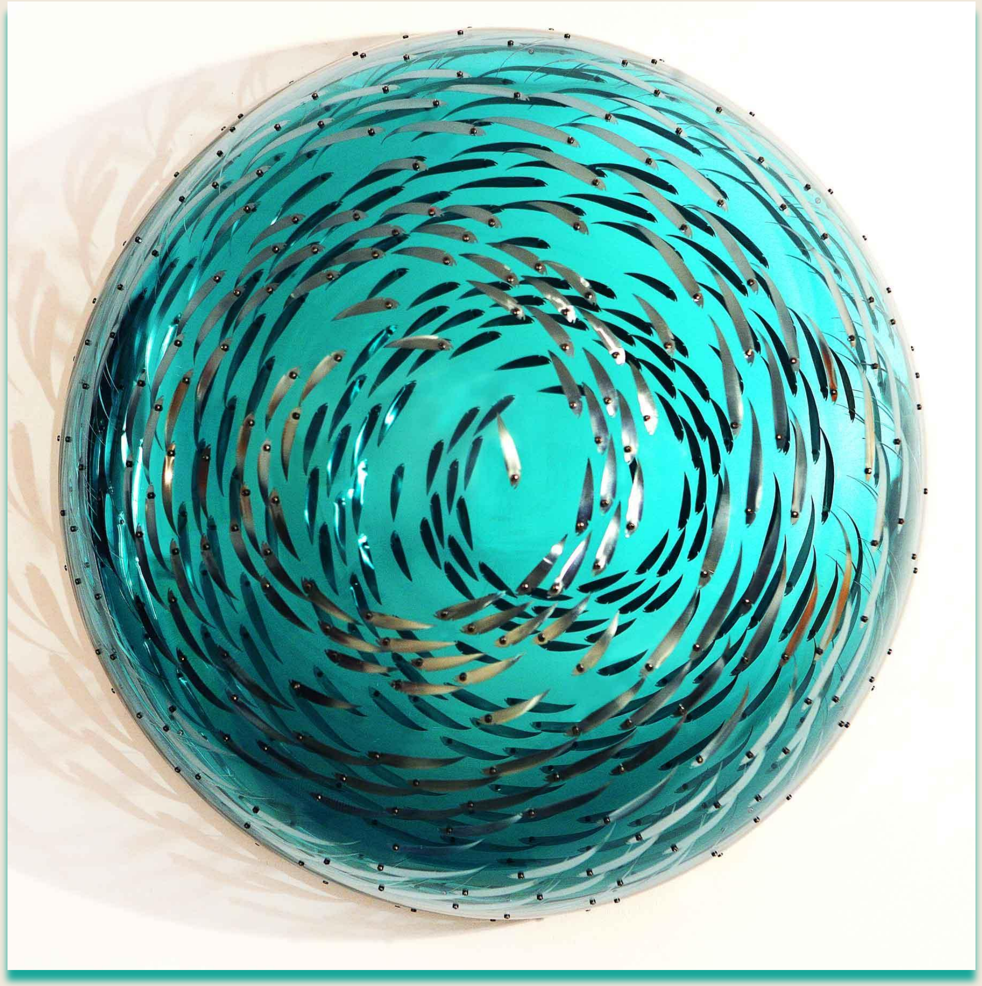
Fast and Efficient Solvers Project



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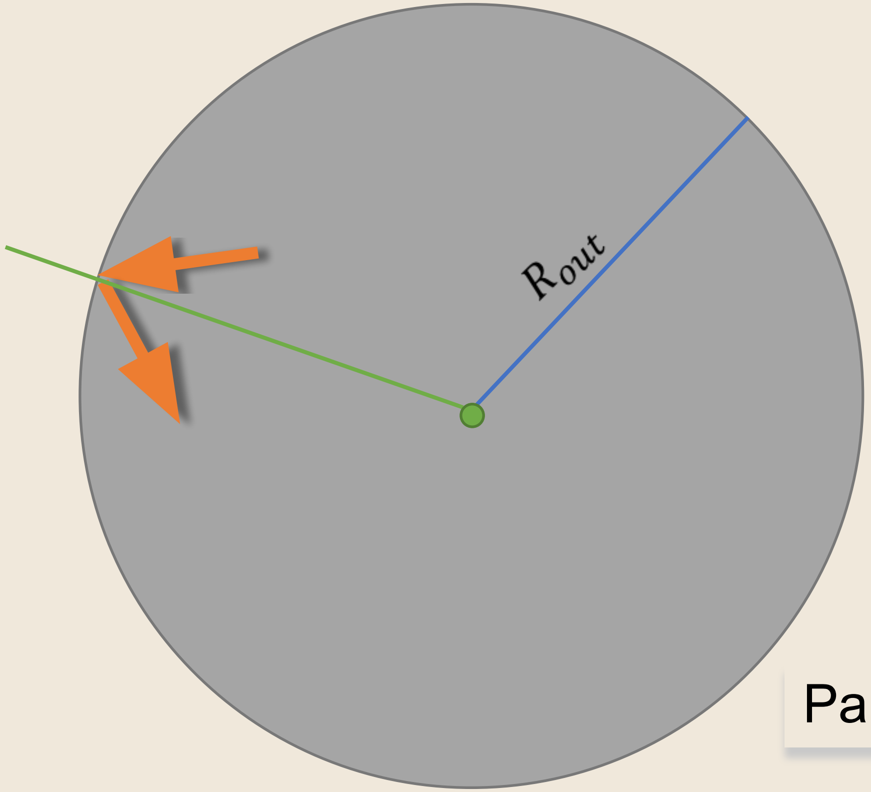
Problem Statement:



