

609 Assignment-1: Ranking NFL Teams

Texas A&M University

Department of Industrial & Systems Engineering

ISEN 609 - Fall 2018

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"On my honor, as an Aggie, I have neither given nor received unauthorized aid on this assignment."

Signature:

Date:

Question:

The National Football League (NFL) is a professional American football league consisting of 32 teams, divided equally between the National Football Conference (NFC) and the American Football Conference (AFC). Both conferences consist of four four-team divisions. Each team plays 16 regular-season games; thus, teams do not play all other teams during a single regular season. The focus of this assignment is the 2007 NFL regular season. All scores, for that season, can be downloaded from <https://www.pro-football-reference.com>. The goal of this assignment is to rank the 32 teams after the regular season using an algorithm based on Markov chains. Each team should be represented by a state in a Markov chain, and team ranks should be based on values that are proportional to the stationary probabilities of the constructed Markov chain. Your task is to convert the available game scores into a transition matrix and provide a justification for such a conversion. Note that game scores serve as proxies for relative team strengths. At least two different versions of the matrix should be proposed. Note that one of the teams won all its 16 games. For the constructed matrices, the corresponding stationary distributions should be computed, and team ranks should be produced.

This report contains three approaches to rank NFL (National Football League) teams from “best” to “worst” using the stationary distribution of a Markov-chain. There are total of 32 teams in the league and each team plays 16 games in a single regular season. The scores of 2007 NFL regular season are taken into consideration to rank the teams. If we take each team as a state, then the ranking of these teams can be done using DTMC.

Let $\{X_0 = i\}$ be the rank of team X initially and we want to find the probability of $\{X_n = j, n > 1\}$. Consider the state-space E made up of all the teams and i, j, k as ranks such that $\{32 \geq i, j, k \geq 1\}$. Here, the ranking of any team at any time is i.i.d and we can find the ranking of teams considering the probabilities of each team at any time using transition matrix multiplication.

$$P[X_n = j / X_0 = i] = \sum_{k \in E} P[X_n = j, X_{n-1} = k / X_0 = i]$$

$$P[X_n = j / X_0 = i] = \sum_{k \in E} P[X_n = j / X_{n-1} = k, X_0 = i] \cdot P[X_{n-1} = k / X_0 = i]$$

$$P[X_n = j / X_0 = i] = \sum_{k \in E} P[X_n = j / X_{n-1} = k] \cdot P[X_{n-1} = k / X_0 = i]$$

$$P[X_n = j / X_0 = i] = \sum_{k \in E} P[k, j] \cdot P[X_{n-1} = k / X_0 = i]$$

$$P[X_n = j / X_0 = i] = P[k, j] \cdot P^{n-1}[i, k]$$

$$P[X_n = j / X_0 = i] = P^n[i, j]$$

In this report, team ranks are based on the values that are proportional to the stationary probabilities of a constructed algorithm. The basic idea of ranking these teams is the transition of points from losing team to winning team and/or winning team to losing team and/or giving points to team by team itself. The final ranking is done based on the number of points each team has. Since, there are two conferences and each team plays only 16 games, there is no team which plays games with every other team. So, the transition of points occurs only between the teams that play together.

```
library(openxlsx)
library(markovchain)

## Package: markovchain
## Version: 0.6.9.12
## Date: 2018-08-23
## BugReport: http://github.com/spedygiorgio/markovchain/issues

options(max.print=9999)
scores <- read.xlsx("C:/Users/Dell/Desktop/609 A1/NFL_Scores.xlsx")
dim(scores)

## [1] 256 4
```

The scores are imported from the Microsoft Excel file- “NFL_Scores.xlsx” and stored into the “scores” data-frame. The “scores” contains 256 rows and 4 columns.

```
head(scores, 16)
```

