Tandon Bridge Program - Extended - Homework 1

Archana Balachandran (ab12864) and Rachel Butler (rb3028)

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Question 1

A Convert the following numbers to their decimal representation

1. 10011011_2
   $(2^7 * 1) + (2^6 * 0) + (2^5 * 0) + (2^4 * 1) + (2^3 * 1) + (2^2 * 0) + (2^1 * 1)$
   $+ (2^0 * 1)$

   $128 + 0 + 0 + 16 + 8 + 0 + 2 + 1$

   155

2. 456_7
   $(7^2 * 4) + (7^1 * 5) + (7^0 * 6)$

   $196 + 35 + 6$

   237

3. $38A_{16}$
   $(16^2 * 3) + (16^1 * 8) + (16^0 * 10)$ [A = 10 in decimal]

   $768 + 128 + 10$

   906

4. 2214_5
   $(5^3 * 2) + (5^2 * 2) + (5^1 * 1) + (5^0 * 4)$

   $250 + 50 + 5 + 4$

   309

B Convert the following numbers to their binary representation

1. 69_{10}

2^6	2^5	2^4	2^3	2^2	2^1	2^0
1	0	0	0	1	0	1

   1000101

2. 485_{10}

2^8	2^7	2^6	2^5	2^4	2^3	2^2	2^1	2^0
1	1	1	1	0	0	1	0	1

   111100101

3. $6D1A_{16}$

   Hexadecimal numbers can be converted to their binary representation by converting each digit to its binary equivalent.

   For 6D1A:
   [6] 0110_2 [D] 1101_2 [1] 0001_2 [A] 1010_2

   Hence 6D1A can be translated to 0110 1101 0001 1010 where every four binary digits represent the corresponding hexadecimal digit in order.

C Convert the following numbers to their hexadecimal representation

1. 1101011_2

   The decimal equivalent of 1101011_2 is calculated using:
   $(2^6 * 1) + (2^5 * 1) + (2^4 * 0) + (2^3 * 1) + (2^2 * 0) + (2^1 * 1) + (2^0 * 1)$
   64+32+0+8+0+2+1
   107

   The highest power of 16 that divides into 107 is 16^1 and it can do so up to 6 times. Hence we have 6 in the 16's place. 107 - (16*6) leaves us with 11 as a remainder, which is represented as B in hexadecimal representation.

   Hence, 1101011_2 is 6B in hexadecimal.

2. 895_{10}
   The highest power of 16 that divides into 895 is 16^2 and it can do so up to 3 times. Hence we have 3 in the 256's place. 895 - (16^2*3) leaves us with 127 as a remainder. The highest power of 16 that divides into 127 is 16^1 and it does so 7 times. Hence we have 7 in the 16's place. Finally, we have a remainder of 15 which is denoted using F in the hexadecimal system.

   Hence, 895_{10} is 37F in hexadecimal.

Question 2

Solve the following, do all calculation in the given base. Show your work.

1. $7566_8 + 4515_8$

Carry over	1	1	1	1	NA
First number	0	7	5	6	6
Second number	0	4	5	1	5
Result	1	4	3	0	3

The addition of 7566_8 and 4515_8 gives 14303_8

2. $10110011_2 + 1101_2$

Carry over	0	1	1	1	1	1	1	NA
First number	1	0	1	1	0	0	1	1
Second number	0	0	0	0	1	1	0	1
Result	1	1	0	0	0	0	0	0

The addition of 10110011_2 and 1101_2 gives 11000000_2

3. $7A66_{16} + 45C5_{16}$

Carry over	1	1	0	0
First number	7	A	6	6
Second number	4	5	C	5
Result	C	0	2	B

The addition of $7A66_{16}$ and $45C5_{16}$ gives $C02B_{16}$

4. 3022_5 - 2433_5

Carry over	1	1	0	0
First number	3	0	2	2
Second number	2	4	3	3
Result	0	0	3	4

Subtracting 2433_5 from 3022_5 gives 34_5

Question 3

A Convert the following numbers to their 8-bits two's complement representation. Show your work.

Solution method
In the k-bit two's complement method, a positive number is written in its (k-1) bit unsigned binary padded with a zero to the left. In this method, the sum of a number and its additive inverse is 2^k. When we have a positive number at hand, say x, we find the binary representation of - x by finding the y such that y = 2^k - x. The resulting y is essentially the binary representation of -x.