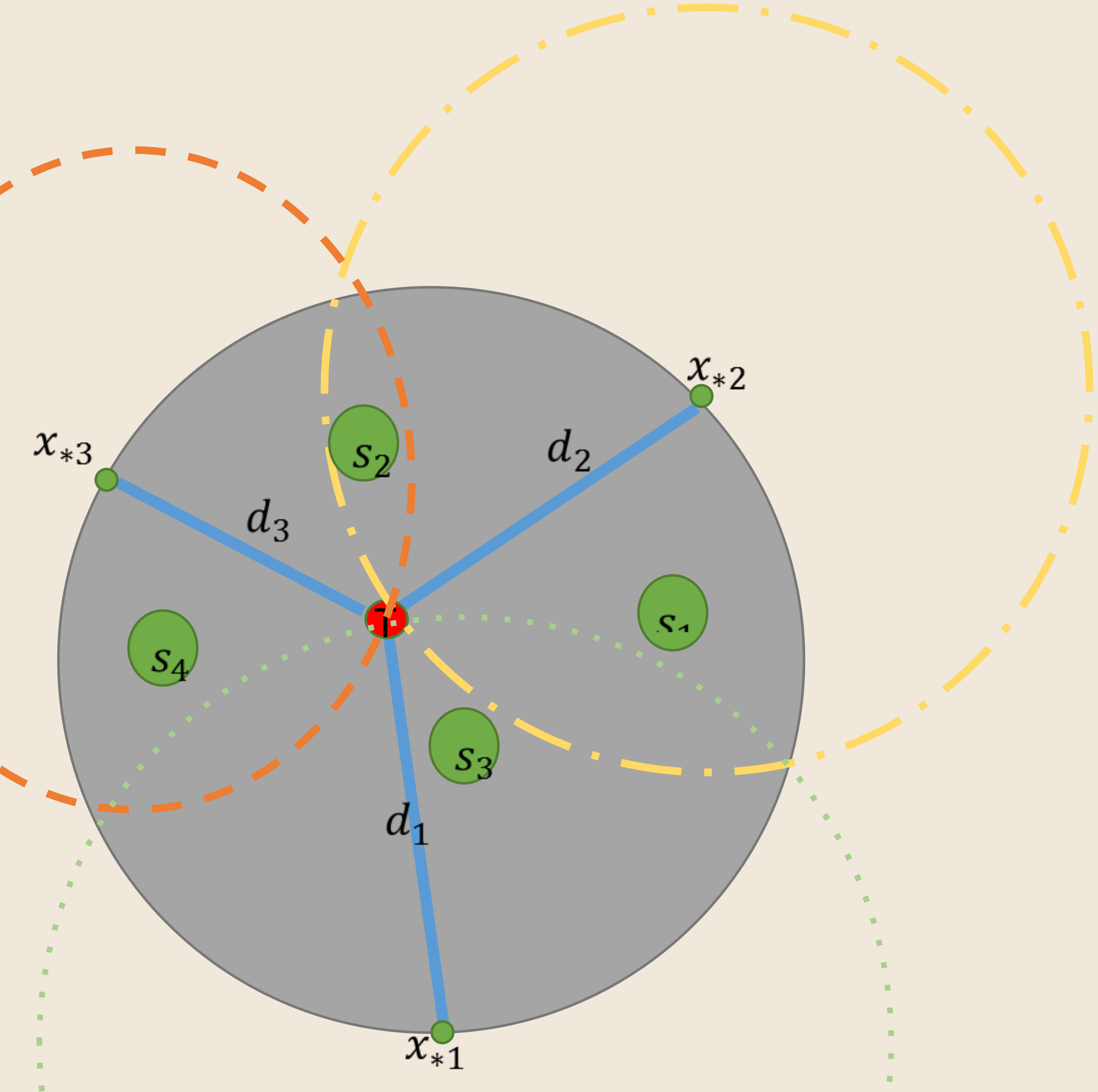
$$\begin{cases} \dfrac{dx_i}{dt} = v_i \\ \dfrac{dv_i}{dt} = (A - B|v_i|^2)v_i + \sum_{j \neq i} F_{i,j} \end{cases}$$

$$x_i = (x_{i,x}, y_{i,y}) \qquad v_i = (v_{i,x}, v_{i,y})$$

Particle interaction force

$$F(\vec{x}_i - \vec{x}_j) = \frac{\vec{x}_i - \vec{x}_j}{\left(\| \vec{x}_i - \vec{x}_j \| + \varepsilon\right)^{\alpha}}$$

Middle Man :



