Machine Learning Course
basic track

Lecture 8:
Intro to Deep Learning

MIPT, Moscow
April 2020

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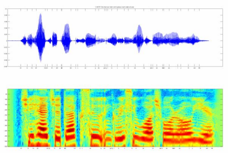
Outline

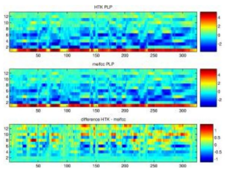
1. Neural Networks in different areas. Historical overview.
2. Backpropagation.
3. More on backpropagation.
4. Activation functions.
5. Playground.

Real world applications

Audio Features

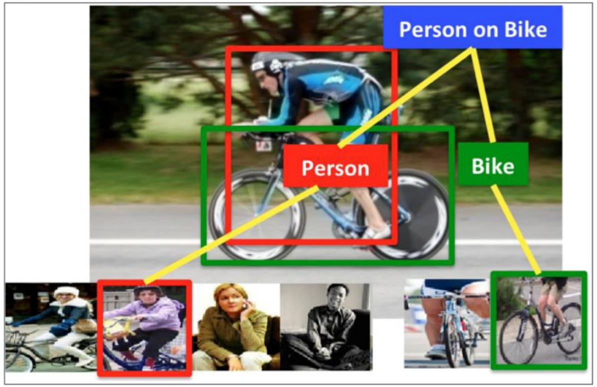


Spectrogram



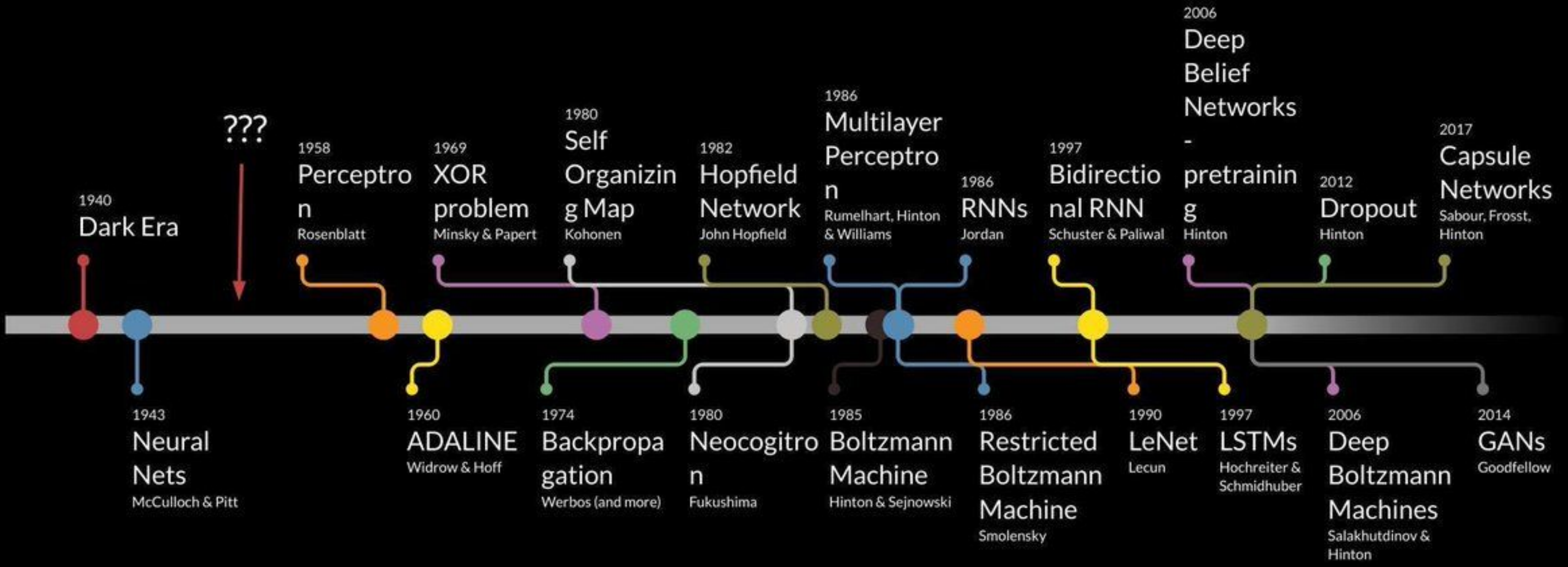
MFCC

- Object detection
- Action classification
- Image captioning
- …



"man in black shirt is playing guitar."

Deep Learning Timeline



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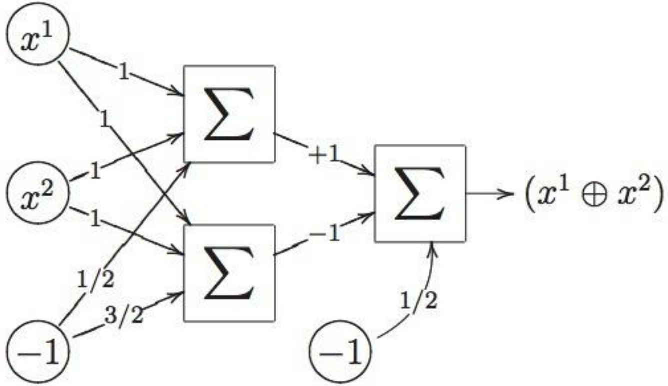
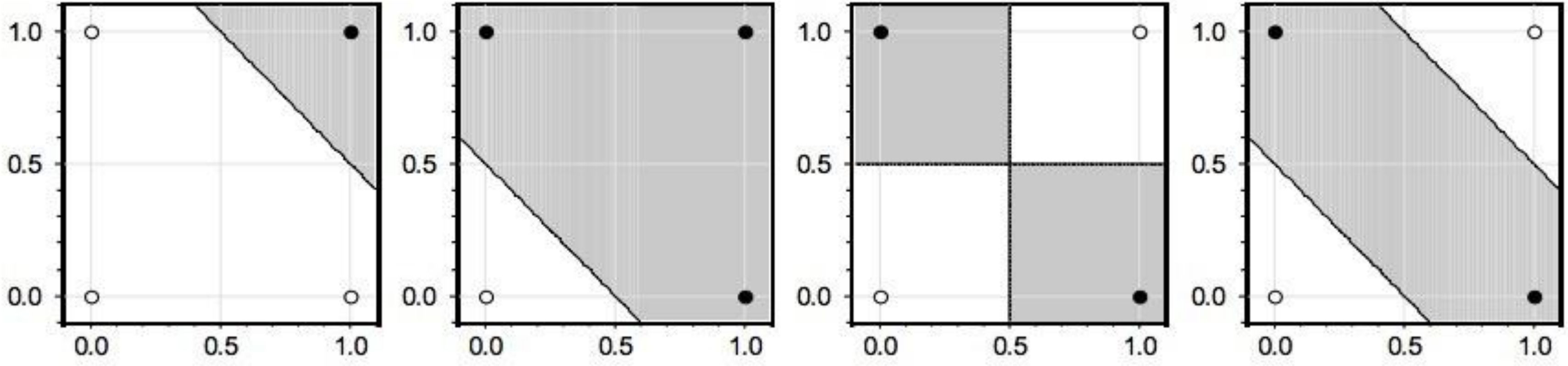
XOR problem



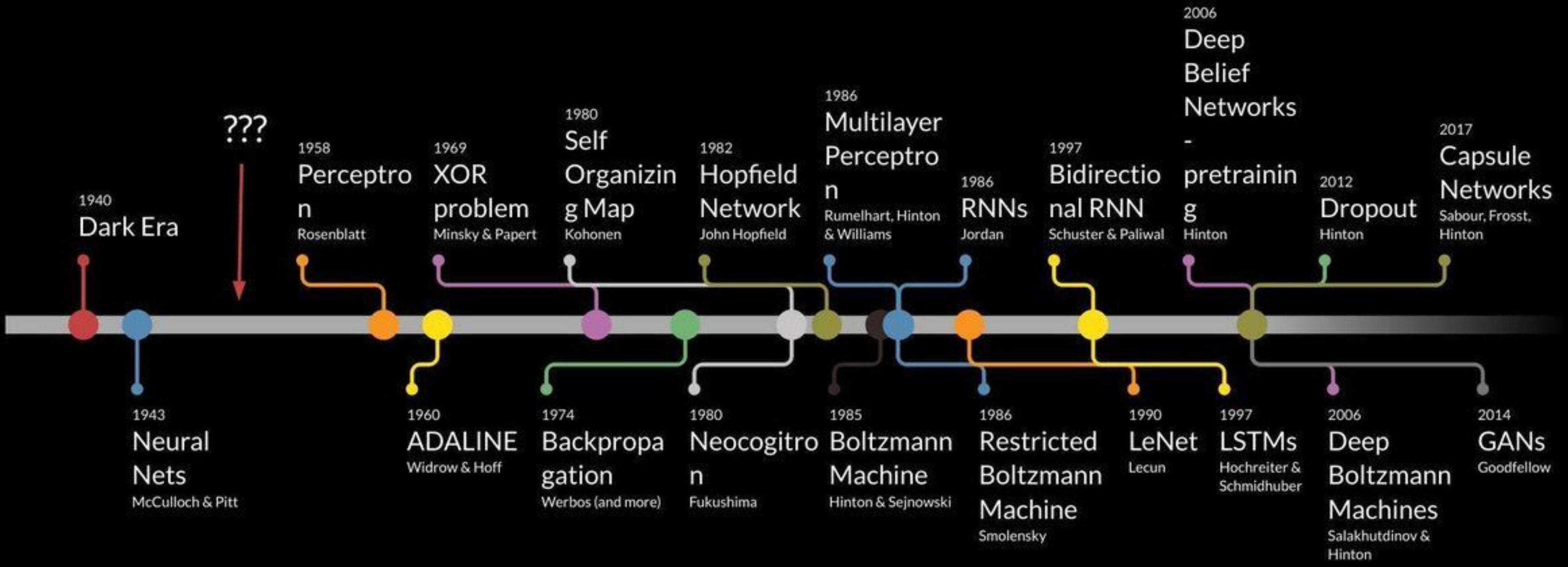
This 2-layer NN (on the left) implements XOR with only x1 and x2 features.

1-layer NN also can succeed, but only with extra feature x1*x2.



AND OR XOR(with x1*x2) XOR

Deep Learning Timeline



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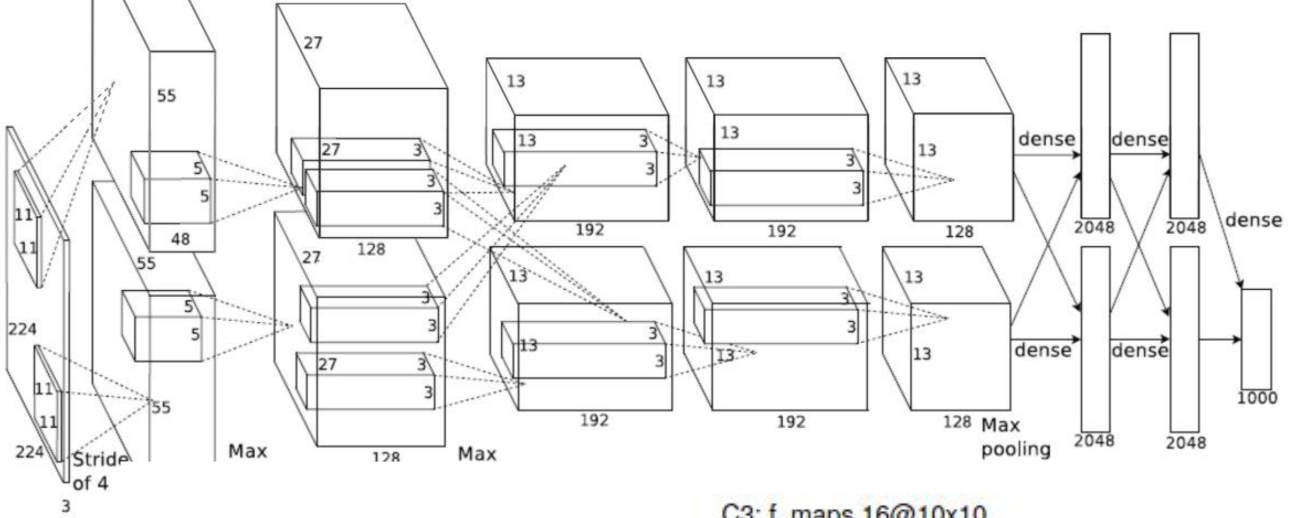
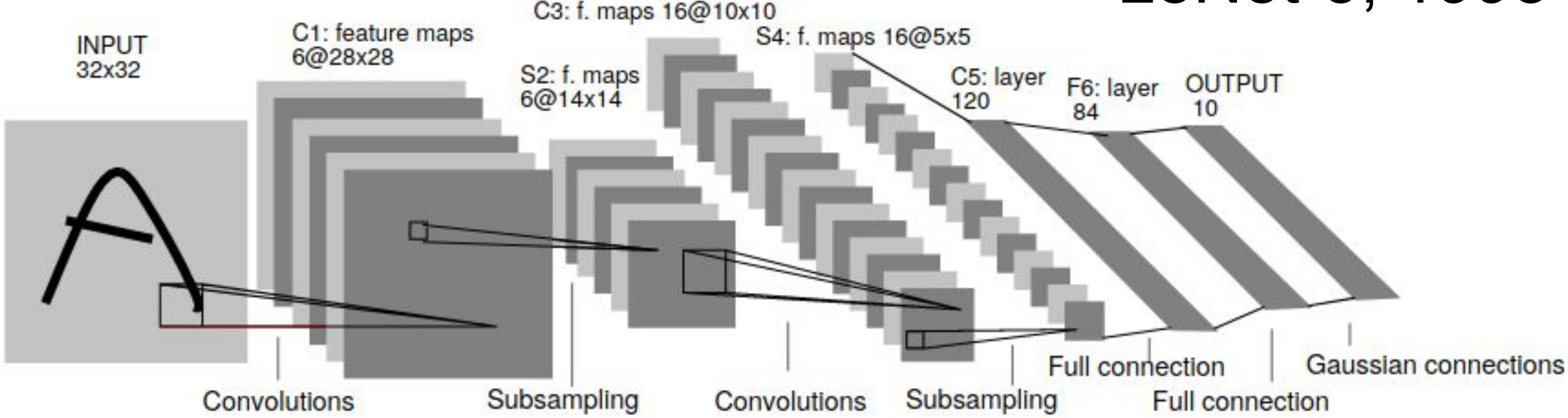
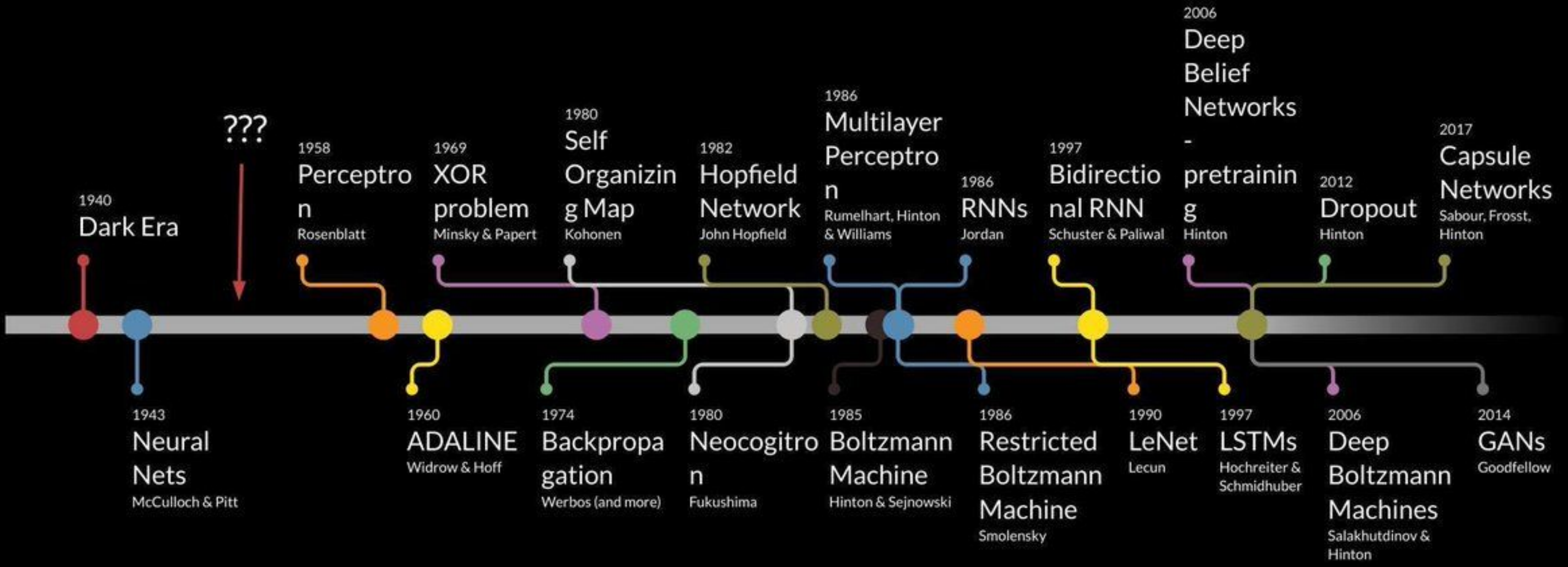
AlexNet, 2012