1. 124_{10}

   Here since we are working with a positive number, we find the 7 (k-1 or 8-1) bit unsigned binary and pad a zero to its left.

   $124 = (2^6 * 1) + (2^5 * 1) + (2^4 * 1) + (2^3 * 1) + (2^2 * 1) + (2^1 * 0) + (2^0 * 0)$

   Hence the (k-1) bit unsigned binary representation of 124_{10} is 1111100_2 and the k-bit two's complement binary representation is 01111100_2.

2. -124_{10}

   We found the 8-bit two's complement of 124 in the previous question. The 8-bit two's complement of -124 is basically the binary number that needs to be added to 01111100_2 to arrive at 100000000_2. This number is 100000000_2 - 01111100_2 = 10000100_2.

3. 109_{10}
   $109 = (2^6 * 1) + (2^5 * 1) + (2^4 * 0) + (2^3 * 1) + (2^2 * 1) + (2^1 * 0) + (2^0 * 1)$
   Since a positive number is written in its (k-1) bit unsigned binary padded with a zero to the left, 109_{10} is 01101101_2

4. -79_{10}
   Following the standard steps for finding the binary of a decimal

base, $79 = (2^6*1) + (2^5*0) + (2^4*0) + (2^3*1) + (2^2*1) + (2^1*1) + (2^0*1)$.

Therefore $79_{10} = 1001111_2$. In the 8-bit two's complement method, this would be 01001111_2.

The corresponding representation for -79_{10} would be $100000000_2 - 01001111_2 = 10110001_2$

B Convert the following numbers (represented as 8-bit two's complement) to their decimal representation. Show your work.

1. $00011110_{8bit2scomp} = 30$
   $\frac{0}{128}\frac{0}{64}\frac{0}{32}\frac{1}{16}\frac{1}{8}\frac{1}{4}\frac{1}{2}\frac{0}{1}$
   $0 + 0 + 0 + 16 + 8 + 4 + 2 + 0 = 16 + 14 = 30$

2. $11100110_{8bit2scomp} =$ -26
   Subtract 1110 0110 from 2^8: 1 0000 0000 - 1110 0110 = 0001 1010
   Convert to base-10: $(2^0*0)+(2^1*1)+(2^2*0)+(2^3*1)+(2^4*1)+(2^5*0)+(2^6*0)+(2^7*0) = 0 + 2 + 0 + 8 + 16 + 0 + 0 + 0 = 26$
   Add -: -26

3. $00101101_{8bit2scomp} = 45$
   $\frac{0}{128}\frac{0}{64}\frac{1}{32}\frac{0}{16}\frac{1}{8}\frac{1}{4}\frac{0}{2}\frac{1}{1}$
   $0 + 0 + 32 + 0 + 8 + 4 + 0 + 1 = 45$

4. $10011110_{8bit2scomp} =$ -98
   Subtract 1001 1110 from 2^8 to get the inverse: 1 0000 0000 - 1001 1110 = 0110 0010
   Convert to base-10: $(2^0*0)+(2^1*1)+(2^2*0)+(2^3*0)+(2^4*0)+(2^5*1)+(2^6*1)+(2^7*0) = 0 + 2 + 0 + 0 + 0 + 32 + 64 + 0 = 98$ Add -: -98

Question 4 Solve the following questions from the Discrete Math zyBook:

1. Exercise 1.2.4: Writing truth tables: sections b,c

   b. $\neg(p \vee q)$

p	q	$p \vee q$	$\neg(p \vee q)$
T	T	T	F
T	F	T	F
F	T	T	F
F	F	F	T

   c. $r \vee (p \wedge \neg q)$

p	q	r	$\neg q$	$(p \wedge \neg q)$	$r \vee (p \wedge \neg q)$
T	T	T	F	F	T
T	T	F	F	F	F
T	F	T	T	T	T
T	F	F	T	T	T
F	T	T	F	F	T
F	T	F	F	F	F
F	F	T	T	F	T
F	F	F	T	F	F