Derived R- Expansion for $F(\vec{x}_i - \vec{x}_j) = \frac{\vec{x}_i - \vec{x}_j}{\left(\|\vec{x}_i - \vec{x}_j\| + \varepsilon\right)^{\alpha}}$

$$F(\vec{x}_i - \vec{x}_j) \approx \Phi_{ij} = F\left(\vec{x}_i - \overrightarrow{x_{*m(i,j)}}\right) \circ \sum_{k=0}^{p} \frac{F^{(k)}\left(\vec{1}\right)}{k!\left(\vec{x}_i - \overrightarrow{x_{*m(i,j)}}\right)^k} \left(\vec{x}_j - \overrightarrow{x_{*m(i,j)}}\right)^k$$

$$\underset{p=1}{\equiv} F\left(\vec{x}_i - \overrightarrow{x_{*m(i,j)}}\right) \circ F\left(\vec{1}\right) \circ \left(\vec{1} + \left(1 - \frac{\alpha}{\sqrt{2}\left(\sqrt{2} + \varepsilon\right)}\right) \circ \left(\vec{x}_j - \overrightarrow{x_{*m(i,j)}}\right)\right)$$

Derived R-expansion constraints:

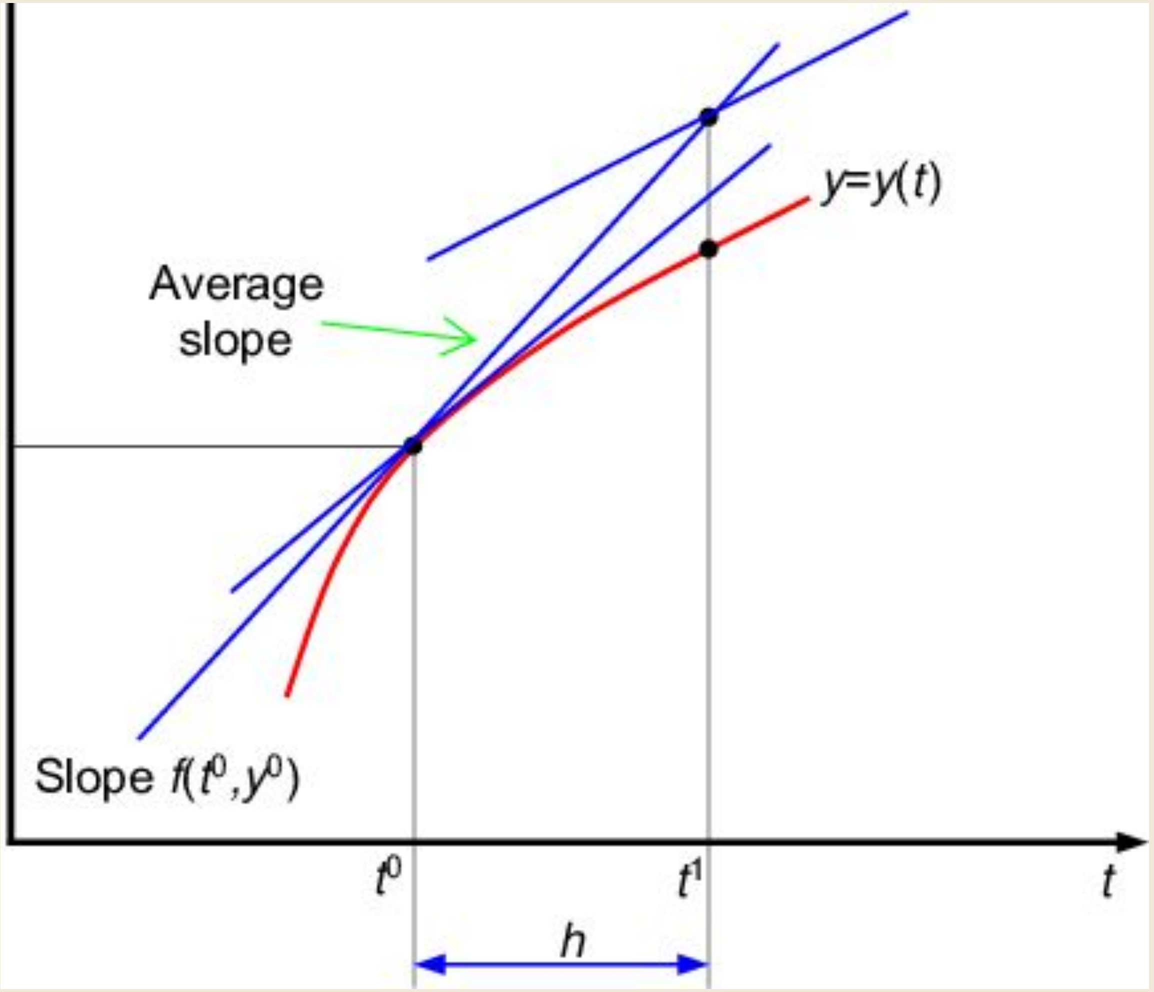
$d_{i,m(i,j)}$ is distance between *m*-th "middle man" $\overrightarrow{x_{*m(i,j)}}$ point and *i*-th target point $\vec{x_i}$

1. $\left\|\vec{x}_i - \overrightarrow{x_{*m(i,j)}}\right\| = d_{i,m(i,j)} < \frac{1}{\sqrt{\varepsilon}}$
2. $\frac{\left\|\vec{x}_j - \overrightarrow{x_{*m(i,j)}}\right\|}{\left\|\vec{x}_i - \overrightarrow{x_{*m(i,j)}}\right\|} < \frac{R_{i,m(i,j)}}{d_{i,m(i,j)}} \le 1,$

we use extreme case: $R_{i,m(i,j)} = d_{i,m(i,j)}$

Second Order Runge-Kutta Method:

