COMPUTATIONAL PHYSICS LAB

(PH49012)

SPRING-2021, IIT KGP

Assignment 10

Q1. A simple electrostatic problem

An empty box (10×10 grid points) has conducting walls, all of which are grounded at 0 volts except for the walls at the top, which is at voltage $V = 1.0$ unit. Solve the equation which is relevant for the given system using Jacobi method. Also show the pseudo color plot of the potential over the specified domain.

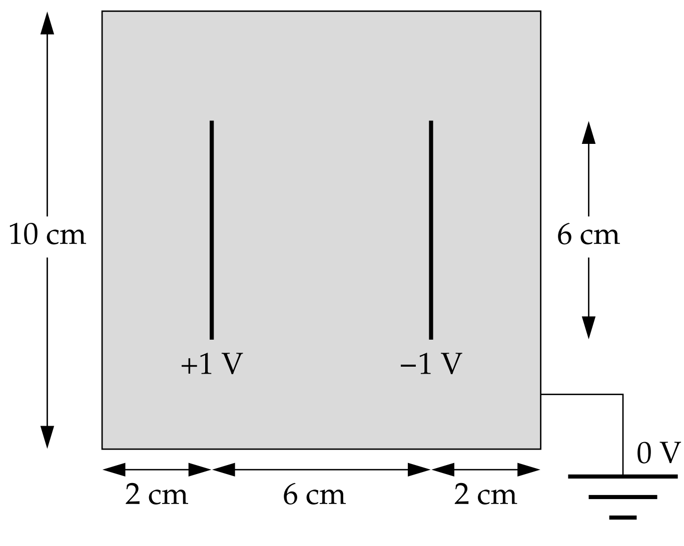
(10 points)

Q2. Write a program, or modify the one from Q1, to solve for the following system:

Consider a square shaped box of length 100 cm. Take 100 points along each axis to discretize the domain. Consider two charges at $(x, y) = (20\text{ cm}, 20\text{ cm})$ and $(80\text{ cm}, 80\text{ cm})$ with $q_1 = 1$ and $q_2 = -1$ respectively. Work in units where $\epsilon_0 = 1$ and continue the iteration until your solution for the electric potential changes by less than $10^{-5}\,V$ per step at every grid point.

(15 points)

Q3. Consider the following simple model of an electronic capacitor, consisting of two flat metal plates enclosed in a square metal box:



For simplicity let us model the system in two dimensions. Using any of the methods we have studied, write a program to calculate the electrostatic potential in the box on a grid of 100 ×100 points, where the walls of the box are at the voltage zero and the two plates (which are of negligible thickness) are at voltages $\pm 1\,V$ as shown. Have your program calculate the value of the potential at each grid point to a precision of 10^{-6} volts and then make a density plot of the result.

Hint: Notice that the capacitor plates are at the fixed voltage, not fixed charge. In effect, the capacitor plates are part of the boundary condition in that case, they behave the same way as the walls of the box, with potentials that are fixed at a certain value and cannot change.

(20 points)

Q4. The heat equation

The flat base of a container made of 1 cm thick stainless steel is initially at a uniform temperature of 20^0C everywhere. The container is placed in a bath of cold water at 0^0C and filled with hot water at 50^0C. Solve the diffusion equation (also called the heat equation in this context) for one dimension:

$$\frac{\partial T}{\partial t} = D\frac{\partial^2 T}{\partial x^2}$$

Let us divide the x-axis into 100 equal grid intervals, meaning that there will be 101 grid points in total, counting the first and last ones. The first and last points have fixed temperatures of 50^0C and 0^0C, respectively, while the intermediate points are initially all at the 20^0C. Take $D = 4.25 \times 10^{-6}\mathrm{m}^2\mathrm{s}^{-1}$. Solve the equation using FTCS method and generate the temperature profiles at $t = 0.01$ s, 0.1 s, 0.4 s and 1 s on a same graph.

(15 points)
