## Case: $h = 0 \mod 3$

| Baseli  |                                     | Baseline No Jacobi cache  Average Jacobi ops to find 1st v(1) |          |                               |                           |                   |                          | Baseline Jacobi cache  Average Jacobi ops to find 1st v(1) |                          |                               |                           |                   |                          |                      | Baseline Cache advantage  Jacobi cache / No Jacobi cache |                               |                           |                   |                          |                      |                          |
|---------|-------------------------------------|---|----------|-------------------------------|---------------------------|-------------------|--------------------------|--|--------------------------|-------------------------------|---------------------------|-------------------|--------------------------|----------------------|--|-------------------------------|---------------------------|-------------------|--------------------------|----------------------|--------------------------|
| [ n, n+ | 1000)                               | h = 3*base_n [ h, h+6000 )                                    |          | search starting at 3          |                           | sorted by v(1)    |                          | reverse sort by freq                                       |                          | search starting at 3          |                           | sorted by v(1)    |                          | reverse sort by freq |  | search starting at 3          |                           | sorted by v(1)    |                          | reverse sort by freq |                          |
| base_n  | n_beyond                            | hase_h  | h_beyond | integer<br>search<br>1st v(1) | odd<br>search<br>1st v(1) | known<br>1st v(1) | odd<br>known<br>1st v(1) | known<br>1st v(1)  | odd<br>known<br>1st v(1) | integer<br>search<br>1st v(1) | odd<br>search<br>1st v(1) | known<br>1st v(1) | odd<br>known<br>1st v(1) | known<br>1st v(1)    | odd<br>known<br>1st v(1)                                 | integer<br>search<br>1st v(1) | odd<br>search<br>1st v(1) | known<br>1st v(1) | odd<br>known<br>1st v(1) | known<br>1st v(1)    | odd<br>known<br>1st v(1) |
| 4194304 | 4195304                             | 12582912  | 12588912 | 9.878                         | 5.900                     | 4.926             | 4.875                    | 4.864  | 4.852                    | 7.559                         | 4.620                     | 4.433             | 4.364                    | 4.369                | 4.350  | 1.3068                        | 1.2771                    | 1.1112            | 1.1171                   | 1.1133               | 1.1154                   |
| 4331116 | 4332116                             | 12993348  | 12999348 | 9.884                         | 5.898                     | 4.929             | 4.875                    | 4.868  | 4.852                    | 7.561                         | 4.618                     | 4.435             | 4.363                    | 4.371                | 4.349  | 1.3072                        | 1.2772                    | 1.1114            | 1.1174                   | 1.1137               | 1.1157                   |
| 4885002 | 4886002                             | 14655006  | 14661006 | 9.873                         | 5.892                     | 4.923             | 4.871                    | 4.862  | 4.847                    | 7.555                         | 4.615                     | 4.431             | 4.359                    | 4.367                | 4.345  | 1.3068                        | 1.2767                    | 1.1110            | 1.1175                   | 1.1134               | 1.1155                   |
| 5209020 | 5210020                             | 15627060  | 15633060 | 9.867                         | 5.891                     | 4.922             | 4.871                    | 4.861  | 4.847                    | 7.551                         | 4.614                     | 4.430             | 4.359                    | 4.366                | 4.345  | 1.3067                        | 1.2768                    | 1.1111            | 1.1175                   | 1.1134               | 1.1155                   |
| 6286862 | 6287862                             | 18860586  | 18866586 | 9.891                         | 5.901                     | 4.930             | 4.877                    | 4.869  | 4.853                    | 7.565                         | 4.620                     | 4.436             | 4.363                    | 4.373                | 4.349  | 1.3075                        | 1.2773                    | 1.1114            | 1.1178                   | 1.1134               | 1.1159                   |
| 7676777 | 7677777                             | 23030331  | 23036331 | 9.878                         | 5.897                     | 4.925             | 4.874                    | 4.863  | 4.849                    | 7.557                         | 4.617                     | 4.432             | 4.361                    | 4.367                | 4.346  | 1.3071                        | 1.2772                    | 1.1112            | 1.1176                   | 1.1136               | 1.1157                   |
| 8388608 | 8389608                             | 25165824  | 25171824 | 9.880                         | 5.901                     | 4.926             | 4.876                    | 4.865  | 4.852                    | 7.559                         | 4.621                     | 4.434             | 4.364                    | 4.370                | 4.350  | 1.3071                        | 1.2770                    | 1.1110            | 1.1173                   | 1.1133               | 1.1154                   |
|         | Standard Deviation                  |   |          |                               | 0.004                     | 0.003             | 0.002                    | 0.003  | 0.003                    | 0.004                         | 0.003                     | 0.002             | 0.002                    | 0.003                | 0.002  | 0.0003                        | 0.0002                    | 0.0002            | 0.0002                   | 0.0002               | 0.0002                   |
| Av      | Average Jacobi ops to find 1st v(1) |   |          |                               | 5.897                     | 4.926             | 4.874                    | 4.865  | 4.850                    | 7.558                         | 4.618                     | 4.433             | 4.362                    | 4.369                | 4.348  | 1.3070                        | 1.2770                    | 1.1112            | 1.1174                   | 1.1134               | 1.1156                   |