2. Exercise 1.3.4: Truth table with conditional operations: sections b,d

   b. $(p \implies q) \implies (q \implies p)$

p	q	$p \implies q$	$q \implies p$	$(p \implies q) \implies (q \implies p)$
T	T	T	T	T
T	F	F	T	T
F	T	T	F	F
F	F	T	T	T

   d. $(p \longleftrightarrow q) \oplus (p \longleftrightarrow \neg q)$

p	q	$p \longleftrightarrow q$	$\neg q$	$(p \longleftrightarrow \neg q)$	$(p \longleftrightarrow q) \oplus (p \longleftrightarrow \neg q)$
T	T	T	F	F	T
T	F	F	T	T	T
F	T	F	F	T	T
F	F	T	T	F	T

Question 5 Solve the following questions from the Discrete Math zyBook

1. Exercise 1.2.7, sections b,c

   b. The applicant must present at least two of the following forms of identification: birth certificate, driver's license, marriage license.
   $(B \wedge D) \vee (D \wedge M) \vee (M \wedge B)$

   c. Applicant must present either a birth certificate or both a driver's license and marriage license.
   $B \vee (D \wedge M)$

2. Exercise 1.3.7, sections b-e

   s: a person is a senior
   y: a person is at least 17 years of age
   p: a person is allowed to park in the school parking log

   b. A person can park in the school parking lot if they are a senior or at least 17 years of age.
   $(s \vee y) \implies p$

   c. Being seventeen years of age is necessary condition for being able to park in the school parking lot.
   $p \implies y$

   d. A person can park in the school parking lot if and only if the person is a senior and at least 17 years of age.
   $p \longleftrightarrow (s \wedge y)$

   e. Being able to park in the school parking lot implies that the person is either a senior or at least 17 years old.
   $p \implies (s \vee y)$

3. Exercise 1.3.9, sections c,d
   y: the applicant is at least 18 years old

p: the applicant has parental permission
c: the applicant can enroll in the course

c. The applicant can enroll in the course only if the applicant has parental permission.
   $c \implies p$

d. Having parental permission is a necessary condition for enrolling in this course.
   $c \implies p$

Question 6

1. Exercise 1.3.6, sections b - d.
   Give an English sentence in the form 'If..then..' that is equivalent to each sentence

   b. Maintaining a B average is necessary for Joe to be eligible for the honors program.
   If Joe is eligible for the program, then he has maintained a B average.

   c. Rajiv can go on the roller coaster only if he is at least four feet tall.
   If Rajiv can go on the roller coaster, then he is at least four feet tall.

   d. Rajiv can go on the roller coaster if he is at least four feet tall.
   If Rajiv is at least four feet tall, then he can go on the roller coaster.

2. Exercise 1.3.10, sections c-f.
   The variable p is true, q is false and the truth value for variable r is unknown. Indicate whether the truth value of each logical expression is true, false or unknown.

   c. $(p \vee r) \longleftrightarrow (q \wedge r)$
   $(T \vee r) \longleftrightarrow (F \wedge r)$
   T $\longleftrightarrow$ F
   F

   d. $(p \wedge r) \longleftrightarrow (q \wedge r)$
   $(T \wedge r) \longleftrightarrow (F \wedge r)$
   $(T \wedge r) \longleftrightarrow$ F
   Unknown

   e. $p \implies (r \vee q)$
   $T \implies (r \vee F)$
   Unknown

f. $(p \wedge q) \implies r$
$(T \wedge F) \implies r$
$F \implies r$
T

Question 7

Exercise 1.4.5, sections b-d

Express each pair of sentences using logical expressions. Then prove whether the two expressions are logically equivalent.

j: Sally got the job
l: Sally was late for her interview
r: Sally updated her resume

b. if Sally did not get the job, then she was late for her interview or did not update her resume.
If sally updated her resume and did not get the job, then she was late for her interview.

Statement 1: $\neg j \implies (l \lor \neg r)$
Statement 2: $(r \land \neg l) \implies \neg j)$

j	l	r	$\neg j$	$\neg l$	$\neg r$	$l \lor \neg r$	$\neg j \implies l \lor \neg r$	$r \land \neg l$	$(r \land \neg l) \implies j$
T	T	T	F	F	F	T	T	F	T
T	T	F	F	F	T	T	T	F	T
T	F	T	F	T	F	F	T	T	T
T	F	F	F	T	T	T	T	F	T
F	T	T	T	F	F	T	T	F	T
F	T	F	T	F	T	T	T	F	T
F	F	T	T	T	F	F	F	T	F
F	F	F	T	T	T	T	T	F	T

The two statements are **logically equivalent** as they have the same truth values.

c. If Sally got the job, then she was not late for the interview.
If Sally did not get the job, then she was not late for her interview.

