Problem 1

→ Balance if I win = \$ W

→ Balance if I lose = \$ L

→ Let probability of winning be = P_W

∴ probability of losing = $1 - P_W$

Expected account balance after participating in the bet

$= W(P_W) + L(P_L)$

$= WP_W + L(1 - P_W)$

In order for one to accept the bet, the expected amount must become greater than the current balance B.

∴ the condition,

Expected balance after wining $> B$ should hold true.

$$\therefore\ W(P_W) + L(1 - P_W) > B$$

$$WP_W + L - LP_W > B$$

$$P_W(W - L) + L > B$$

$$P_W(W - L) > B - L$$

$$P_W > \frac{B - L}{W - L}$$

So in order for me to accept the bet,

$$\boxed{P_W > \frac{B-L}{W-L}}$$

Problem 2

Green wallet : 6 Pennies 4 Dimes

Black wallet : 8 Pennies 2 Dimes.

→ Probability of picking 1 Dime & 2 Pennies from Green wallet :-

$$\text{probability} = \frac{\text{number of favourable outcomes}}{\text{sample space}} \quad \text{—— (1)}$$

→ calculating sample space =

all possible ways of picking any 3 coins from Green wallet with 10 coins.

$$= \binom{10}{3} \quad \text{—— (2)}$$

$$= \frac{10!}{3!(10-3)!}$$

→ calculating favourable outcomes =

number of ways to pick 1 Dime & 2 Pennies from green wallet with 4 Dimes & 6 Pennies

$$= \binom{4}{1} \cdot \binom{6}{2}$$

$$= \frac{4!}{1!\,(4-1)!} \times \frac{6!}{2!\,(6-2)!} \quad \text{— (3)}$$

→ Probability of getting 1 dime & 2 pennies from green wallet,

from (1), (2) & (3) we have

$$P(1D, 2P) = \frac{\dfrac{4!}{1!\,(4-1)!} \times \dfrac{6!}{2!\,(6-2)!}}{\dfrac{10!}{3!\,(10-3)!}}$$

$$= \frac{\dfrac{\cancel{4}\times\cancel{3!}}{\cancel{3!}\times 1} \times \dfrac{\overset{3}{\cancel{6}}\times\overset{1}{\cancel{5}}\times\cancel{4!}}{\cancel{4!}\times\cancel{2}\times 1}}{\dfrac{\overset{2}{\cancel{10}}\times\overset{3}{\cancel{9}}\times\overset{4}{\cancel{8}}\times\cancel{7!}}{\cancel{7!}\times\cancel{3}\times\cancel{2}\times 1}}$$

$$= \frac{1}{2} \qquad \text{—— (4)}$$

$$= 0.5$$

Similarly probability of picking 1 Dime & 2 Pennies from black wallet would be.

$$P(1D\,2P) = \frac{\binom{2}{1}\binom{8}{2}}{\binom{10}{3}}$$

$$= \frac{\frac{2!}{1!(2-1)!} \times \frac{8!}{2!(8-2)!}}{\frac{10!}{3!(10-3)!}}$$

$$= \frac{\frac{\cancel{2}\times 1}{1} \times \frac{\cancel{8}\times 7\times \cancel{6!}}{\cancel{2}\times 1\times \cancel{6!}}}{\frac{\overset{5}{\cancel{10}}\times \overset{3}{\cancel{9}}\times \cancel{8}\times \cancel{7!}}{\cancel{3}\times \cancel{2}\times 1\times \cancel{7!}}} = \frac{7}{15} \quad \text{— (5)}$$

Steps involved to solve the problem:

Part I] Considering,

P(Picking Green) = P(Picking Black)

Part II] It is Given that,

P(Picking Green wallet) = 4 × P(Picking Black wallet)

Part III] Probability that the previous question was answered wrong (probability of error)

Part I] We consider the p(picking Green) is equal to p(picking black)

ie,

$$P(\text{Picking green}) = P(\text{picking black}) = \frac{1}{2}$$

$\rightarrow$ $P(\text{Green wallet} \mid \text{1 Dime 2 Pennies})$

$$= \frac{P(\text{1 Dime, 2 Pennies} \mid \text{Green wallet}) \times P(\text{green wallet})}{P(\text{1 Dime, 2 Pennies} \mid \text{Green Wallet}) \times P(\text{Green w}) + P(\text{1 Dime, 2 Pennies} \mid \text{Black wallet}) \times P(\text{Black wallet})} \quad \text{— (6)}$$

from (4), (5) & (6)

$\therefore$ $P(\text{Green wallet} \mid \text{1 Dime, 2 Pennies})$

$$= \frac{\frac{1}{2} \times \frac{1}{2}}{\frac{1}{2} \times \frac{1}{2} + \frac{7}{15} \times \frac{1}{2}} = \frac{15}{29} = 0.5172 \quad \text{— (7)}$$

