Support Vector Machine

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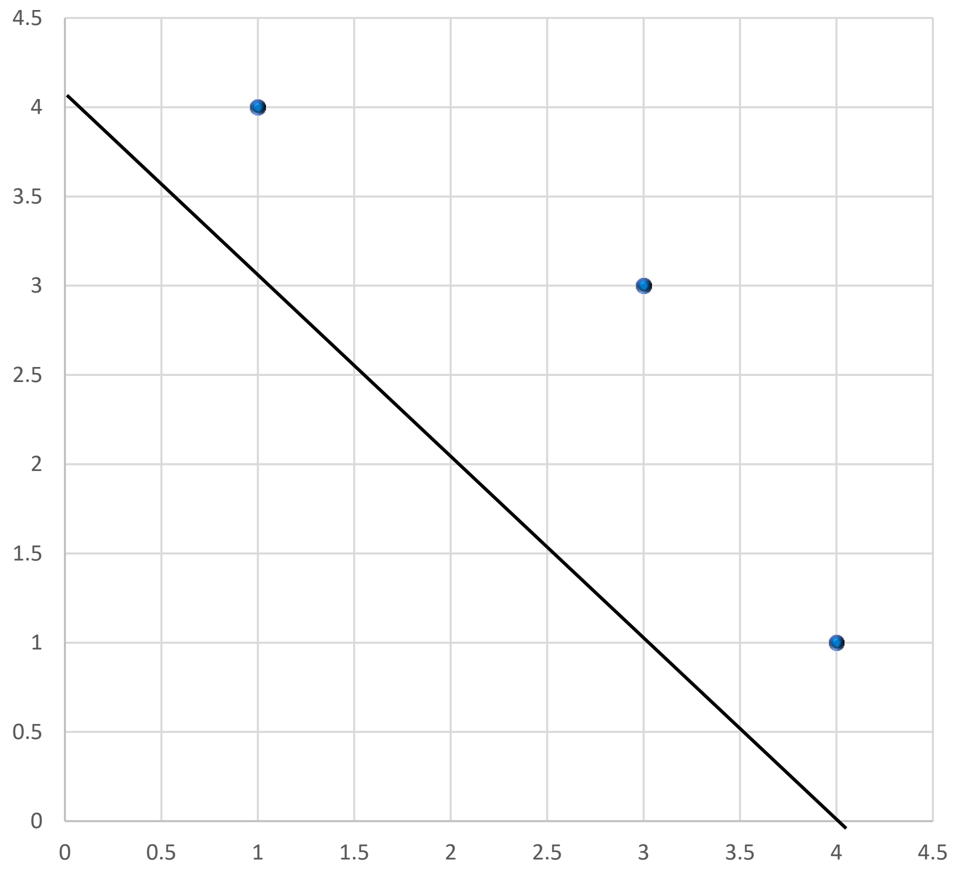

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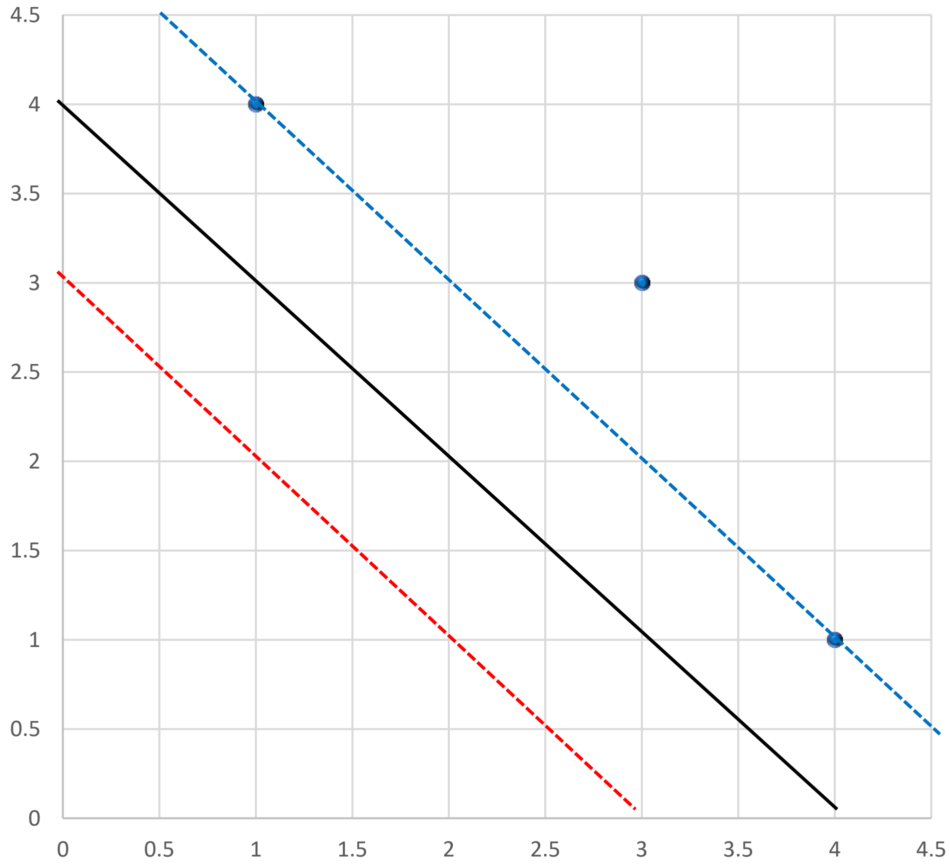