LeNet-5, 1998



Deep Learning Timeline



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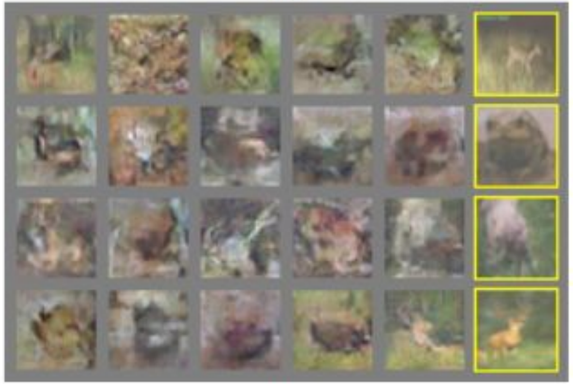
GANs, 2014+

a)

b)

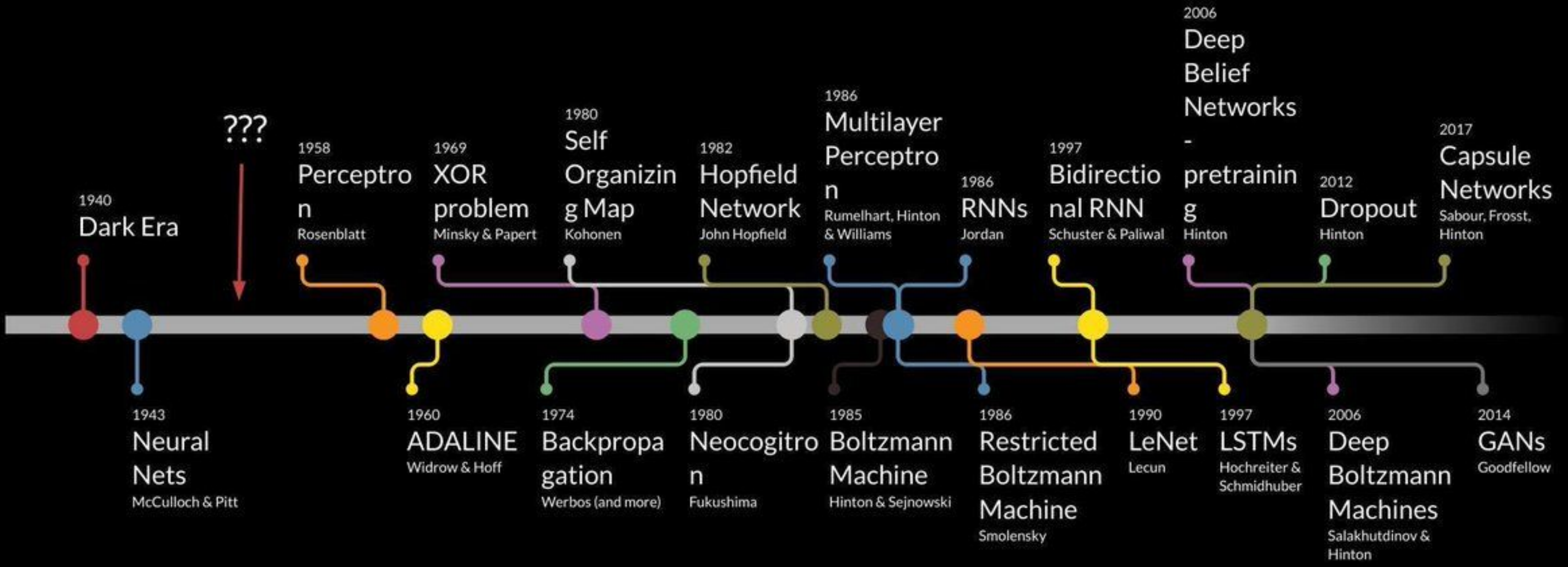
c)



d)

https://thispersondoesnotexist.com/

Deep Learning Timeline



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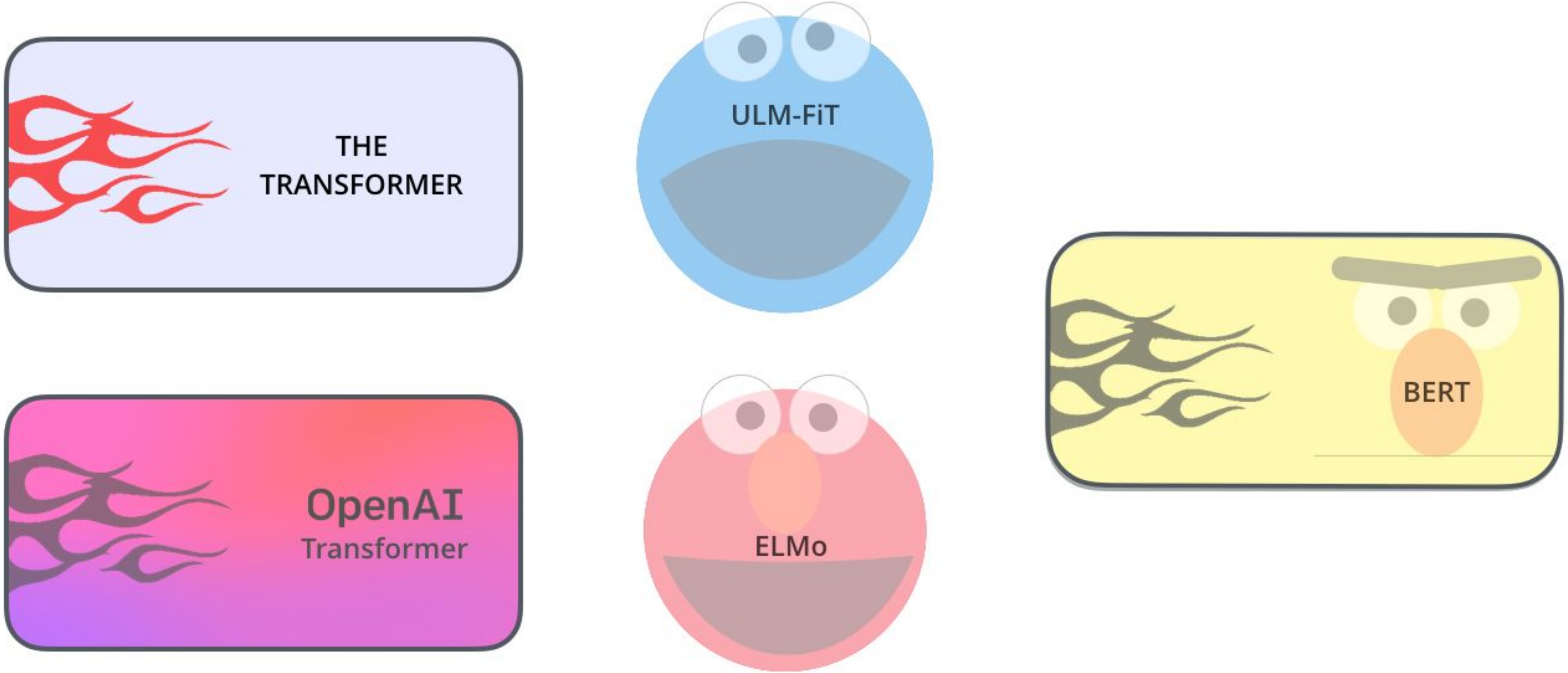
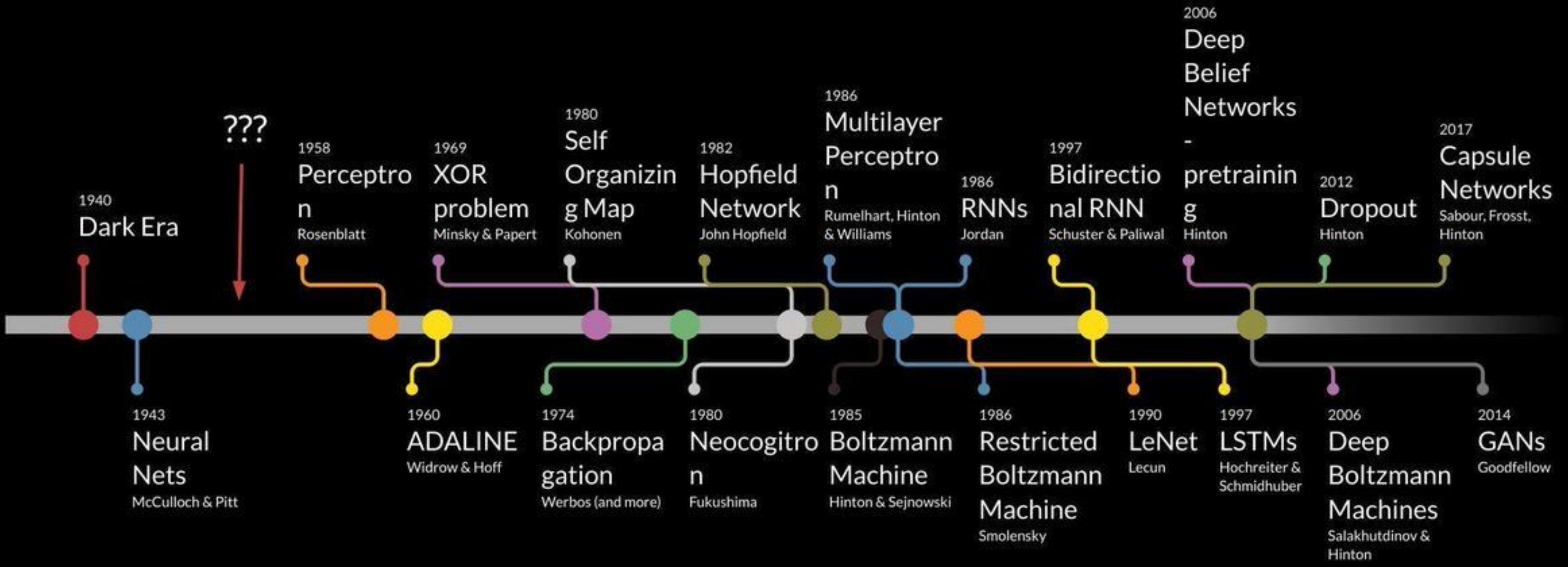
Transformer, BERT, GPT-2 and more, 2017+



Deep Learning Timeline



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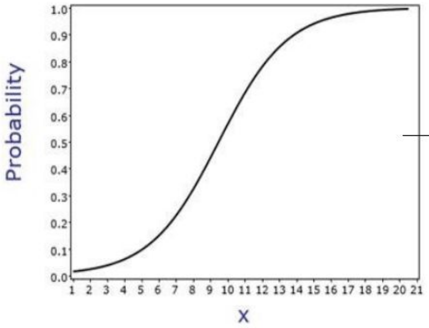
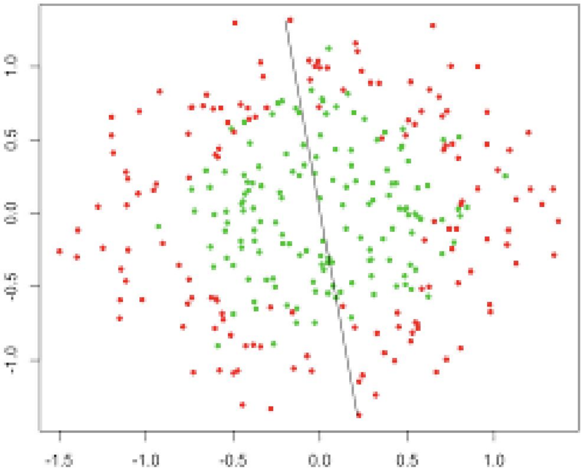
Logistic regression

X → Wx +b →  → P(y)

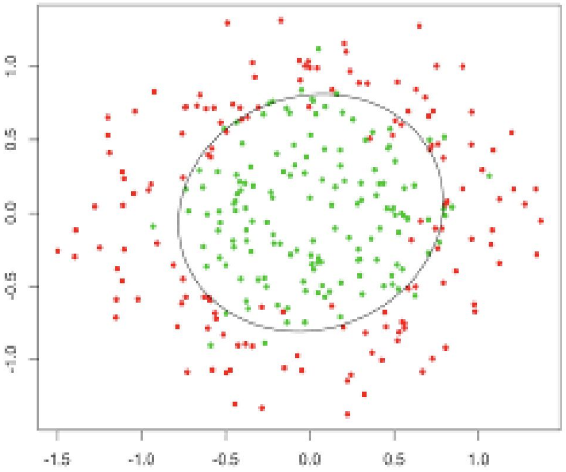
$$P(y|x) = \sigma(w \cdot x + b)$$

$$L = -\sum_i y_i \log P(y|x_i) + (1 - y_i) \log(1 - P(y|x_i))$$

Problem: nonlinear dependencies



What we have

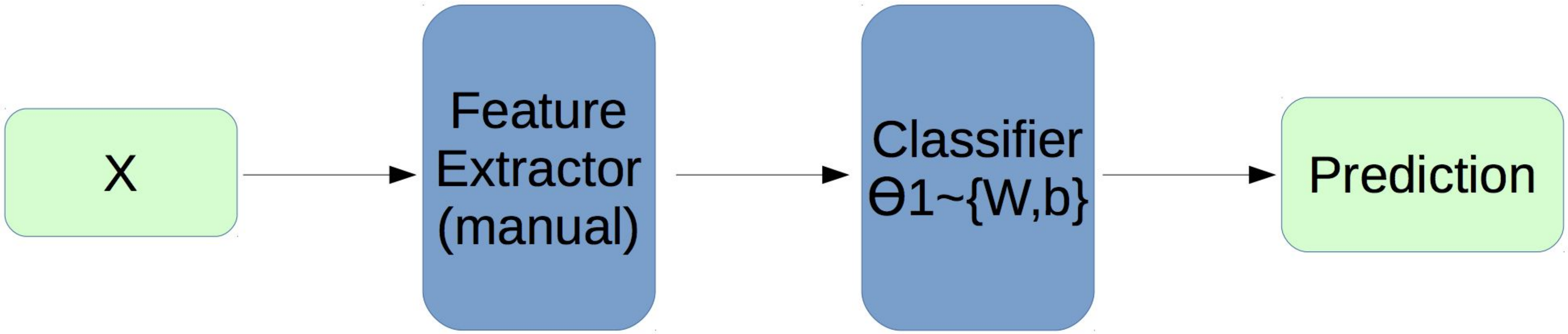


What we want

Logistic regression (generally, linear model) need feature engineering to show good results.

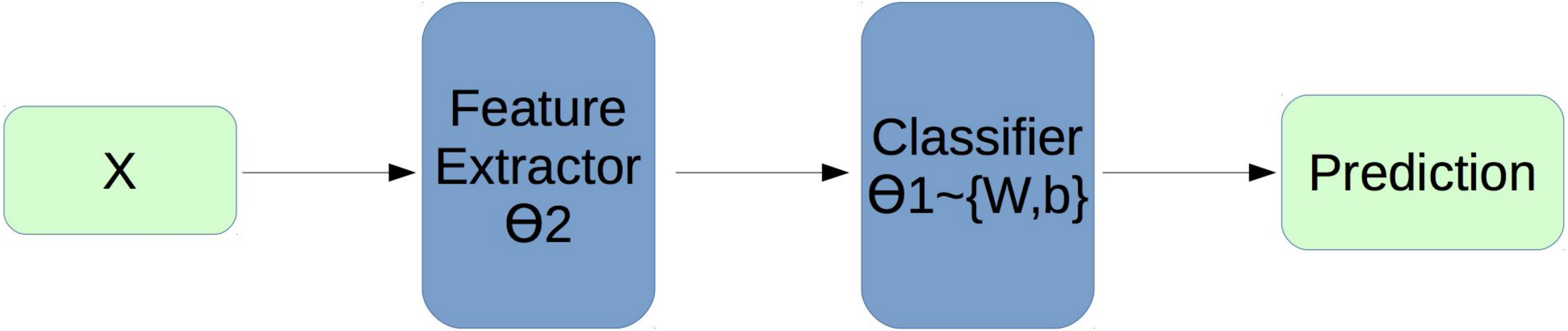
And feature engineering is an *art*.

Classic pipeline



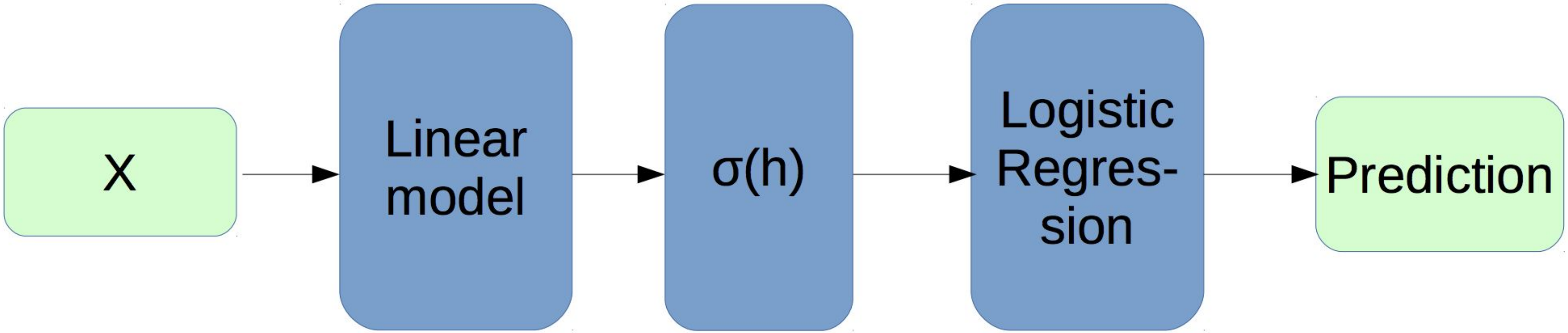
Handcrafted features, generated by experts.

