The homomorphic image of a PID is a PID

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pf:

Let R be a PID,

$$\phi : R \to S$$

be a ring homomorphism.

Let I be an ideal of S.

Then $J = \phi^{-1}(I)$ is an ideal of R, so

$J = \langle g \rangle \quad g \in R$.

Let $p \in I \subseteq S$.

Then $\phi^{-1}(p) = kg \quad k, g \in R$

so $p = \phi(k)\,\phi(g)$

so $J = \langle \phi(g) \rangle$

$\square$