

CS 559: Homework Set 1

Collaboration Policy. Homeworks will be done individually: each student must hand in their own answers. Use of partial or entire solutions obtained from others or online is strictly prohibited.

Late Policy. No late submissions will be allowed without consent from the instructor. If urgent or unusual circumstances prohibit you from submitting a homework assignment in time, please e-mail me explaining the situation.

Submission Format. Electronic submission on Canvas is mandatory.

Problem 1. (30 points) You have $\$B$ in your bank account. You are asked if you would like to participate in a bet in which, if you win, your bank account will become $\$W$. However, if you lose, your bank account will contain only $\$L$. You win the bet with probability p_w . How large should p_w be to accept the bet? (I assume that you would rather have a higher expected value of the amount of money in your account.)

Problem 2. (30 points) Assume you have a green and a black wallet. The green wallet contains 6 pennies and 4 dimes. The black wallet contains 8 pennies and 2 dimes. Which wallet were you more likely to have picked if you pulled a dime followed by two pennies from it? Based on past experience, you use the green wallet 4 times more often than the black one.

What is the probability that the optimal answer you gave in the previous question was wrong?

Note: Answering the second part without attempting the first part will receive 0 credit.

Problem 3. (40 points)

Part 1. In Matlab, or the programming language of your choice, do the following:

- Generate N observations from a normal distribution: $data = randn(N, 1)$; This will generate N 1-D samples with mean equal to 0 and variance equal to 1.
- Estimate the mean and variance of the data for $N = 10, 100, 1000$ etc.
- Modify the code so that the generated data have mean and variance equal to user-specified parameters $mean$ and var .

Submit a function that accepts $mean$, var and N and generates data as in the last step above. No explanations required.

Part 2. Using the function above, generate two datasets: one with $N_1 = 2000$, $\mu_1 = 1$, $\sigma_1^2 = 4$ and one with $N_2 = 1000$, $\mu_2 = 4$, $\sigma_2^2 = 9$. Derive theoretically the mean and variance of the combined dataset that includes all 3000 samples. Verify experimentally that your theoretical estimates are correct.

Submit the derivations for the mean and variance of the combined dataset. Make sure it is written with $mean_i$, var_i and N_i as parameters. No code required.