Known 1st v(1) sorted by v(1) - even in BOLD

Odd Known 1st v(1) sorted by v(1)

Known 1st v(1) rev sorted by freq - out of order in red

Odd Known 1st v(1) rev sorted by freq - out of order in red

Validated primes 1st v(1) rev sorted by freq - out of order in red

Sample Jacobi line

| More Riesel test numbers h*2^n-1    |                     |                          |                         |   | More No Jacobi cache  Average Jacobi ops to find 1st v(1)  search starting at 3 sorted by v(1) reverse sort by freq |                             |                             |                                 |   |                               | More Jacobi cache  Average Jacobi ops to find 1st v(1) |                             |                          |                                |   |                               |                           | More Cache advantage  Jacobi cache / No Jacobi cache |                                     |                   |   |  |  |
|-------------------------------------|---------------------|--------------------------|-------------------------|---|---|-----------------------------|-----------------------------|---------------------------------|---|-------------------------------|--|-----------------------------|--------------------------|--------------------------------|---|-------------------------------|---------------------------|--|-------------------------------------|-------------------|---|--|--|
| [ n, n-                             | -1000 )<br>n_beyond | h = 3*base_n [<br>hase_h | h, h+6000 )<br>h_beyond | search sta<br>integer<br>search<br>1st v(1) | odd<br>search<br>1st v(1)   | sorted<br>known<br>1st v(1) | by v(1)  odd known 1st v(1) | reverse so<br>known<br>1st v(1) | ort by freq<br>odd<br>known<br>1st v(1) | integer<br>search<br>1st v(1) | odd<br>search<br>1st v(1)                              | sorted<br>known<br>1st v(1) | odd<br>known<br>1st v(1) | reverse s<br>known<br>1st v(1) | ort by freq<br>odd<br>known<br>1st v(1) | integer<br>search<br>1st v(1) | odd<br>search<br>1st v(1) | sorted<br>known<br>1st v(1)                          | by v(1)<br>odd<br>known<br>1st v(1) | known<br>1st v(1) | ort by freq<br>odd<br>known<br>1st v(1) |  |  |
| 1391827                             | 1392827             | 4175481                  | 4181481                 | 9.865                                       | 5.889   | 4.920                       | 4.869                       | 4.861                           | 4.845                                   | 7.550                         | 4.612  | 4.428                       | 4.357                    | 4.365                          | 4.343                                   | 1.3066                        | 1.2769                    | 1.1111   | 1.1175                              | 1.1136            | 1.1156                                  |  |  |
| 3727058                             | 3728058             | 11181174                 | 11187174                | 9.892                                       | 5.903   | 4.930                       | 4.877                       | 4.868                           | 4.852                                   | 7.566                         | 4.622  | 4.437                       | 4.364                    | 4.373                          | 4.350                                   | 1.3074                        | 1.2772                    | 1.1111   | 1.1176                              | 1.1132            | 1.1154                                  |  |  |
| 5718259                             | 5719259             | 17154777                 | 17160777                | 9.888                                       | 5.903   | 4.929                       | 4.877                       | 4.866                           | 4.852                                   | 7.564                         | 4.621  | 4.436                       | 4.365                    | 4.370                          | 4.350                                   | 1.3072                        | 1.2774                    | 1.1111   | 1.1173                              | 1.1135            | 1.1154                                  |  |  |
| 12776050                            | 12777050            | 38328150                 | 38334150                | 9.849                                       | 5.876   | 4.915                       | 4.861                       | 4.854                           | 4.838                                   | 7.541                         | 4.604  | 4.423                       | 4.351                    | 4.359                          | 4.337                                   | 1.3061                        | 1.2763                    | 1.1112   | 1.1172                              | 1.1136            | 1.1155                                  |  |  |
| 23059373                            | 23060373            | 69178119                 | 69184119                | 9.849                                       | 5.882   | 4.914                       | 4.864                       | 4.855                           | 4.841                                   | 7.540                         | 4.609  | 4.422                       | 4.354                    | 4.361                          | 4.340                                   | 1.3062                        | 1.2762                    | 1.1113   | 1.1171                              | 1.1133            | 1.1154                                  |  |  |