	Winner/tie	Loser/tie	PtsW	PtsL
## 1	Indianapolis Colts	New Orleans Saints	41	10
## 2	Carolina Panthers	St. Louis Rams	27	13
## 3	Minnesota Vikings	Atlanta Falcons	24	3
## 4	Denver Broncos	Buffalo Bills	15	14
## 5	Green Bay Packers	Philadelphia Eagles	16	13
## 6	Washington Redskins	Miami Dolphins	16	13
## 7	Pittsburgh Steelers	Cleveland Browns	34	7
## 8	New England Patriots	New York Jets	38	14
## 9	Tennessee Titans	Jacksonville Jaguars	13	10
## 10	Houston Texans	Kansas City Chiefs	20	3
## 11	Detroit Lions	Oakland Raiders	36	21
## 12	San Diego Chargers	Chicago Bears	14	3
## 13	Seattle Seahawks	Tampa Bay Buccaneers	20	6
## 14	Dallas Cowboys	New York Giants	45	35
## 15	Cincinnati Bengals	Baltimore Ravens	27	20
## 16	San Francisco 49ers	Arizona Cardinals	20	17

The first 16 entries of the “scores” can be seen above. As we can see all 16 winning teams and all 16 losing teams are different. We can combine 1-16 winning teams and 1-16 losing teams to make a vector containing all 32 teams. So, the vector – “Teams” contains all the 32 teams and the order of teams in this vector can be seen below. This vector containing all the team names is fed as states to create Markov chain later.

```
Team1<-scores$Winner/tie[1:16]
Team2<-scores$Loser/tie[1:16]
Teams <- c(Team1,Team2)
Teams
```

```
## [1] "Indianapolis Colts" "Carolina Panthers" "Minnesota Vikings"
## [4] "Denver Broncos" "Green Bay Packers" "Washington Redskins"
## [7] "Pittsburgh Steelers" "New England Patriots" "Tennessee Titans"
## [10] "Houston Texans" "Detroit Lions" "San Diego Chargers"
## [13] "Seattle Seahawks" "Dallas Cowboys" "Cincinnati Bengals"
## [16] "San Francisco 49ers" "New Orleans Saints" "St. Louis Rams"
## [19] "Atlanta Falcons" "Buffalo Bills" "Philadelphia Eagles"
## [22] "Miami Dolphins" "Cleveland Browns" "New York Jets"
## [25] "Jacksonville Jaguars" "Kansas City Chiefs" "Oakland Raiders"
## [28] "Chicago Bears" "Tampa Bay Buccaneers" "New York Giants"
## [31] "Baltimore Ravens" "Arizona Cardinals"
```

Approach (1)

The concept behind this approach is that every team would give points proportional to its score to itself and proportional of the opponent team's score to the opponent team. In this approach if team A beats team B, then there is a high probability of points transitioning from B to A and small probability of points transitioning from A to B. The final ranking would be based on the points each team has at steady state.

If we consider only two teams A and B, we can form the transition matrix M such that,

$$M = \begin{matrix} & \begin{matrix} A & B \end{matrix} \\ \begin{matrix} A \\ B \end{matrix} & \begin{bmatrix} AA & AB \\ BA & BB \end{bmatrix} \end{matrix}$$

Where,

AA = Team A score / Total score

BB = Team B score / Total score

AB = Team B score / Total score

BA = Team A score / Total score

Since, each team is playing 16 games, the addition of all the elements of each row would be 16. So, to normalize this transition matrix, whole transition matrix is divided by 16.

#Matrix Generation

```
TransMatrix1 = matrix(0, nrow = 32, ncol=32)
for(i in 1:32){
  for(j in 1:256){
    if(scores$Winner/tie[j] == Teams[i]){
      TransMatrix1[i,i] = TransMatrix1[i,i] + (scores$PtsW[j]/(scores$PtsW[j]
+ scores$PtsL[j]))
    }
  }
}
```

```

        index <- which(Teams == scores$Loser/tie[j])
        TransMatrix1[i,index] = TransMatrix1[i,index] + (scores$PtsL[j]/(scores
$PtsW[j] + scores$PtsL[j]))
    }

    if(scores$Loser/tie[j] == Teams[i]){
        TransMatrix1[i,i] = TransMatrix1[i,i] + (scores$PtsL[j]/(scores$PtsW[j]
+ scores$PtsL[j]))
    }

    index <- which(Teams == scores$Winner/tie[j])
    TransMatrix1[i,index] = TransMatrix1[i,index] + (scores$PtsW[j]/(scores
$PtsW[j] + scores$PtsL[j]))
}
}
}

