Algebra 1: Tutorial 10

When you answer these questions practise your proof writing.
Be a group, ring, or module.

Question 1: Presentation Matrices

What is the presentation matrix of the following abelian groups:

- The abelian group generated by x, y, with the single relation $19x+13y = 0$;
- The abelian group generated by x, y, z, with the single relation $19x + 13y = 0$.

Question 2: Presentation Matrices

Identify the $\mathbb{Z}$-module presented by the following presentation matrices:

$$\begin{bmatrix} 2 & 5 \\ 4 & 10 \end{bmatrix}; \qquad \begin{bmatrix} 2 & -6 & 0 \\ -6 & 12 & 0 \end{bmatrix}.$$

Question 3: A Non-Noetherian Ring

Consider the ring $C(\mathbb{R})$ of continuous functions $f : \mathbb{R} \to \mathbb{R}$, where addition and multiplication of functions is performed pointwise, i.e. $(f+g)(x) = f(x)+g(x)$, $(fg)(x) = f(x)g(x)$. Show that it is not a Noetherian ring.

Question 4: $\mathbb{Z}[i]$-modules

Let M be an abelian group. Show that M has the structure of a $\mathbb{Z}[i]$ module, if and only if, then there exists a group homomorphism $\varphi : M \to M$ such that $\varphi \circ \varphi$ is the identity homomorphism on M.