NN pipeline



Automatically extracted features.

NN pipeline: example



E.g. two logistic regressions one after another.

Actually, it's a neural network.

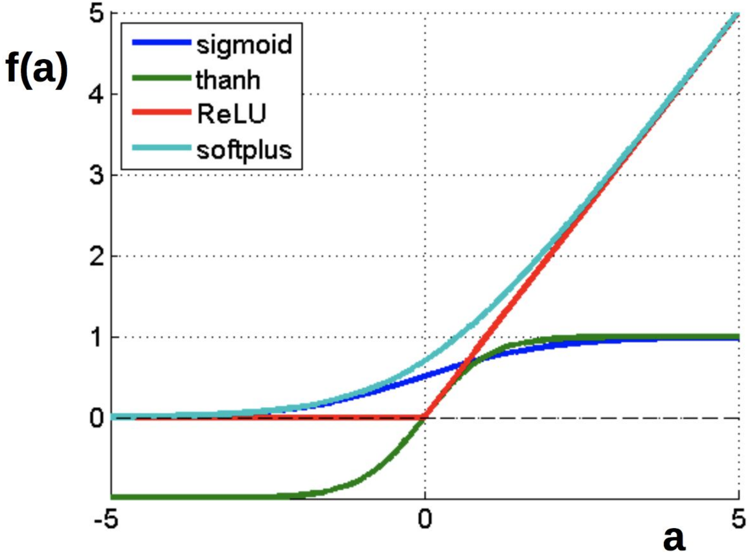
Activation functions: nonlinearities

$$f(a) = \frac{1}{1 + e^{a}}$$

$$f(a) = \tanh(a)$$

$$f(a) = \max(0, a)$$

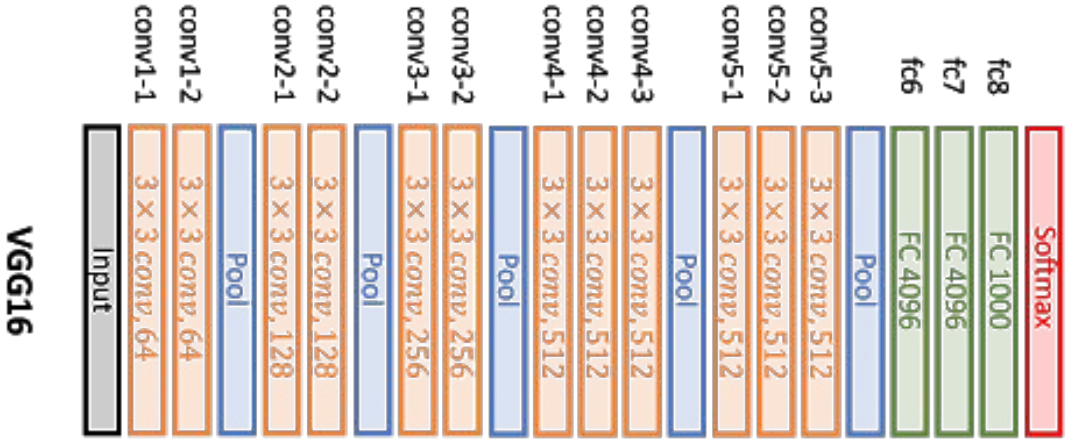
$$f(a) = \log(1 + e^{a})$$



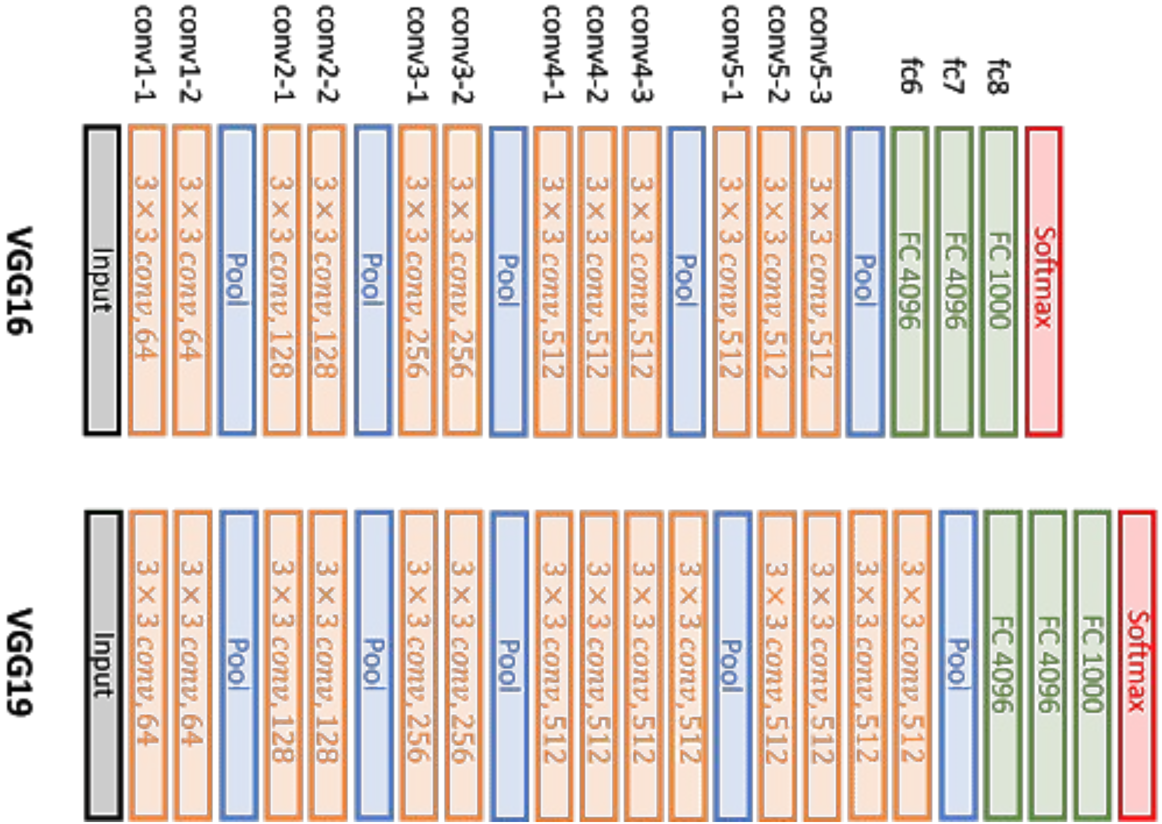
Some generally accepted terms

- Layer – a building block for NNs :
  - Dense/Linear/FC layer: $f(x) = Wx+b$
  - Nonlinearity layer: $f(x) = \sigma(x)$
  - Input layer, output layer
  - A few more we will cover later
- Activation function – function applied to layer output
  - Sigmoid
  - tanh
  - ReLU
  - Any other function to get nonlinear intermediate signal in NN
- Backpropagation – a fancy word for "chain rule"

Actually, networks can be deep



And deeper...



conv1-1
conv1-2
conv2-1
conv2-2
conv3-1
conv3-2
conv4-1
conv4-2
conv4-3
conv5-1
conv5-2
conv5-3
fc6
fc7
fc8
VGG16
Input
3 × 3 conv, 64
3 × 3 conv, 64
Pool
3 × 3 conv, 128
3 × 3 conv, 128
Pool
3 × 3 conv, 256
3 × 3 conv, 256
Pool
3 × 3 conv, 512
3 × 3 conv, 512
3 × 3 conv, 512
Pool
3 × 3 conv, 512
3 × 3 conv, 512
3 × 3 conv, 512
Pool
FC 4096
FC 4096
FC 1000
Softmax
VGG19
Input
3 × 3 conv, 64
3 × 3 conv, 64
Pool
3 × 3 conv, 128
3 × 3 conv, 128
Pool
3 × 3 conv, 256
3 × 3 conv, 256
Pool
3 × 3 conv, 512
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3 × 3 conv, 512
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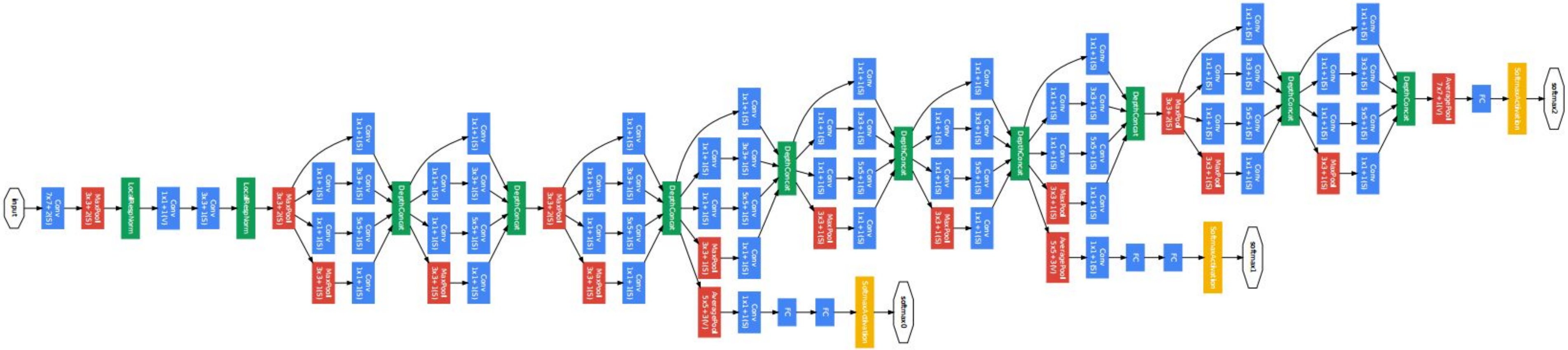
Pool
FC 4096
FC 4096
FC 1000
Softmax

Much deeper...

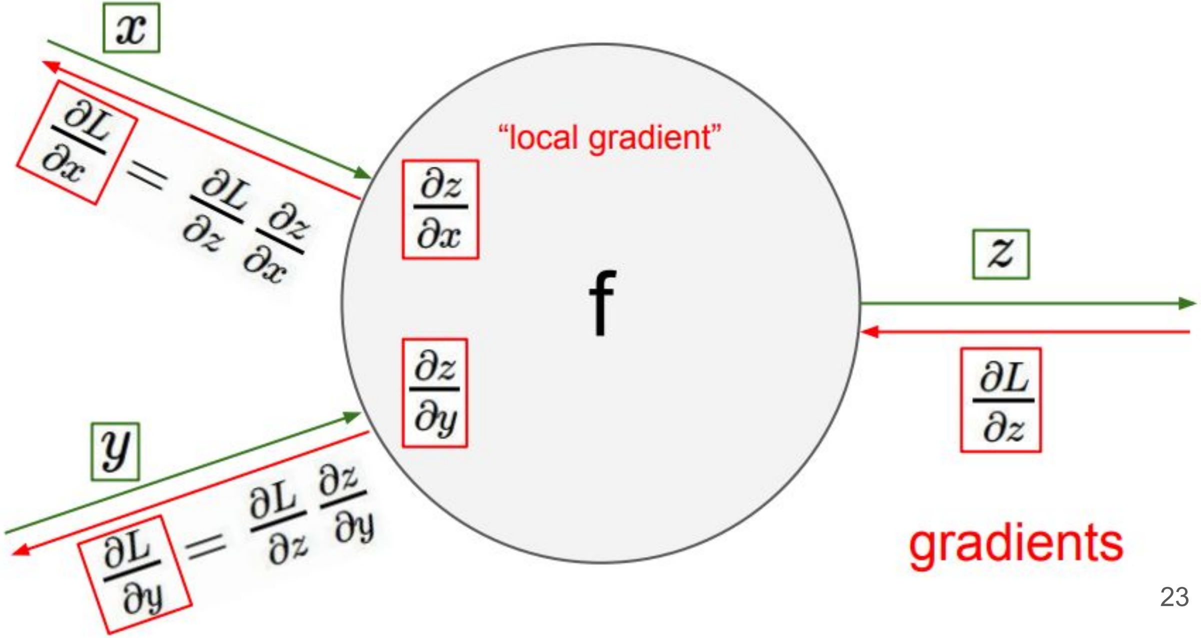


How to train it?

Backpropagation and chain rule

Chain rule is just simple math: $$\frac{\partial L}{\partial x} = \frac{\partial L}{\partial z}\frac{\partial z}{\partial x}$$

Backprop is just way to use it in NN training.



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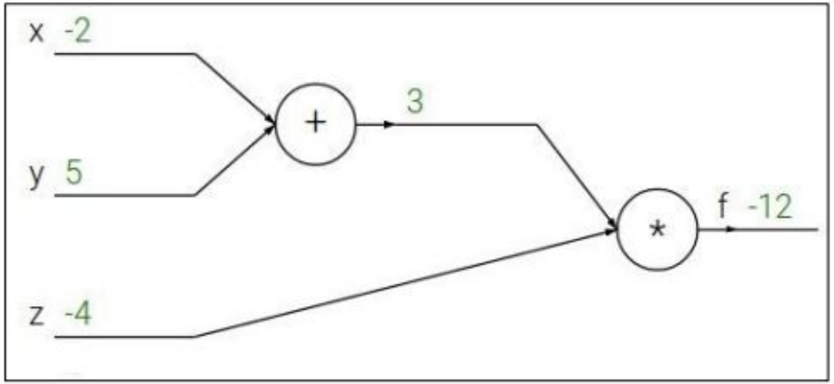
Backpropagation example

$$f(x, y, z) = (x + y)z$$

e.g. x = -2, y = 5, z = -4



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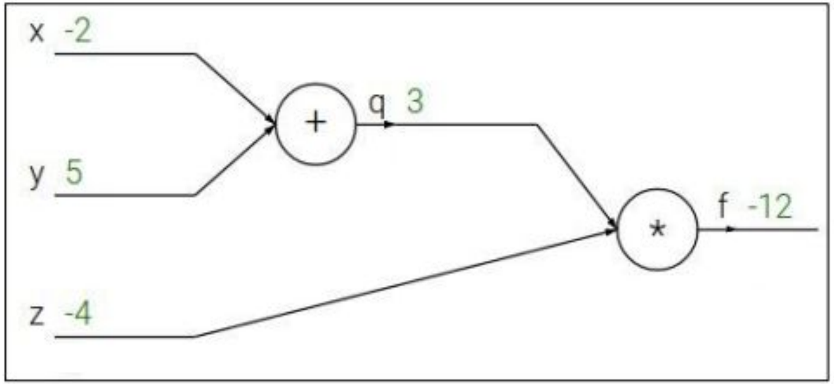
Want: $\frac{\partial f}{\partial x}, \frac{\partial f}{\partial y}, \frac{\partial f}{\partial z}$



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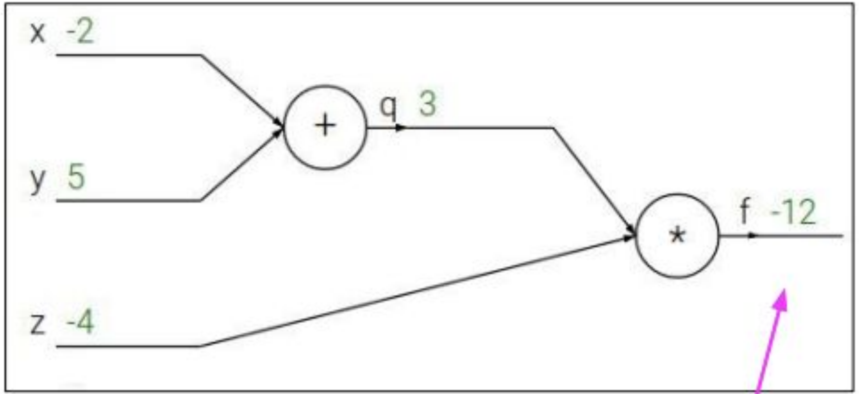

$$\frac{\partial f}{\partial f}$$

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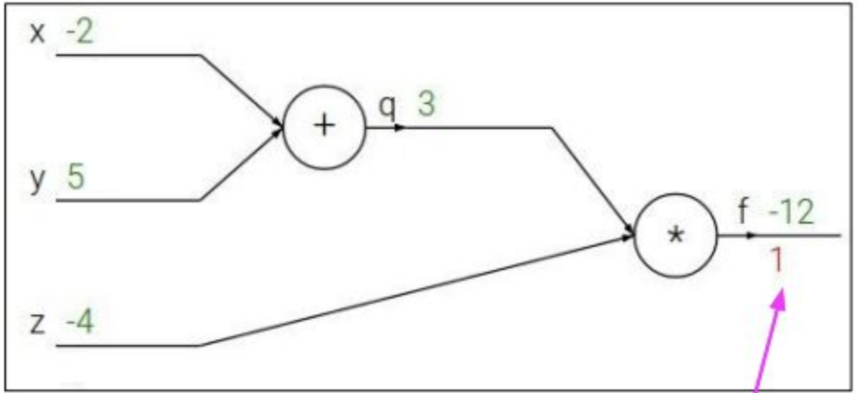

