Quiz 2: Probability Theory (Part 2) Note

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1. Q1:

Using one-hot encoding to represent the ground truth y_i:

$$\hat{y}_1 = [0.3\ 0.5\ 0.2\ ] \qquad y_1 = [1\ 0\ 0\ ]$$

$$L_1 = -\ (1 \times log(0.3)\ +\ 0 \times log(0.5)\ +\ 0 \times log(0.2))\ =\ -\ log(0.3)$$

$$\hat{y}_2 = [0.1\ 0.1\ 0.8\ ] \qquad y_2 = [0\ 1\ 0\ ]$$

$$L_2 = -\ (0 \times log(0.1)\ +\ 1 \times log(0.1)\ +\ 0 \times log(0.8))\ =\ -\ log(0.1)$$

$$\hat{y}_3 = [0.3\ 0.4\ 0.4\ ] \qquad y_3 = [0\ 0\ 1\ ]$$

$$L_3 = -\ (0 \times log(0.3)\ +\ 0 \times log(0.4)\ +\ 1 \times log(0.4))\ =\ -\ log(0.4)$$

$$L\ =\ L_1\ +\ L_2\ +\ L_3\ =\ -\ (log(0.3)\ +\ log(0.1)\ +\ log(0.4))$$

2. Q2:

Class 1:

$$H_p(q) = -\ p(x)log(q(x)) - (1\ -\ p(x))log(1\ -\ q(x))$$
$$=\ -\ 1 \times log(0.5)\ -\ (1\ -\ 1) \times log(1\ -\ 0.5)$$
$$= -\ log(0.5)$$

Class 2:

$$H_p(q) = -\ p(x)log(q(x)) - (1\ -\ p(x))log(1\ -\ q(x))$$
$$=\ -\ 1 \times log(0.9)\ -\ (1\ -\ 1) \times log(1\ -\ 0.9)$$
$$= -\ log(0.9)$$

Class 3:

$$H_p(q) = -\ p(x)log(q(x)) - (1\ -\ p(x))log(1\ -\ q(x))$$
$$=\ -\ 0 \times log(0.4)\ -\ (1\ -\ 0) \times log(1\ -\ 0.4)$$
$$= -\ log(0.6)$$

Total:

$$-\ log(0.5)\ -\ log(0.9)\ -\ log(0.6)$$

3. Q3:

P1:

$$H(p) = -\sum_{x \in X} p(x) log(p(x))$$

$$= -0.2 \times log(0.2) - 0.2 \times log(0.2) - 0.6 \times log(0.6)$$

$$\approx 0.9503$$

P2:

$$H(p) = -\sum_{x \in X} p(x) log(p(x))$$

$$= -0.25 \times log(0.25) - 0.25 \times log(0.25) - 0.5 \times log(0.5)$$

$$\approx 1.0397$$

P3:

$$H(p) = -\sum_{x \in X} p(x) log(p(x))$$

$$= -0.05 \times log(0.05) - 0.05 \times log(0.05) - 0.9 \times log(0.9)$$

$$\approx 0.3944$$

4. Q4:

q1:

$$H(p, q) = -\sum_{x \in X} p(x) log(q(x))$$

$$= -0.1 \times log(0.3) - 0.8 \times log(0.2) - 0.1 \times log(0.5)$$

$$\approx 1.4773$$

q2:

$$H(p, q) = -\sum_{x \in X} p(x) log(q(x))$$

$$= -\ 0.1 \times log(0.6)\ -\ 0.8 \times log(0.3)\ -\ 0.1 \times log(0.1)$$

$$\approx 1.2445$$

q3:

$$H(p,\ q) = \ -\sum_{x \in X} p(x) log(q(x))$$

$$= -\ 0.1 \times log(0.2)\ -\ 0.8 \times log(0.7)\ -\ 0.1 \times log(0.1)$$

$$\approx 0.6765$$

5. Q5:

$$D_{KL}(P_1 || P_2) =$$

$$= -\ 0.1 \times log(\tfrac{0.2}{0.1})\ -\ 0.8 \times log(\tfrac{0.6}{0.8})\ -\ 0.1 \times log(\tfrac{0.2}{0.1})$$

$$\cong 0.0915$$

$$D_{KL}(P_2 || P_3) =$$

$$= -\ 0.2 \times log(\tfrac{0.4}{0.2})\ -\ 0.6 \times log(\tfrac{0.2}{0.6})\ -\ 0.2 \times log(\tfrac{0.4}{0.2})$$

$$\cong 0.3819$$