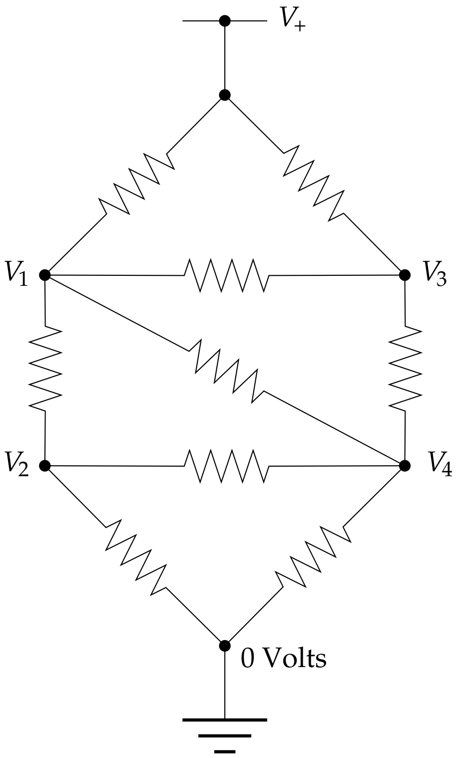
Computational Physics Lab

(PH49012)

Spring-2021, IIT KGP

Assignment 09

Q1. A circuit of resistors
Consider the following circuit of resistors:



All the resistors have the same resistance R. The power rail at the top is at voltage $V_+ = 5\,\mathrm{V}$. What are the other four voltages, V_1 to V_4?

To answer this question we use Ohm's law and the Kirchhoff current law, which says that the total net current flow out of (or into) any junction in a circuit must be zero. Thus for the junction at voltage V_1, for instance, we have

$$\frac{V_1 - V_2}{R} + \frac{V_1 - V_3}{R} + \frac{V_1 - V_4}{R} + \frac{V_1 - V_+}{R} = 0,$$

or equivalently

$$4V_1 - V_2 - V_3 - V_4 = V_+.$$

Write similar equations for the other three junctions with unknown voltages. Write a program to solve the four resulting equations using Gaussian elimination method and hence find the four voltages.

(15 points)

Q2. Gaussian elimination with backsubstitution
Let us consider the following set of linear equations given in the form $\mathbf{Ax} = \mathbf{b}$

$$\begin{bmatrix} 2 & 1 & 4 & 1 \\ 3 & 4 & -1 & -1 \\ 1 & -4 & 1 & 5 \\ 2 & -2 & 1 & 3 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \end{bmatrix} = \begin{bmatrix} -4 \\ 3 \\ 9 \\ 7 \end{bmatrix}$$

Write a code to solve the system of equations using Gaussian elimination with backsubstitution i.e, first convert the matrix $\mathbf{A}$ into an upper triangular form, and apply the backsubstitution to get the results.

(10 points)

Q3. Gauss-Jordan method

Solve the following system of equations given in the form $\mathbf{A}\mathbf{x} = \mathbf{b}$ using Gauss-Jordan algorithm. Partial pivoting may help you write the solver.

$$\begin{bmatrix} 0 & 2 & 0 & 1 \\ 2 & 2 & 3 & 2 \\ 4 & -3 & 0 & 1 \\ 6 & 1 & -6 & -5 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \end{bmatrix} = \begin{bmatrix} 0 \\ -2 \\ -7 \\ 6 \end{bmatrix}$$

(15 points)

Q1 and Q2 have been taken from Computational Physics by Mark Newman