, the pixel values in one group each are as close as possible to the others in the same group.
- The algorithm is exhaustive, it tries every threshold and calculates the variances of the classes until it finds the threshold with the least variance.
- The algorithm was published in 1979, since then several alternatives have been invented which make the computation faster. OpenCV combines it with Canny Edge Detection in its function to maximize results.

Otsu’s Algorithm

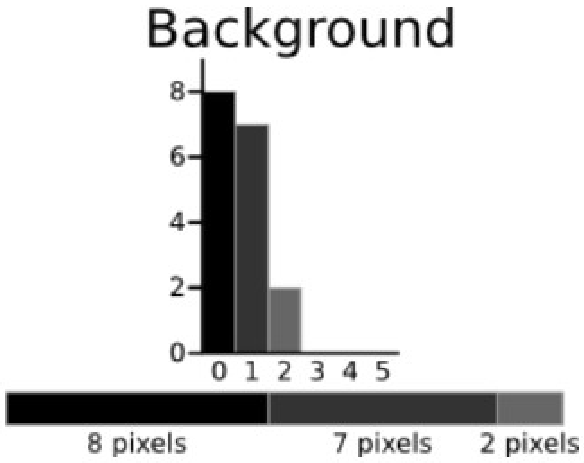
- The method involves of iterating through all the possible threshold values and calculating a measure of spread for the pixel levels on each side of the threshold (foreground or background).
- The method’s objective is minimizing the intra-class variance, which is just a weighted sum of the variance between the foreground and background.
- The first step is segmenting the grey-scale levels into classes like in a histogram.
- Weights ω_0 and ω_1 are the probabilities of the two classes separated by a threshold t, and σ^2_0 and σ^2_1 are the variances of the two classes.





$$\sigma_w^2(t) = \omega_0(t)\sigma_0^2(t) + \omega_1(t)\sigma_1^2(t)$$

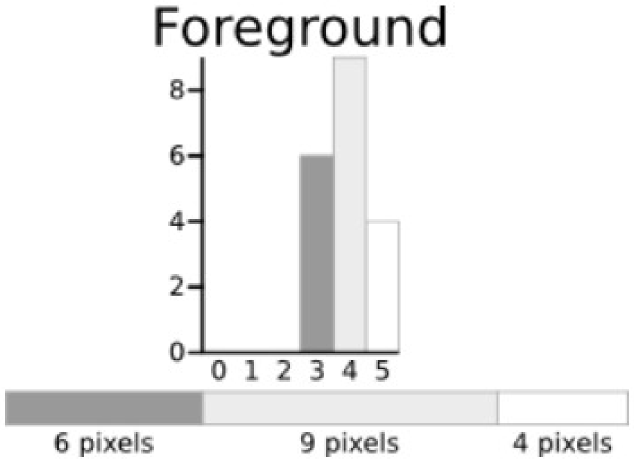
Calculating the weights and variance



$$\text{Weight} \quad W_b = \frac{8+7+2}{36} = 0.4722$$

$$\text{Mean} \quad \mu_b = \frac{(0\times 8)+(1\times 7)+(2\times 2)}{17} = 0.6471$$

$$\begin{aligned}\text{Variance} \quad \sigma_b^2 &= \frac{((0-0.6471)^2\times 8)+((1-0.6471)^2\times 7)+((2-0.6471)^2\times 2)}{17} \\ &= \frac{(0.4187\times 8)+(0.1246\times 7)+(1.8304\times 2)}{17} \\ &= 0.4637\end{aligned}$$

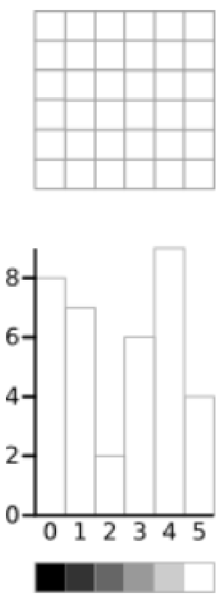
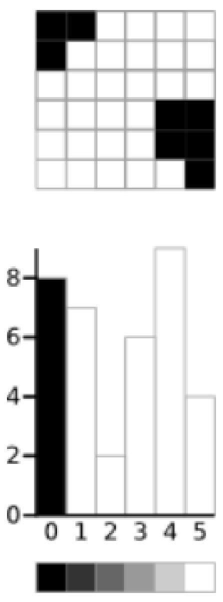
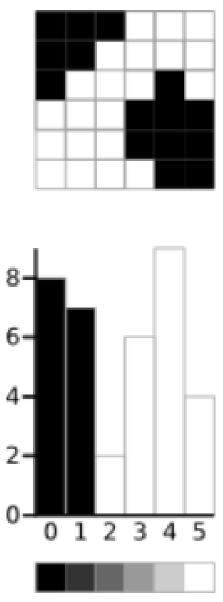
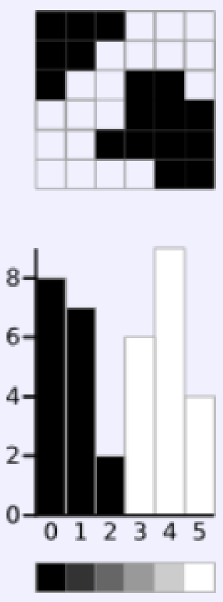
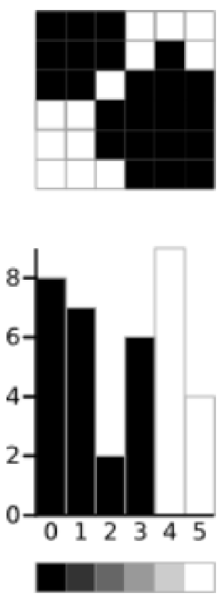
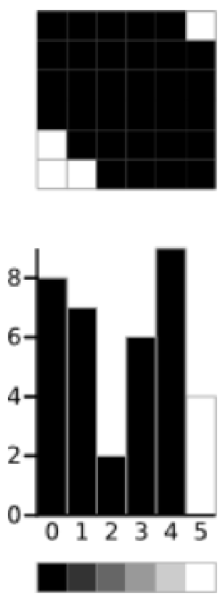


$$\text{Weight} \quad W_f = \frac{6+9+4}{36} = 0.5278$$

$$\text{Mean} \quad \mu_f = \frac{(3\times 6)+(4\times 9)+(5\times 4)}{19} = 3.8947$$

$$\begin{aligned}\text{Variance} \quad \sigma_f^2 &= \frac{((3-3.8947)^2\times 6)+((4-3.8947)^2\times 9)+((5-3.8947)^2\times 4)}{19} \\ &= \frac{(4.8033\times 6)+(0.0997\times 9)+(4.8864\times 4)}{19} \\ &= 0.5152\end{aligned}$$

Thresholds

Threshold	T=0	T=1	T=2	T=3	T=4	T=5
						
Weight, Background	$W_b = 0$	$W_b = 0.222$	$W_b = 0.4167$	$W_b = 0.4722$	$W_b = 0.6389$	$W_b = 0.8889$
Mean, Background	$M_b = 0$	$M_b = 0$	$M_b = 0.4667$	$M_b = 0.6471$	$M_b = 1.2609$	$M_b = 2.0313$
Variance, Background	$\sigma^2_b = 0$	$\sigma^2_b = 0$	$\sigma^2_b = 0.2489$	$\sigma^2_b = 0.4637$	$\sigma^2_b = 1.4102$	$\sigma^2_b = 2.5303$
Weight, Foreground	$W_f = 1$	$W_f = 0.7778$	$W_f = 0.5833$	$W_f = 0.5278$	$W_f = 0.3611$	$W_f = 0.1111$
Mean, Foreground	$M_f = 2.3611$	$M_f = 3.0357$	$M_f = 3.7143$	$M_f = 3.8947$	$M_f = 4.3077$	$M_f = 5.000$
Variance, Foreground	$\sigma^2_f = 3.1196$	$\sigma^2_f = 1.9639$	$\sigma^2_f = 0.7755$	$\sigma^2_f = 0.5152$	$\sigma^2_f = 0.2130$	$\sigma^2_f = 0$
Within Class Variance	$\sigma^2_W = 3.1196$	$\sigma^2_W = 1.5268$	$\sigma^2_W = 0.5561$	$\boldsymbol{\sigma^2_W = 0.4909}$	$\sigma^2_W = 0.9779$	$\sigma^2_W = 2.2491$

A faster approach

- A very similar calculation to the within class or intra-class variance is the between class variance. This method is much faster and it has a perfect correlation with the within class variance , its calculated in the following way

$$
\begin{aligned}
&\text{Within Class Variance} \quad \sigma_W^2 = W_b\,\sigma_b^2 + W_f\,\sigma_f^2 \quad \text{(as seen above)}\\
&\text{Between Class Variance} \quad \sigma_B^2 = \sigma^2 - \sigma_W^2\\
&\qquad = W_b(\mu_b - \mu)^2 + W_f(\mu_f - \mu)^2 \quad (\text{where } \mu = W_b\,\mu_b + W_f\,\mu_f)\\
&\qquad = W_b\,W_f\,(\mu_b - \mu_f)^2
\end{aligned}
$$

Threshold	T=0	T=1	T=2	T=3	T=4	T=5
Within Class Variance	$\sigma^2_W = 3.1196$	$\sigma^2_W = 1.5268$	$\sigma^2_W = 0.5561$	$\boldsymbol{\sigma^2_W = 0.4909}$	$\sigma^2_W = 0.9779$	$\sigma^2_W = 2.2491$
Between Class Variance	$\sigma^2_B = 0$	$\sigma^2_B = 1.5928$	$\sigma^2_B = 2.5635$	$\boldsymbol{\sigma^2_B = 2.6287}$	$\sigma^2_B = 2.1417$	$\sigma^2_B = 0.8705$

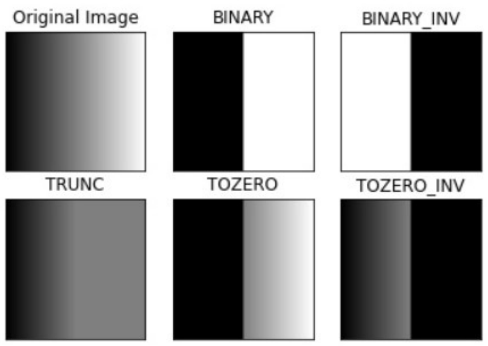
openCV

- OpenCV offers many thresholding methods with their function:

cv2.threshold(image, threshold, blackvalue, method).

