### I- Graphical Models: Representation

- 1- Directed Graphical Models: Bayesian Networks
- 2- Undirected Graphical Models: Markov Random Fields
- & Conditional Random Fields
- 3- Factor Graphs
- 4- Mixture Models (Discrete Hidden State)
  - Mixture of Multinomials
  - Mixture of Gaussians
- 5- Factor Analysis (Continuous Hidden State)
- 6- Sequantial Models

State Space Models (SSM can be thought of as a sequential Factor Analysis or continuous state HMM)

- Online Inference or Filtering (Ex. Kalman Filter) Analogue to the Forward Algorithm for HMM
- Offline Inference or Smoothing (Ex. Rauch-Tung-Strievel algorithm) Analogue to the Forward-Backward Algorithm for HMM

### **Switching State-Space Models**

Discrete Hidden State (HMM vs.  $\operatorname{CRF}$ ) - Mixture Models are the Building Blocks

Factorial Hidden Markov Models

# II- Graphical Models: Exact Inference and Parameter Learning

### 1- Exact Inference

- Variable Elimination
- Sum-Product Belief Propagation (Marginal Inference)
- Max-Product Belief Propagation (MAP Inference)
- MAP as a Linear Optimization Problem (LP Relaxation) (Or via Mixed Integer Linear Programming (MILP))
- Junction tree (must satisfy Running Intersection Property (RIP)): Junction Tree Algorithm
- (non-tree structure) graphs: Belief Propagation in Loopy Graphs (Approximate Inference Algorithm and Special Case of Variational Inference Algorithms)

- Hidden Markov Models/CRF:
  - Forward Algorithm (Used to calculate a 'belief state')
  - Forward Backward as Sum-Product Belief Propagation (Marginal Inference)
  - Viterbi Algorithm as Max-Product Belief Propagation (MAP Inference)

### 2- Parameter Learning

### Learning Fully Observable Bayesian Networks

- Generalized Linear Models
- Maximum Likelihood Estimation

### Learning Partially Observed Bayesian Networks

- Expectation–maximization algorithm
- Baum-Welch algorithm (HMM)

## Learning Fully Observable MRFs and CRFs (maximum-likelihood learning reduces to inference)

### MLE of UGM with Discrete RV (MRFs with tabular potentials)

- MLE by Inspection (Decomposable Model)
- Iterative Proportional Fitting (IPF)

## MLE of UGM with Continuous RV (MRFs with features based potentials)

- Generalized Iterative Scaling (Feature-Based Models/Log-Linear Parametrization)
- Gradient-based Methods

## III- Graphical Models: Structure learning

### 1- Causal discovery

### Constraint-based approach

- PC Algorithm
- FCI (Fast Causal Inference)

### Score-based approach

- Bayesian scoring
- Non-Bayesian scoring
- GES (Greedy Equivalence Search)

### Functional causal model-based approach

### 2- Causality-based learning

### **IV-** Graphical Models: Structured Prediction

• Structured SVM (Max-Margin Markov Networks)

## V- Graphical Models: Approximate Inference

## 1- Approximate Inference: Stochastic Simulation / Sampling Methods

**Markov Chains** 

#### Monte Carlo

- Rejection sampling
- Importance sampling

### Markov Chains Monte Carlo (MCMC)

- Metropolis Algorithm
- Metropolis-Hastings (M-H) Algorithm
- Gibbs Sampling
- Slice Sampling
- Hamiltonian Monte Carlo
- Variational MCMC
- Langevin dynamics
- Sequential Monte Carlo
- Sequential Markov Chain Monte Carlo (SMCMC)

### 2- Approximate Inference: Variational Methods

- Variational EM
- Loopy Belief Propagation
- Mean Field Approximation
- Coordinate Ascent Variational Inference
- Expectation Propagation
- Bethe Approximation, Kikuchi Approximation and Generalized Belief Propagation
- Black Box Variational Inference
- Amortized Vartiational Inference
- Stochastic variational inference
- Structured Stochastic Variational Inference
- Automatic Differentiation Variational Inference
- Variational Sequential Monte Carlo

• Automatic structured variational inference

### 3- Approximate Inference: Models

### **Bayesian Models**

- Bayesian Linear Regression
- Gaussian Mixture Model (GMM) with:
  - Gibbs Sampler
  - Mean-field Variational Inference
  - Expectation-Maximization
- Bayesian Dark Knowledge (SGLD + Distillation)

#### Bayesian Nonparametric Models

- Gaussian Process Regression
- GMM with CRP prior for Infinite Mixture Model
- Generative stories:
  - Chinese Restaurant Process (CRP)
  - Stick Breaking Construction
  - Indian Buffet Process (IBP)

### Dirichlet Process Mixture Model (DPMM)

### Hierarchical Dirichlet Process (HDP)

### LDA (App: Probabilistic Topic Models)

- With Gibbs Sampler
- With Collapsed Gibbs Sampler
- With Mean-field Variational Inference

### **Deep Exponential Families**

### Deep Generative Models

- Variational Autoencoders
- Normalizing Flow Models
- Generative Adversarial Networks

### Deep Neural Networks (DNNs)

### Deep Belief Networks (DBNs)

- Sigmoid Belief Network (SBN)
- Contrastive Divergence Learning (an Approximate Maximum-Likelihood Learning Algorithm)

### Deep Boltzman Machines (DBMs)

- Boltzman Machines
- Restricted Boltzman Machines

## VI- Graphical Models: RL and Control

- 1- Reinforcement Learning as Inference
  - Structured Prediction and Reinforcement Learning
- 2- Control as Inference

VII- Graphical Models: Causality and Causal Inference