Assignment-2

Statistical Physics-II (Autumn 2022-23)

Full-Marks - 10

Consider the Ising model on a 2D square lattice with nearest neighbour interactions

$$\mathcal{H} = -J \sum_{\langle ij \rangle} S_i S_j,$$

where $S_i = \pm 1$ are the Ising spins, while $\langle ij \rangle$ represents nearest neighbours. Fix the value for $J = 1$.

1. Write down a computer program to simulate the 2D Ising model (with periodic boundary conditions), using the Metropolis algorithm. Write the program such that the no. of lattice points may be easily changed. Use some optimally chosen value for equilibration time and auto-correlation time (you do not have to compute auto-correlations explicitly).

2. Perform the simulation for different values of temperature T. Use an interval of 0.1, for a range of T which comfortably contains the transition temperature T_c. At this stage you may estimate the transition temperature using mean-field-theory.

3. Plot the equilibrium distribution of 'spontaneous magnetization' and 'total energy' for individual distributions. Using Histogram plot, with an optimally chosen bin size for presenting the data clearly. In the same graph, show these plots for two values of temperature, one slightly below T_c and the other slightly above. Qualitatively explain why you expect the results of this plot. Study how the variance of these plots changes, if you increase the system size (no. of lattice points). Explain the observation.

   [Note that the quality and nature of plots here compared to your expectations will provide a check of whether the chosen values of equilibration time and auto-correlation time have been optimal.]

4. Plot the 'spontaneous magnetization' per unit spin, 'magnetic susceptibility' and 'specific heat' as a function of temperature. The averaging is to be performed over all samples collected after equilibration. Use finite size scaling on these plots to compute the transition temperature more precisely.

5. Use your result in (4) above (use the largest value of no. of lattice points) to estimate the scaling coefficients α, β and γ. Employ an appropriate curve fitting on the zoomed on log-log plot near T_c. Find the coefficients both above and below T_c. From, these values, estimate the value of the scaling coefficients δ, η and ν, using scaling relations for $D = 2$. Make a table of all critical coefficients representing calculated values and exact values (available from the literature).

6. Consider a temperature close to the critical point (as close as your chosen discretisation of temperature allows). At this temperature, display the log-log plot of 'average' 2-point function (see notes for definition), as a function of r, for large r. Read off the scaling coefficient η from this plot, by appropriately fitting the curve. You may use the lattice with the largest number of points, that have been used in these simulation.