- They also offer local thresholding under the name ‘adaptive’, but it is not needed for high quality scans as there is a clear distinction between foreground and background.
- For OTSU’s method the thresl / value as its really ca unct nethods for OSTU’s

- cv.THRESH_BINARY
- cv.THRESH_BINARY_INV
- cv.THRESH_TRUNC
- cv.THRESH_TOZERO
- cv.THRESH_TOZERO_INV



Tests

OTSU’s Method

PODER JUDICIAL

JUZGADO TERCERO CIVIL DE SAN JOSÉ

OFICIO PARA EL TRÁMITE DE EMBARGO O CAPTURA

EMBARGO ☐ CAPTURA ☒ Lugar: San José Fecha 27 de setiembre del año 2017

Señor: Dirección General del Tránsito Ministerio de Seguridad Pública Ministerio de Gobernación y Policía

Suma embargable ¢XX $XX

Cantidad en letras: XX

SÍRVASE EJECUTAR LO INDICADO EN LA CASILLA SEÑALADA SEGUIDAMENTE

☒ Proceder a **CAPTURAR** los vehículos placas números SJB 8246, SJB 8241, SJB 6704, SJB 3276, SJB 3384, SJB 3386, SJB 5727, SJB 5728, SJB 3171, SJB 3197, SJB 3277, SJB 3278, SJB 3279, SJB 3280, SJB 3360, SJB 4841, SJB 4842, SJB 4904, SJB 4515, SJB 11232, SJB 11983, SJB 12116, SJB 12117, SJB 12269, SJB 12271, SJB 12935, SJB 12936, SJB 12939, SJB 12940, SJB 10503, SJB 9818, SJB 10886, SJB 10201, LB 126, CB 6378, CL 228107, 340790, 594003, 631894 y 775684 y ponerlos a la orden de la Delegación de Tránsito o de la Fuerza Pública más cercana. **Una vez practicado el embargo y nombrado el depositario judicial, la orden quedará sin efecto.-**

☐ Levantar orden de captura del vehículo placas Nº **XX** Oficio Nº **XX** Fecha XX.

LO INDICADO ANTERIORMENTE ES APLICABLE AL(A LOS) DEMANDADO(S) SIGUIENTE(S)

DEMANDADO(A)	CÉDULA
CAMPOS CAMPOS FRANCISCO RAFAEL ANGEL	0203530977
DISCAR SA	3101010067

Actor(a) **INDIANNA CLACHAR BARBARA** Expediente Nº **17-000222-0182-CI**

Autoridad Judicial **JUZGADO TERCERO CIVIL DE SAN JOSÉ**

Observaciones -

Firma autorizada

LORENA SABORIO NUÑEZ

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Manually set 127 Threshold

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Results

- The overall results are very similar with every method CV2 has to offer, as the binarization results are dilated so the contour detection doesn't depend too much on it.

OTSU's Method

Manually set 127 Threshold

Adaptive Thresholding

- This method can be fine-tuned to obtained the desired results but it is not necessary when dealing with a recognizable difference between the foreground and background.

Gaussian

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Mean

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Suma embargable ¢XX $XX

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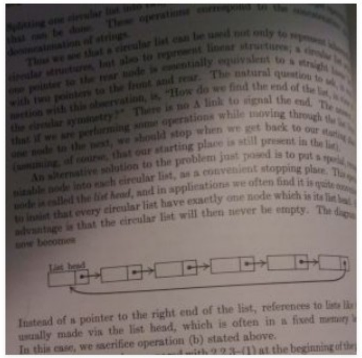
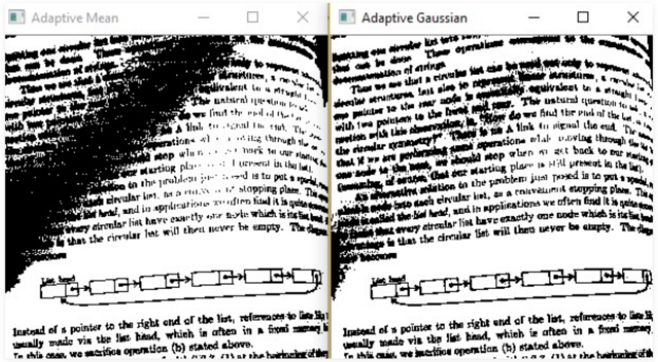
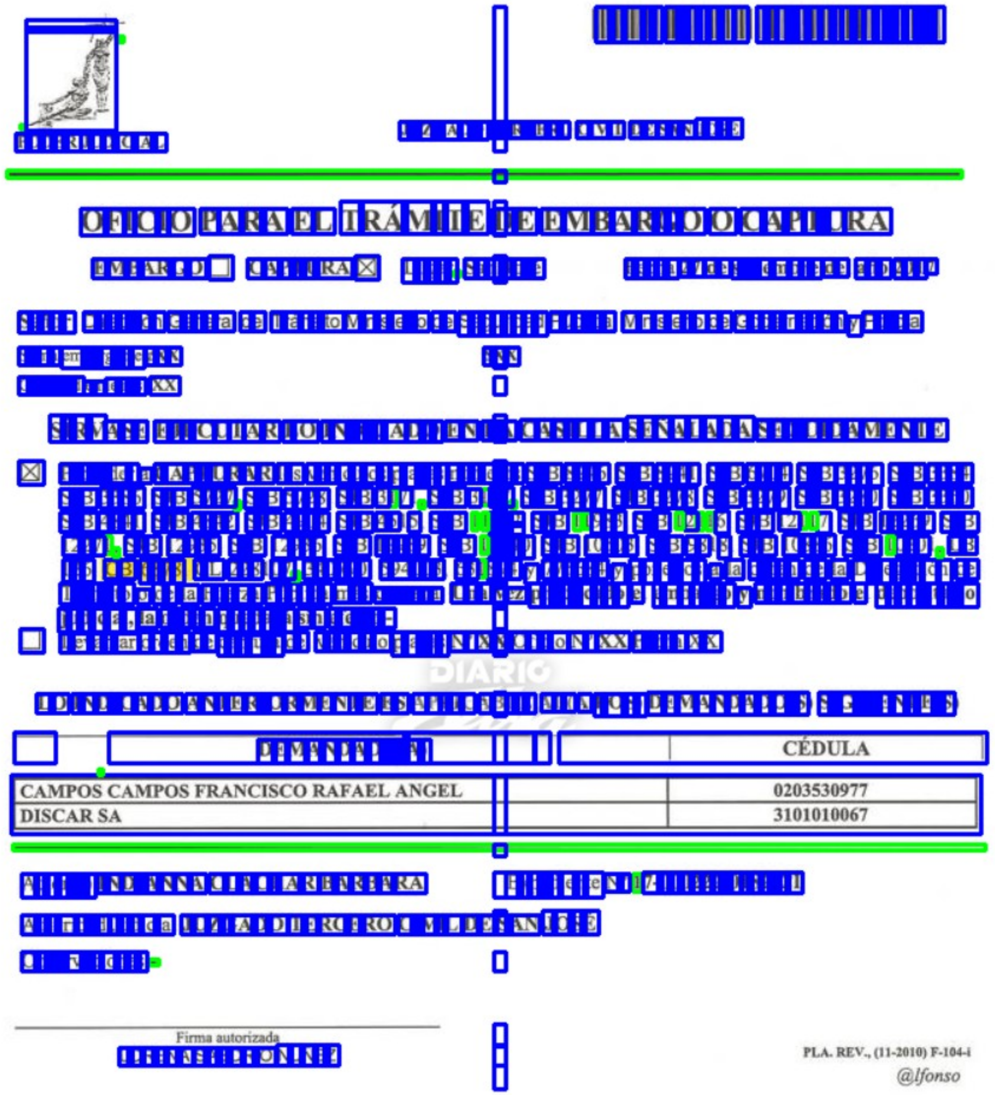
Adaptive Results

- The results for both the gaussian and mean adaptive thresholding are the same, they don't have perfect parametrization which causes the middle boxes but it is not needed anyway for high quality