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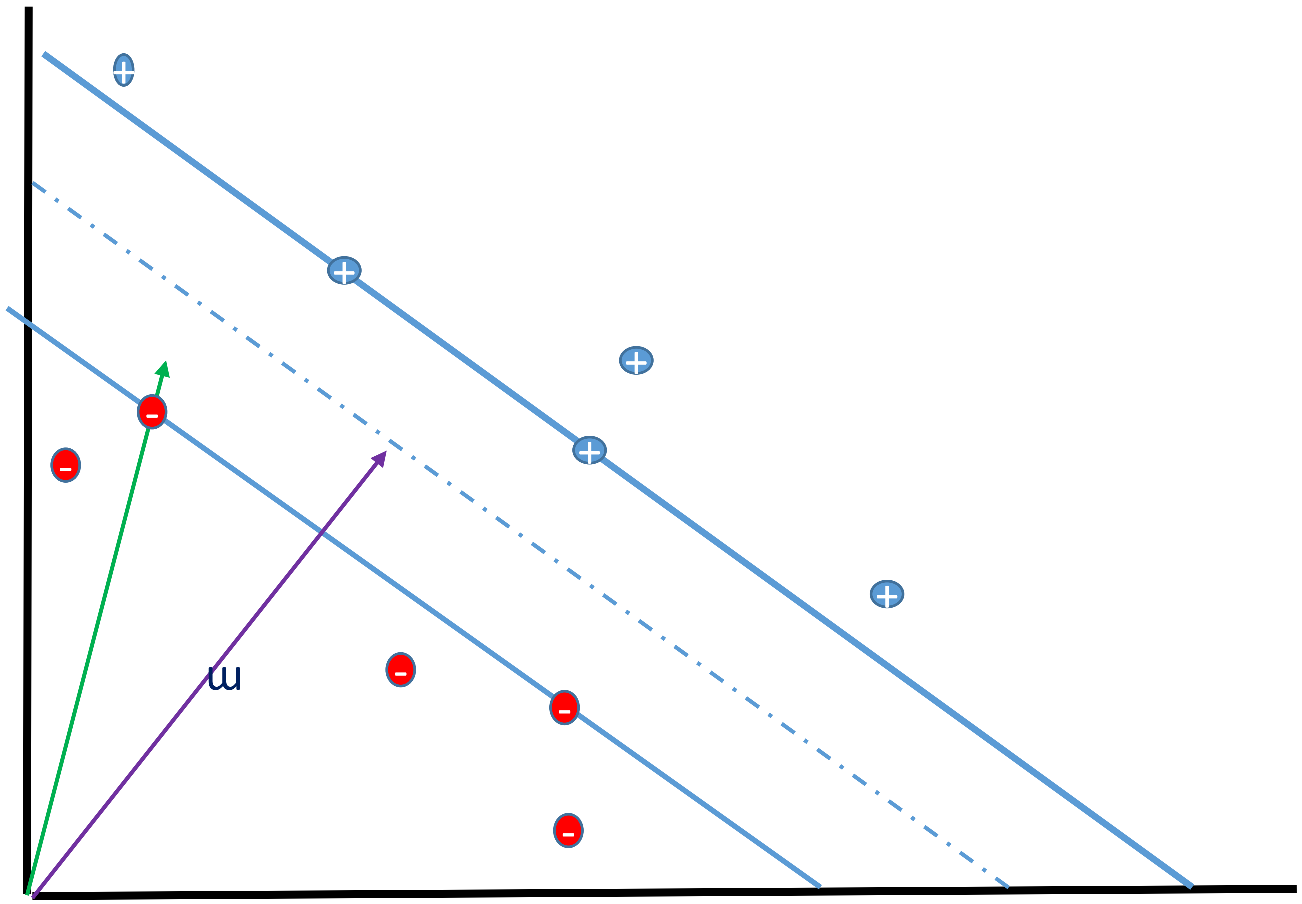