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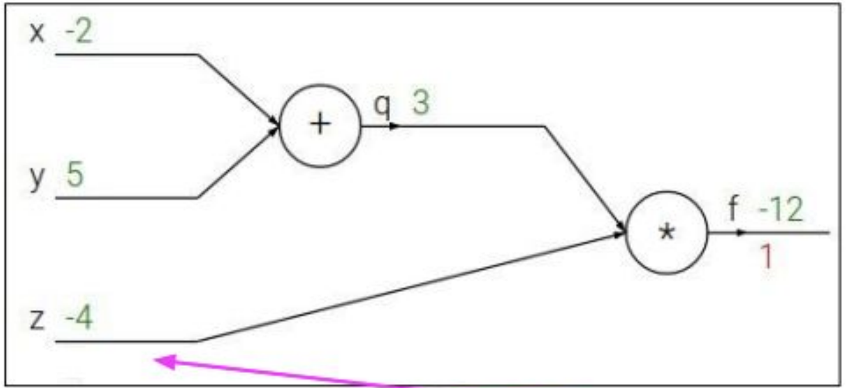

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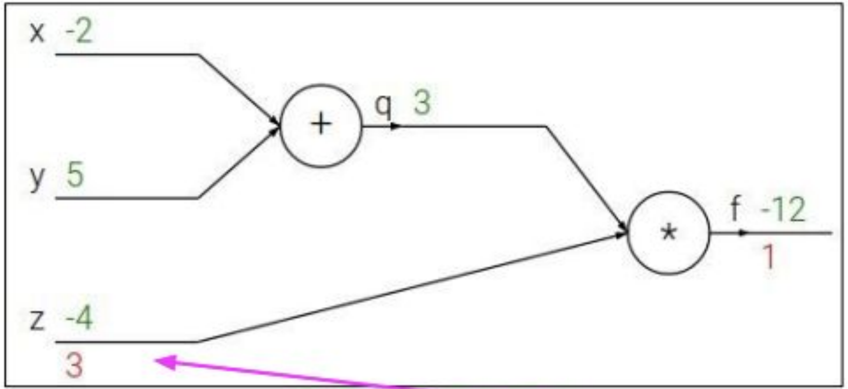

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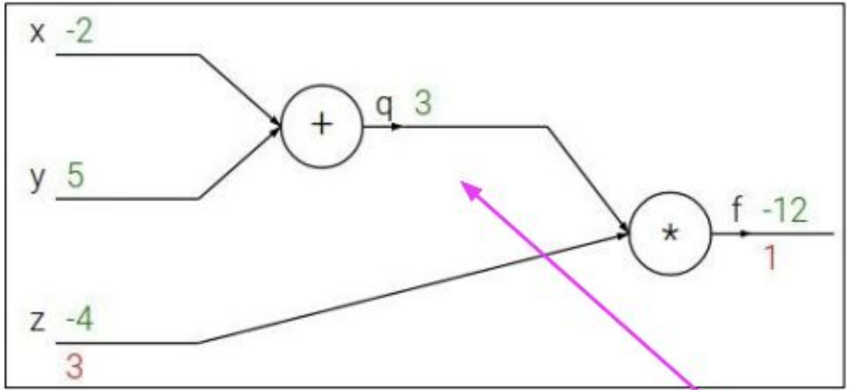

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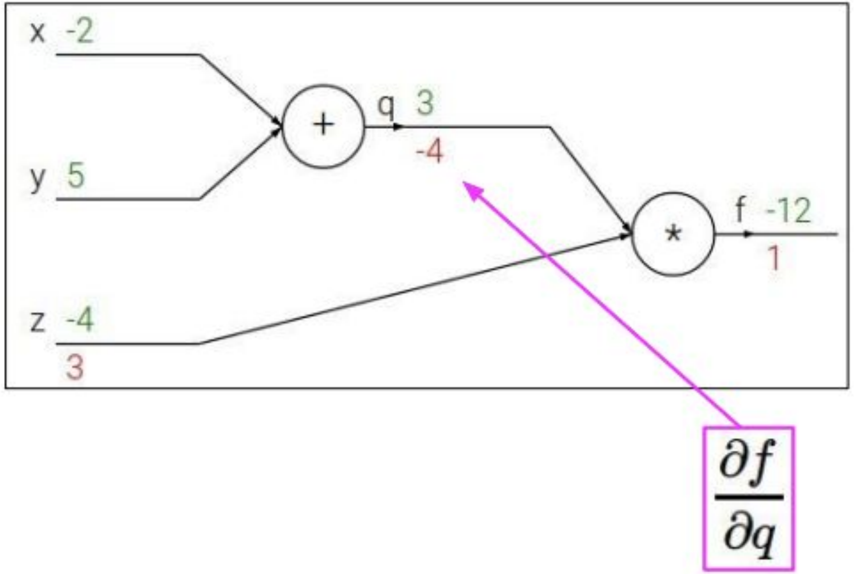

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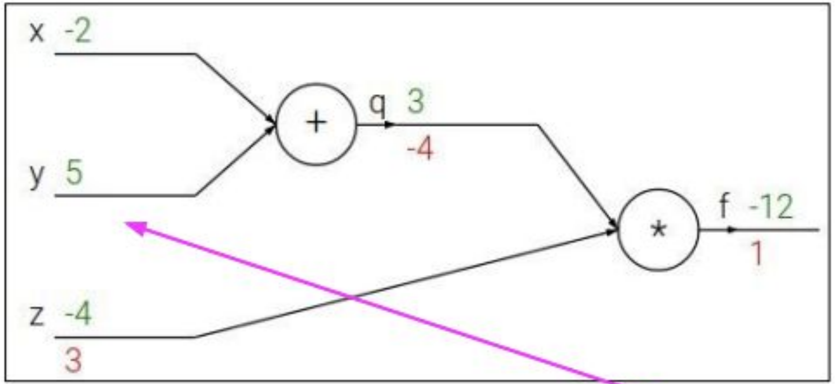

$\boxed{\frac{\partial f}{\partial y}}$

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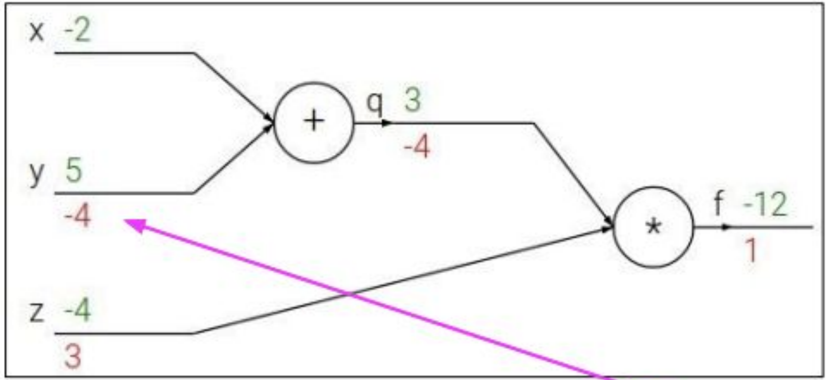

$$\frac{\partial f}{\partial y}$$

Chain rule:

$$\frac{\partial f}{\partial y} = \frac{\partial f}{\partial q} \frac{\partial q}{\partial y}$$

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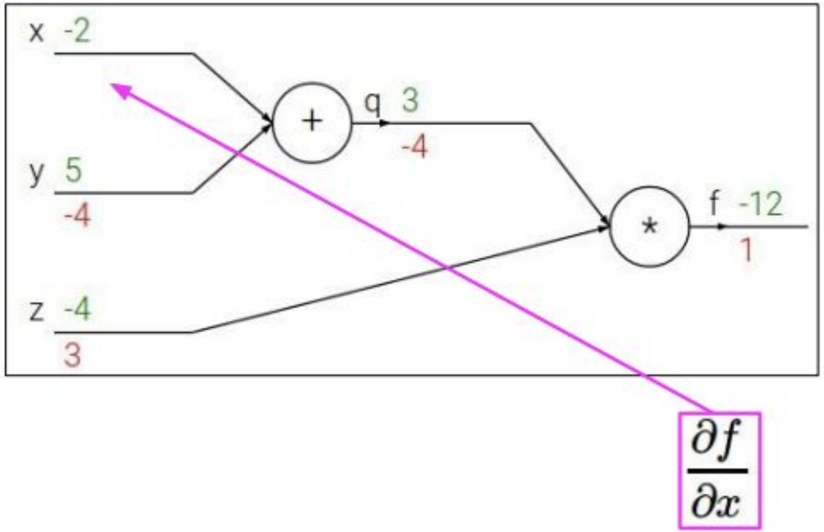

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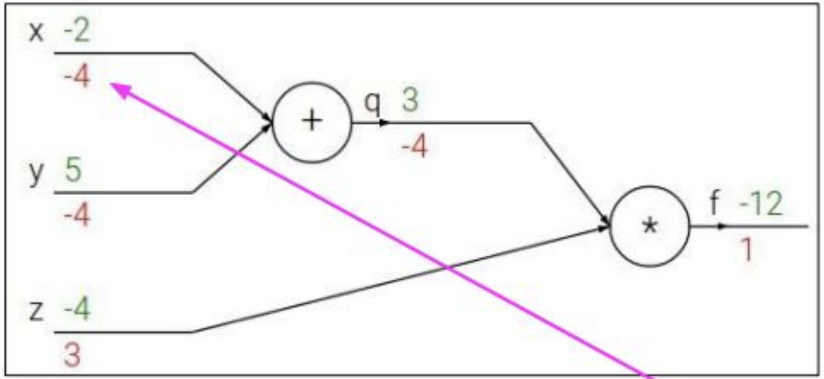

$$\frac{\partial f}{\partial x}$$

Chain rule:

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Backpropagation and chain rule

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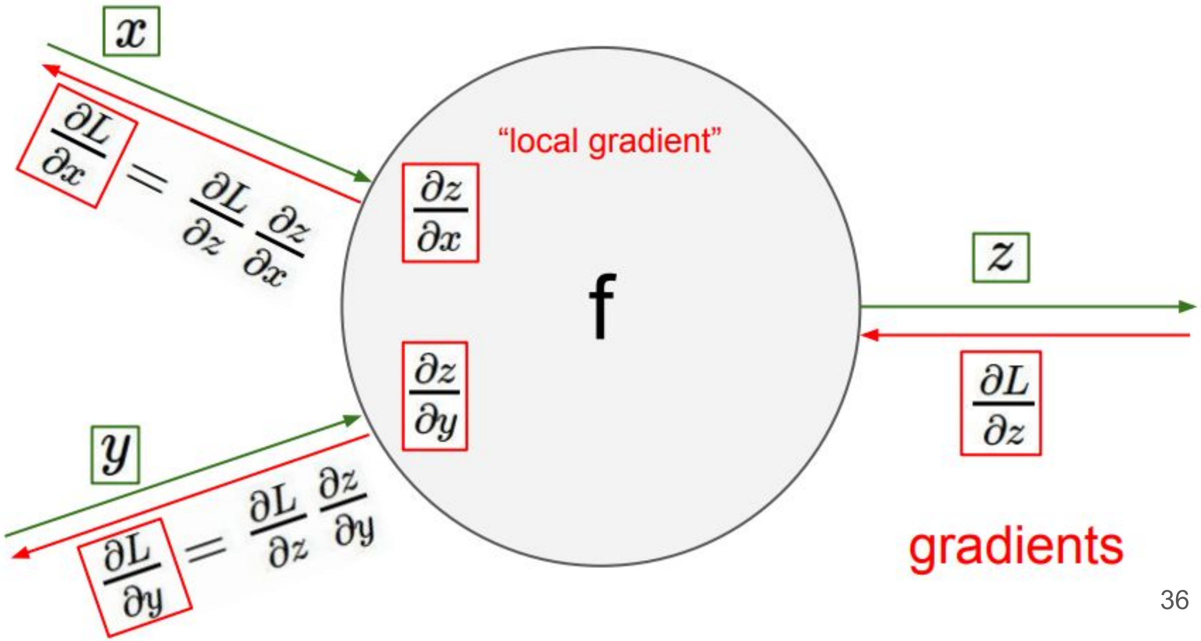
Backprop is just way to use it in NN training.



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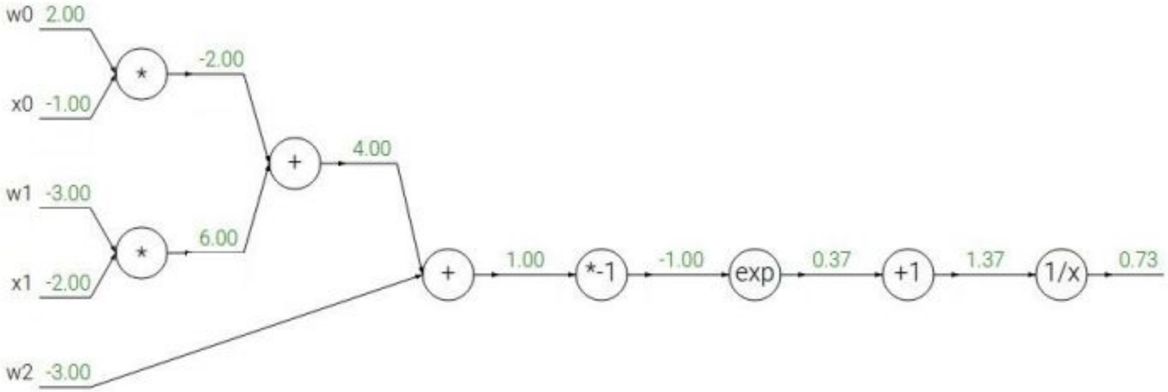
Backpropagation example

Another example: $f(w,x) = \frac{1}{1+e^{-(w_0x_0+w_1x_1+w_2)}}$



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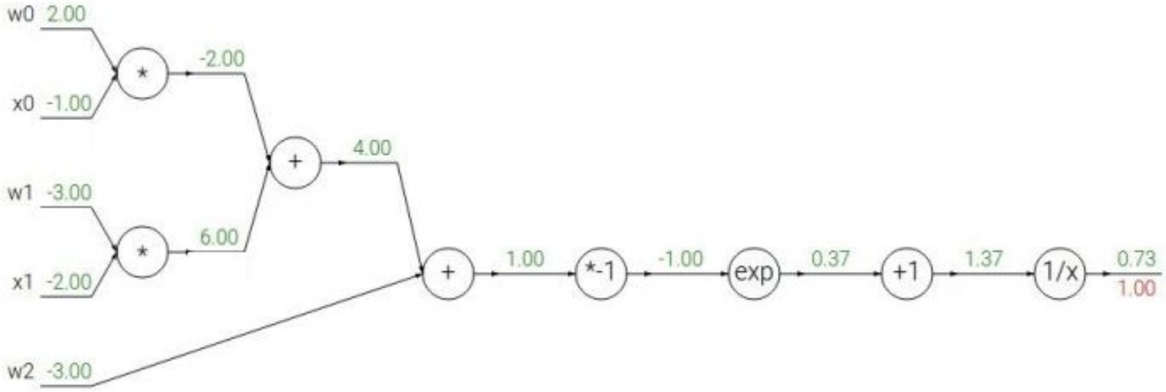

$$f(x) = e^x \quad \rightarrow \quad \frac{df}{dx} = e^x \qquad\qquad f(x) = \frac{1}{x} \quad \rightarrow \quad \frac{df}{dx} = -1/x^2$$

$$f_a(x) = ax \quad \rightarrow \quad \frac{df}{dx} = a \qquad\qquad f_c(x) = c + x \quad \rightarrow \quad \frac{df}{dx} = 1$$

source: http://cs231n.github.io

Backpropagation example

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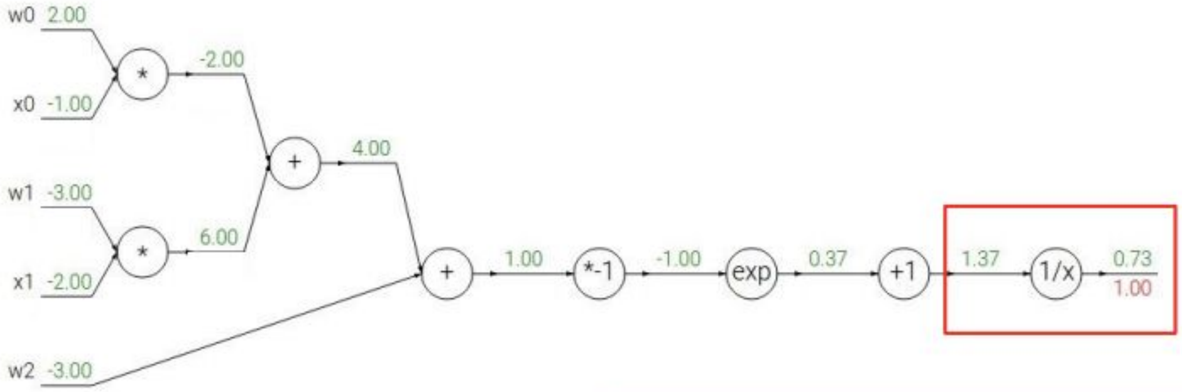

