COL 702

Practice Problems

Prove that the following problems are NP-complete (you can reduce from Independent Set, Vertex Cover, Hamiltonian CyclePath, Partition, Subset Sum, 3-SAT, Clique):

1. Given an undirected graph G, does G contain a simple path that visits all but 17 vertices?

2. Given an undirected graph $G = (V, E)$ and a number k, is there is subset of at least k vertices S such that at most 342 edges in E have both endpoints in S?

3. Let $G = (V, E)$ be a graph. A dominating set in G is a subset S of the vertices such that every vertex in G is either in S or adjacent to a vertex in S. The Dominating Set Problem is defined as follows: given a graph G and an integer k, does G contain a dominating set of size k ?

4. A subset S of vertices in an undirected graph G is triangle-free if, for every triple of vertices $u, v, w \in S$, at least one of the three edges $(u, v), (u, w), (v, w)$ is absent from G. Given a graph G and a parameter k, does G have a triangle-free subset of size at least k ?

5. You have a set of friends F whom you're considering to invite, and you're aware of a set of k project groups, $S_1, \ldots, S_k$, among these friends (these sets need not be disjoint). The problem is to decide if there is a set of n of your friends whom you could invite so that not all members of any one group are invited.

6. Given an undirected graph $G = (V, E)$, a feedback set is a set $X \subseteq V$ with the property that $G - X$ has no cycles. The undirected feedback set problem asks: given G and k, does G contain a feedback set of size at most k?

7. Consider the following problem. You are given positive integers $x_1, \ldots, x_n$, and numbers k and B. You want to know whether it is possible to partition the numbers $\{x_i\}$ into k sets $S_1, \ldots, S_k$ so that the squared sums of the sets add up to at most B:

$$\sum_{i=1}^{k} \left(\sum_{x_j \in S_i} x_j \right)^2 \leq B.$$