Slope $f(t^0 + h, y^0 + h)$



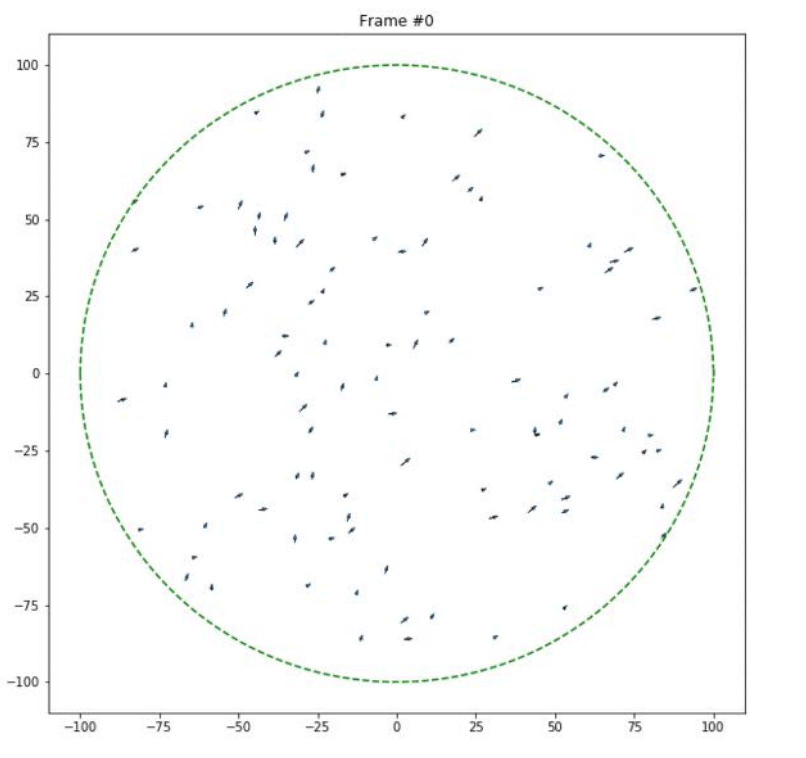
$$k_1 = hf(y_n, t_n)$$

$$k_2 = hf(y_n + k_1, t_n + h)$$

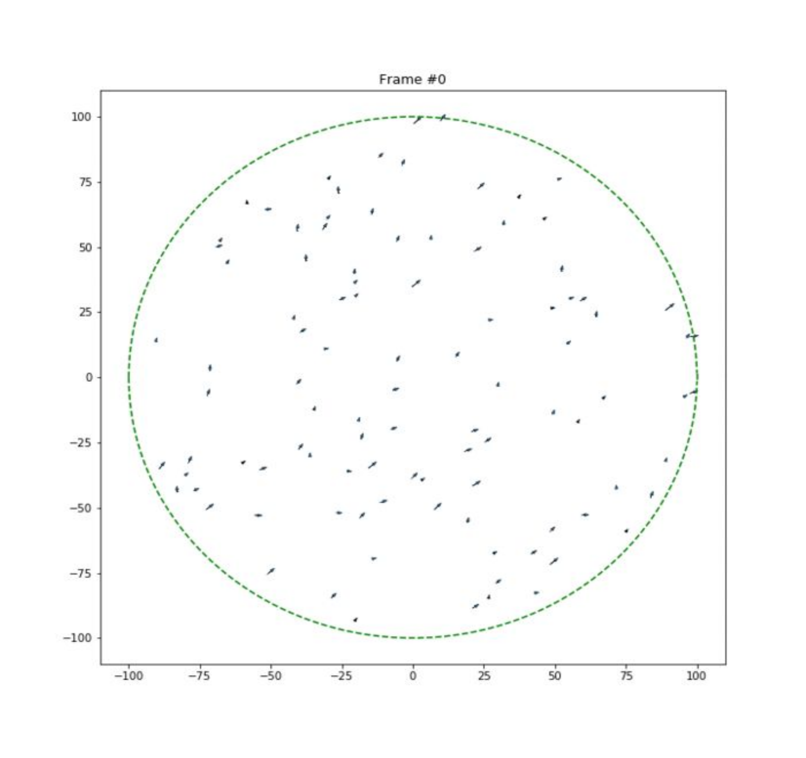
$$y_{n+1} = \mathrm{y_n} + (\mathrm{k_1} + \mathrm{k_2})/2$$

Representation of the results:

