In 2-dimensional space we are interested in dynamics and equilibrium state. The system of equations looks very simple:

$$\begin{aligned} \frac{d\mathbf{x_i}}{dt} &= \mathbf{v_i}, \\ \frac{d\mathbf{v_i}}{dt} &= \left(A - B|\mathbf{v_i}|^{\mathbf{2}}\right)\mathbf{v_i} + \sum_{\mathbf{j}\neq\mathbf{i}} \mathbf{F_{i,j}}, \end{aligned} \tag{1}$$

where $\mathbf{x_i} = (\mathbf{x_{i,x}}, \mathbf{x_{i,y}})$, $\mathbf{v_i} = (\mathbf{v_{i,x}}, \mathbf{v_{i,y}})$ and the particles' interaction force is defined by generalized gravitational potential

$$F_{i,j} = \frac{\mathbf{x_j} - \mathbf{x_i}}{(|\mathbf{x_j} - \mathbf{x_i}| + \varepsilon)^{\alpha}}. \tag{2}$$

This system of equations differs from classical Newton equations in several important details:

- Each particle has "self-speed" with modulus $\sqrt{A/B}$
- The generalized gravity is not singular and changes the classical gravity at small distance level.

In addition we assume that the particles "live" inside a **circle** of radii R_{out} and follow the reflection law at the outer boundary.

Those two small details allow us to ask the following question: which kind of steady-state (equilibrium state) can exist in such kind of systems? Will all the particles "run" into the state of black hole or self-organize into some structures?

Task: provide a numerical investigation of possible solutions for such equations. Parameters A, B should vary in $[0, 10]$, $\alpha \in [1; 4]$, $\varepsilon \in [10^{-4}, 10^{-1}]$. N of particles in system should vary from 100 to 10^4

Which fast methods: you have to utilize ideas from fast multipole method for efficient evaluation of right-hand side.

Consultations: from Sergey Matveev and possibly from prof. Nikolai Brilliantov.

Deadlines: October 19 – pre-submission of project results. At last lecture day there will be a day with presentations.