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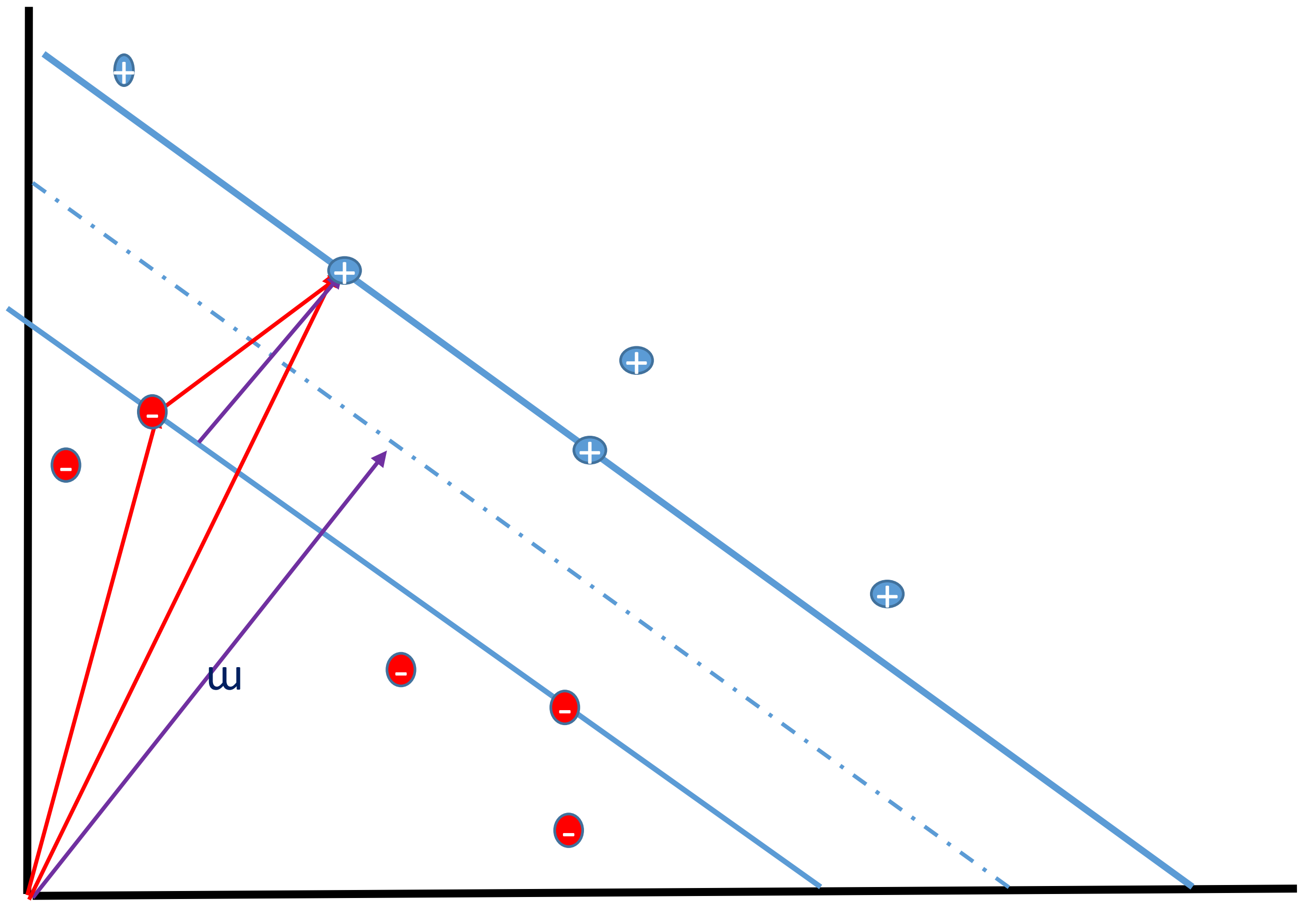