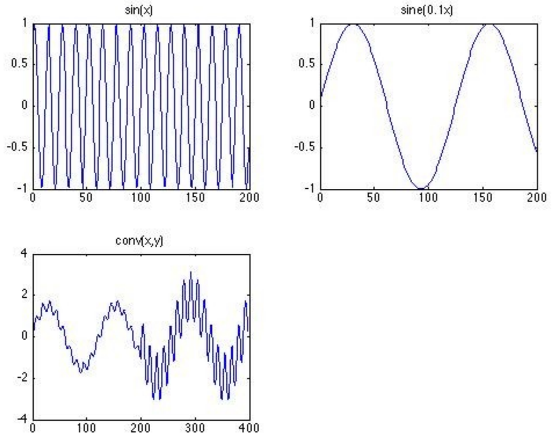


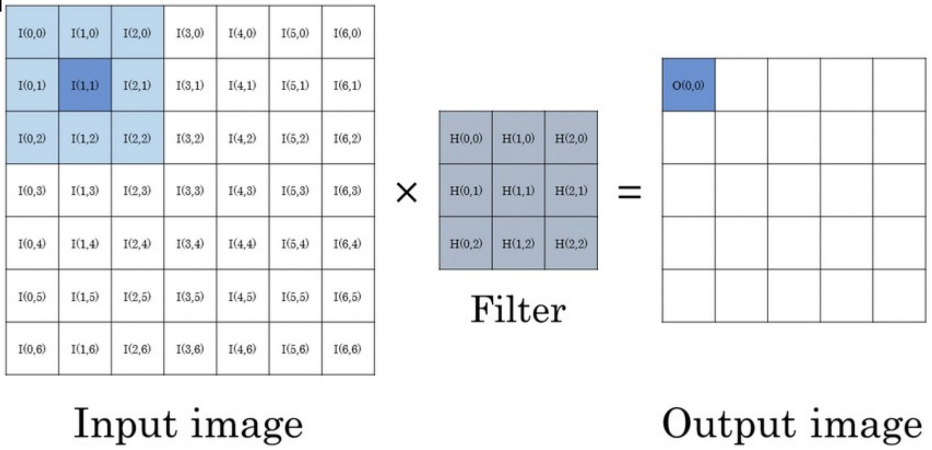




Convolution

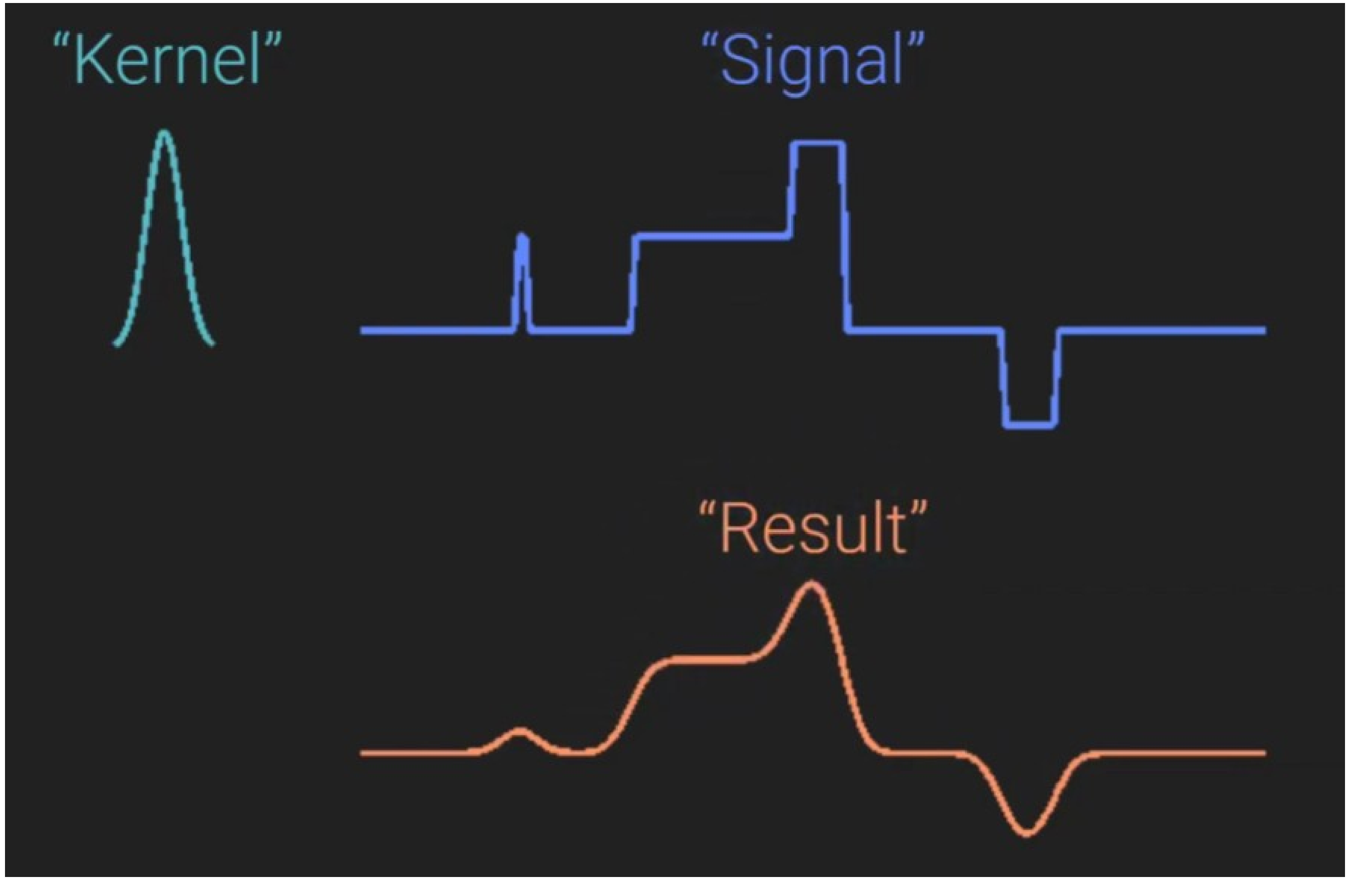
- Convolution is a combination by multiplication of two functions to form a third output function. This output function expresses how the shape of one function is modified by the other.
- It is the core of digital signal processing but also has applications in many other areas, including image p





Kernel

- A kernel is the function that is combined with another function in a convolution to generate the output.
- It can be thought of as the “filter” that the signal/function is put through.
- The original signal adopts the kernel’s characteristics.



Convolution Functio

$$(f^{*}g)(t) \triangleq \int_{-\infty}^{\infty} f(\tau)g(t-\tau)\tau'$$

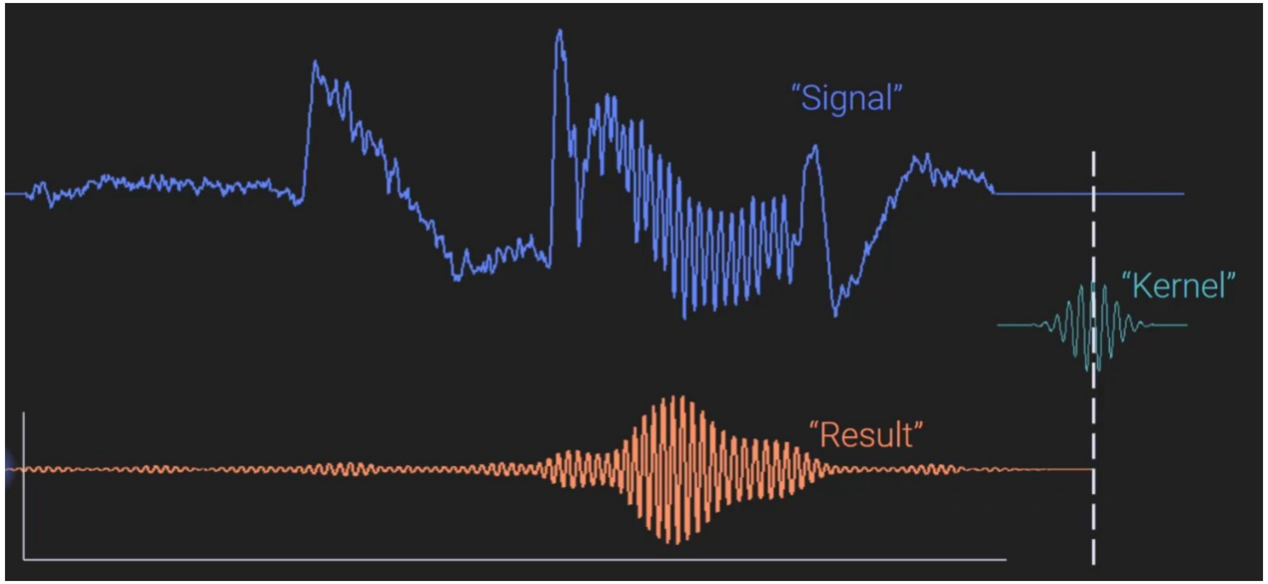
$(f^{*}g)(t)$ = functions that are being convoluted

t = real number variable of functions f and g

$g(\tau)$ = convolution of the function f(t)

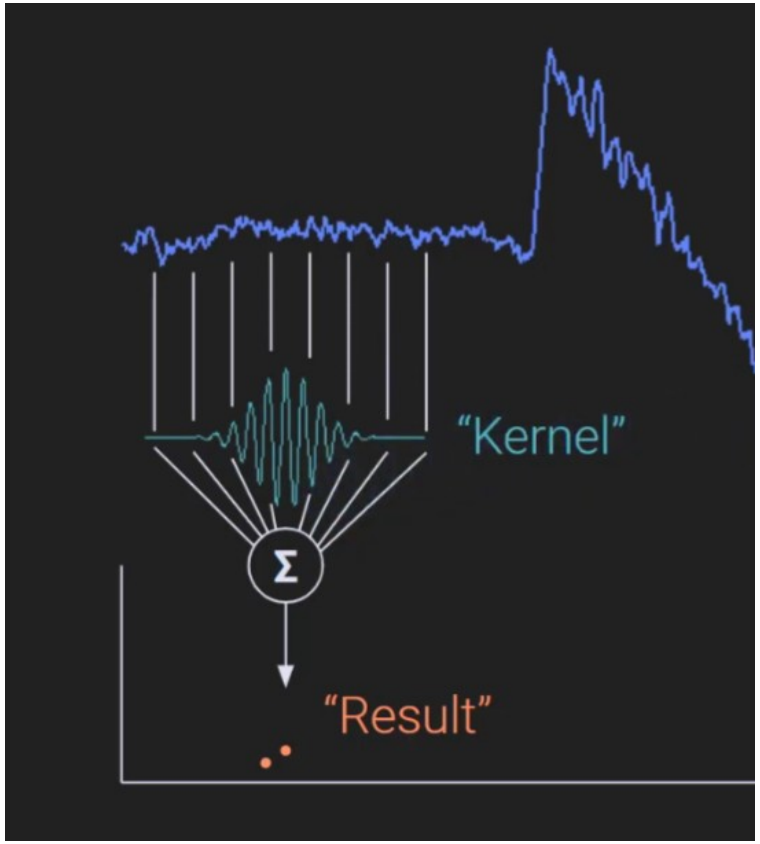
τ' = first derivative of g(tau) function

- To convolute two functions, one must be uniformly multiplied across the entirety of the other.
- Graphically, one function is shifted through the other and while computing the result of their multiplications.
- In continuous functions, the step size of the shift is reduced to an infinitesimal amount dτ and the convolution is the integral over the larger signal of the multiplication of the functions.

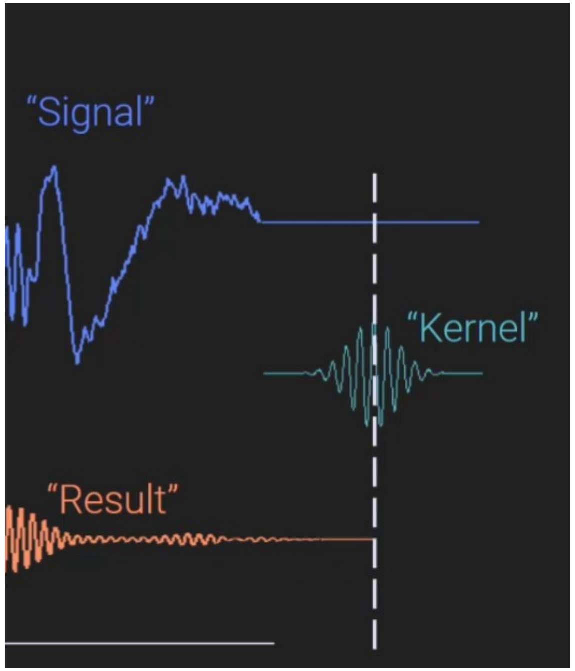


Graphical Intuition to Convolution

- When convolving two functions, one of the signals is swept across the other one and at each point the dot product of the two functions is taken, and the length of the multiplied values is mapped to a single point in the result.
- The smaller function which commonly does the sweeping is called the kernel, it is the ‘filter’ that is applied to the original signal, namely the input signal.
- At the beginning and end of the function the input signal is padded with zeroes so that the result can have the same length as the input signal.