```

#Normalization

```
TransMatrix1 = TransMatrix1/16
```

#Solving Transition Matrix

```

dtmc1 <- new("markovchain", states = Teams, transitionMatrix = TransMatrix1,
name = "Ranking1")
t(steadyStates(dtmc1))

```

```

##                                [,1]
## Indianapolis Colts    0.05105737
## Carolina Panthers    0.02040019
## Minnesota Vikings    0.03296356
## Denver Broncos       0.02372360
## Green Bay Packers    0.04513173
## Washington Redskins  0.03803848
## Pittsburgh Steelers  0.04667978
## New England Patriots 0.06855865
## Tennessee Titans     0.03332510
## Houston Texans       0.03136654
## Detroit Lions        0.02576864
## San Diego Chargers   0.04711369
## Seattle Seahawks     0.03243077
## Dallas Cowboys       0.04267364
## Cincinnati Bengals   0.02818755
## San Francisco 49ers  0.01458686
## New Orleans Saints   0.02573810
## St. Louis Rams       0.01496042
## Atlanta Falcons      0.01633589
## Buffalo Bills        0.02135179
## Philadelphia Eagles   0.03554460
## Miami Dolphins       0.01838538
## Cleveland Browns     0.03118659

```

```

## New York Jets      0.02259028
## Jacksonville Jaguars 0.04122230
## Kansas City Chiefs 0.02209334
## Oakland Raiders    0.02154983
## Chicago Bears       0.03026679
## Tampa Bay Buccaneers 0.03241109
## New York Giants     0.03602619
## Baltimore Ravens    0.02318024
## Arizona Cardinals   0.02515103

#sorting the probabilities and Ranking

R1 <- steadyStates(dtmc1)
FinalRank1 <- data.frame(R1[1,], rank(-R1))
names(FinalRank1)<-c("Probs1", "Ranks1")
FinalRank1[order(FinalRank1$Ranks1),]

##              Probs1 Ranks1
## New England Patriots 0.06855865      1
## Indianapolis Colts    0.05105737      2
## San Diego Chargers    0.04711369      3
## Pittsburgh Steelers   0.04667978      4
## Green Bay Packers     0.04513173      5
## Dallas Cowboys        0.04267364      6
## Jacksonville Jaguars   0.04122230      7
## Washington Redskins   0.03803848      8
## New York Giants       0.03602619      9
## Philadelphia Eagles    0.03554460     10
## Tennessee Titans      0.03332510     11
## Minnesota Vikings     0.03296356     12
## Seattle Seahawks      0.03243077     13
## Tampa Bay Buccaneers   0.03241109     14
## Houston Texans        0.03136654     15
## Cleveland Browns      0.03118659     16
## Chicago Bears         0.03026679     17
## Cincinnati Bengals    0.02818755     18
## Detroit Lions         0.02576864     19
## New Orleans Saints     0.02573810     20
## Arizona Cardinals     0.02515103     21
## Denver Broncos        0.02372360     22
## Baltimore Ravens      0.02318024     23
## New York Jets         0.02259028     24
## Kansas City Chiefs    0.02209334     25
## Oakland Raiders       0.02154983     26
## Buffalo Bills         0.02135179     27
## Carolina Panthers     0.02040019     28
## Miami Dolphins        0.01838538     29
## Atlanta Falcons       0.01633589     30
## St. Louis Rams        0.01496042     31
## San Francisco 49ers    0.01458686     32

```

Approach (2)

The concept behind this approach is that every losing team would give points to the winning team. We can understand this by the analogy of a bet. Consider there are equal number of bettors betting on each team at the starting of the season and if team A wins against team B then the certain number of bettors betting on team B, will start betting on team A. We can take this number of bettors as the difference of the points made by winning team and the points made by losing team.

