Mathematical Sciences Institute
MATH2322/3104/6118: Algebra 1

Name:
Uni Id:

Week 4 Algebra 1 Tutorial Add-on Questions

Exercise 1: Show that the canonical homomorphism $\mathbb{Z} \to \mathbb{Z}/5\mathbb{Z}$ does not preserves order of elements.

Exercise 2: Suppose $\phi; G \to G'$ is a group homomorphism. Show that, for all $g \in G$, the order of $\phi(g)$ divides the order of g. Furthermore, prove that they are equal if ϕ is injective.

Exercise 3: Prove that in any group, ab is conjugate to ba.

Exercise 4: Prove that every subgroup of an abelian group is a normal subgroup.

Exercise 5: Show that if A is a subgroup of G and B is a normal subgroup, then $AB = \{x \mid x = ab \text{ for } a \in A, b \in B\}$ is a subgroup of G.

Exercise 6: An index 2 subgroup of a group is normal. (Third way to prove A_n is normal!)

Exercise 7: Show that if a group G has just one subgroup H, then H is normal.

Exercise 8: Show that any group of order 125 has a subgroup of order 5.

Exercise 9: Suppose $|G| = 25$. If G has only a subgroup of order 5, then G is cyclic.

Exercise 10: Suppose $|G| = 55$. Show that G must have an element of order 5.

Exercise 11: Prove that every group of order 35 has elements of order 5 and order 7.

Exercise 12: Prove that a group of even order must possesses at least one element of order 2.