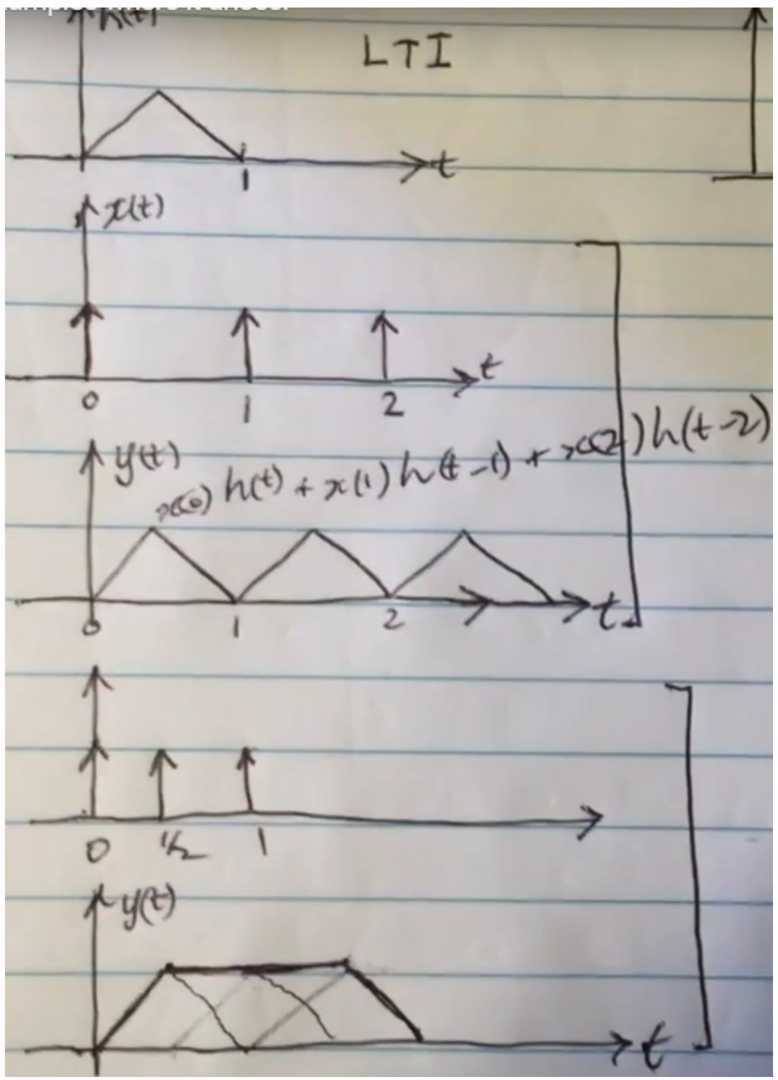


$$a \cdot b = \sum_{i=1}^{n} a_i b_i$$



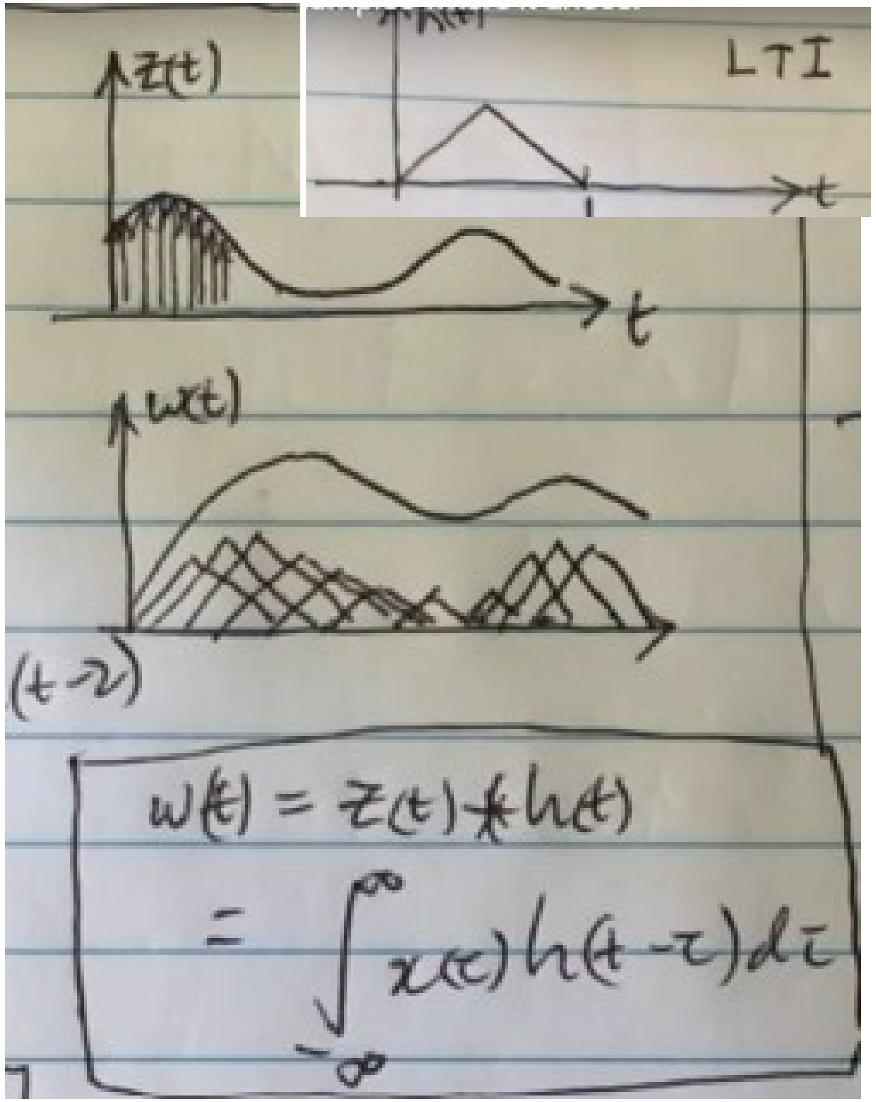
Linear Time Invariant Systems

- Linear systems are systems whose outputs for a linear combination of inputs are the same as a linear combination of individual responses to those inputs. Time-invariant systems are systems where the output does not depend on when an input was applied.
- So, when a function is convoluted in an LTI this same function can be multiplied across the second without any changes independent of time.
- In linear time invariant systems there is generally an impulse function which will express the inputs through time and an impulse response function that describes the occurrence, convolving the two gives the occurrences over time.
- In this example, since the system is LTI the same triangle function is multiplied with the impulse functions producing the third function.
- If this triangle function was convoluted with the fourth function the fifth function is obtained, the same multiplication occurs but the inner slopes of the first and third triangle cancel out with the middle triangle resulting in the trapezoid figure.



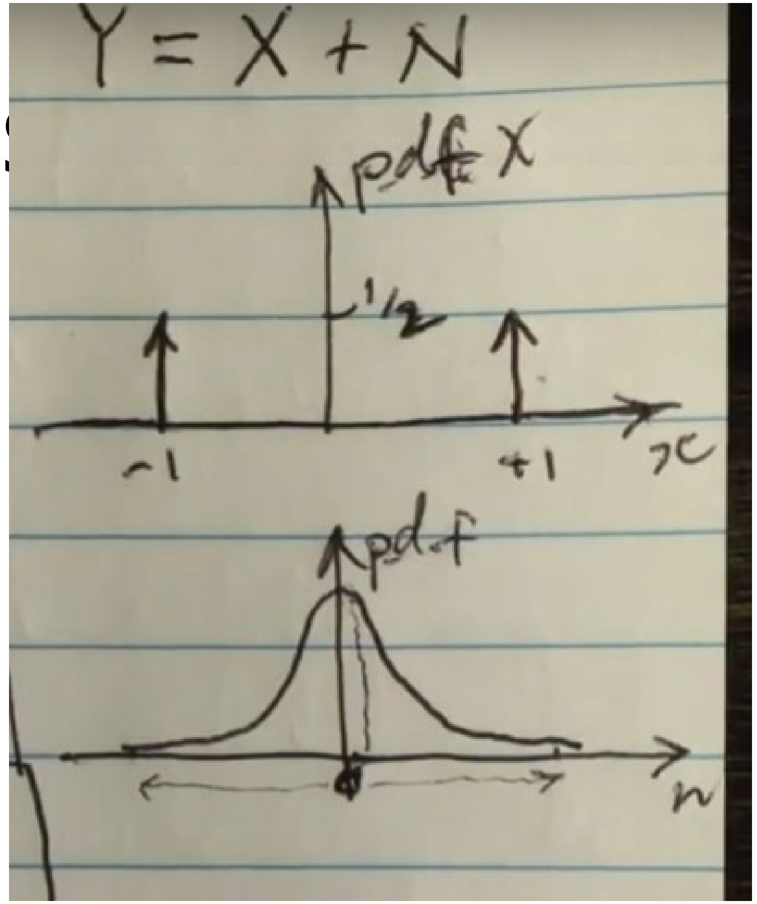
Asymmetric Continuous Functions

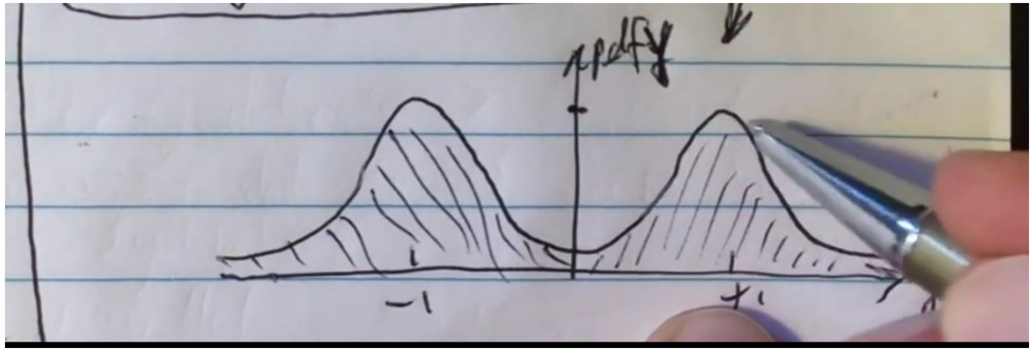
- When functions aren’t discrete the convolution sweep becomes an integral sliding across the function from multiplying both signals.
- In this example the same triangle function is convolved but with z(t), a continuous polynomial function.
- Z(t) can be viewed as an infinite series of impulses, each with an amplitude going up to the value of the function.
- Then, taking the convolution of these two functions would result in w(t), where the triangle is multiplied by each impulse; resulting in a lot of overlapping triangles, whose addition yields the smoothed out continuous curve seen over them, which is the actual result of the convolution.



Digital Communications

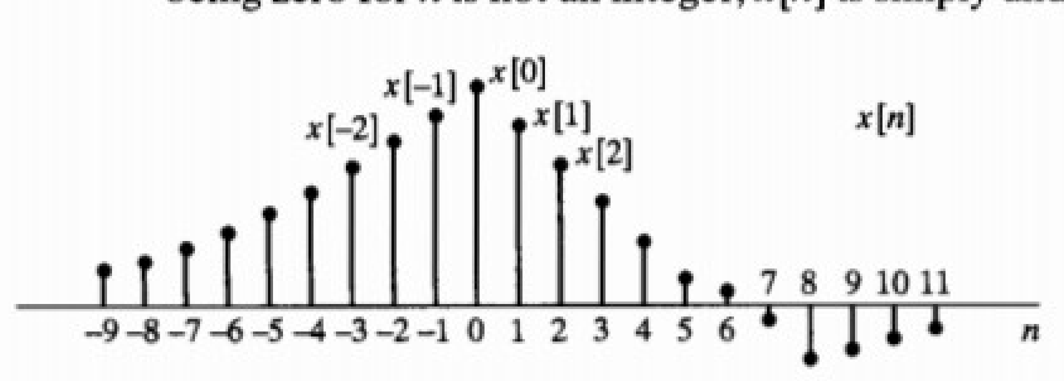
- In digital communications, the output of a system is the signal plus some noise. The signal can be 1 or -1, each with a probability of a half, and the noise has a probability density function of a gaussian curve.
- Taking the convolution of these two functions would yield the actual probability density function of the system, which would be two gaussian curves at points -1 and 1.