$\rightarrow$ P(Black wallet | 1 Dime, 2 Pennies)

$$= \frac{P(\text{1 Dime, 2 Pennies} \mid \text{Black wallet}) \times P(\text{Black wallet})}{P(\text{1 Dime, 2 Pennies} \mid \text{Green wallet}) \times P(\text{Green w}) + P(\text{1 Dime, 2 Pennies} \mid \text{Black wallet}) \times P(\text{Black wallet})}$$

$$= \frac{\frac{7}{15} \times \frac{1}{2}}{\frac{1}{2} \times \frac{1}{2} + \frac{7}{15} \times \frac{1}{2}} = \frac{14}{29} = 0.4828 \quad \text{—(8)}$$

$\rightarrow$ Since, $\frac{15}{29} > \frac{14}{29}$ i.e,

P(Picking Green wallet | 1 Dime, 2 Pennies)

is greater than,

P(Picking Black wallet | 1 Dime, 2 Pennies)

It is more likely that I have picked,

$\boxed{\text{Green Wallet}}$

Part II] : Given that, from prior knowledge,

$p(\text{picking green}) = 4 \times p(\text{picking black})$

$\therefore \quad p(\text{picking green wallet}) = \frac{4}{5}$

&

$p(\text{picking black wallet}) = \frac{1}{5}$

$\rightarrow P(\text{Green wallet} \mid 1 \text{ Dime } 2 \text{ Pennies})$

$$= \frac{P(1 \text{ Dime}, 2 \text{ Pennies} \mid \text{Green wallet}) \times P(\text{green wallet})}{P(1 \text{ Dime}, 2 \text{ Pennies} \mid \text{Green Wallet}) \times P(\text{Green w}) + P(1 \text{ Dime}, 2 \text{ Pennies} \mid \text{Black wallet}) \times P(\text{Black wallet})}$$

$$= \frac{\frac{1}{2} \times \frac{4}{5}}{\frac{1}{2} \times \frac{4}{5} + \frac{7}{15} \times \frac{1}{5}} = \frac{\frac{4}{10}}{\frac{74}{150}} = \frac{30}{37}$$

$\rightarrow P(\text{Black wallet} \mid 1 \text{ Dime}, 2 \text{ Pennies})$

$$= \frac{P(1 \text{ Dime}, 2 \text{ Pennies} \mid \text{Black wallet}) \times P(\text{Black wallet})}{P(1 \text{ Dime}, 2 \text{ Pennies} \mid \text{Green wallet}) \times P(\text{Green w}) + P(1 \text{ Dime}, 2 \text{ Pennies} \mid \text{Black wallet}) \times P(\text{Black wallet})}$$

$$= \frac{\frac{7}{15} \times \frac{1}{5}}{\frac{1}{2} \times \frac{4}{5} + \frac{7}{15} \times \frac{1}{5}} = \frac{\frac{7}{75}}{\frac{74}{150}} = \frac{7}{37}$$

Since, $\frac{30}{37} > \frac{7}{37}$, i.e,

ie $p(\text{picking Green}) > p(\text{picking black})$

considering the <u>prior</u> knowledge.

I would Pick up <u>Green wallet</u>

Part III] Probability that the optimal answer given for the previous question is wrong is =

probability of Error of picking up the wrong wallet.

→ Probability of Error = minimum (Probabilities of picking wallets)

Calculations from part II shows that

p(picking black) < p(picking Green)

since $\frac{7}{37} < \frac{30}{37}$

∴ the Probability Of Error i.e, probability that the optimal answer given for the previous question is wrong = $\boxed{\frac{7}{37} \approx 0.18919}$

Problem 3 - Part 2

Dataset 1;

$N_1 = 2000$

$\mu_1 = 1$

$\sigma_1^2 = 4$

Dataset 2:

$N_2 = 1000$

$\mu_2 = 4$

$\sigma_2^2 = 9$

Calculating combine mean using,

$$\mu_{combined} = \frac{N_1 \mu_1 + N_2 \mu_2}{N_1 + N_2}$$

$$= \frac{2000 \times 1 + 1000 \times 4}{2000 + 1000}$$

$$= \frac{6000}{3000} = 2$$

$$\therefore \boxed{\mu_c = 2}$$

Calculating combined variance,

$$\sigma^2_{combined} = \frac{N_1\left[S_1^2 + (\mu_1 - \bar{\mu}_c)^2\right] + N_2\left[S_2^2 + (\mu_2 - \bar{\mu}_c)^2\right]}{N_1 + N_2}$$

$$= \frac{2000\left[4 + (1-2)^2\right] + 1000\left[9 + (4-2)^2\right]}{2000 + 1000}$$

$$= \frac{2000\left[4+1\right] + 1000\left[9+4\right]}{3000}$$

$$= \frac{10000 + 13000}{3000}$$

$$= \frac{23000}{3000} = 7.6667$$

$$\boxed{\therefore \sigma^2_{combined} = 7.6667}$$