Graph theory

Carlos Vigil Vásquez

Contents

1 Metrics

1.1 Centrality

In graph theory and network analysis, indicators of cenrtality identify the most important vertices within a graph. This kind of metrix try to answer the question of "What characterizes an important vertex?", through the use of real-valued functions on the vertices of the graph that are expected to provide a ranking which identiies the most important nodes. There are 2 categorization schemes proposed to identify the "importance" of a node in the graph:

- "Importance" in relation to a type of **flow or transfer** accross the network.
- "Importance" in relation to the involvement in the **cohesiveness** of the network.

This implies that a centrality which is appropiate for one category will often "get it wrong" when applied to a different category.

1.1.1 Degree centrality

The **degree centrality** is defines as the number of of links incident upon a node (i.e., the number of ties that the node has). This centrality can be interpreted in terms of the inmidiate risk of a node for catching whatever is flowing through the network.

In directed networks, we usually define 2 separete measures of degree entrality, namely indegree and outdegree.

1.1.2 Closeness centrality

The normalized closeness centrality (or closeness) of a node is the average lenghth of the shortest path between the node and all other nodes in the graph. Thus, the more central a node is, the closer it is to all other nodes.

1.1.3 Betweenness centrality

Betweenness is a centrality measure of a node within a graph that quantifies the number of times a node acts as a bridge along the shortest path between two other nodes. Vertices that have a high probability to occur on a randomly chosen shortest path between two randomly chosen vertices have a high betweenness.

1.1.4 Eigenvector centrality

Eigenvector centrality or eigencentrality is a measure of the influence of a node in a network. It assigns a relative score to all nodes in the network based on the concept that connections to high-scoring nodes contribute more to the score of the node in quention than qeual connections to low-scoring nodes.

1.1.5 Katz centrality

Ketz centrality is a generalization of degree centrality. This metric measures the number of all nodes that can be connected trough a path, while the contribution of distant nodes are penalized.