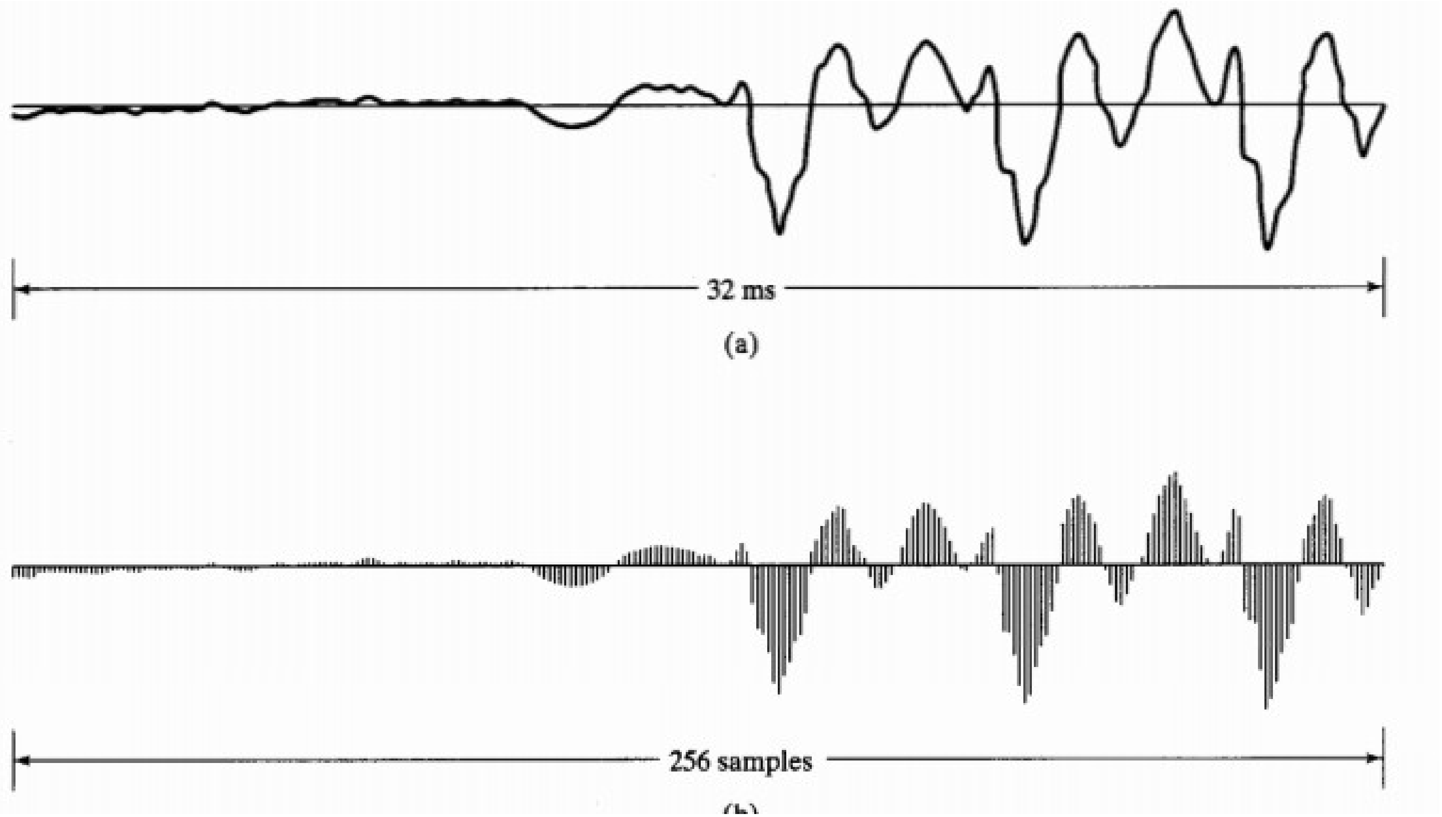
Introduction to Discrete Time Signals

- Digital signals have discrete time and amplitude, while discrete time signals can have continuous amplitude.
- Discrete time signal analysis can be done by treating signals as analog with impulse trains (dirac delta functions).
- Discrete time signals are represented mathematically as a sequence of numbers, take $x[n]$ for $-\infty < n < \infty$. ∞



Analog to Discrete

- To convert analog signals into discrete time signals the signal is divided into samples depending on a step size or timestamp that is defined.
- At values where there is no peak, the functon is undefined.
- It is constituted of impulse functions on each sample representing the ampltude of the original signal at that time.



Simple Discrete Operations

- The most basic manipulational operations on disrete signals are arithmetic operations.
- Signals can be scaled, multiplied and added together by just taking the sample by sample operations.
- Signals an also be shifted by an integer n

$$y[n] = x[n - n_0],$$

- The unit sample sequence is what's used in the representation of discrete signals, it is similar to the dirac delta function except it is 1 [illegible] nt instead of infinity.

$$\delta[n] = \begin{cases} 0, & n \neq 0, \\ 1, & n = 0. \end{cases}$$

Mathematical Signal Representation

- Using the step funtions a signal can be represented as a sum of scaled, delayed inpulses; where the delays are what place each impulse at a specific moment in the time domain

$$p[n] = a_{-3}\delta[n+3] + a_1\delta[n-1] + a_2\delta[n-2] + a_7\delta[n-7].$$

- More generally

$$x[n] = \sum_{k=-\infty}^{\infty} x[k]\delta[n-k].$$

Other expressions for sequences

- Exponential sequences are represented as:

$$x[n] = \begin{cases} A\alpha^n, & n \geq 0, \\ 0, & n < 0. \end{cases}$$

A much simpler expression is $x[n] = A\alpha^n u[n]$.

- Complex exponentials (phased)

$$x[n] = |A|e^{j(\omega_0 n+\phi)} = |A|\cos(\omega_0 n + \phi) + j|A|\sin(\omega_0 n + \phi);$$

- Sinusoidal signals:

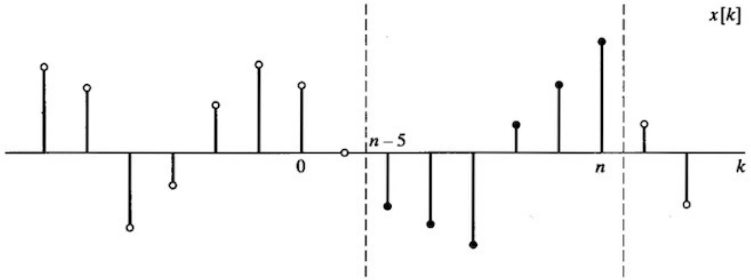
Unlike continuous sinusoidal signals, these signals’ periods don’t decrease with higher frequencies. On the contrary, they also increase because timesteps are discrete, and since the domain is made of integer indices each period with higher frequency requires more steps than lower frequencies.

$$x[n] = A\cos(\omega_0 n + \phi), \qquad \text{for all } n,$$

Moving Average

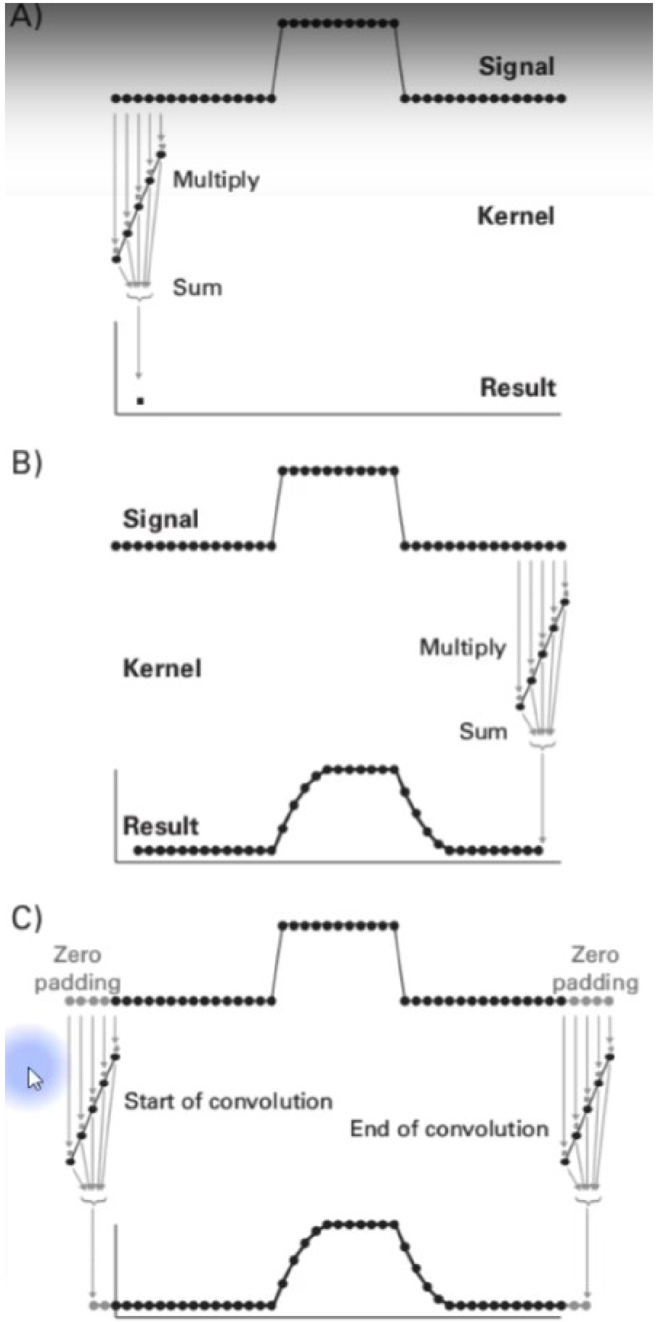
$$
\begin{aligned}
y[n] &= \frac{1}{M_1 + M_2 + 1} \sum_{k=-M_1}^{M_2} x[n-k] \\
&= \frac{1}{M_1 + M_2 + 1} \{x[n + M_1] + x[n + M_1 - 1] + \cdots + x[n] \\
&\quad + x[n-1] + \cdots + x[n - M_2]\}.
\end{aligned}
$$

- The moving average of a sequence takes an unweighted average on a certain domain range.
- It can be used to



Discrete Convolution

- Convolution in discrete signals is very similar to continuous functions. The same kernel sliding occurs over the input function where at each point the dot product is taken and the sum is a resulting point in the new signal.
- In this example, what happens when a sloped line is convoluted with a step function is a smoothening of the step, because when the line is multiplied with the part of the step function near where the steps occur, the zeroes and the ones from the step even out with the lower and higher points of the sloped line, resulting in a smooth curve in the step regions.



Function Flip

- The first step in convolution is flipping the function that will be shifted through the other function. Flipping one of the functions ensures that the domain times in one function will be aligned with the other properly.
- Every action in the impulse response function needs to happen relative to the time of activation not the system time.
- Take as an example two functions with a time frame of 5 seconds.

At $t=5 \rightarrow i(t)*r(t) = i(0)*r(5) + i(1)*r(4) + i(2)*r(3) + i(3)*r(2) + i(4)*r(1) + i(5)*r(0)$

- Flipping any of the two functions work, but intuitively it makes more sense to flip the response function; the function that dictates the action that happens, generally known as the kernel.

