Solution 1]

A function D is a distance function, if it has the following properties :

1. Distance is always non-negative

D(x, y) >= 0 and D(x ,y) = 0, if and only if x = y

1. Commutative, distance from “x to y” is distance from “y to x”

D(x, y) = D(y ,x)

1. Triangle inequality holds, distance from “x to z” must be less than or equal to distance from “x to y to z”

D(x, z) <= d(x, y) + d(y, z)

Given distance formula

d(x, y) = ∑(|xi - yi|^3)

Let, the points be p(0,0,0), q(0,1,0), r(0,1,1) and s(1,1,1)

From the given function,

Distance from p to q:

(|0-0| + |0-1| + |0-0|)^3 = 1

Distance from q to r:

(|0-0| + |1-1| + |0-1|)^3 = 1

Distance from r to s:

(|0-1| + |1-1| + |1-1|)^3 = 1

Distance from s to p:

(|1-0| + |1-0| + |1-0|)^3 = 27

Distance from p to r:

(|0-0| + |0-1| + |0-1|)^3 = 8

Distance from q to s:

(|0-0| + |1-1| + |0-1|)^3 = 8

The given distance function should satisfy all the three properties of a distance function.

According to triangle inequality,

d(q, s) <= d(q, r) + d(r, s) and

d(p, r) <= d(p, q) + d(q, r)

should hold true

d(q, s) = 8

d(q, r) + d(r, s) = 1 + 1 = 2

8 > 2,

Which means,

d(q, s) > d(q, r) + d(r, s)

also,

d(p, r) = 8

d(p, q) + d(q, r) = 1 + 1 = 2

8 > 2

Which means,

d(p, r) > d(p, q) + d(q, r)

Since the given function fails to satisfy the triangle inequality property, it is not a proper distance function.