$$L = -\frac{1}{n}\sum_{i=1}^{n}\left(y\log\hat{y} + (1-y)\log(1-\hat{y})\right) + \lambda\sum_{j=1}^{d} w_j^2$$

$$\hat{y} = \sigma(xw+b).$$

Let $xw+b = a \quad \therefore \hat{y} = \sigma(a).$

$$\frac{\partial L}{\partial w} = \frac{\partial L}{\partial \hat{y}} \cdot \frac{\partial \hat{y}}{\partial a} \cdot \frac{\partial a}{\partial w} + \frac{\partial L_2}{\partial w}$$

$$\frac{\partial L}{\partial \hat{y}} = -\frac{1}{n}\sum_{i=1}^{n}\left(\frac{\partial L}{\partial \hat{y}}(y\log\hat{y}) + \frac{\partial L}{\partial \hat{y}}(1-y)\log(1-\hat{y})\right)$$

$$= -\frac{1}{n}\sum_{i=1}^{n}\left(\frac{y}{\hat{y}} - \frac{(1-y)}{(1-\hat{y})}\right)$$

$$\frac{\partial L}{\partial \hat{y}} = \frac{1}{n}\sum_{i=1}^{n}\left(\frac{-y}{\hat{y}} + \frac{(1-y)}{(1-\hat{y})}\right)$$

$$\frac{\partial \hat{y}}{\partial a} = \frac{\partial \sigma(a)}{\partial a} = \sigma(a)(1-\sigma(a))$$

$$= \hat{y}(1-\hat{y})$$

$$\frac{\partial a}{\partial w} = \frac{\partial(xw+b)}{\partial w} = x$$

$$\therefore \frac{\partial L}{\partial w} = \frac{1}{n}\sum_{i=1}^{n}\left(\frac{-y}{\hat{y}} + \frac{(1-y)}{(1-\hat{y})}\right)\hat{y}(1-\hat{y})\cdot x + \_\_$$

$$= \frac{1}{n}\sum_{i=1}^{n}\left(-y(1-\hat{y}) + \hat{y}(1-y)\right)\cdot x + \_\_$$

$$= \frac{1}{n}\sum_{i=1}^{n}\left(-y + \cancel{y\hat{y}} + \hat{y} - \cancel{y\hat{y}}\right)\cdot x$$

$$= \frac{1}{n}\sum_{i=1}^{n}\left(\hat{y} - y\right)\cdot x + \frac{\partial}{\partial w}\lambda\sum_{j=1}^{d} w_j^2$$

$$\frac{\partial L}{\partial w} = \frac{1}{n}\sum_{i=1}^{n}(\hat{y}-y)x + \lambda\sum_{j=1}^{d} 2w_j$$

$$\frac{\partial L}{\partial b} = \frac{1}{n}\sum_{i=1}^{n}(\hat{y}-y)\cdot 1 + 0.$$