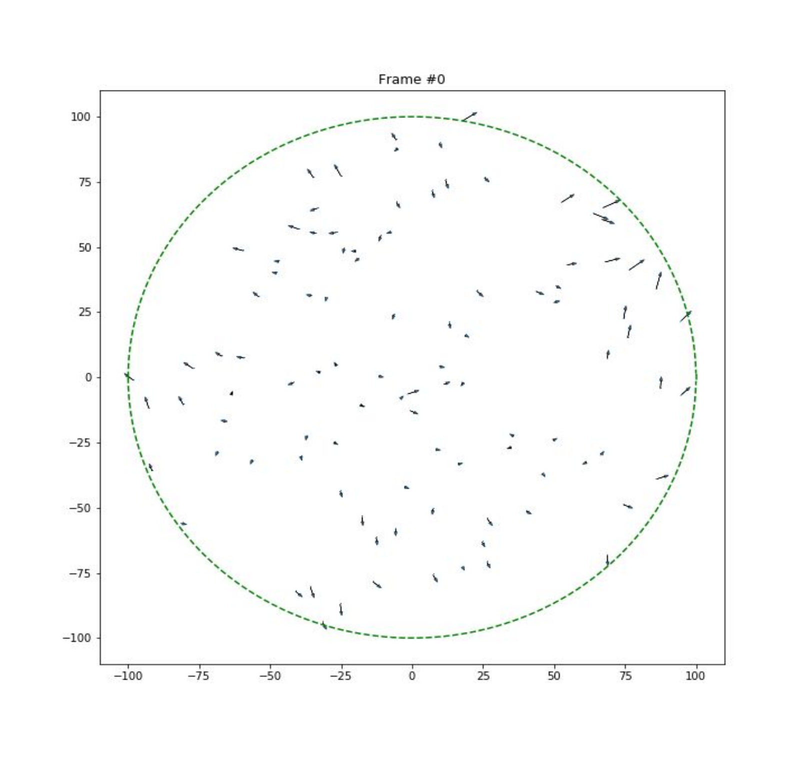
N = 100, A = B = 5, epsilon = 0.0001. Forces are inverse. 1 reflecting.