$\bar{w}\cdot\bar{u} \geq c$ $\quad b = -c$

$\bar{w}\cdot\bar{u} + b \geq 0$

$\bar{w}\cdot\bar{x}_i^+ + b \geq 1$

$\bar{w}\cdot\bar{x}_i^- + b \leq -1$

Define $y_i = 1$ for x_i^+

$y_i = -1$ for x_i^-

$$\begin{cases} y_i(\bar{w}\cdot x_i^+ + b) \geq 1 \\ y_i(\bar{w}\cdot x_i^- + b) \geq 1 \end{cases}$$

for x_i in the gutter

$$\begin{cases} y_i(\bar{w}\ x_i^+ + b) - 1 = 0 \\ y_i(\bar{w}\ x_i^- + b) - 1 = 0 \end{cases}$$

width of the street

$$(x_i^+ - x_i^-)\cdot\frac{\bar{w}}{\|w\|} = \frac{\bar{w}\cdot x^+ - \bar{w}\,x^-}{\|w\|}$$

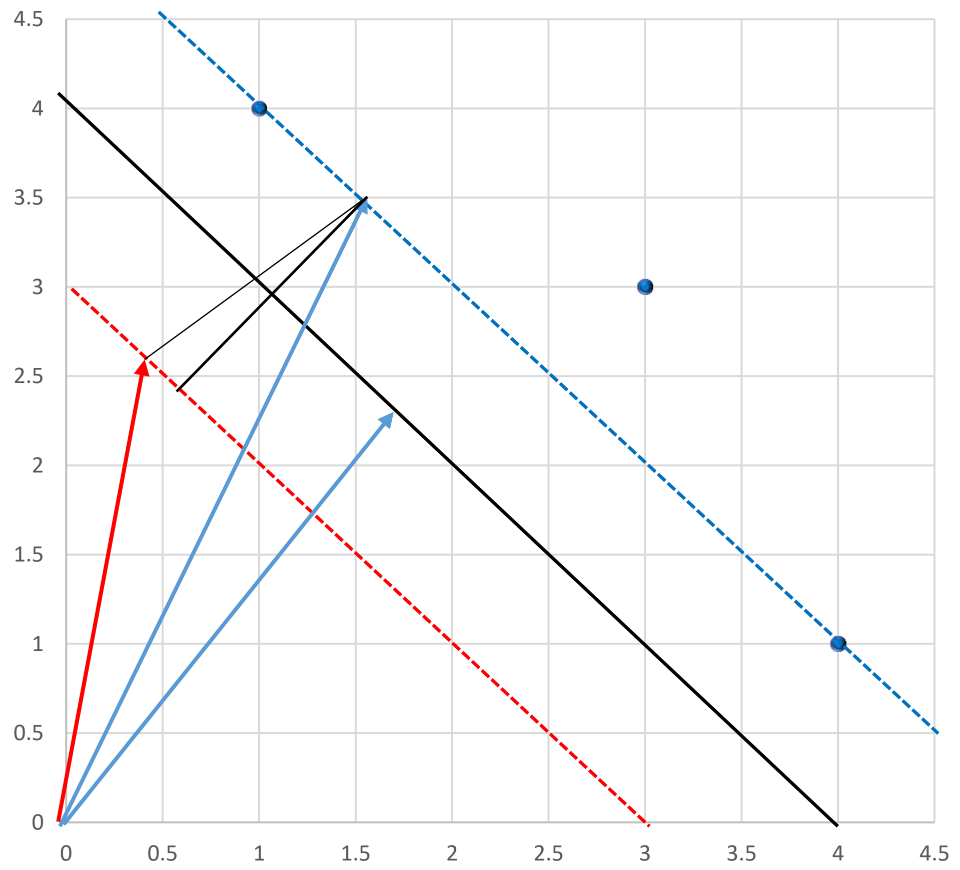
for x^+ $y_i = 1$ then $\bar{w}\,x^+ = 1-b$

for x^- $y_i = -1 - b$

$$\frac{\bar{w}\,x^+ - \bar{w}\,x^-}{\|w\|} = \frac{(1-b)-(-1-b)}{\|w\|} = \frac{2}{\|w\|}$$ = width of the street

Max $\frac{2}{\|w\|}$ s.t. $y_i(\bar{w}\,x_i + b) - 1 = 0$

$\Rightarrow$ Min $\|w\|$ s.t. $y_i(\bar{w}\,x_i + b) - 1 = 0$



$$\Rightarrow \text{Min } \frac{1}{2} \|w\|^2$$

$$\text{s.t.} = \; y_i(\bar{w} x_i + b) - 1 = 0 \quad \text{for all } i$$

$$\Rightarrow L = \text{Min } \frac{1}{2} \|w\|^2 - \sum_i \alpha_i \left[y_i (\bar{w} x_i + b) - 1 \right]$$

$$\begin{cases} \dfrac{\partial L}{\partial w} = w - \sum_i \alpha_i y_i x_i = 0 \Rightarrow \boxed{w = \sum_i \alpha_i \cdot y_i x_i} \\[2ex] \dfrac{\partial L}{\partial b} = \boxed{\sum \alpha y_i = 0} \end{cases}$$

$$\text{Max } L = \frac{1}{2} \sum_i \alpha_i y_i x_i \sum_j \alpha_j y_j x_j - \sum_i \alpha_i y_i x_i \sum_j \alpha_j y_j x_j$$