Statement 1: $j \implies \neg l$
Statement 2: $\neg j \implies l$

j	l	$\neg j$	$\neg l$	$j \implies \neg l$	$\neg j \implies l$
T	T	F	F	F	T
T	F	F	T	T	T
F	T	T	F	T	T
F	F	T	T	T	F

The two statements are **not logically equivalent** as they do not have the same truth values.

d. If Sally updated her resume or she was not late for her interview, then she got the job.
If Sally got the job, then she updated her resume and was not late for her interview.
Statement 1: $(r \vee \neg l) \implies j$
Statement 2: $j \implies (r \wedge \neg l)$

j	l	r	$\neg l$	$r \vee \neg l$	$r \wedge \neg l$	$(r \vee \neg l) \implies j$	$j \implies (r \wedge \neg l)$
T	T	T	F	T	F	T	F
T	T	F	F	F	F	T	F
T	F	T	T	T	T	T	T
T	F	F	T	T	F	T	F
F	T	T	F	T	F	F	T
F	T	F	F	F	F	T	T
F	F	T	T	T	T	F	T
F	F	F	T	T	F	F	T

The two statements are **not logically equivalent** as their truth values are not equal.

Question 8

1. Exercise 1.5.2, sections c,f,i
   Use the laws of propositional logic to prove the following:

   c. $(p \implies q) \land (p \implies r) \equiv p \implies (q \land r)$

   Starting with the left hand side or LHS:
   $(p \implies q) \land (p \implies r)$
   $(\neg p \lor q) \land (\neg p \lor r)$ (Conditional identity)
   $\neg p \lor (q \land r)$ (Distributive law)
   $\neg\neg p \implies (q \land r)$ (Conditional identity)
   p $\implies (q \land r)$ (Double negation law)

   Thus we have arrived at the given right hand side or RHS.

   f. $\neg(p \lor (\neg p \land q)) \equiv \neg p \land \neg q$

   Starting with the left hand side or LHS:
   $\neg(p \lor (\neg p \land q))$
   $\neg p \land \neg\ (\neg p \land q)$ (De Morgan's law)
   $\neg p \land (\neg\neg p \lor \neg q)$ (De Morgan's law)
   $\neg p \land (p \lor \neg q)$ (Double negation law)
   $(\neg p \land p) \lor (\neg p \land \neg q)$ (Distributive law)
   F $\lor (\neg p \land \neg q)$ (Complement law)
   $(\neg p \land \neg q) \lor$ F (Commutative law)
   $(\neg p \land \neg q)$ (Identity law)

   Thus we have arrived at the given right hand side or RHS.

   i. $(p \land q) \implies r \equiv (p \land \neg r) \implies \neg q$

   Starting with the left hand side or LHS:
   $(p \land q) \implies r$
   $\neg\ (p \land q) \lor$ r (Conditional identity)
   $(\neg p\lor$ q) $\lor$ r (De Morgan's law)
   $(\neg p \lor r) \lor \neg q)$ (Associative and commutative law)
   $\neg(p \land \neg r) \lor \neg q$ (De Morgan's law)

$(p \wedge \neg r) \implies \neg q$ (Conditional identity)

Thus we have arrived at the given right hand side or RHS.

2. Exercise 1.5.3, sections c, d
   Use the laws of propositional logic to prove that each statement is a tautology

   c. $\neg r \vee (\neg r \implies p)$
      $\neg r \vee (\neg\neg r \vee p)$ (Conditional identity)
      $\neg r \vee (r \vee p)$ (Double negation law)
      $(\neg r \vee r) \vee$ p (Associative law)
      T $\vee$ p (Complement law)
      T (Domination law)

   d. $\neg(p \implies q) \implies \neg q$
      $\neg\neg(p \implies q) \vee \neg q$ (Conditional identity)
      $(p \implies q) \vee \neg q$ (Double negation law)
      $(\neg p \vee q) \vee \neg q$ (Conditional identity)
      $\neg p \vee (q \vee \neg q)$ (Associative law)
      $\neg p \vee T$ (Complement law)
      T (Domination law)

Question 9

1. Exercise 1.6.3 c, d
   Consider the following statements in English. Write a logical expression with the same meaning. The domain is the set of all real numbers.