h = 0.0001



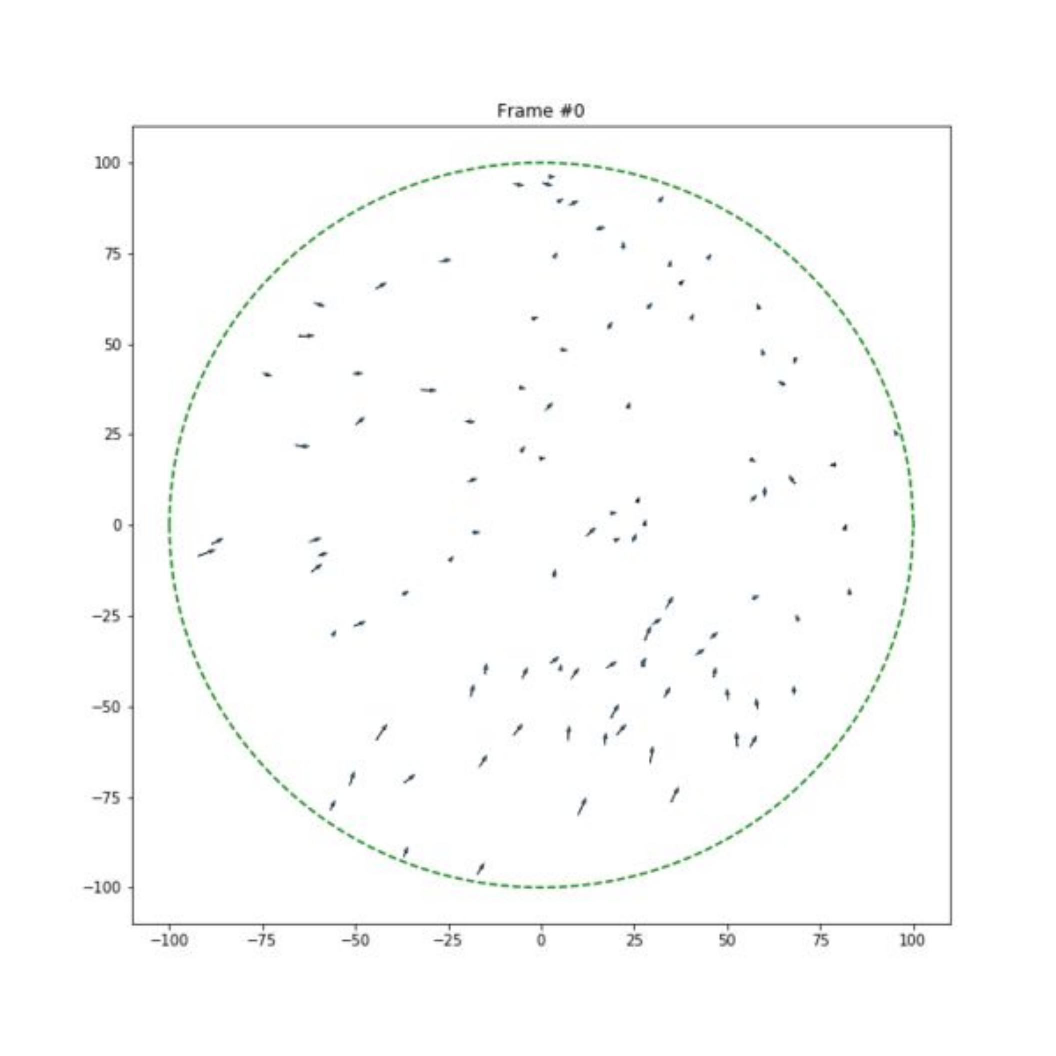
h=0.001



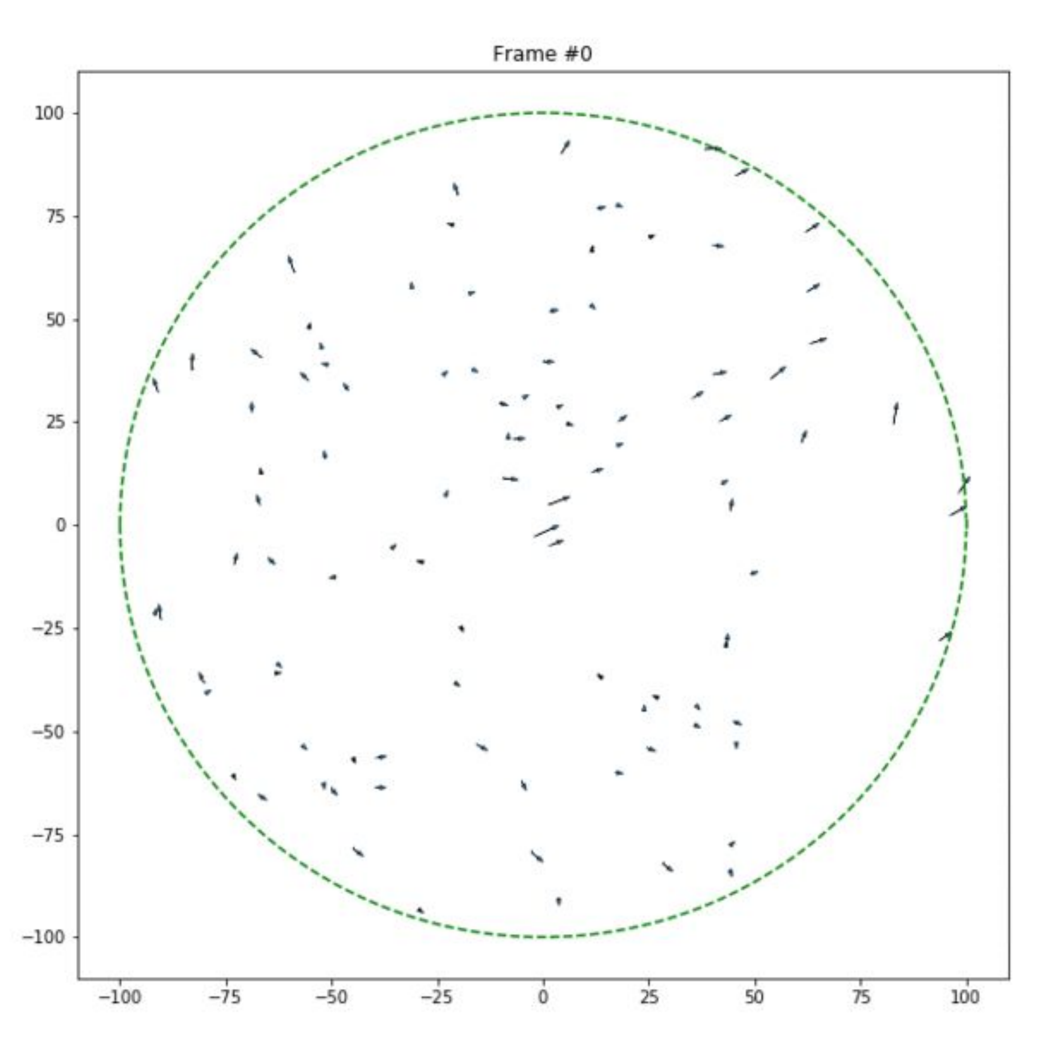
h=0.01

A = B = 5; N = 100; epsilon = 0.0001; alpha = 1; h = 0.01. Two reflectings?

Forces are direct (Black hole)