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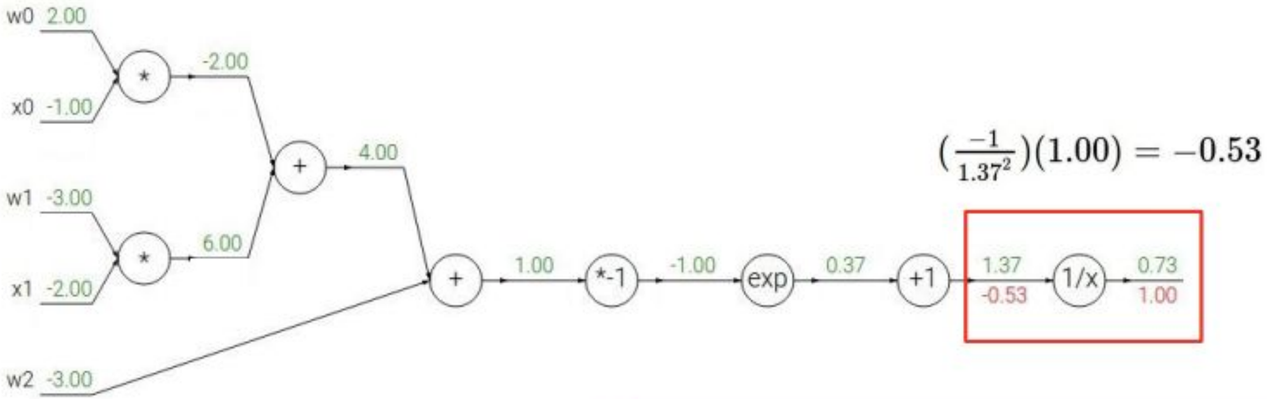

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$$\boxed{f(x) = \frac{1}{x} \quad \rightarrow \quad \frac{df}{dx} = -1/x^2}$$

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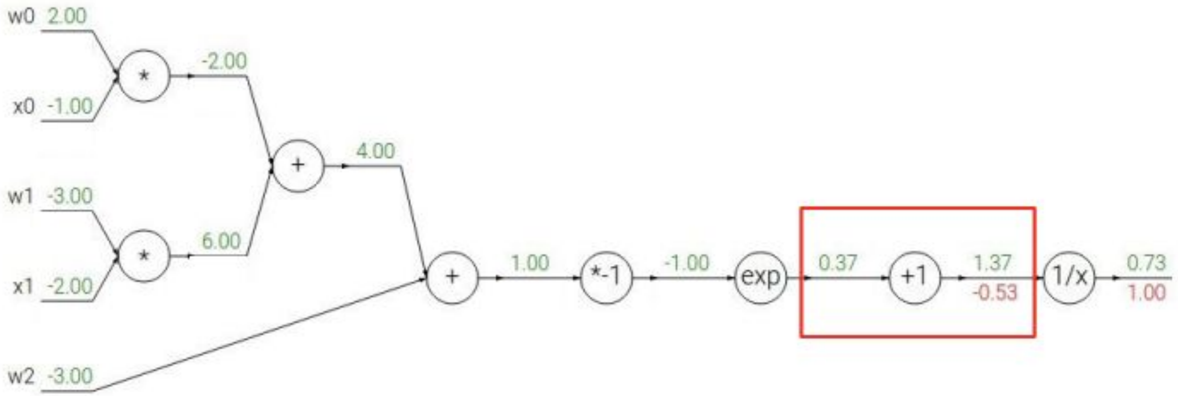
source: http://cs231n.github.io

Backpropagation example

Another example: $$f(w,x) = \frac{1}{1+e^{-(w_0x_0+w_1x_1+w_2)}}$$



$$f(x) = e^x \quad \rightarrow \quad \frac{df}{dx} = e^x$$

$$f_a(x) = ax \quad \rightarrow \quad \frac{df}{dx} = a$$

$$f(x) = \frac{1}{x} \quad \rightarrow \quad \frac{df}{dx} = -1/x^2$$

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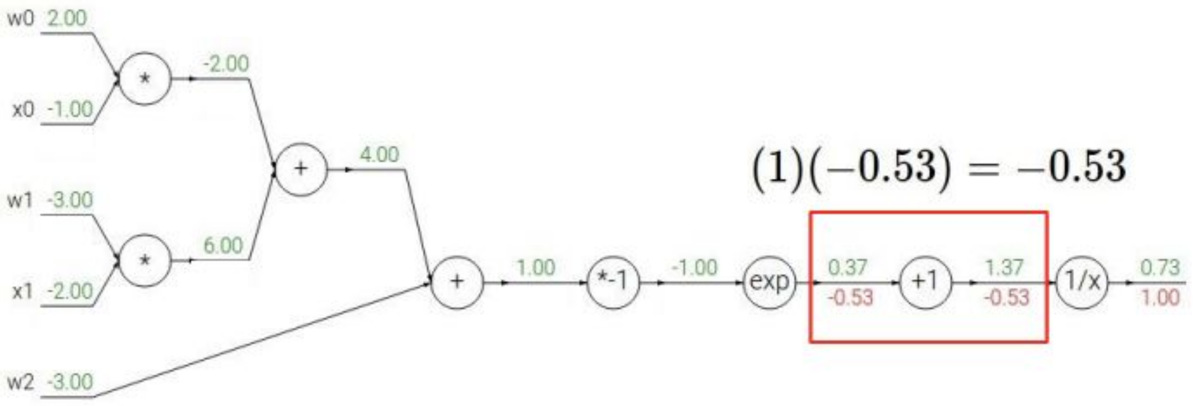

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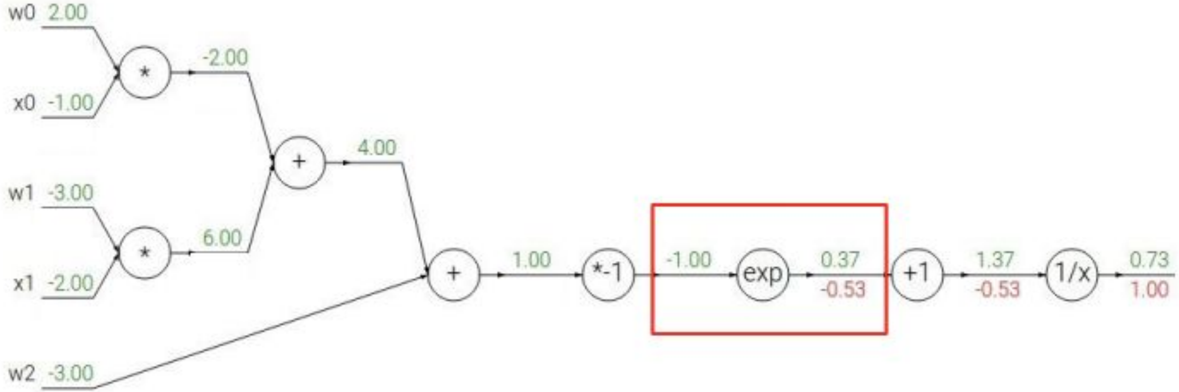

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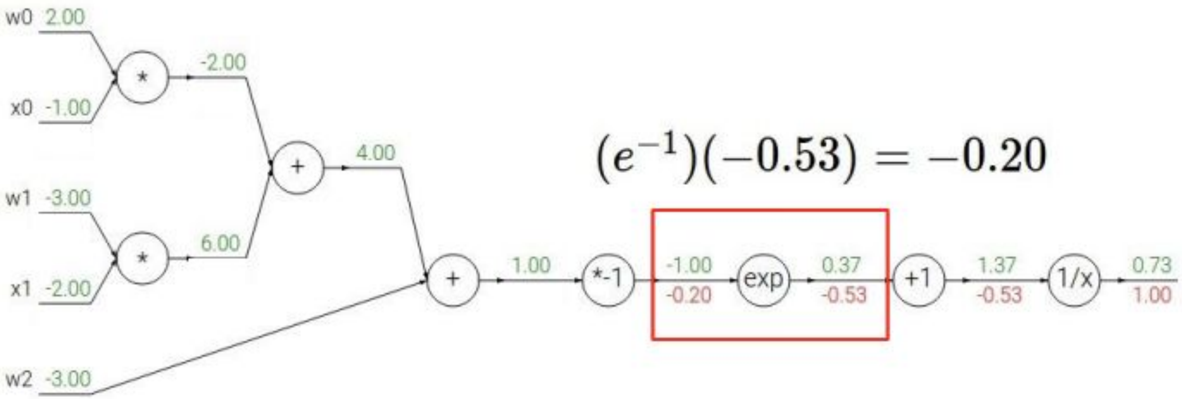

$$(e^{-1})(-0.53) = -0.20$$

$$\boxed{f(x) = e^x \quad \to \quad \frac{df}{dx} = e^x}$$

$$f_a(x) = ax \quad \to \quad \frac{df}{dx} = a$$

$$f(x) = \frac{1}{x} \quad \to \quad \frac{df}{dx} = -1/x^2$$

$$f_c(x) = c + x \quad \to \quad \frac{df}{dx} = 1$$

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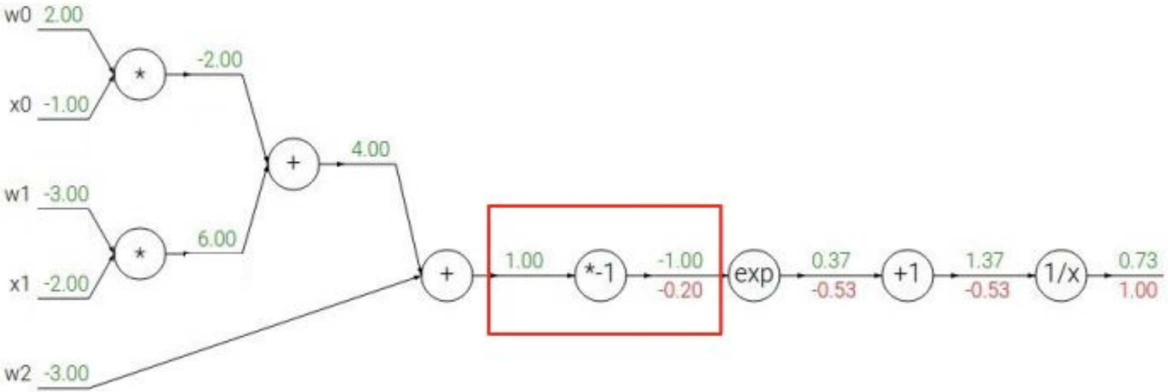

$$f(x)=e^x \quad\rightarrow\quad \frac{df}{dx}=e^x$$

$$\boxed{f_a(x)=ax \quad\rightarrow\quad \frac{df}{dx}=a}$$

$$f(x)=\frac{1}{x} \quad\rightarrow\quad \frac{df}{dx}=-1/x^2$$

$$f_c(x)=c+x \quad\rightarrow\quad \frac{df}{dx}=1$$

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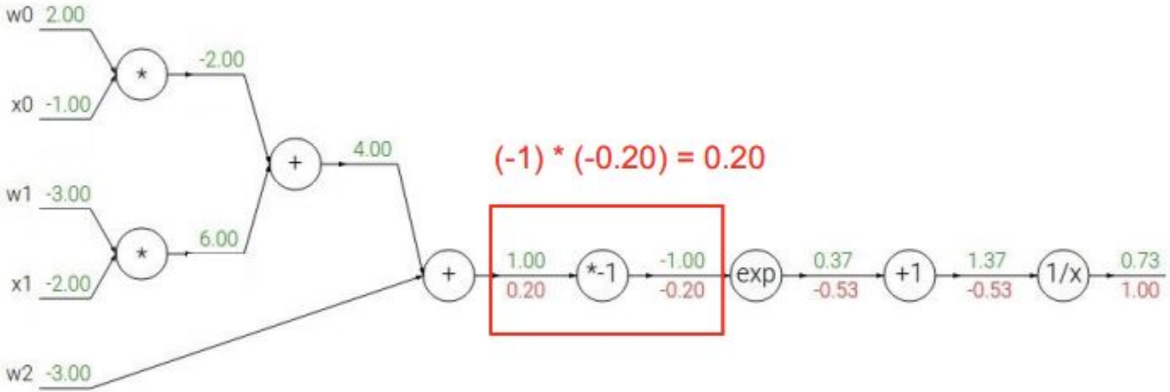

$$f(x) = e^x \quad \rightarrow \quad \frac{df}{dx} = e^x$$

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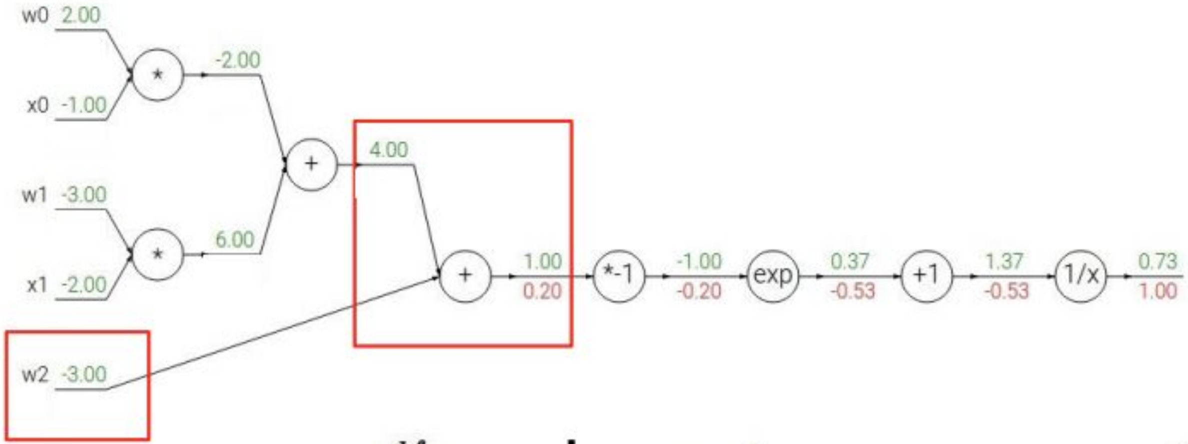

$f(x) = e^x \quad \rightarrow \quad \frac{df}{dx} = e^x$ | $f(x) = \frac{1}{x} \quad \rightarrow \quad \frac{df}{dx} = -1/x^2$

$f_a(x) = ax \quad \rightarrow \quad \frac{df}{dx} = a$ | $f_c(x) = c + x \quad \rightarrow \quad \frac{df}{dx} = 1$

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Backpropagation example

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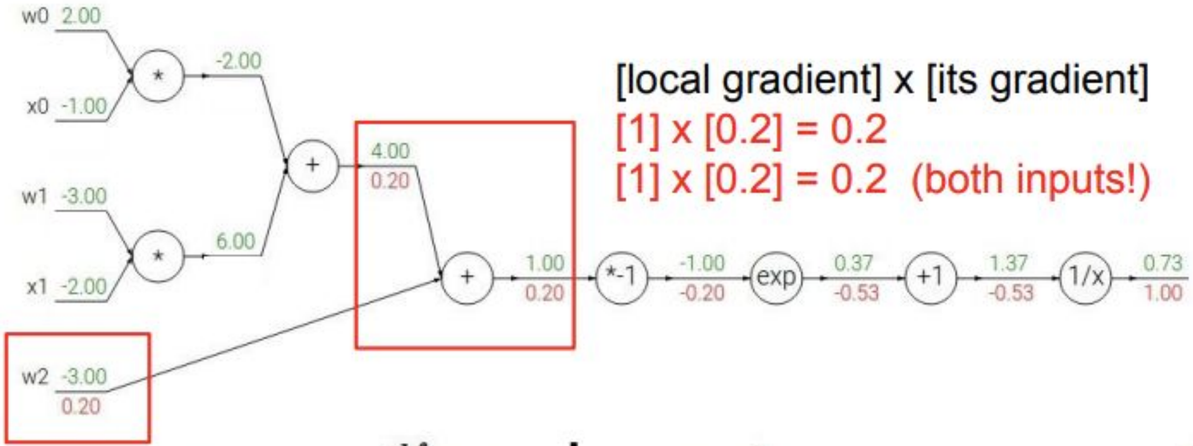


[local gradient] x [its gradient]

[1] x [0.2] = 0.2

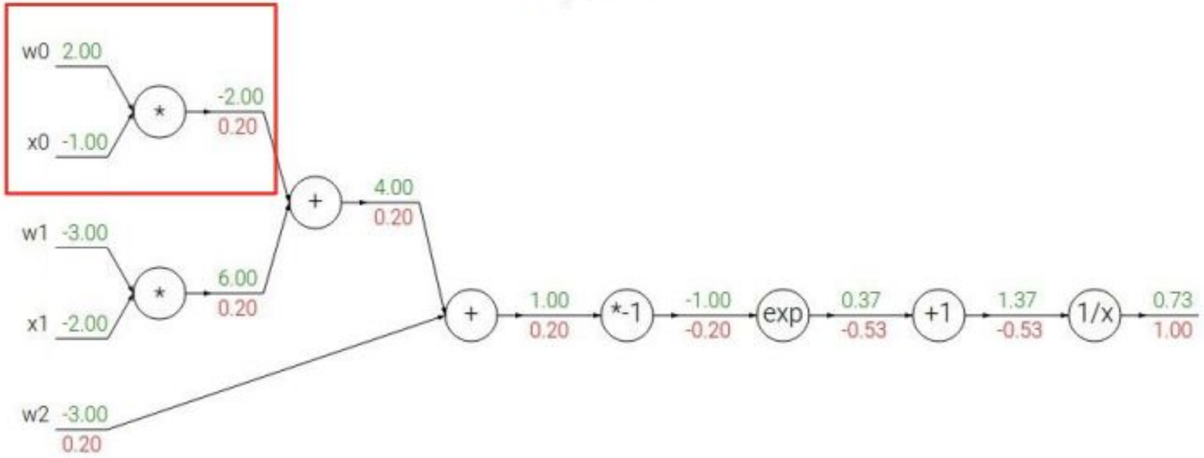
[1] x [0.2] = 0.2 (both inputs!)