   (c) There is a number that is equal to its square.

   Logic: $\exists x(x = x^2)$

   (d) Every number is less than or equal to its square plus 1.

   Logic: $\forall x(x \leq x^2 + 1)$

2. Exercise 1.7.4, sections b - d
   Translate the following English statements into a logical expression with the same meaning.

   (b) Everyone was well and went to work yesterday.

   Logic: $\forall x(\neg S(x) \wedge W(x))$

   (c) Everyone who was sick yesterday did not go to work.

   Logic: $\forall x(S(x) \rightarrow \neg W(x)$

   (d) Yesterday someone was sick and went to work.

   Logic: $\exists x(S(x) \wedge W(x))$

Question 10

1. Exercise 1.7.9, sections c - i

   (c) $\exists x((x = c) \rightarrow P(x))$
   False. P(c) is false, so the statement is false.

   (d) $\exists x(Q(x) \wedge R(x))$
   True. When x=e, both are true, so the statement is true.

   (e) $Q(a) \wedge P(d)$
   True. Both are true, so the statement is true.

   (f) $\forall x((x \neq b) \rightarrow Q(x))$
   True. Q(x) is true in all cases except when x=b, so the statement is true.

   (g) $\forall x(P(x) \vee R(x))$
   False. When x=c, both P(c) and R(c) are false, so the statement is false.

   (h) $\forall x(R(x) \rightarrow P(x))$
   True. R(x) is false for x=a-d, so the statement is true for all those cases. For the final x=e, R(e) and P(e) is true.

   (i) $\exists x(Q(x) \vee R(x))$
   True. An example: when x=a, Q(a) is true, so the statement is true.

2. Exercise 1.9.2, sections b - i

   (b) $\exists x \forall y Q(x, y)$
   True. Example: x=2.

   (c) $\exists y \forall x P(x, y)$
   True. Example: y=1.

   (d) $\exists x \exists y S(x, y)$
   False.

   (e) $\forall x \exists y Q(x, y)$
   False. Counter: x=1.

   (f) $\forall x \exists y P(x, y)$
   True. Examples: (1,1), (2,1), (3,1).

(g) $\forall x \forall y P(x, y)$
False. Counter: (2,2).

(h) $\exists x \exists y Q(x, y)$
True. An example: (2,2).

(i) $\forall x \forall y \neg S(x, y)$
True.

Question 11

1. Exercise 1.10.4, sections c - g

   (c) There are two numbers whose sum is equal to their product.
   Logic: $\exists x \exists y (x + y = x * y)$

   (d) The ratio of every two positive numbers is also positive.
   Logic: $\forall x \forall y ((x > 0) \wedge (y > 0)) \to (x/y > 0)$

   (e) The reciprocal of every positive number less than one is greater than one.
   Logic: $\forall x (0 < x < 1) \to (1/x) > 1)$

   (f) There is no smallest number.
   Logic: $\forall x \exists y (x > y)$

   (g) Every number other than 0 has a multiplicative inverse.
   Logic: $\forall x \exists y (x \neq 0 \to y = 1/x)$

2. Exercise 1.10.7, sections c - f
   Give a logical expression for each of the following sentences.

   (c) There is at least one new employee who missed the deadline.
   Logic: $\exists x (N(x) \wedge D(x))$

   (d) Sam knows the phone number of everyone who missed the deadline.
   Logic: $\forall y (D(y) \to P(Sam, y)$

   (e) There is a new employee who knows everyone's phone number.
   Logic: $\exists x \forall y (N(x) \wedge P(x, y)$

   (f) Exactly one new employee missed the deadline.
   Logic: $\forall x \exists y (N(y) \wedge D(y)) \wedge (x \neq y \wedge N(x) \to \neg D(x))$

3. Exercise 1.10.10, sections c – f

   (c) Every student has taken at least one class other than Math 101.
   Logic: $\forall x \exists y (T(x, y) \wedge y \neq Math101)$

   (d) There is a student who has taken every math class other than Math 101.
   Logic: $\exists x \forall y (T(x, y) \wedge y \neq Math101)$

(e) Everyone other than Sam has taken at least two different math classes.
Logic: $\forall x \exists y \exists z((x \neq Sam) \rightarrow T(x,y) \wedge T(x,z) \wedge y \neq z)$

(f) Sam has taken exactly two math classes.
Logic: $\forall x \exists y \exists z((y \neq z) \wedge T(Sam,y) \wedge T(Sam,z) \wedge (x \neq y \wedge x \neq z \rightarrow \neg T(Sam,x)))$

Question 12

1. Exercise 1.8.2, sections b – e

   (b) Every patient was given the medication or the placebo or both.

