# Algebra 1: Tutorial 3

When you answer these questions practise your proof writing.
**Be clear, concise, and complete.**

## Question 1: Question

Question text

a) Easy part of question

b) A slightly harder part of question

c) A much harder, but doable part of question

## Question 2: Abelian Groups of Order 12

The cyclic group $(\mathbb{Z}/12\mathbb{Z})$ and the product groups $(\mathbb{Z}/3\mathbb{Z}) \times (\mathbb{Z}/4\mathbb{Z})$ and $(\mathbb{Z}/2\mathbb{Z}) \times (\mathbb{Z}/6\mathbb{Z})$ all have order 12. Are any of these groups isomorphic?

## Question 3: Correspondence Theorem – Cyclic Groups

Let $G$ be the cyclic group of order 12 generated by $g$. Define a group homomorphism $\varphi : \mathbb{Z} \to G$ which maps 1 to $g$. Verify that $\varphi$ is surjective. What is the exact correspondence given by the correspondence theorem?

## Question 4: Extremely creative question title

Let $G, G'$ be groups with subgroups $H, H'$. Show that $H \times H'$ is a subgroup of $G \times G'$. Furthermore, show that if $H$ and $H'$ are both normal, then $H \times H'$ is normal. Are all subgroups of $G \times G'$ of the form $H \times H'$ for some $H \subset G$, $H' \subset G'$?

## Question 5: Not so creative question title

Question text