$f(x) = e^x \quad \rightarrow \quad \frac{df}{dx} = e^x$ | $f(x) = \frac{1}{x} \quad \rightarrow \quad \frac{df}{dx} = -1/x^2$

$f_a(x) = ax \quad \rightarrow \quad \frac{df}{dx} = a$ | $f_c(x) = c + x \quad \rightarrow \quad \frac{df}{dx} = 1$

source: http://cs231n.github.io

Another example: $$f(w,x) = \frac{1}{1+e^{-(w_0x_0+w_1x_1+w_2)}}$$



$$f(x) = e^x \quad \rightarrow \quad \frac{df}{dx} = e^x$$

$$f_a(x) = ax \quad \rightarrow \quad \frac{df}{dx} = a$$

$$f(x) = \frac{1}{x} \quad \rightarrow \quad \frac{df}{dx} = -1/x^2$$

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Backpropagation example

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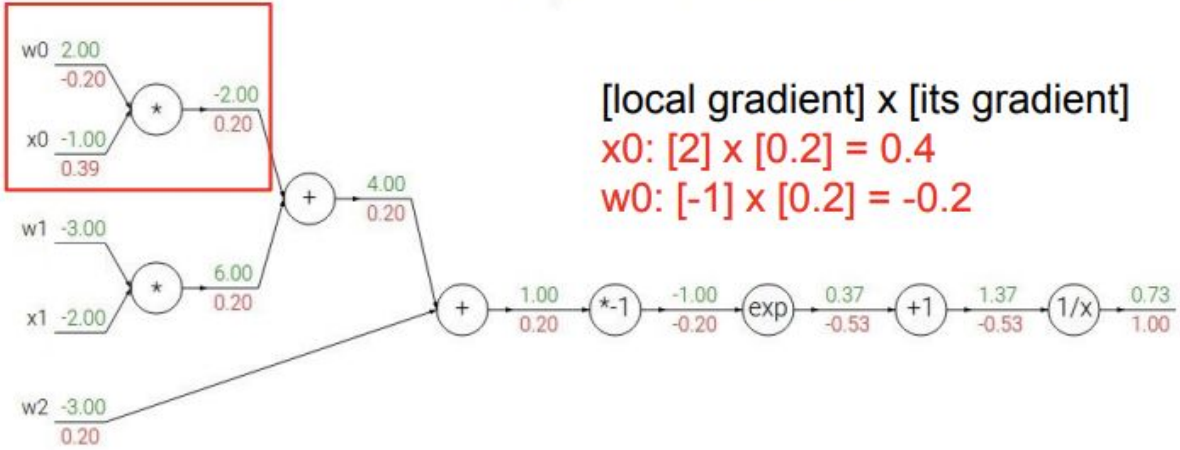


[local gradient] x [its gradient]

x0: [2] x [0.2] = 0.4

w0: [-1] x [0.2] = -0.2

$$f(x) = e^x \quad \rightarrow \quad \frac{df}{dx} = e^x$$

$$f_a(x) = ax \quad \rightarrow \quad \frac{df}{dx} = a$$

$$f(x) = \frac{1}{x} \quad \rightarrow \quad \frac{df}{dx} = -1/x^2$$

$$f_c(x) = c + x \quad \rightarrow \quad \frac{df}{dx} = 1$$

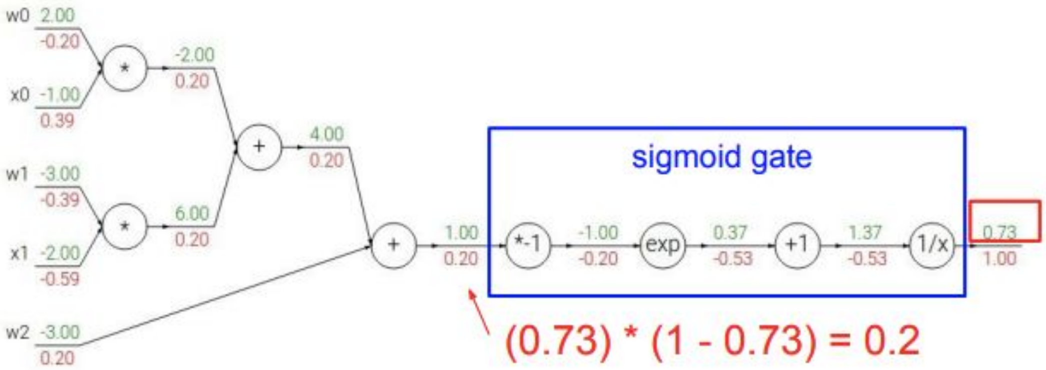
source: http://cs231n.github.io

Backpropagation example

$$f(w,x) = \frac{1}{1+e^{-(w_0x_0+w_1x_1+w_2)}}$$

$$\boxed{\sigma(x) = \frac{1}{1+e^{-x}}}$$ sigmoid function

$$\frac{d\sigma(x)}{dx} = \frac{e^{-x}}{(1+e^{-x})^2} = \left(\frac{1+e^{-x}-1}{1+e^{-x}}\right)\left(\frac{1}{1+e^{-x}}\right) = (1-\sigma(x))\,\sigma(x)$$

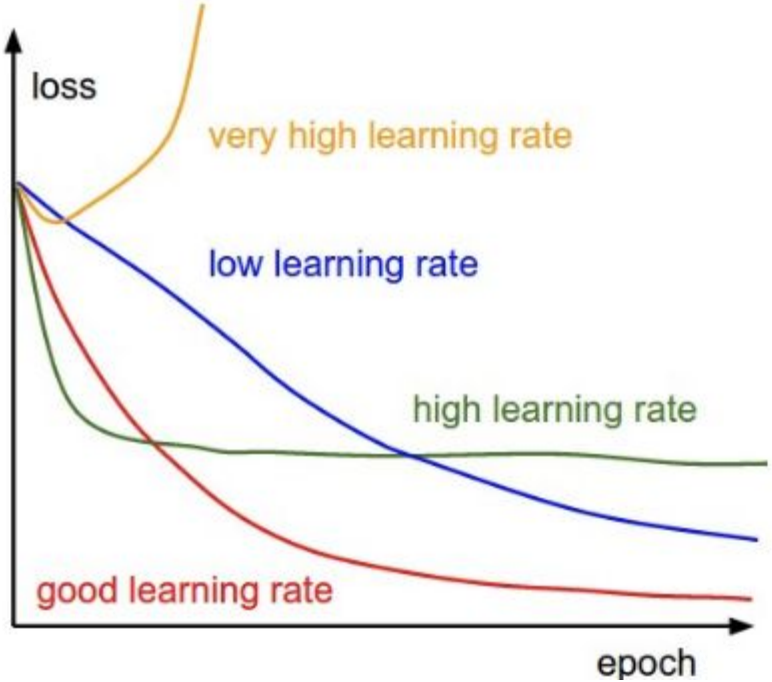


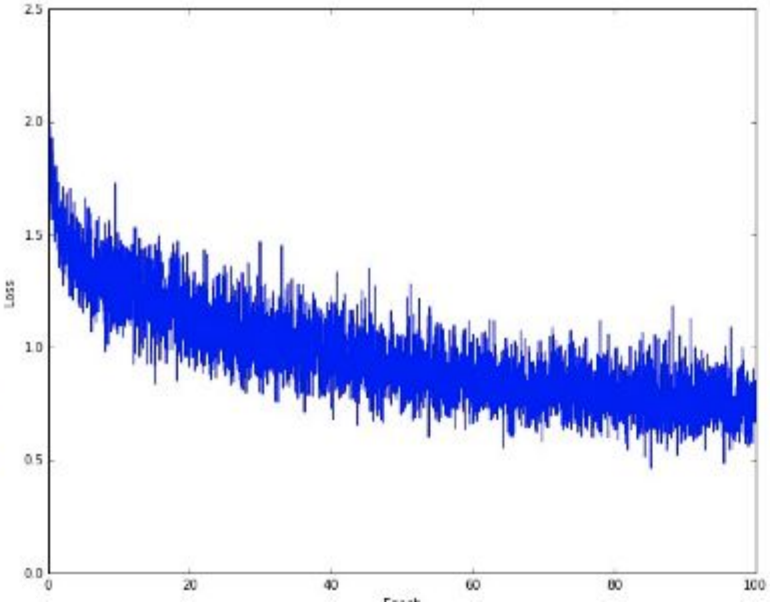
source: http://cs231n.github.io

Gradient optimization

Stochastic gradient descent (and variations) is used to optimize NN parameters.

$$x_{t+1} = x_t - \text{learning rate} \cdot dx$$





source: http://cs231n.github.io/neural-networks-3/

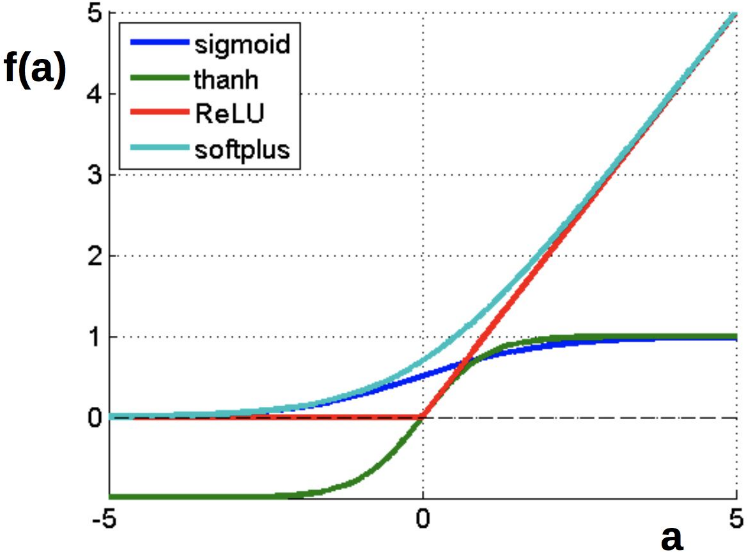
Once more: nonlinearities

$$f(a) = \frac{1}{1+e^{a}}$$

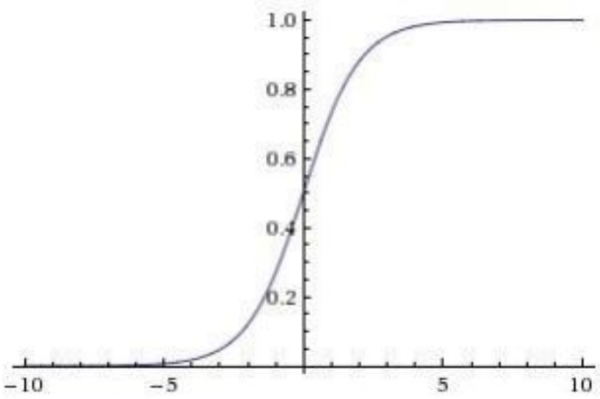
$$f(a) = \tanh(a)$$

$$f(a) = \max(0, a)$$

$$f(a) = \log(1+e^{a})$$



Activation functions



Sigmoid

$$f(a) = \frac{1}{1+e^{a}}$$

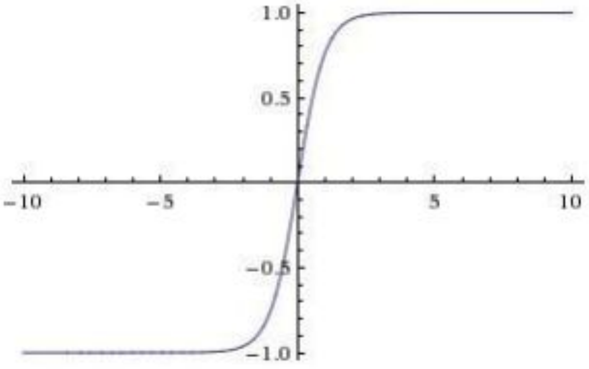
- Squashes numbers to range [0,1]
- Historically popular since they have nice interpretation as a saturating "firing rate" of a neuron

3 problems:

1. Saturated neurons "kill" the gradients
2. Sigmoid outputs are not zero-centered
3. exp() is a bit compute expensive

source: http://cs231n.github.io

Activation functions



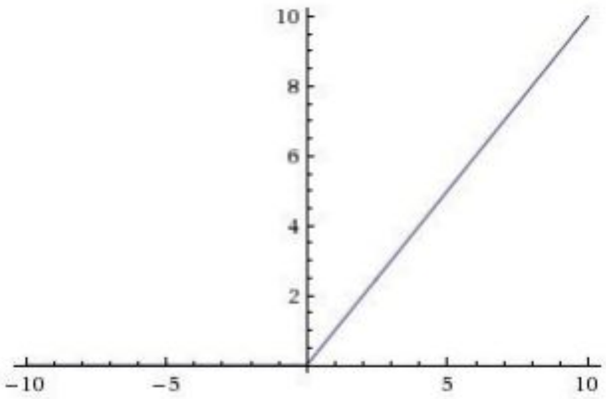
tanh(x)

- Squashes numbers to range [-1,1]
- zero centered (nice)
- still kills gradients when saturated :(

$$f(a) = \tanh(a)$$

source: http://cs231n.github.io

Activation functions



ReLU
(Rectified Linear Unit)

$$f(a) = \max(0, a)$$

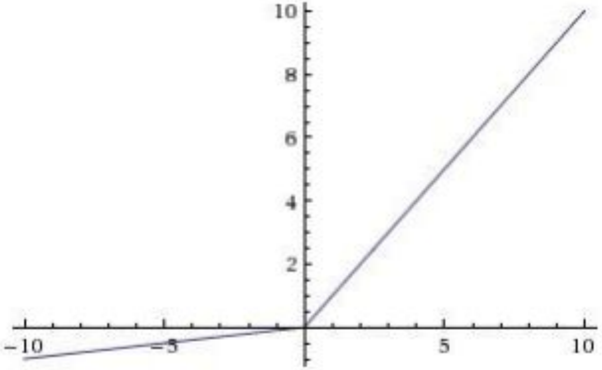
- Does not saturate (in +region)
- Very computationally efficient
- Converges much faster than sigmoid/tanh in practice (e.g. 6x)

- Not zero-centered output
- An annoyance:

hint: what is the gradient when x < 0?

source: http://cs231n.github.io

Activation functions



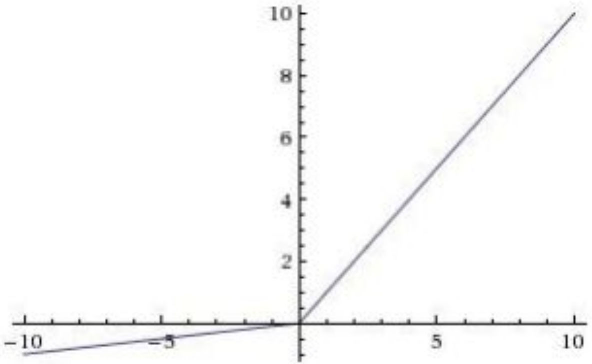
- Does not saturate
- Computationally efficient
- Converges much faster than sigmoid/tanh in practice! (e.g. 6x)
- **will not “die”.**

Leaky ReLU

$$f(x) = \max(0.01x, x)$$

source: http://cs231n.github.io

Activation functions



- Does not saturate
- Computationally efficient
- Converges much faster than sigmoid/tanh in practice! (e.g. 6x)
- **will not “die”.**

Leaky ReLU

$$f(x) = \max(0.01x, x)$$

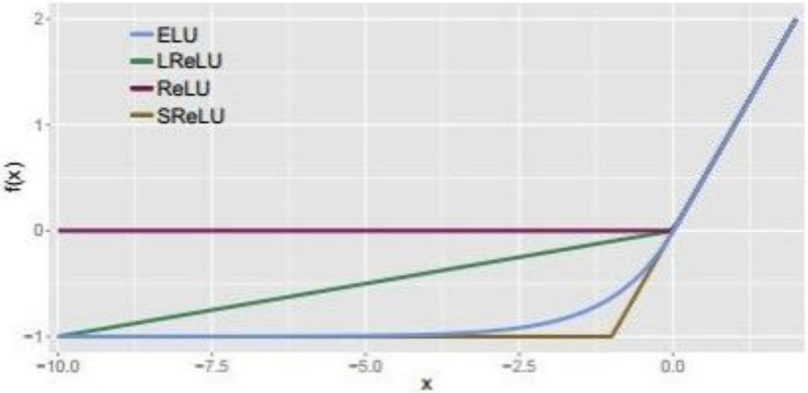
Parametric Rectifier (PReLU)

$$f(x) = \max(\alpha x, x)$$

backprop into \alpha
(parameter)

source: http://cs231n.github.io

Exponential Linear Units (ELU)



$$f(x) = \begin{cases} x & \text{if } x > 0 \\ \alpha\,(\exp(x) - 1) & \text{if } x \le 0 \end{cases}$$