If we consider only two teams A and B and if team A wins against team B, we can form the transition matrix M such that,

$$M = \begin{matrix} & \begin{matrix} A & B \end{matrix} \\ \begin{matrix} A \\ B \end{matrix} & \begin{bmatrix} AA & AB \\ BA & BB \end{bmatrix} \end{matrix}$$

Where,

$$AA = 0$$

$$BB = 0$$

$$AB = 0$$

$$BA = \text{Team A score} - \text{Team B score}$$

To make a transition matrix we must normalize each row.

We can define (i, j)th entry of TransMatrix2 as,

$$\text{TransMatrix2}_{(i,j)} = \frac{w_{ij}}{\sum w_{ij}}$$

Since, the team “New England Patriots” remains undefeated during the whole season, there is a zero row against the state- New England Patriots. In this case we must convert the zero row into a row with entries equal to $1/n$ where n is equal to the total number of teams which is 32 in our case. The explanation of this conversion can be that on any match day, there is an equal probability of this team to lose against any team.

#Matrix Generation

```
TransMatrix2 = matrix(0, nrow = 32, ncol=32)
for(i in 1:256){
  indexW <- which(Teams == scores$`Winner/tie`[i])
  indexL <- which(Teams == scores$`Loser/tie`[i])
```



```

    TransMatrix2[indexL,indexW] = TransMatrix2[indexL,indexW] + scores$PtsW[i]
- scores$PtsL[i]
}

```

#Normalization

```

sum = 0
for(j in 1:32){
  for(k in 1:32){
    sum = sum + TransMatrix2[j,k]
  }
  if(sum != 0){
    TransMatrix2[j,] = TransMatrix2[j,]/sum
  }
  else{
    rowNumber = j
  }
  sum = 0
}

```

#Finding the zero row and replacing it. Zero row means undefeated team.

```

for(j in 1:32){
  TransMatrix2[rowNumber,j]= 1/32
}

```

#Solving Transition Matrix

```

dtmc2 <- new("markovchain", states = Teams, transitionMatrix = TransMatrix2,
name = "Ranking2")
t(steadyStates(dtmc2))

```

```

##                                [,1]
## Indianapolis Colts      0.055070372
## Carolina Panthers      0.016259069
## Minnesota Vikings      0.037739920
## Denver Broncos         0.024173069
## Green Bay Packers      0.050418698
## Washington Redskins    0.054649405
## Pittsburgh Steelers    0.038521106
## New England Patriots   0.138622802
## Tennessee Titans       0.044721618
## Houston Texans         0.029301449
## Detroit Lions          0.026448758
## San Diego Chargers     0.049489209
## Seattle Seahawks       0.028063226
## Dallas Cowboys         0.059674428
## Cincinnati Bengals     0.027116249
## San Francisco 49ers    0.009854448
## New Orleans Saints     0.030577128
## St. Louis Rams         0.007356663

```

```
## Atlanta Falcons      0.010077405
## Buffalo Bills       0.010608141
## Philadelphia Eagles  0.028991175
## Miami Dolphins      0.004800550
## Cleveland Browns    0.014462354
## New York Jets       0.008007175
## Jacksonville Jaguars 0.040599487
## Kansas City Chiefs  0.017916438
## Oakland Raiders     0.007779364
## Chicago Bears       0.049489844
## Tampa Bay Buccaneers 0.023703807
## New York Giants     0.025448843
## Baltimore Ravens    0.011402292
## Arizona Cardinals   0.018655510
```

#Sorting and Ranking

```
R2 <- steadyStates(dtmc2)
FinalRank2 <- data.frame(R2[1,], rank(-R2))
names(FinalRank2)<-c("Probs2","Ranks2")
FinalRank2[order(FinalRank2$Ranks2), ]
```

```
##              Probs2 Ranks2
## New England Patriots 0.138622802      1
## Dallas Cowboys       0.059674428      2
## Indianapolis Colts   0.055070372      3
## Washington Redskins  0.054649405      4
## Green Bay Packers    0.050418698      5
## Chicago Bears        0.049489844      6
## San Diego Chargers   0.049489209      7
## Tennessee Titans     0.044721618      8
## Jacksonville Jaguars  0.040599487      9
## Pittsburgh Steelers  0.038521106     10
## Minnesota Vikings    0.037739920     11
## New Orleans Saints   0.030577128     12
## Houston Texans       0.029301449     13
## Philadelphia Eagles   0.028991175     14
## Seattle Seahawks     0.028063226     15
## Cincinnati Bengals   0.027116249     16
## Detroit Lions        0.026448758     17
## New York Giants      0.025448843     18
## Denver Broncos       0.024173069     19
## Tampa Bay Buccaneers 0.023703807     20
## Arizona Cardinals    0.018655510     21
## Kansas City Chiefs   0.017916438     22
## Carolina Panthers    0.016259069     23
## Cleveland Browns     0.014462354     24
## Baltimore Ravens     0.011402292     25
## Buffalo Bills        0.010608141     26
## Atlanta Falcons      0.010077405     27
```