Convolution Exan

- Because of the simplicity of the step function, to convolute these two signals the integral can be broken down into the different cases that depend on how the two functions are interacting. The procedure below is followed.

$$(f * g)(t) \stackrel{\text{def}}{=} \int_{-\infty}^{\infty} f(\tau)g(t-\tau)\, d\tau$$

To convolve a kernel with an input signal:
flip the signal, move to the desired time,
and accumulate every interaction with the kernel

Sections 2-3 to 2-5: Convolution

2.10 Functions $x(t)$ and $h(t)$ are both rectangular pulses, as shown in **Fig. P2.10**. Apply graphical convolution to determine $y(t) = x(t) * h(t)$ given the following data.

(a) $A = 1,\ B = 1,\ T_1 = 2$ s, $T_2 = 4$ s

(b) $A = 2,\ B = 1,\ T_1 = 4$ s, $T_2 = 2$ s

*__(c)__ $A = 1,\ B = 2,\ T_1 = 4$ s, $T_2 = 2$ s.

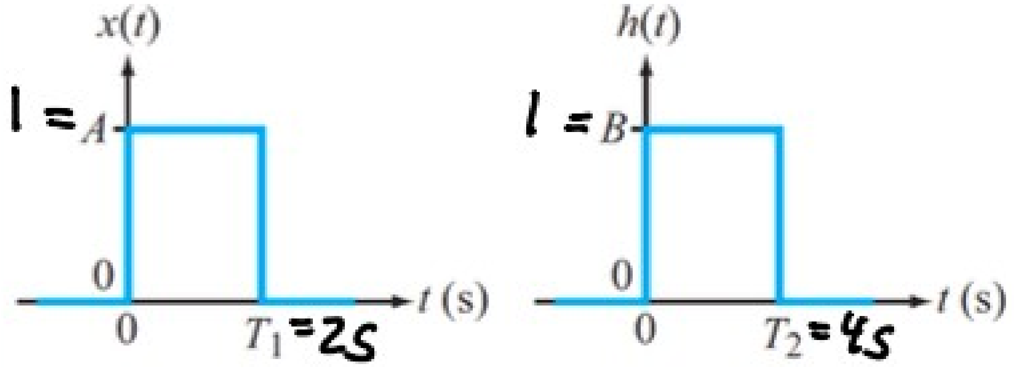
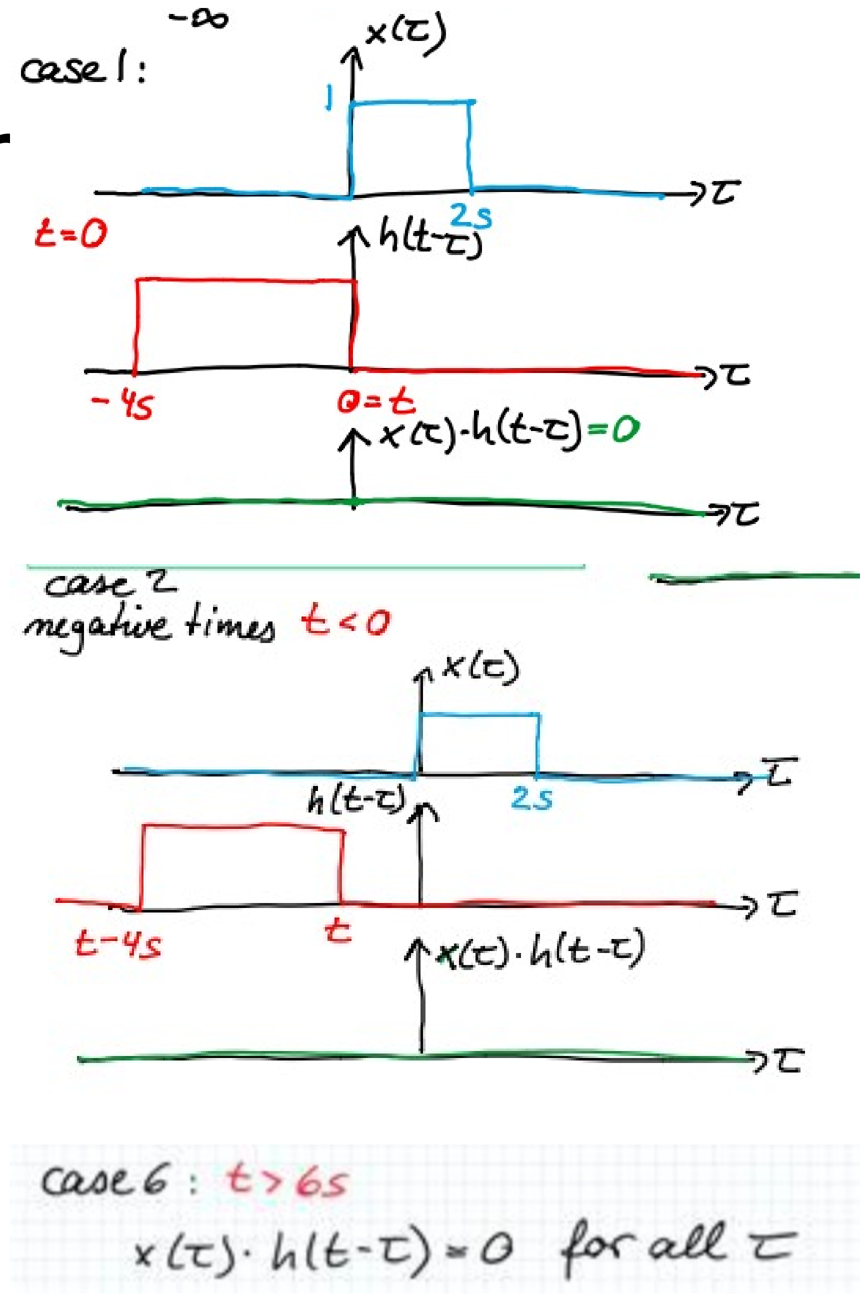


Figure P2.10: Waveforms of $x(t)$ and $h(t)$.

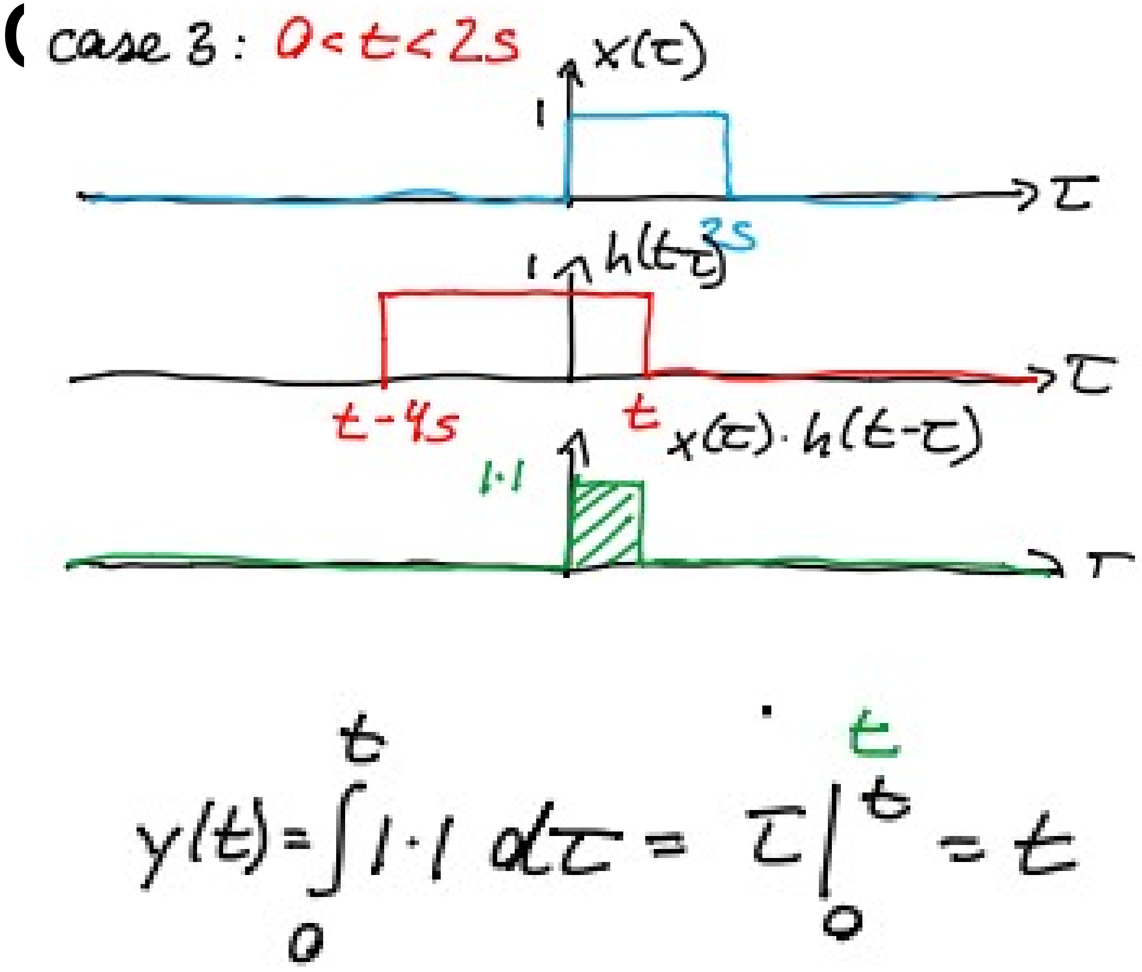
Signal Example Cor

- To evalute the convolution, the signals are plotted over the tao domain, this doesn't change the shape in any way.
- As the kernel function h(t) becomes h(t-tao) it is mirrored on the y axis in the tao domain.
- This kernel functon is what will be graphically moving through x(tao) along the integral.
- This first case denotes when t=0, the time at which the positive steps on the functions aren't lined up so their values of 0 cancel eachother out, seen by the output function in green below.
- The same is true for case 2 in which t is negative and case 6 where t>6, both cases where the behaviour is the same as case 1, the functions cancel out when multiplied because their positive values don't overlap.