- All benefits of ReLU
- Does not die
- Closer to zero mean outputs

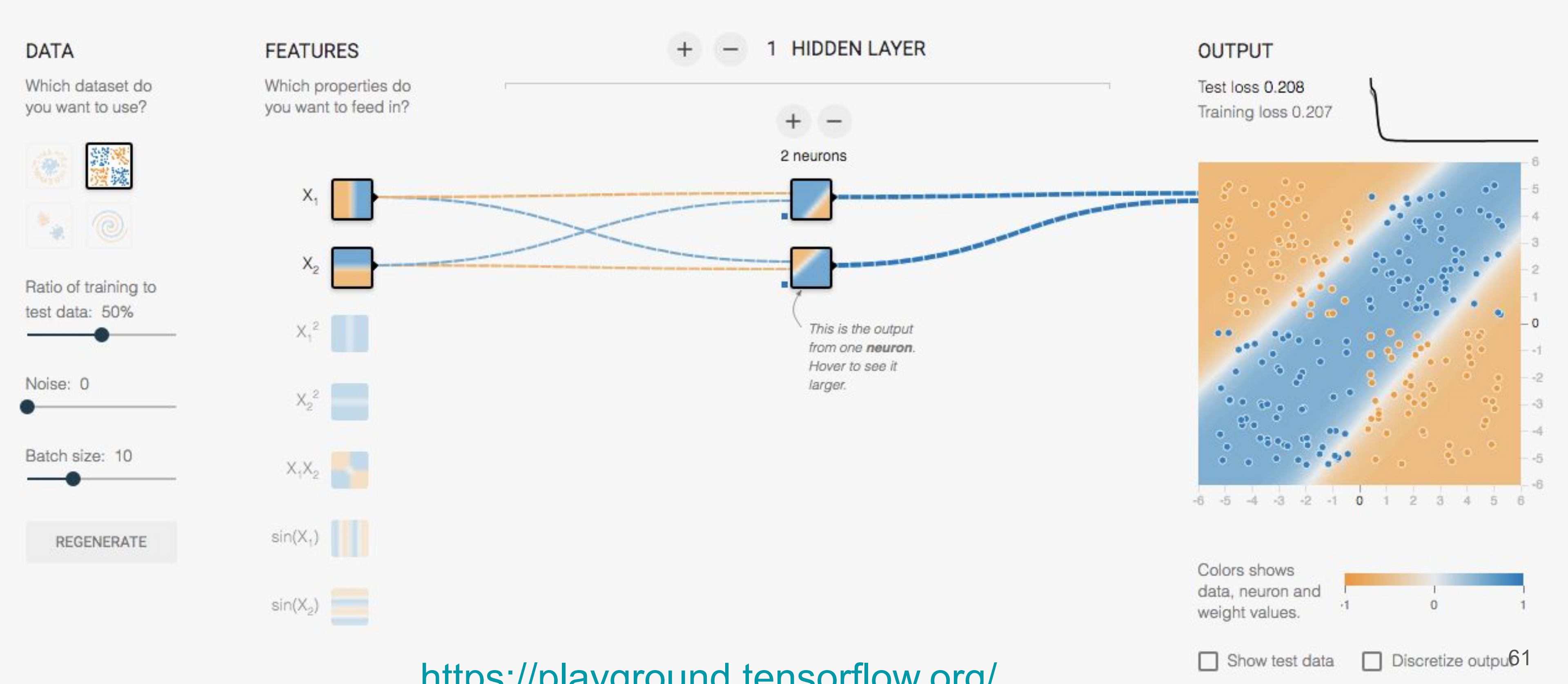
- Computation requires exp()

source: http://cs231n.github.io

Activation functions: sum up

- Use ReLU as baseline approach
- Be careful with the learning rates
- Try out Leaky ReLU or ELU
- Try out tanh but do not expect much from it
- Do not use Sigmoid

Don’t miss the interactive playground



https://playground.tensorflow.org/

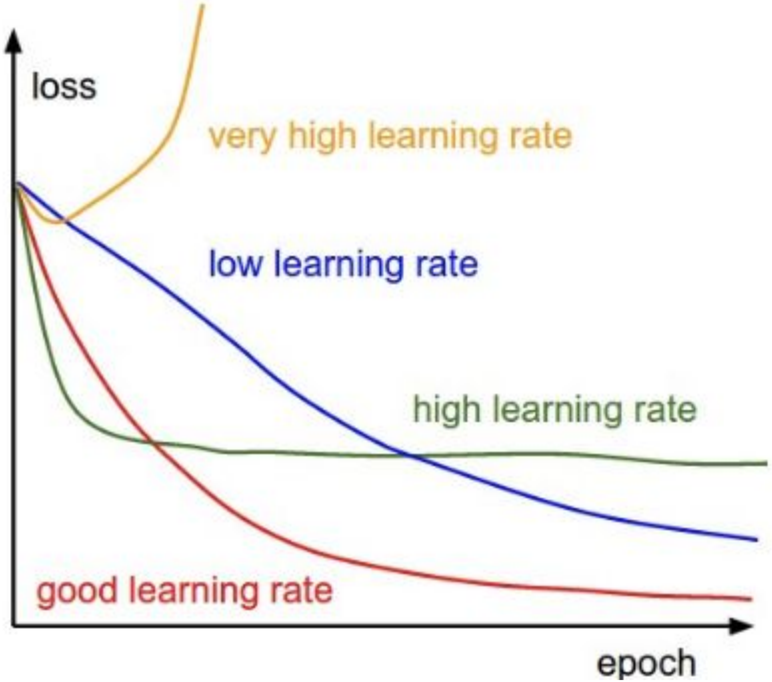

WHO'S AWESOME?
You're Awesome

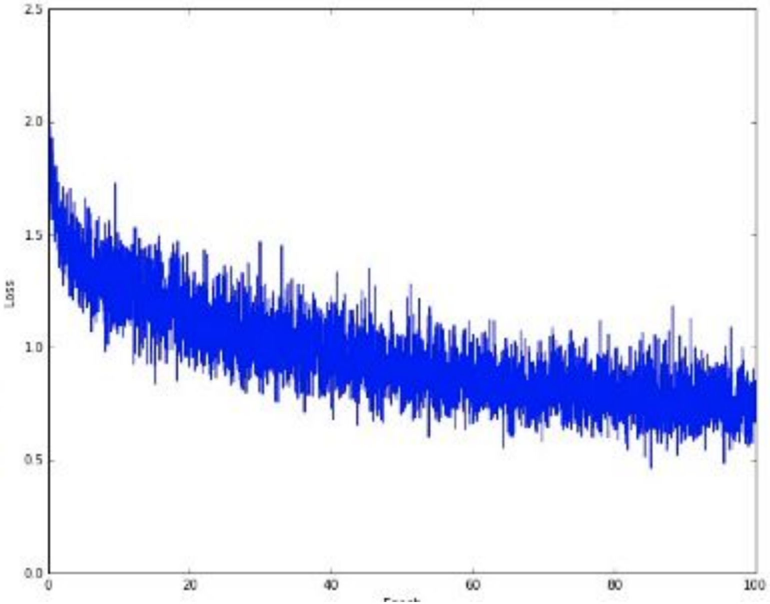
Backup

Optimizers

Stochastic gradient descent is used to optimize NN parameters.

$$x_{t+1} = x_t - \text{learning rate} \cdot dx$$



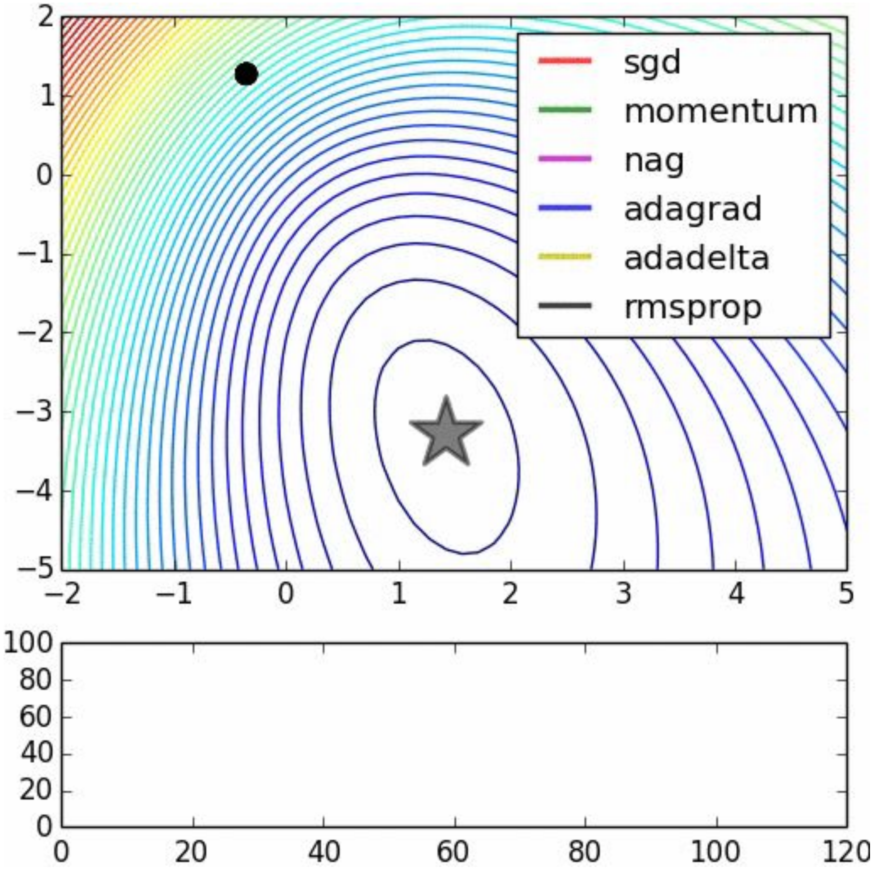


source: http://cs231n.github.io/neural-networks-3/

Optimizers

There are much more optimizers:

- Momentum
- Adagrad
- Adadelta
- RMSprop
- Adam
- …
- even other NNs

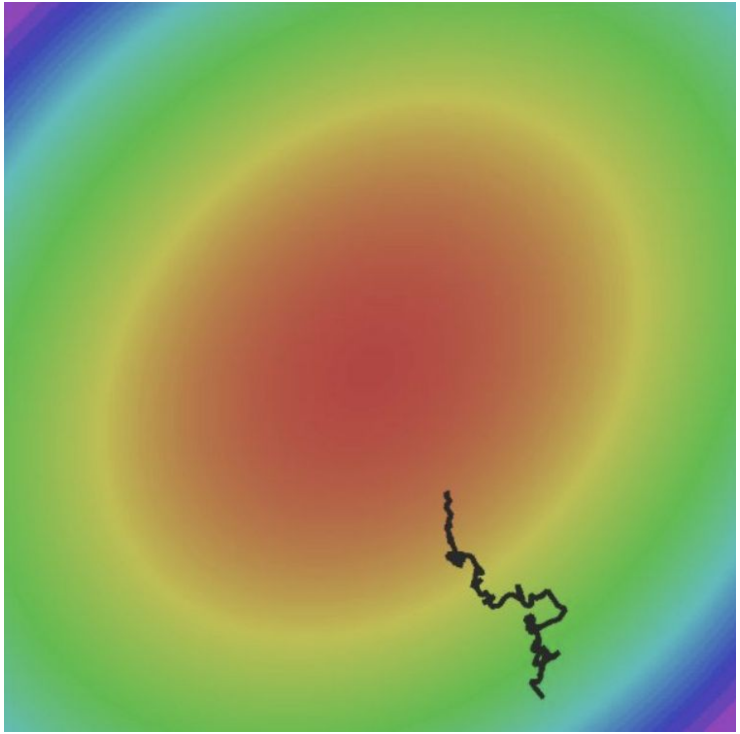


Optimization: SGD

$$L(W) = \frac{1}{N}\sum_{i=1}^{N} L_i(x_i, y_i, W)$$

$$\nabla_W L(W) = \frac{1}{N}\sum_{i=1}^{N} \nabla_W L_i(x_i, y_i, W)$$

Averaging over minibatches ---> noisy gradient



source: http://cs231n.stanford.edu/slides/2017/cs231n_2017_lecture7.pdf

First idea: momentum

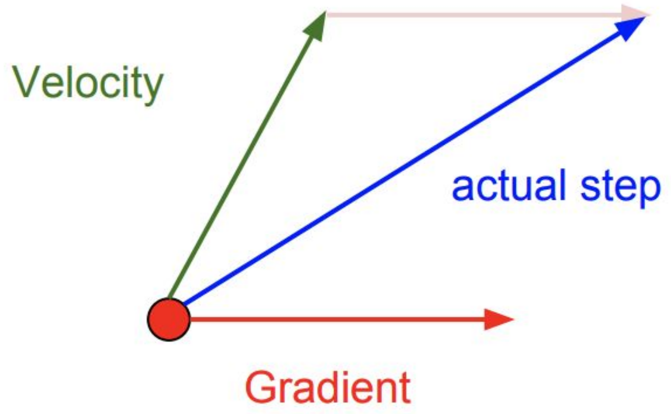
Simple SGD

$$x_{t+1} = x_t - \alpha \nabla f(x_t)$$

SGD with momentum

$$v_{t+1} = \rho v_t + \nabla f(x_t)$$
$$x_{t+1} = x_t - \alpha v_{t+1}$$

Momentum update:



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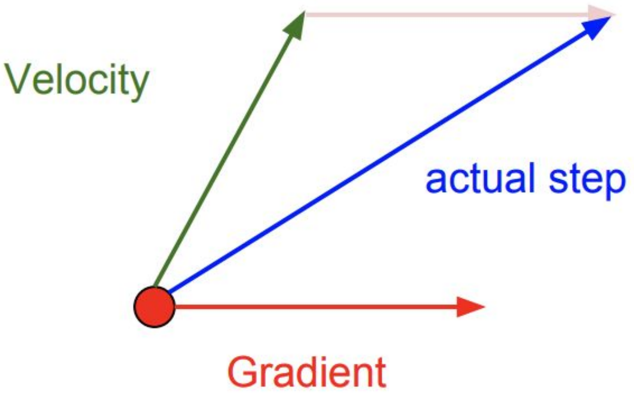
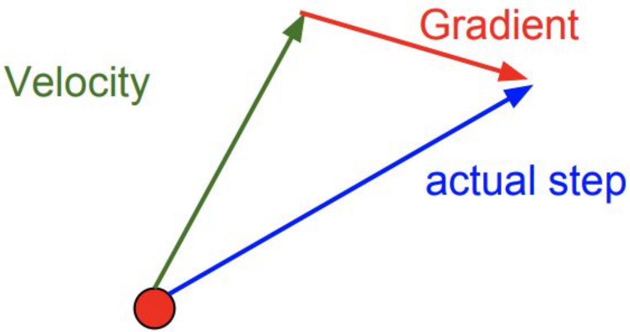
Nesterov momentum

Momentum update:



$$v_{t+1} = \rho v_t + \nabla f(x_t)$$
$$x_{t+1} = x_t - \alpha v_{t+1}$$

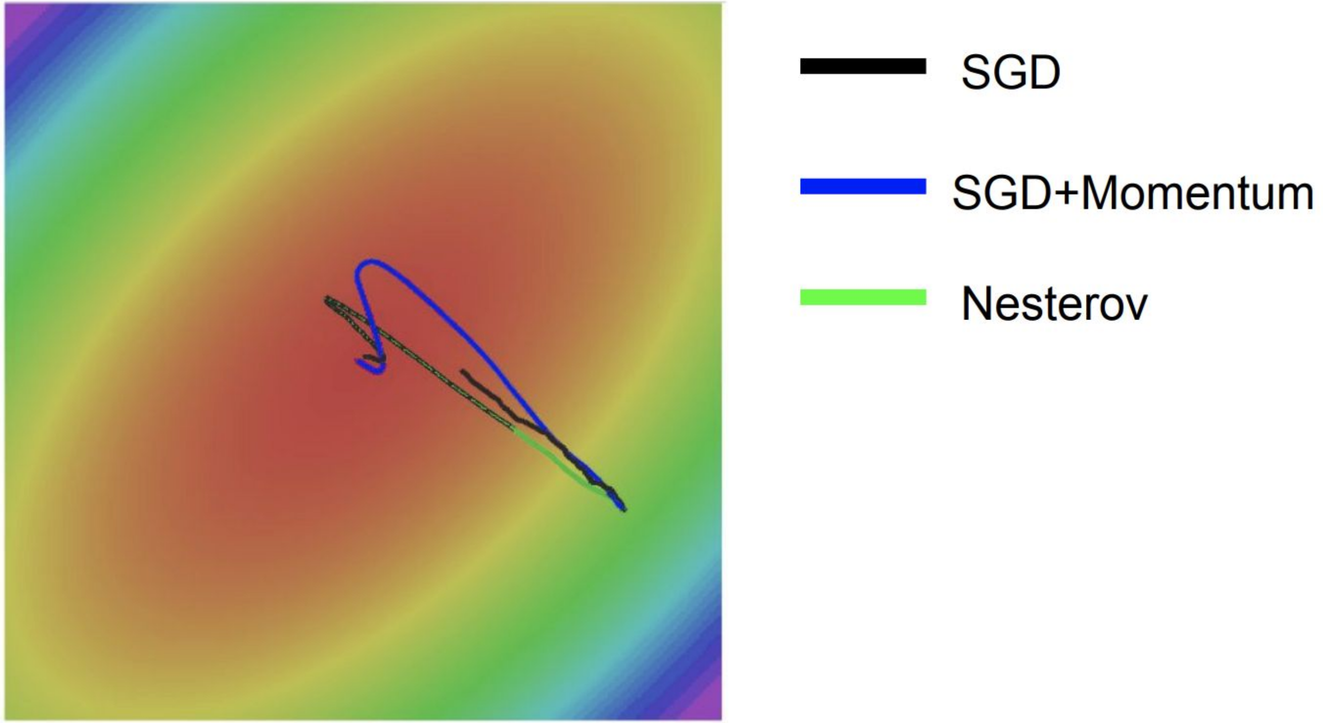
Nesterov Momentum



$$v_{t+1} = \rho v_t - \alpha \nabla f(\boxed{x_t + \rho v_t})$$
$$x_{t+1} = x_t + v_{t+1}$$

source: http://cs231n.stanford.edu/slides/2017/cs231n_2017_lecture7.pdf

Comparing momentums



source: http://cs231n.stanford.edu/slides/2017/cs231n_2017_lecture7.pdf

Second idea: different dimensions are different

Adagrad: SGD with cache

$$\text{cache}_{t+1} = \text{cache}_t + (\nabla f(x_t))^2$$

$$x_{t+1} = x_t - \alpha \frac{\nabla f(x_t)}{\text{cache}_{t+1} + \varepsilon}$$