   - Original logic statement: $\forall x(P(x) \vee D(x))$
   - Negate: $\neg\forall x(P(x) \vee D(x))$
   - Apply DeMorgan's Law: $\exists x \neg P(x) \wedge \neg D(x)$
   - English: There is a patient who did not receive the placebo and did not get the medication.

   (c) There is a patient who took the medication and had migraines.

   - Original logic statement: $\exists x(D(x) \wedge M(x))$
   - Negate: $\neg\exists x(D(x) \wedge M(x))$
   - Apply DeMorgan's Law: $\forall x \neg D(x) \vee \neg M(x)$
   - English: Every patient either did not receive the medication or did not have migraines or both.

   (d) Every patient who took the placebo had migraines.

   - Original logic statement: $\forall x(P(x) \rightarrow M(x))$
   - Conditional: $\forall x(\neg P(x) \vee M(x))$
   - Negate: $\neg\forall x(\neg(P(x) \vee M(x))$
   - Apply DeMorgan's Law: $\exists x(\neg\neg P(x) \wedge \neg M(x))$
   - Double Negation: $\exists x(P(x) \wedge \neg M(x))$
   - English: There is a patient who received the placebo and did not have migraines.

   (e) There is a patient who had migraines and was given the placebo.

   - Original logic statement: $\exists x(M(x) \wedge P(x))$
   - Negate: $\neg\exists x(M(x) \wedge P(x))$
   - Apply DeMorgan's Law: $\forall x \neg M(x) \vee \neg P(x)$
   - English: Every patient either did not have migraines or did not receive the placebo.

2. Exercise 1.9.4, sections c - e
   Write the negation of each of the following logical expressions so that all negations immediately precede predicates. In some cases, it may be necessary to apply one or more laws of propositional logic.

(c) $\exists x \forall y(P(x,y) \to Q(x,y))$

- Conditional: $\exists x \forall y(\neg P(x,y) \vee Q(x,y))$
- Negate: $\neg \exists x \forall y(\neg P(x,y) \vee Q(x,y))$
- Apply DeMorgan's Law: $\forall x \exists y(\neg\neg P(x,y) \wedge \neg Q(x,y))$
- Double Negation: $\forall x \exists y(P(x,y) \wedge \neg Q(x,y))$

(d) $\exists x \forall y(P(x,y) \iff P(y,x))$

- Simplify: $\exists x \forall y((P(x,y) \implies P(y,x)) \wedge (P(y,x) \implies P(x,y))$
- Conditional: $\exists x \forall y((\neg P(x,y) \vee P(y,x)) \wedge (\neg P(y,x) \vee P(x,y)))$
- Negate: $\neg \exists x \forall y((\neg P(x,y) \vee P(y,x)) \wedge (\neg P(y,x) \vee P(x,y)))$
- Apply DeMorgan's Law: $\forall x \exists y\ (\neg(\neg P(x,y) \vee P(y,x)) \vee \neg(\neg P(y,x) \vee P(x,y)))$
- Apply DeMorgan's Law again: $\forall x \exists y\ ((\neg\neg P(x,y) \wedge \neg P(y,x)) \vee (\neg\neg P(y,x) \wedge \neg P(x,y)))$
- Double Negation: $\forall x \exists y(P(x,y) \wedge \neg P(y,x)) \vee (P(y,x) \wedge \neg P(x,y))$

(e) $\exists x \exists y P(x,y) \wedge \forall x \forall y Q(x,y)$

- Negate: $\neg(\exists x \exists y P(x,y) \wedge \forall x \forall y Q(x,y))$
- Negate: $\neg(\exists x \exists y P(x,y)) \vee \neg(\forall x \forall y Q(x,y))$
- Apply DeMorgan's Law: $\forall x \forall y \neg P(x,y) \vee \exists x \exists y \neg Q(x,y)$