Neural Network Analysis

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1 Network Specifications

- **Architecture**: 4-3-2 (Input-Hidden-Output)
- **Activation Function**: Sigmoid

$$\sigma(x) = \frac{1}{1 + e^{-x}} \tag{1}$$

- **Total Parameters**: 20 (12 weights + 6 weights + 2 biases)
- **Network Type**: Feedforward, fully connected

1.1 Forward Propagation Equations

For a general feedforward network, the forward pass is calculated as:

$$\mathbf{z}^{(l)} = \mathbf{W}^{(l)}\mathbf{a}^{(l-1)} + \mathbf{b}^{(l)} \tag{2}$$

$$\mathbf{a}^{(l)} = \sigma(\mathbf{z}^{(l)}) \tag{3}$$

Where:

- $\mathbf{z}^{(l)}$ is the pre-activation vector for layer l
- $\mathbf{W}^{(l)}$ is the weight matrix for layer l
- $\mathbf{a}^{(l)}$ is the activation vector for layer l
- $\mathbf{b}^{(l)}$ is the bias vector for layer l

2 Input Data and Parameters

2.1 Input Vector

The input vector is defined as:

$$\mathbf{x} = \begin{pmatrix} 2 \\ 3 \\ -1 \\ 2 \end{pmatrix} \tag{4}$$

2.2 Weight Matrices

Input to Hidden Layer ($\mathbf{W}^{(1)}$):

$$\mathbf{W}^{(1)} = \begin{pmatrix} 2 & 3 & 0 & 1 \\ 4 & 3 & 2 & 0 \\ 1 & 1 & 1 & 2 \end{pmatrix}_{3\times4} \tag{5}$$

Hidden to Output ($\mathbf{W}^{(2)}$):

$$\mathbf{W}^{(2)} = \begin{pmatrix} 1 & 7 & 1 \\ 7 & 1 & 4 \end{pmatrix}_{2\times3} \tag{6}$$

2.3 Bias Vector

$$\mathbf{b} = \begin{pmatrix} -6 \\ -2 \end{pmatrix} \tag{7}$$

3 Forward Propagation Calculation

3.1 Step 1: Input to Hidden Layer

Pre-activation calculation:

$$\mathbf{z}^{(1)} = \mathbf{W}^{(1)}\mathbf{x} \tag{8}$$

Detailed calculations:

$$z_1^{(1)} = 2 \cdot 2 + 3 \cdot 3 + 0 \cdot (-1) + 1 \cdot 2 = 4 + 9 + 0 + 2 = 15 \tag{9}$$
$$z_2^{(1)} = 4 \cdot 2 + 3 \cdot 3 + 2 \cdot (-1) + 0 \cdot 2 = 8 + 9 - 2 + 0 = 15 \tag{10}$$
$$z_3^{(1)} = 1 \cdot 2 + 1 \cdot 3 + 1 \cdot (-1) + 2 \cdot 2 = 2 + 3 - 1 + 4 = 8 \tag{11}$$

Therefore: $\mathbf{z}^{(1)} = \begin{pmatrix} 15 \\ 15 \\ 8 \end{pmatrix}$

Activation calculation:

$$a_1^{(1)} = \sigma(15) = \frac{1}{1 + e^{-15}} \approx 1.000000 \tag{12}$$
$$a_2^{(1)} = \sigma(15) = \frac{1}{1 + e^{-15}} \approx 1.000000 \tag{13}$$
$$a_3^{(1)} = \sigma(8) = \frac{1}{1 + e^{-8}} \approx 0.999665 \tag{14}$$

Hidden layer output: $\mathbf{a}^{(1)} = \begin{pmatrix} 1.000000 \\ 1.000000 \\ 0.999665 \end{pmatrix}$

3.2 Step 2: Hidden to Output

Pre-activation calculation:

$$\mathbf{z}^{(2)} = \mathbf{W}^{(2)}\mathbf{a}^{(1)} + \mathbf{b} \tag{15}$$

Detailed calculations:

$$\begin{aligned} z_1^{(2)} &= 1 \cdot 1.000000 + 7 \cdot 1.000000 + 1 \cdot 0.999665 + (-6) && (16)\\ &= 1 + 7 + 0.999665 - 6 = 2.999665 && (17)\\ z_2^{(2)} &= 7 \cdot 1.000000 + 1 \cdot 1.000000 + 4 \cdot 0.999665 + (-2) && (18)\\ &= 7 + 1 + 3.99866 - 2 = 9.99866 && (19) \end{aligned}$$

Therefore: $\mathbf{z}^{(2)} = \begin{pmatrix} 2.999665 \\ 9.99866 \end{pmatrix}$

Final activation calculation:

$$a_1^{(2)} = \sigma(2.999665) = \frac{1}{1 + e^{-2.999665}} \approx 0.952559 \tag{20}$$

$$a_2^{(2)} = \sigma(9.99866) = \frac{1}{1 + e^{-9.99866}} \approx 0.999955 \tag{21}$$

Final network output: $\mathbf{a}^{(2)} = \begin{pmatrix} 0.952559 \\ 0.999955 \end{pmatrix}$

4 Results Analysis

4.1 Summary of Computational Results

Layer	**Pre-activation**	**Post-activation**
Hidden 1	15.000	1.000000
Hidden 2	15.000	1.000000
Hidden 3	8.000	0.999665
Output 1	2.999665	0.952559
Output 2	9.998656	0.999955

Table 1: Summary of Activations by Layer