
Algebra 1: Tutorial 1

When you answer these questions practise your proof writing.

Be clear, concise, and complete.

Question 1: Examples of Subgroups

Which of the following H are subgroups of the given group G :

- (a) $G := \mathbb{C}^*$ and $H := \{\pm 1, \pm i\}$
- (b) $G := \mathbb{Z}$ and $H := \mathbb{N}$
- (c) $G := \mathrm{GL}_2(\mathbb{C})$ and $H := \mathrm{SL}_2(\mathbb{C}) := \{A \in \mathrm{GL}_2(\mathbb{C}) \mid \det(A) = 1\}$
- (d) $G := \mathbb{Z}$ and $H := \{0\}$

Question 2: New Groups From Old

Let G_1 and G_2 be groups. Show that there is a “natural” group structure on the set theoretic product $G_1 \times G_2$. If $|G_1| = n$ and $|G_2| = m$, what is $|G_1 \times G_2|$?

Question 3: Permutations

Let $\sigma = \begin{pmatrix} 1 & 2 & 3 & 4 & 5 & 6 & 7 \\ 2 & 4 & 7 & 3 & 1 & 6 & 5 \end{pmatrix}$ be a permutation on the set $\{1, \dots, 7\}$

- (a) write σ in cycle notation
- (b) write σ as composition of transpositions. What is $\mathrm{sgn}(\sigma)$?

Question 4: Classification of Subgroups of \mathbb{Z}

Give an example of a subgroup of \mathbb{Z} . Is it cyclic? Can you list *all* subgroups of \mathbb{Z} ? Are they all cyclic?

Question 5: Group Homomorphism Example

What is a homomorphism of groups? What is an isomorphism? Find a homomorphism

$$\varphi : \mathbb{C}^* \rightarrow \mathrm{GL}_2(\mathbb{R})$$

Is the homomorphism (a) injective? (b) surjective? (c) an isomorphism? [Hint: In order to construct a homomorphism, think about where 1 and i must be mapped to]

Question 6: Cosets

Let $H := 4\mathbb{Z} \subseteq \mathbb{Z}$ be a subgroup of integers.

- (a) are 3 and 7 in the same coset?
- (b) are 3 and 6 in the same coset?
- (c) are 0, 4 and 24 in the same coset?