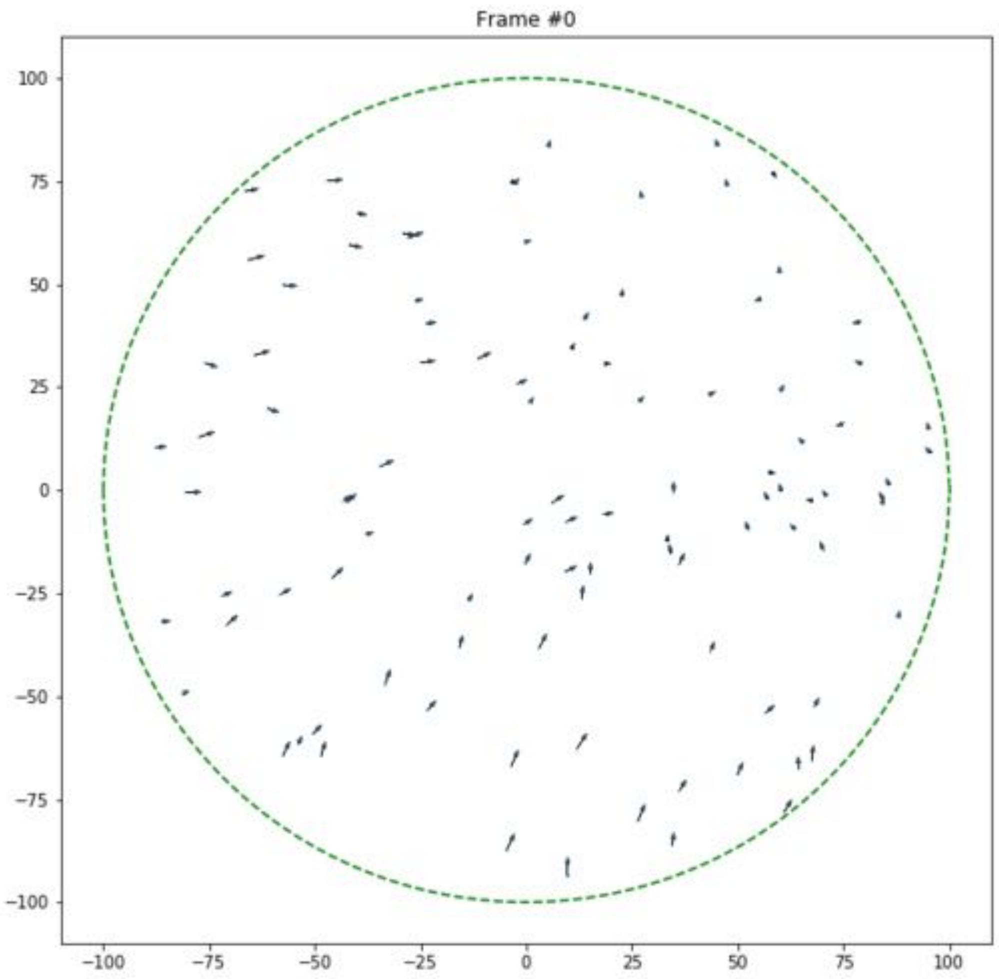


Forces are inverse

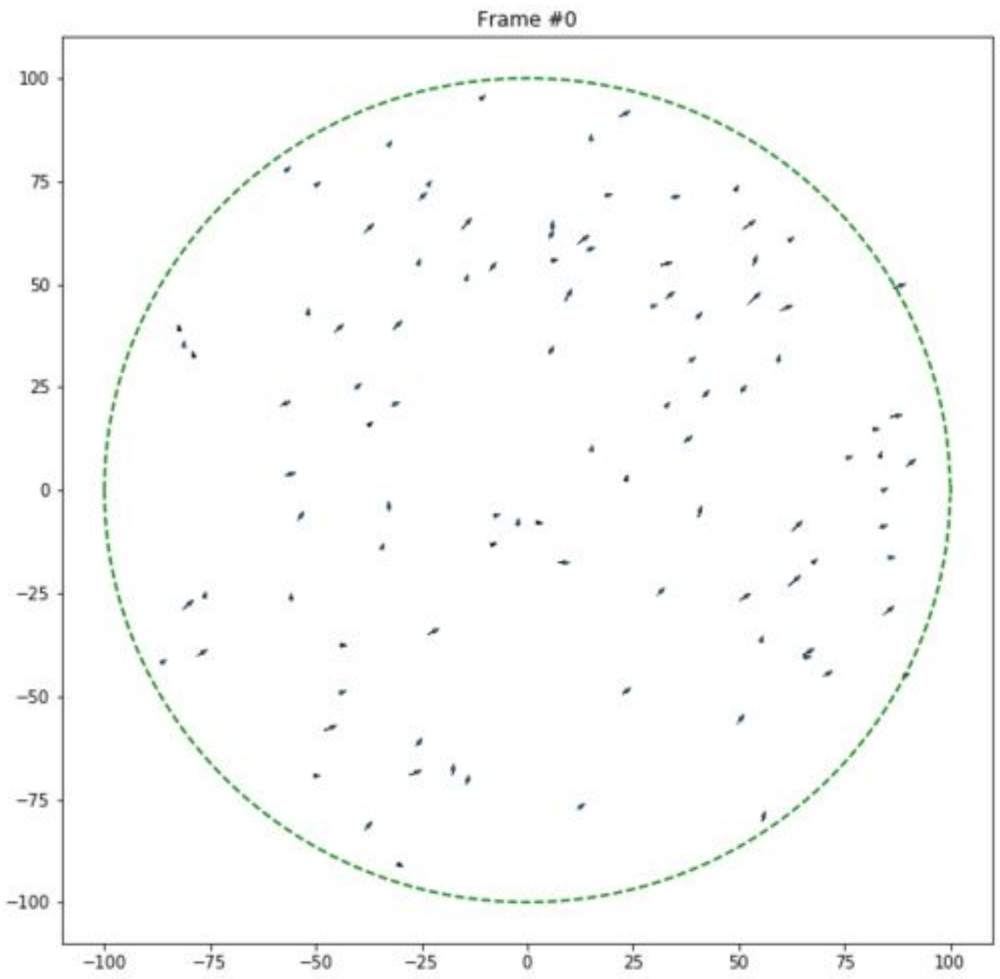


A = 0; B = 10; N = 100; epsilon = 0.0001; alpha = 1; with double reflection

Forces are direct; h = 0.01. (Black hole)