## San Francisco 49ers	0.009854448	28
## New York Jets	0.008007175	29
## Oakland Raiders	0.007779364	30
## St. Louis Rams	0.007356663	31
## Miami Dolphins	0.004800550	32

Approach (3)

The concept behind this approach is that each team would give points to its opponent according to the score of the opponent. We can understand this approach by the analogy of a bet. Consider there are equal number of people betting on each team at the starting of the season. If team A wins against team B, certain number of bettors previously betting on B would start betting on A and certain number of people previously betting on A would start betting on B depending upon the scores. The number of bettors transitioning from A to B will be less compared to the number of bettors transitioning from B to A.

If we consider only two teams A and B, we can form the transition matrix M such that,

$$M = \begin{matrix} & \begin{matrix} A & B \end{matrix} \\ \begin{matrix} A \\ B \end{matrix} & \begin{bmatrix} AA & AB \\ BA & BB \end{bmatrix} \end{matrix}$$

Where,

$$AA = 0$$

$$BB = 0$$

$$AB = \text{Team B score}$$

$$BA = \text{Team A score}$$

To make a transition matrix we must normalize each row.

We can define (i, j)th entry of TransMatrix2 as,

$$\text{TransMatrix2}_{(i,j)} = \frac{w_{ij}}{\sum w_{ij}}$$

#Matrix Generation

```
TransMatrix3 = matrix(0, nrow = 32, ncol=32)
for(i in 1:256){
  indexW <- which(Teams == scores$`Winner/tie`[i])
  indexL <- which(Teams == scores$`Loser/tie`[i])
  TransMatrix3[indexL,indexW] = TransMatrix3[indexL,indexW] + scores$PtsW[i]
  TransMatrix3[indexW,indexL] = TransMatrix3[indexW,indexL] + scores$PtsL[i]
}
```

#Normalization

```
sum = 0
for(j in 1:32){
  for(k in 1:32){
```

```

    sum = sum + TransMatrix3[j,k]
  }
  if(sum != 0){
    TransMatrix3[j,] = TransMatrix3[j,]/sum
  }
  sum = 0
}

```

#Solving Transition Matrix

```

dtmc3 <- new("markovchain", states = Teams, transitionMatrix = TransMatrix3,
name = "Ranking3")
t(steadyStates(dtmc3))

```

```

##                                [,1]
## Indianapolis Colts    0.04113256
## Carolina Panthers    0.02182697
## Minnesota Vikings    0.03356159
## Denver Broncos       0.02991381
## Green Bay Packers    0.03922541
## Washington Redskins  0.03458882
## Pittsburgh Steelers  0.03082364
## New England Patriots 0.05582468
## Tennessee Titans     0.02810602
## Houston Texans       0.03535301
## Detroit Lions        0.03398943
## San Diego Chargers   0.03762890
## Seattle Seahawks     0.02807106
## Dallas Cowboys       0.04506343
## Cincinnati Bengals   0.03084870
## San Francisco 49ers  0.01714713
## New Orleans Saints    0.03090227
## St. Louis Rams       0.02128674
## Atlanta Falcons      0.02171786
## Buffalo Bills        0.02436825
## Philadelphia Eagles   0.03496819
## Miami Dolphins       0.02710426
## Cleveland Browns     0.03272741
## New York Jets        0.02533123
## Jacksonville Jaguars  0.03680030
## Kansas City Chiefs   0.02253871
## Oakland Raiders      0.02591833
## Chicago Bears        0.03323930
## Tampa Bay Buccaneers 0.02562127
## New York Giants      0.03819844
## Baltimore Ravens     0.02698499
## Arizona Cardinals    0.02918728