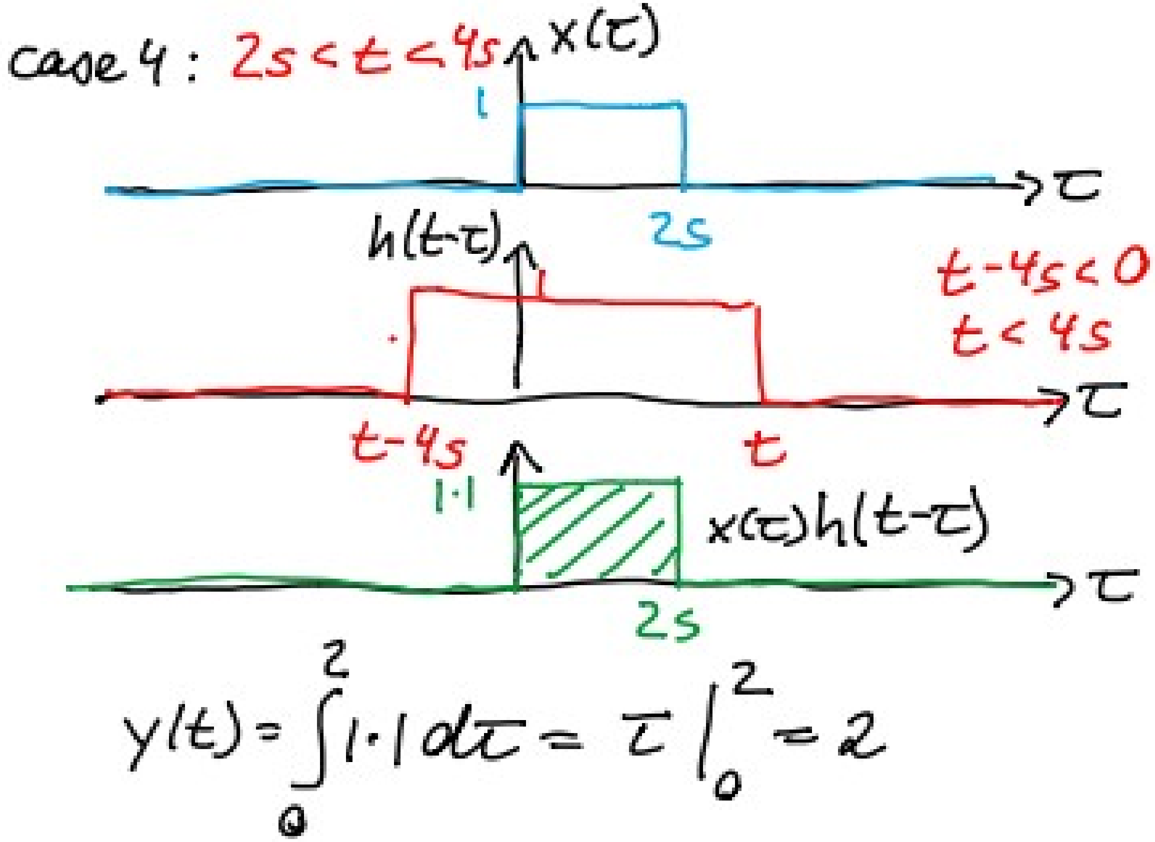
Signal Example Co

- In the third case the integral is being evaluated over a domain space where the kernel has positive values as well as input function.
- This case results in the following integral, which is simply the multiplication of the two functions from time=0 to time=t, it results in the variable t, which represents a 1/1 slope in the range 0-2s.
- The function in green represents the function inside the integral, it is the intersection of both functions.



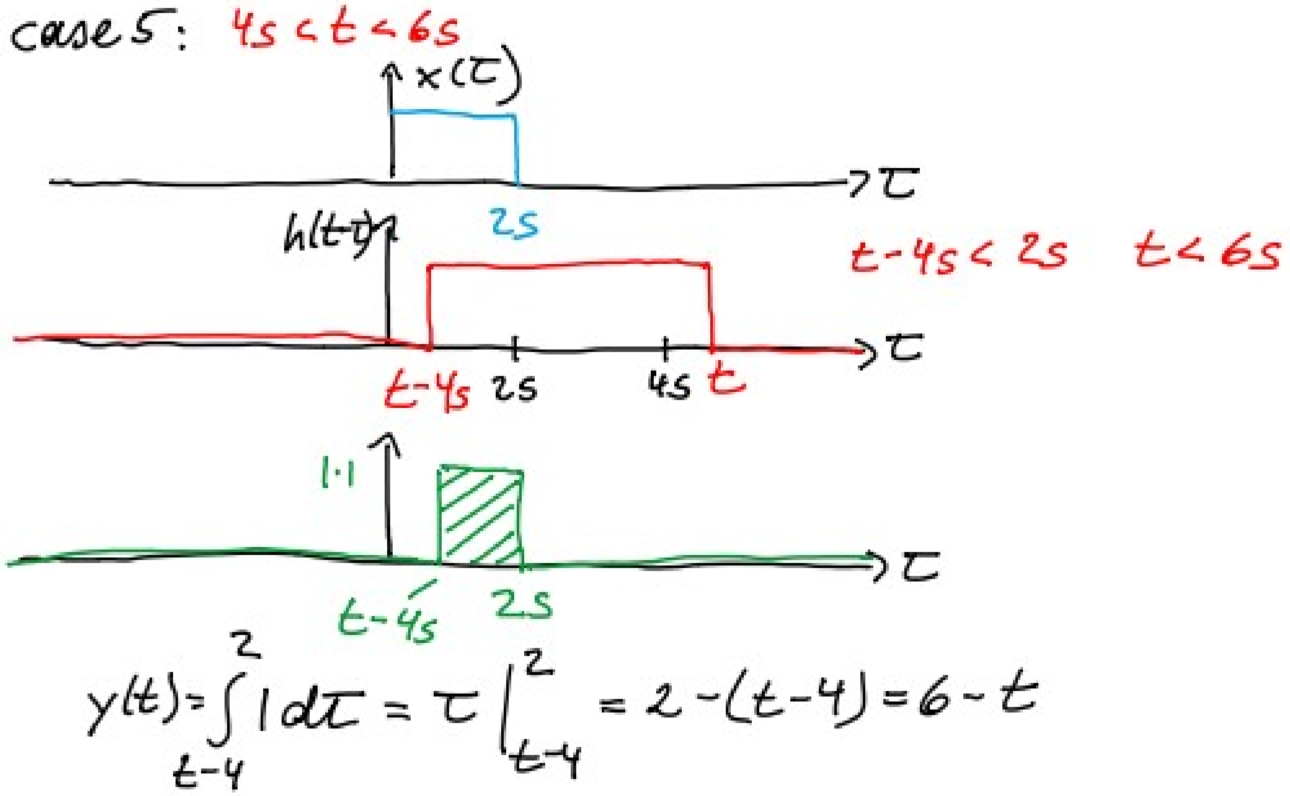
Signal Example Cont.

- This case has the kernel completely overlapping the original function.
- That results in an integral from 0s to 2s which are the intersection values between the functions where they’re both having a value of 1 yielding 1*1 whose integral is tao, giving the output a steady value of 2.



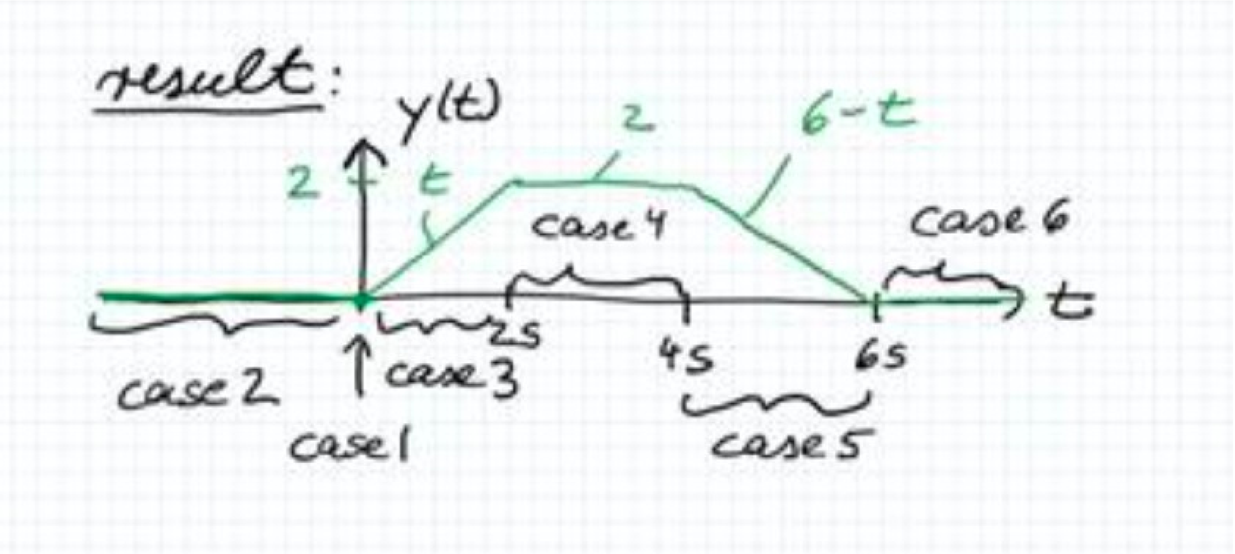
Signal Example Cont.

- This case has the kernel partly overlapping the original function. It is very similar to case 3 but the output function is the opposite.
- The overlaping of the funtions (in green) shows that the integral must be computed from t-4s to 2s.
- The integral results as 6-t, a line with a downward slope of 1 that hits the horizontal axis at t=6, a mirrored function of case 3 along t=3,



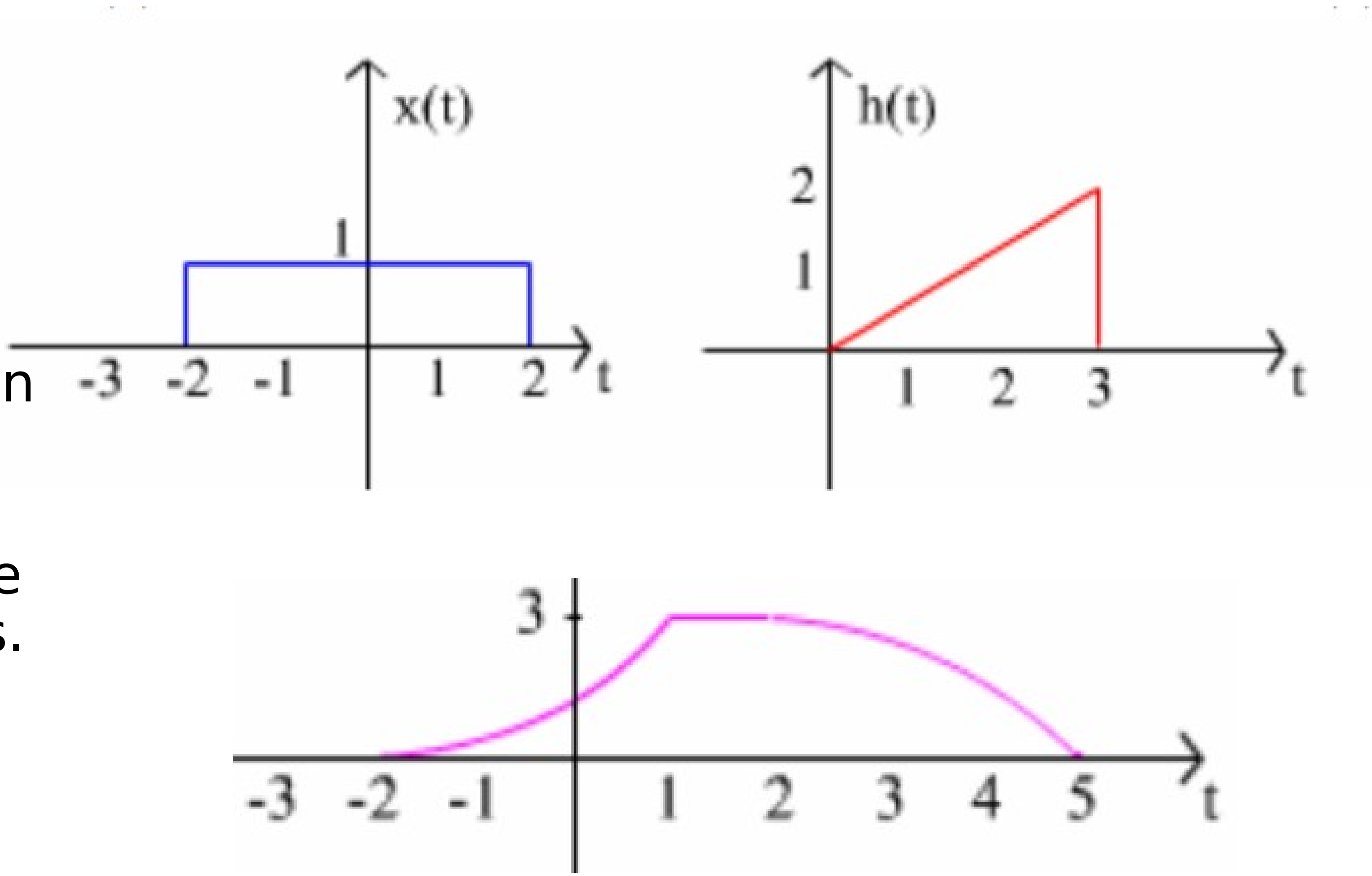
Signal Example Result

- The result of the convolution is the function in green which is 0 everywhere except from times 0s-6s, 6 being the sum of the length of the two signals.
- The output is 2 where both signals fully overlap from 2s to 4s, this region's length is given from the original function's length.
- From 0s-2s and 4s-6s slopes of 1 and -1 are seen. This is expected as the original function has a length of only two so basically the kernel function's overextend on each side is reflected as those slopes where it doesn't match directly with the input signal.



