Quiz: Differentiation

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Question Notes:

1. Q1:

$$\frac{\partial y}{\partial t} = \frac{\partial y}{\partial z}\frac{\partial z}{\partial t}$$

$$\frac{\partial y}{\partial z} = 1 - \frac{(e^x - e^{-x})^2}{(e^x + e^{-x})^2}$$
$$= 1 - tanh^2(x) = 1 - y^2$$

Because:

$$x_1 = \frac{1}{t}, x_2 = 2t$$

Therefore:

$$z = \frac{2}{t} - 12t^2$$
$$\frac{z}{t} = -\frac{2}{t^2} - 24t$$

$$\frac{\partial y}{\partial t} = (1 - y^2)(-\frac{2}{t^2} - 24t)$$

2. Q2:

$$\frac{\partial \sigma(z)}{\partial z} = \frac{e^{-z}}{(1 + e^{-z})^2}$$
$$= \frac{1}{1 + e^{-z}}\frac{e^{-z}}{1 + e^{-z}}$$
$$= \frac{1}{1 + e^{-z}}(1 - \frac{1}{1 + e^{-z}})$$
$$= \sigma(z)(1 - \sigma(z))$$

3. Q3:

$$\frac{\partial C}{\partial w_1} = \frac{\partial C}{\partial \sigma(z)}\frac{\partial \sigma(z)}{\partial z}\frac{\partial z}{\partial w_1}$$

Because:

$$\frac{\partial C}{\partial \sigma(z)} = 2(\sigma(z) - y) = 2 \times (0.8 - 0.5) = 0.6$$
$$\frac{\partial \sigma(z)}{\partial z} = \sigma(z)(1 - \sigma(z)) = 0.8 \times (1 - 0.8) = 0.16$$
$$\frac{\partial z}{\partial w_1} = x_1 = -2$$

Therefore:

$$\frac{\partial C}{\partial w_1} = 0.6 \times 0.16 \times -2 = -0.192$$

4. Q4: Based on previous question:

$$\frac{\partial C}{\partial w_1} = -0.192, \frac{\partial C}{\partial z} = 0.096$$

Therefore:

$$\frac{\partial C}{\partial w_2} = -1 \times 0.096 = -0.096$$

5. Q5:

$$\frac{\partial L}{\partial Z} = \frac{\partial L}{\partial \hat{y}} \frac{\partial \hat{y}}{\partial z}$$

Because:

$$\frac{\partial L}{\partial \hat{y}} = -\frac{y}{\hat{y}} + \frac{1-y}{1-\hat{y}}$$

$$\frac{\partial \hat{y}}{\partial z} = \sigma^{'}(z) = \hat{y}(1-\hat{y}) = \sigma(z)(1-\sigma(z))$$

Therefore:

$$\frac{\partial L}{\partial Z} = \hat{y} - y = -y(1-\sigma(z)) + (1-y)\sigma(z) = -\frac{y\sigma^{'}(z)}{\hat{y}} + \frac{(1-y)\sigma^{'}(z)}{1-\hat{y}}$$

6. Q6:

$$\frac{\partial f}{\partial y} = 2(y-z) = 10$$

$$\frac{\partial f}{\partial z} = -2(y-z) = -10$$

Because:

$$\frac{\partial z}{\partial x_1} = 2, \frac{\partial f}{\partial z} = -10$$

Therefore:

$$\frac{\partial f}{\partial x_1} = \frac{\partial f}{\partial z} \frac{\partial z}{\partial x_1} = -20$$

Because:

$$\frac{\partial z}{\partial x_2} = 6x_2 = 6, \frac{\partial f}{\partial z} = -10$$

Therefore:

$$\frac{\partial f}{\partial x_2} = \frac{\partial f}{\partial z} \frac{\partial z}{\partial x_2} = -60$$

7. Q8: pass

8. Q10: A:

$$\begin{aligned}\frac{\partial z_4}{\partial w_2} &= \frac{\partial f_3}{\partial x} \frac{\partial (w_3 z_3 + b_3)}{\partial z_3} \frac{\partial f_2}{\partial x} \frac{\partial (w_2 z_2 + b_2)}{\partial w_2} \\ &= \frac{\partial f_3}{\partial x} * w_3 * \frac{\partial f_2}{\partial x} * z_2 \\ &= 0.04 \times 0.3 \times 0.6 \times 0.02\end{aligned}$$

B:

$$\begin{aligned}\frac{\partial z_4}{\partial w_2} &= \frac{\partial f_3}{\partial x} \frac{\partial (w_3 z_3 + b_3)}{\partial z_3} \frac{\partial f_2}{\partial x} \frac{\partial (w_2 z_2 + b_2)}{\partial z_2} \frac{\partial f_1}{\partial x} \frac{\partial (w_1 z_1 + b_1)}{\partial w_1} \\ &= 0.04 \times 0.3 \times 0.6 \times -0.03 \times 0.4 \times 0.25\end{aligned}$$

(Very small number)

C: Because:

$$\frac{\partial L}{\partial z_4} = -2(y - z_4) = -2 \times 1.2$$

and from [A]:

$$\frac{\partial z_4}{\partial w_2} == 0.04 \times 0.3 \times 0.6 \times 0.02$$

Therefore:

$$\frac{\partial L}{\partial z_2} = -2 \times 1.2 \times 0.04 \times 0.3 \times 0.6 \times 0.02$$

D:

$$\frac{\partial z_4}{\partial z_2} = \frac{\frac{\partial z_4}{\partial w_1}}{\frac{\partial z_2}{\partial w_1}}$$

$$= \frac{0l04 \times 0.3 \times 0.6 \times -0.03 \times 0.4 \times 0.25}{0.4 \times 0.25}$$

$$= 0.04 \times 0.3 \times 0.6 \times -0.05$$