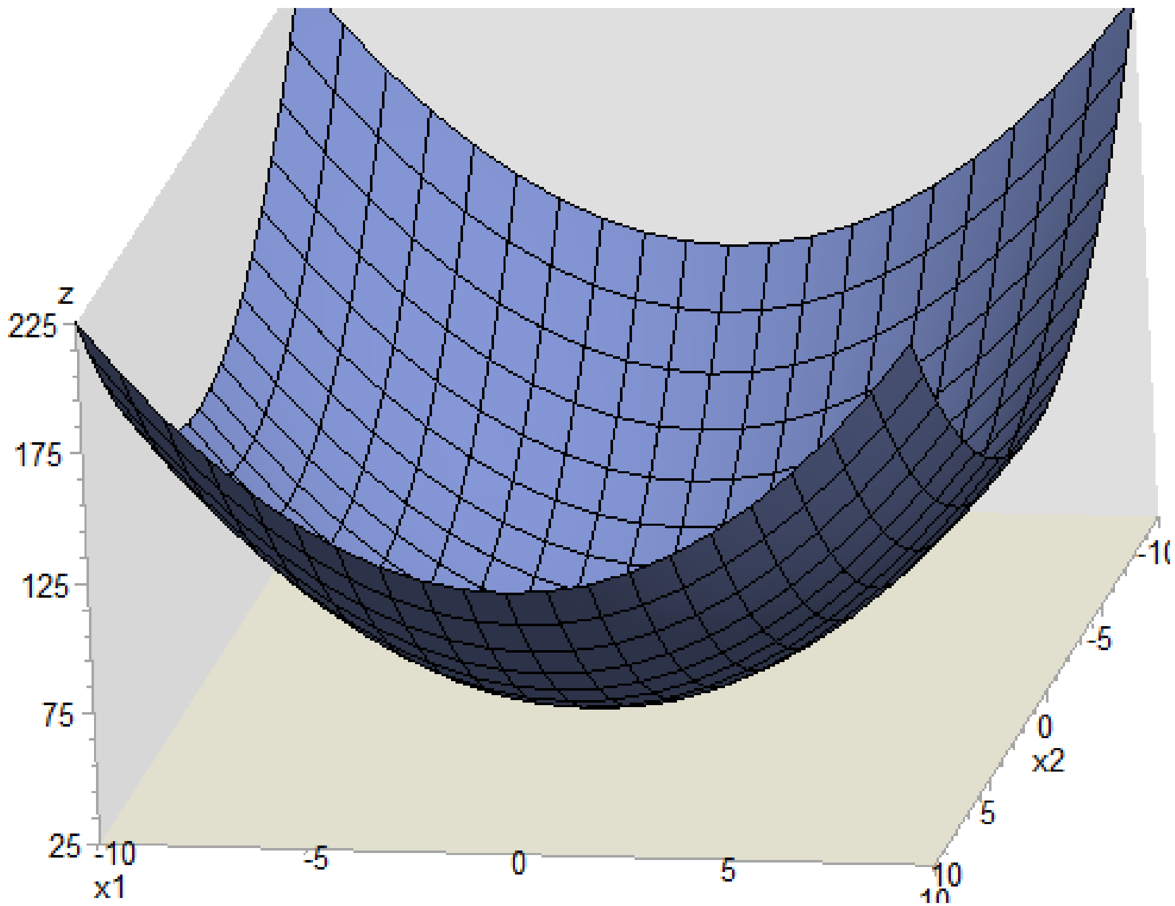
$$- \cancel{\sum_i \alpha_i y_i}^{\,0} b + \sum_i \alpha_i$$

$$\text{Max } L = \sum_i \alpha_i - \frac{1}{2} \sum_i \sum_j \alpha_i \alpha_j y_i y_j x_i x_j$$

$$\text{Define } H_{ij} = y_i x_i \cdot y_j x_j$$

$$\Rightarrow \underset{}{\text{Max}}\; L = \sum_i \alpha_i - \frac{1}{2} \sum_i \sum_j \alpha_i H_{ij} \alpha_j$$

$$\max_{\alpha} L = \sum \alpha_i - \frac{1}{2} \alpha^T H \alpha$$


z
225
175
125
75
25
-10
-5
0
5
10
x1
-5
0
x2
5