Suppose R and S are commutative unital rings and let $\Phi : R \to S$ be a ring homomorphism.

Let $I \triangleleft S$ be an ideal.

(i) If I is prime then $\phi^{-1}(I)$ is prime in R.

(ii) If I is maximal in S then $\phi^{-1}(I)$ is maximal in R.

(i) Assume I is prime in S.

let $B = \phi^{-1}(I)$ an ideal in R. let $a, b \in R$ s.t. $ab \in B$. Then there exists a $c \in I \triangleleft S$ s.t.

$$\phi(ab) = \phi(a)\phi(b) = s$$

By primality of I, $\phi(a) \in I$ or $\phi(b) \in I$

so $b \in B$ or $a \in B$ $\square$

(ii) Assume I is maximal in S.

let $B = \phi^{-1}(I)$ an ideal in R.

Let $\mathbb{F}_S = S/I$, a field since I is maximal in S.

Consider the natural homomorphism

$$\Theta : R \longrightarrow \mathbb{F}_S$$

$$\Theta(a) = \phi(a) + I$$

$$\ker(\Theta) = \phi^{-1}(I) = B.$$

So by the First Isomorphism Theorem

$$R/B \cong \Theta(R)$$

$$\Theta(R) = \mathbb{F}_S \;\ldots ??$$

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