| 56126460                            | 56127460            | 168379380                | 168385380               | 9.876                                       | 5.892   | 4.924                       | 4.872                       | 4.862                           | 4.849                                   | 7.556                         | 4.615  | 4.431                       | 4.360                    | 4.366                          | 4.346                                   | 1.3070                        | 1.2767                    | 1.1113   | 1.1174                              | 1.1136            | 1.1157                                  |  |  |
| 132174368                           | 132175368           | 396523104                | 396529104               | 9.876                                       | 5.893   | 4.925                       | 4.872                       | 4.865                           | 4.849                                   | 7.557                         | 4.616  | 4.433                       | 4.361                    | 4.369                          | 4.348                                   | 1.3069                        | 1.2766                    | 1.1110   | 1.1172                              | 1.1135            | 1.1152                                  |  |  |
| Standard Deviation                  |                     |                          |                         | 0.017                                       | 0.010   | 0.006                       | 0.006                       | 0.005                           | 0.005                                   | 0.010                         | 0.006  | 0.006                       | 0.005                    | 0.005                          | 0.005                                   | 0.0005                        | 0.0004                    | 0.0001   | 0.0002                              | 0.0002            | 0.0002                                  |  |  |
| Average Jacobi ops to find 1st v(1) |                     |                          | 9.871                   | 5.891                                       | 4.922   | 4.870                       | 4.862                       | 4.847                           | 7.553                                   | 4.614                         | 4.430  | 4.359                       | 4.366                    | 4.345                          | 1.3068                                  | 1.2768                        | 1.1112                    | 1.1173   | 1.1135                              | 1.1155            |   |  |  |
|                                     |                     |                          |                         |   |   |                             |                             |                                 | •                                       |                               |  |                             |                          |                                |   |                               |                           |  |                                     |                   |   |  |  |
| Small Validated Riesel primes       |                     |                          | 10.138                  | 6.069                                       | 5.045   | 4.939                       | 4.955                       | 4.922                           | 7.636                                   | 4.660                         | 4.537  | 4.418                       | 4.451                    | 4.415                          | 1.3277                                  | 1.3024                        | 1.1120                    | 1.1179   | 1.1132                              | 1.1148            |   |  |  |
| Large Validated Riesel primes       |                     |                          |                         | 10.615                                      | 6.308   | 5.215                       | 5.093                       | 5.115                           | 5.076                                   | 7.910                         | 4.792  | 4.673                       | 4.538                    | 4.576                          | 4.536                                   | 1.3420                        | 1.3164                    | 1.1160   | 1.1223                              | 1.1178            | 1.1190                                  |  |  |
| All Validated Riesel primes         |                     |                          |                         | 10.404                                      | 6.202   | 5.140                       | 5.025                       | 5.044                           | 5.008                                   | 7.789                         | 4.660  | 4.612                       | 4.484                    | 4.520                          | 4.415                                   | 1.3357                        | 1.3104                    | 1.1145   | 1.1207                              | 1.1159            | 1.1174                                  |  |  |