Second idea: different dimensions are different

Adagrad: SGD with cache

$$\text{cache}_{t+1} = \text{cache}_t + (\nabla f(x_t))^2$$
$$x_{t+1} = x_t - \alpha \frac{\nabla f(x_t)}{\text{cache}_{t+1} + \varepsilon}$$

Problem: gradient fades with time

Second idea: different dimensions are different

Adagrad: SGD with cache

$$\text{cache}_{t+1} = \text{cache}_t + (\nabla f(x_t))^2$$

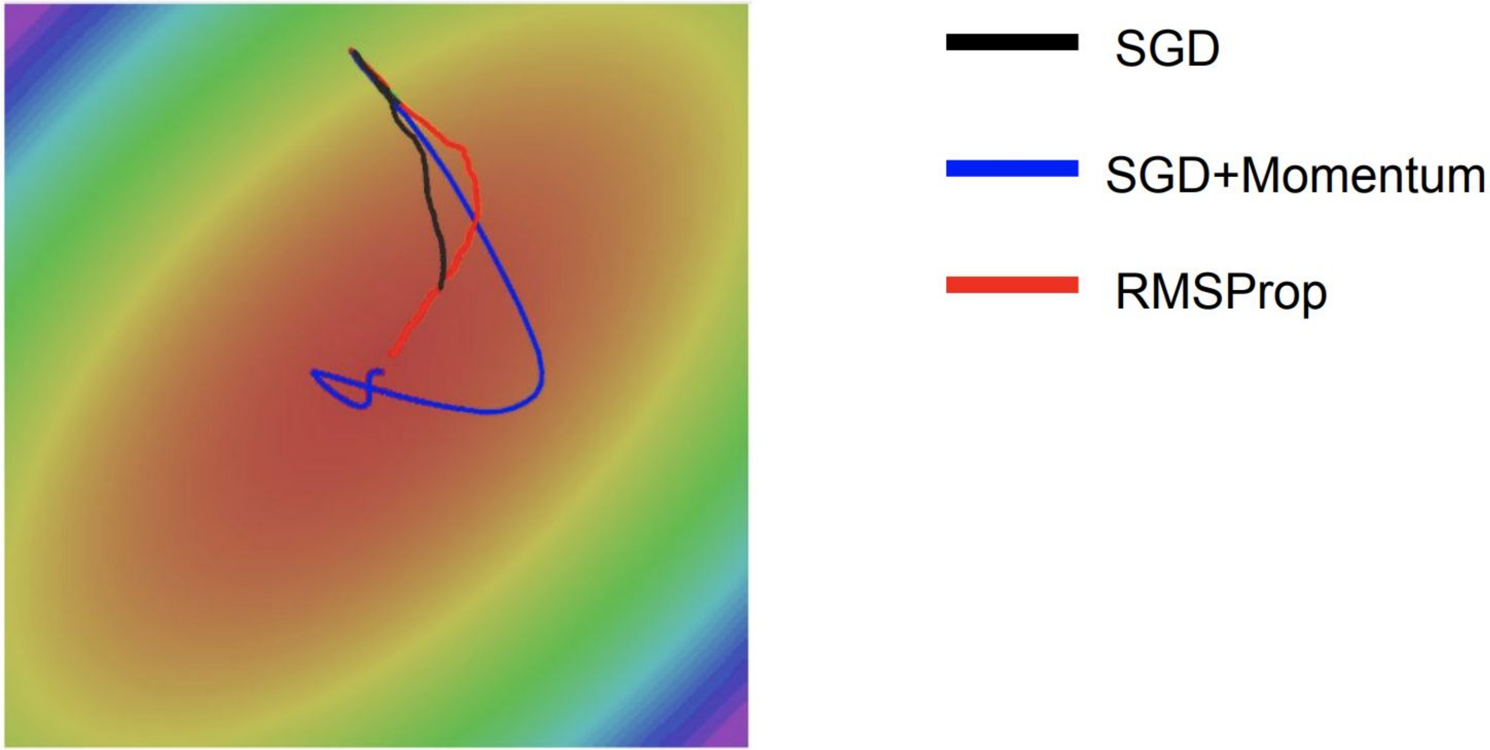
$$x_{t+1} = x_t - \alpha \frac{\nabla f(x_t)}{\text{cache}_{t+1} + \varepsilon}$$

RMSProp: SGD with cache with exp. Smoothing

$$\text{cache}_{t+1} = \beta \text{cache}_t + (1 - \beta)(\nabla f(x_t))^2$$

$$x_{t+1} = x_t - \alpha \frac{\nabla f(x_t)}{\text{cache}_{t+1} + \varepsilon}$$

Slide 29 Lecture 6 of Geoff Hinton's Coursera class
http://www.cs.toronto.edu/~tijmen/csc321/slides/lecture_slides_lec6.pdf



source: http://cs231n.stanford.edu/slides/2017/cs231n_2017_lecture7.pdf

Adam

Let’s combine the momentum idea and RMSProp normalization:

$$v_{t+1} = \gamma v_t + (1-\gamma)\nabla f(x_t)$$

$$\text{cache}_{t+1} = \beta \text{cache}_t + (1-\beta)(\nabla f(x_t))^2$$

$$x_{t+1} = x_t - \alpha \frac{v_{t+1}}{\text{cache}_{t+1} + \varepsilon}$$

Adam full form involves bias correction term. See http://cs231n.github.io/neural-networks-3/ for more info.

Adam

Let's combine the momentum idea and RMSProp normalization:

$$v_{t+1} = \gamma v_t + (1-\gamma)\nabla f(x_t)$$

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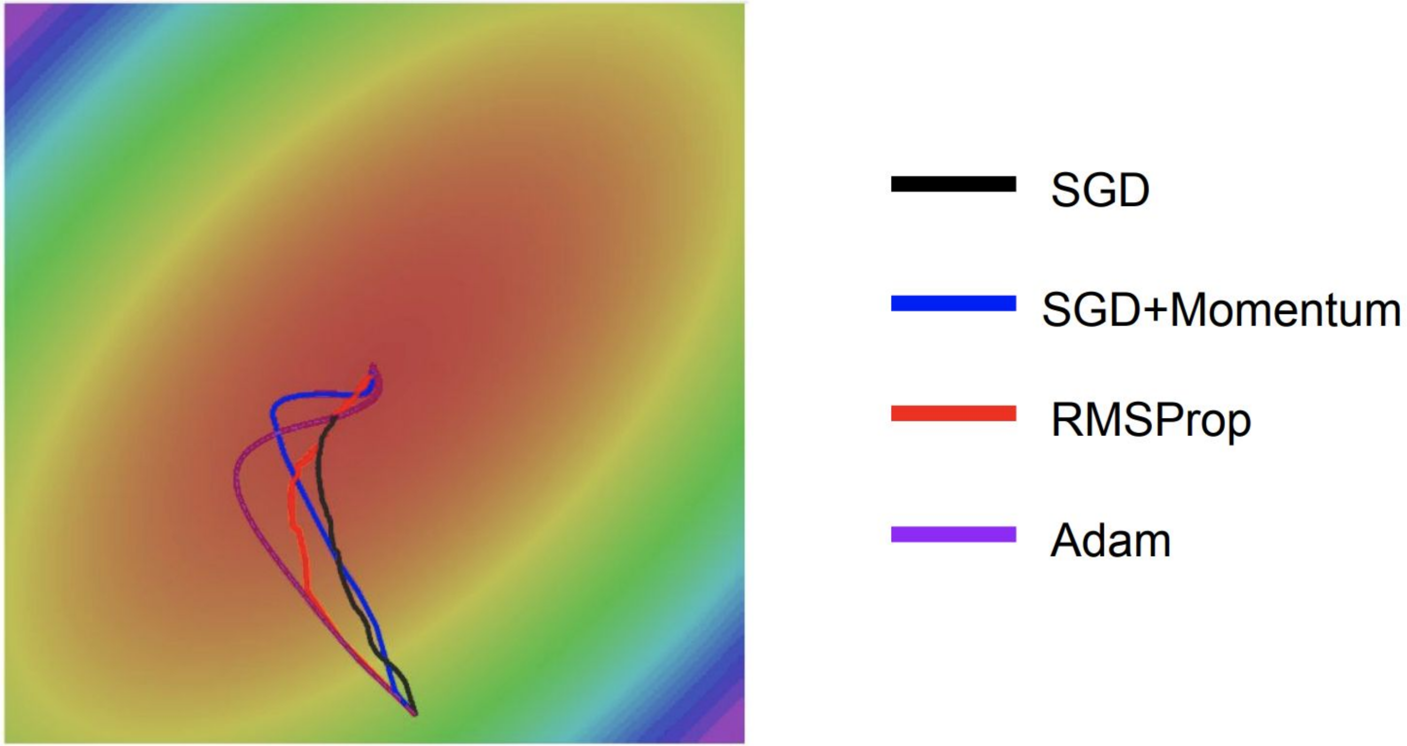
$$x_{t+1} = x_t - \alpha \frac{v_{t+1}}{\text{cache}_{t+1} + \varepsilon}$$

Actually, that's not quite Adam.

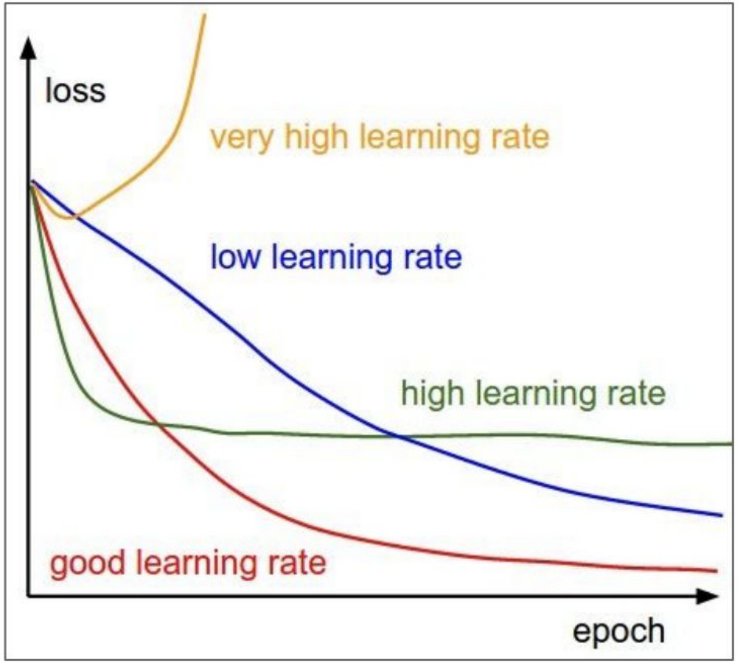
Adam full form involves bias correction term. See http://cs231n.github.io/neural-networks-3/ for more info.

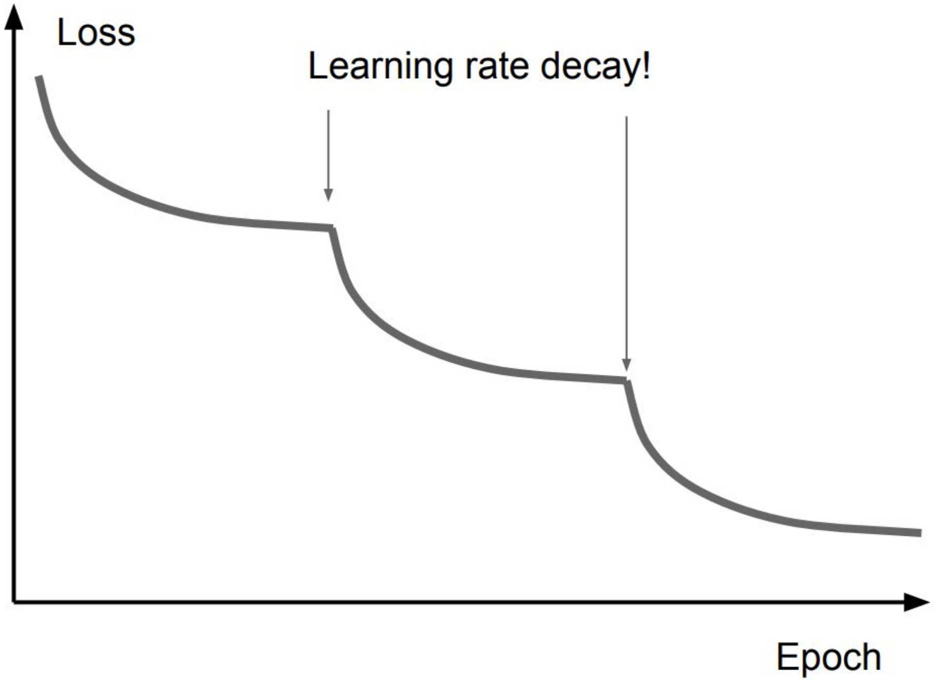
Comparing optimizers



source: http://cs231n.stanford.edu/slides/2017/cs231n_2017_lecture7.pdf

Once more: learning rate

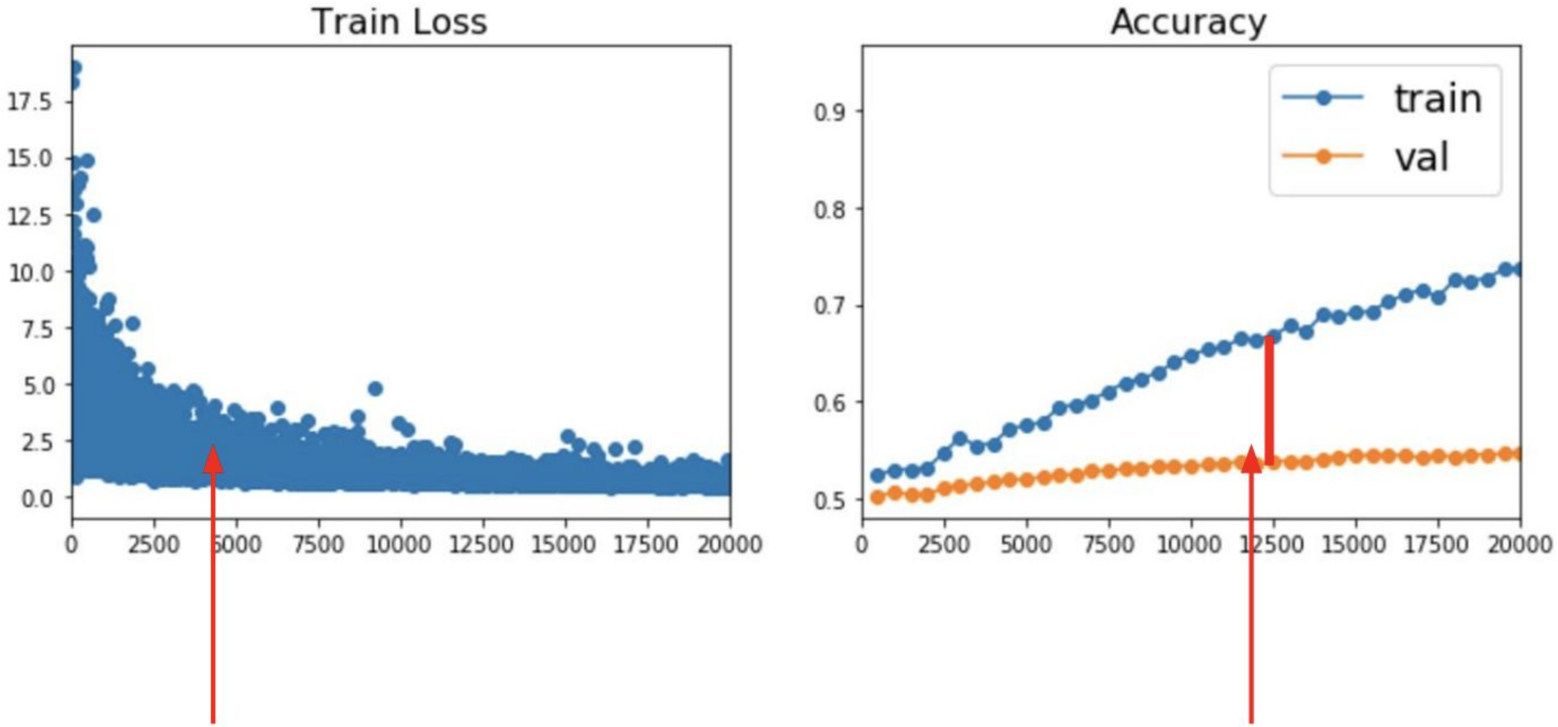




source: http://cs231n.stanford.edu/slides/2017/cs231n_2017_lecture7.pdf

Sum up: optimization

- Adam is great basic choice
- Even for Adam/RMSProp learning rate matters
- Use learning rate decay
- Monitor your model quality



Better optimization algorithms help reduce training loss

But we really care about error on new data - how to reduce the gap?

source: http://cs231n.stanford.edu/slides/2017/cs231n_2017_lecture7.pdf