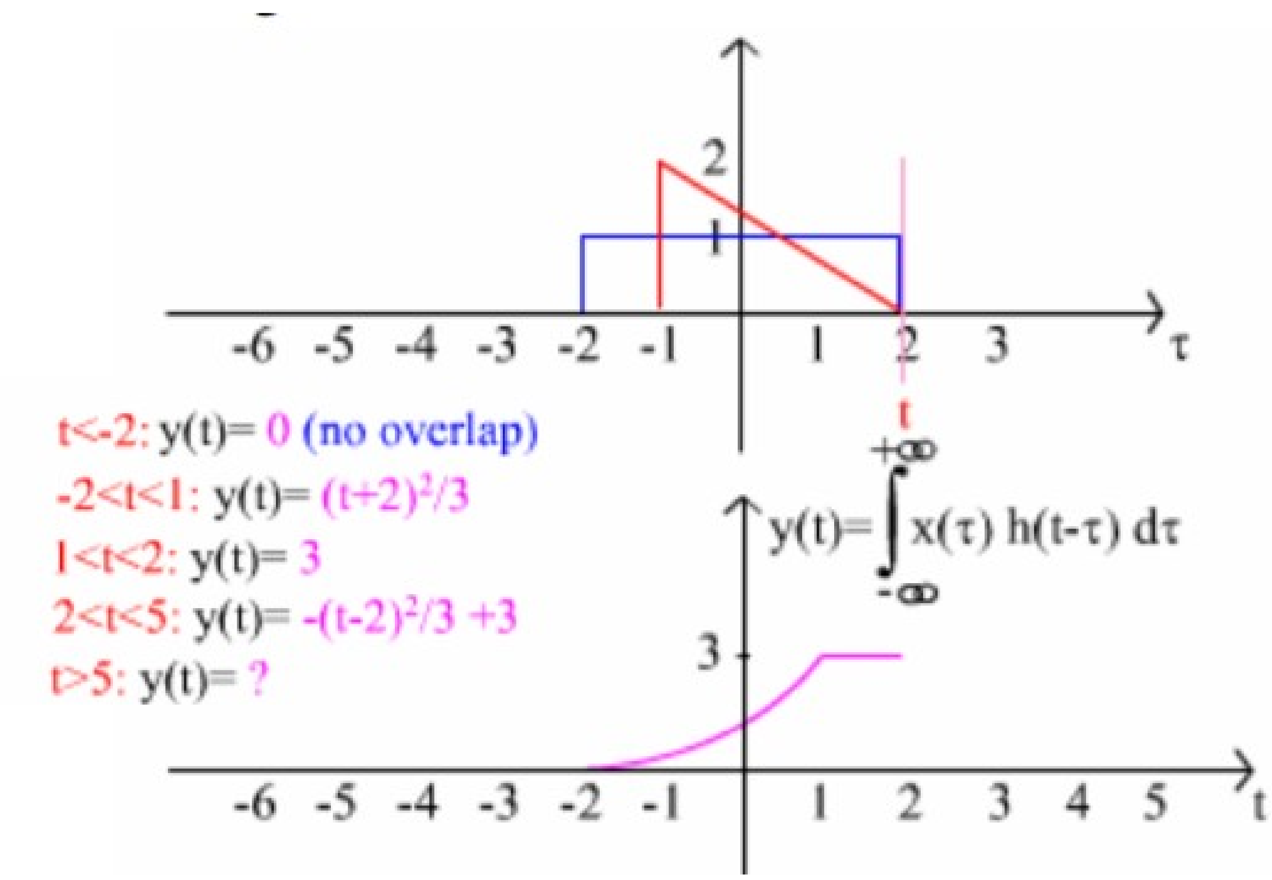
Non-Symmetric Example

- Take the blue and red signals which when convoluted will produce the result on the bottom.
- The output signal has a max height of 3, which is the addition of both heights.
- It has a range from -2, the earliest functions start; to 5, the concatenation of both functions.
- The smooth slope shapes are caused by the averaging of the triangle’s slope with nearby values in the rectangle.



Triangle Flip

- An intuitive understanding to the flipping of one of the functions is the interaction between the functions when one is slid across.
- We want the result's values at the beginning to be a combination of the beginning of the triangle with the beginning of the rectangle, and the same applies at the end (end of rectangle with end of triangle).
- This is what causes the result to be a combination of both functions.



Mathematical Meaning

- Convolution is the sum of the dot products of two functions.
- For discrete functions, a simple summation of the multiplication is taken on each timestamp.
- For continuous functions the same dot product is taken over an integral with infinitesimal steps dτ, evaluated from -∞< τ < ∞.
- The t- τ in g represents a shift in the equation by t and the -τ represents a flip along the y axis.

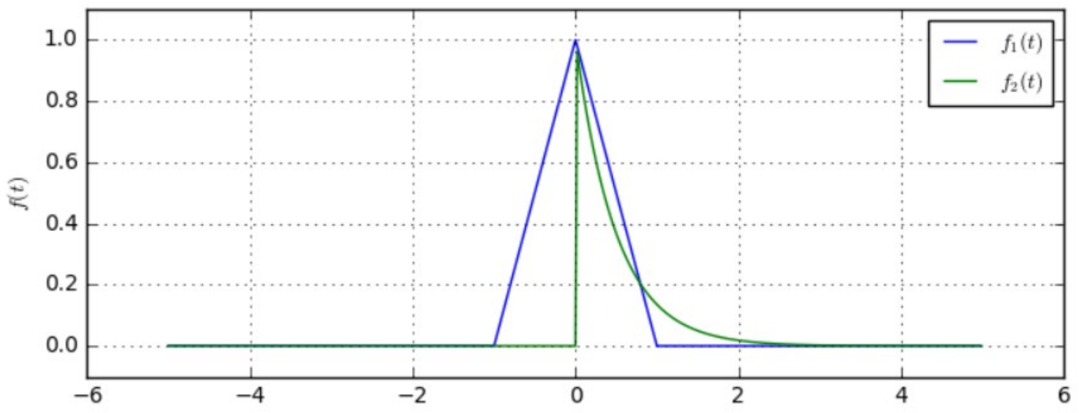
$$y[n] = \sum_{k=-\infty}^{\infty} x[k]h[n-k].$$

$$(f^*g)(t) \triangleq \int_{-\infty}^{\infty} f(\tau)g(t-\tau)\tau'$$

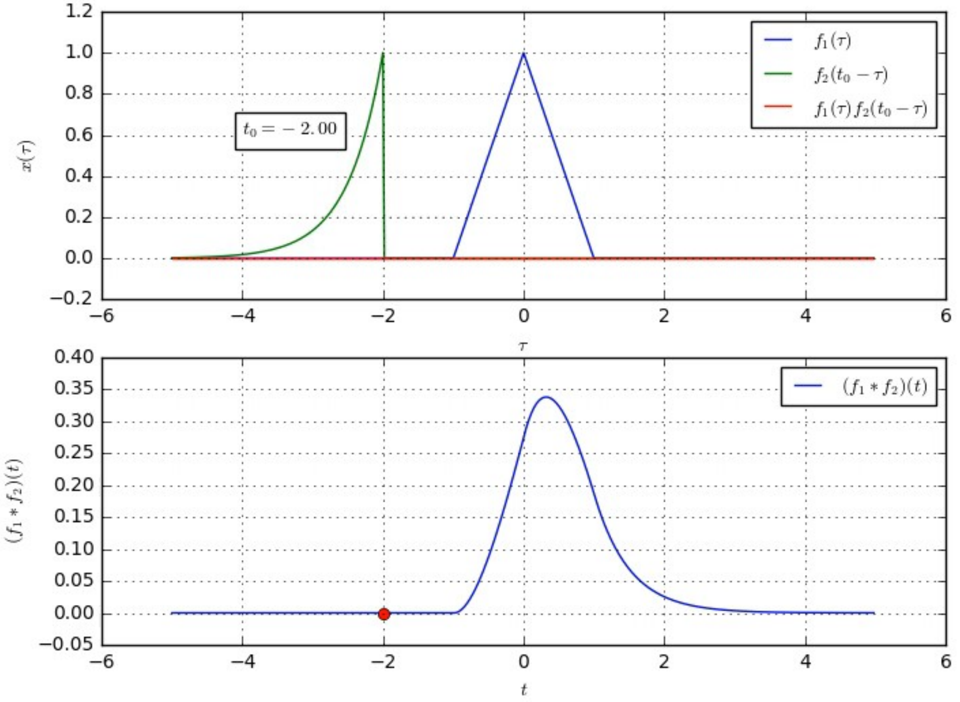
$(f^*g)(t)$	= functions that are being convoluted
t	= real number variable of functions f and g
$g(\tau)$	= convolution of the function f(t)
τ'	= first derivative of g(tau) function

Refs

- https://lpsa.swarthmore.edu/Convolution/CI.html
- https://math.stackexchange.com/questions/1731597/what-is-the-convolution-of-sinbx-and-e-ax
- https://dspillustrations.com/pages/posts/misc/convolution-examples-and-the-convolution-integral.html
- http://www.ceri.memphis.edu/people/smalley/ESCI7355/Ch6_Linear_Systems_Conv.pdf



$f_1(t)$
$f_2(t)$
$f(t)$



$f_1(\tau)$
$f_2(t_0-\tau)$
$f_1(\tau)f_2(t_0-\tau)$
$t_0 = -2.00$
$x(\tau)$
τ
$(f_1 * f_2)(t)$
t

Kernel

- A kernel is a matrix that represents a transformation which will be done to an image.