```

#Sorting and Ranking

```
R3 <- steadyStates(dtmc3)
FinalRank3 <- data.frame(R3[1,], rank(-R3))
names(FinalRank3)<-c("Probs3", "Ranks3")
FinalRank3[order(FinalRank3$Ranks3),]
```

##	Probs3	Ranks3
## New England Patriots	0.05582468	1
## Dallas Cowboys	0.04506343	2
## Indianapolis Colts	0.04113256	3
## Green Bay Packers	0.03922541	4
## New York Giants	0.03819844	5
## San Diego Chargers	0.03762890	6
## Jacksonville Jaguars	0.03680030	7
## Houston Texans	0.03535301	8
## Philadelphia Eagles	0.03496819	9
## Washington Redskins	0.03458882	10
## Detroit Lions	0.03398943	11
## Minnesota Vikings	0.03356159	12
## Chicago Bears	0.03323930	13
## Cleveland Browns	0.03272741	14
## New Orleans Saints	0.03090227	15
## Cincinnati Bengals	0.03084870	16
## Pittsburgh Steelers	0.03082364	17
## Denver Broncos	0.02991381	18
## Arizona Cardinals	0.02918728	19
## Tennessee Titans	0.02810602	20
## Seattle Seahawks	0.02807106	21
## Miami Dolphins	0.02710426	22
## Baltimore Ravens	0.02698499	23
## Oakland Raiders	0.02591833	24
## Tampa Bay Buccaneers	0.02562127	25
## New York Jets	0.02533123	26
## Buffalo Bills	0.02436825	27
## Kansas City Chiefs	0.02253871	28
## Carolina Panthers	0.02182697	29
## Atlanta Falcons	0.02171786	30
## St. Louis Rams	0.02128674	31
## San Francisco 49ers	0.01714713	32

Results

We, can compare ranks achieved by all the three approaches and comparison can done below. Any approach can be considered to rank the teams.

```
compareRanks<- cbind.data.frame(Teams,FinalRank1[,2],FinalRank2[,2],FinalRank3[,2])
```

```
names(compareRanks)<-c("Teams", "Approach1", "Approach2", "Approach3")
compareRanks
```

##	Teams	Approach1	Approach2	Approach3
## 1	Indianapolis Colts	2	3	3
## 2	Carolina Panthers	28	23	29
## 3	Minnesota Vikings	12	11	12
## 4	Denver Broncos	22	19	18
## 5	Green Bay Packers	5	5	4
## 6	Washington Redskins	8	4	10
## 7	Pittsburgh Steelers	4	10	17
## 8	New England Patriots	1	1	1
## 9	Tennessee Titans	11	8	20
## 10	Houston Texans	15	13	8
## 11	Detroit Lions	19	17	11
## 12	San Diego Chargers	3	7	6
## 13	Seattle Seahawks	13	15	21
## 14	Dallas Cowboys	6	2	2
## 15	Cincinnati Bengals	18	16	16
## 16	San Francisco 49ers	32	28	32
## 17	New Orleans Saints	20	12	15
## 18	St. Louis Rams	31	31	31
## 19	Atlanta Falcons	30	27	30
## 20	Buffalo Bills	27	26	27
## 21	Philadelphia Eagles	10	14	9
## 22	Miami Dolphins	29	32	22
## 23	Cleveland Browns	16	24	14
## 24	New York Jets	24	29	26
## 25	Jacksonville Jaguars	7	9	7
## 26	Kansas City Chiefs	25	22	28
## 27	Oakland Raiders	26	30	24
## 28	Chicago Bears	17	6	13
## 29	Tampa Bay Buccaneers	14	20	25
## 30	New York Giants	9	18	5
## 31	Baltimore Ravens	23	25	23
## 32	Arizona Cardinals	21	21	19