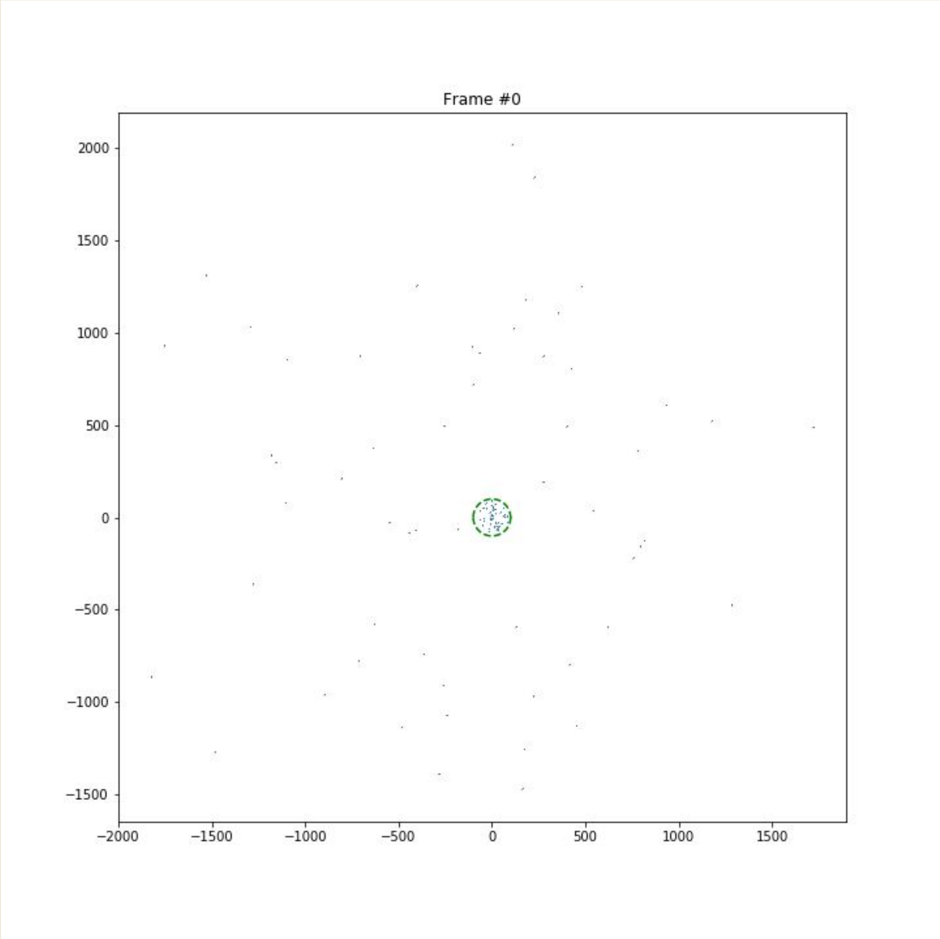


Forces are inverse; h = 0.001.

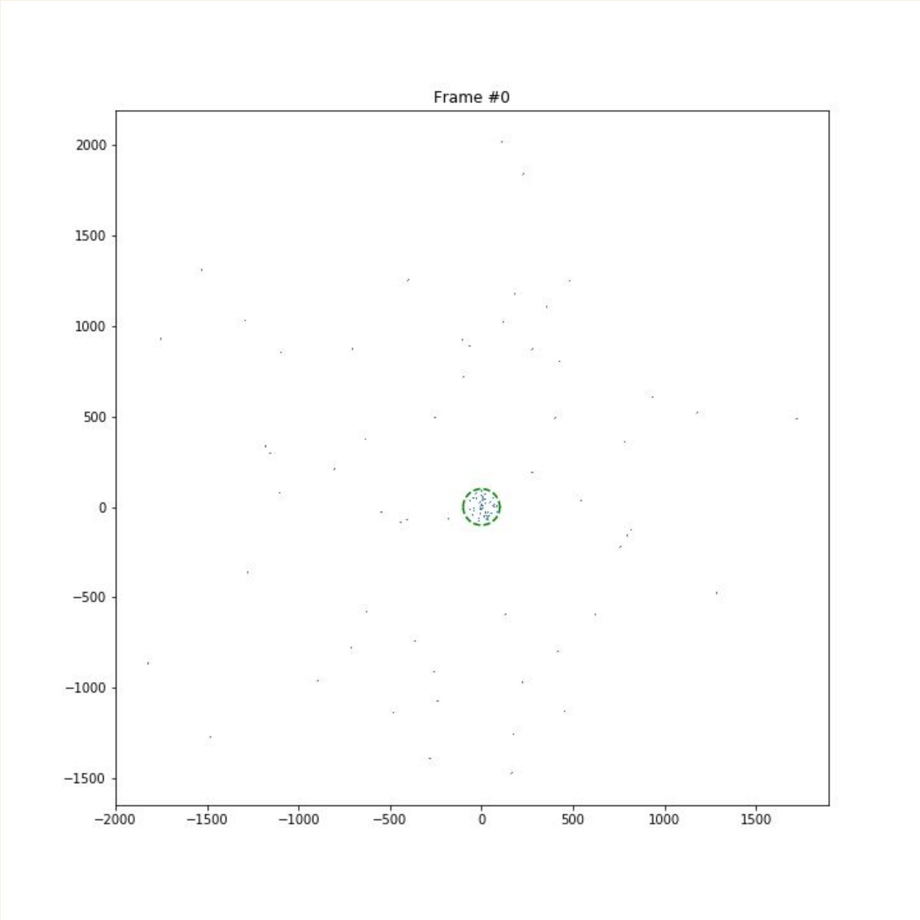


A = 10; B = 0; N = 100; epsilon = 0.0001; alpha = 1; with double reflection

Forces are direct; h = 0.01.

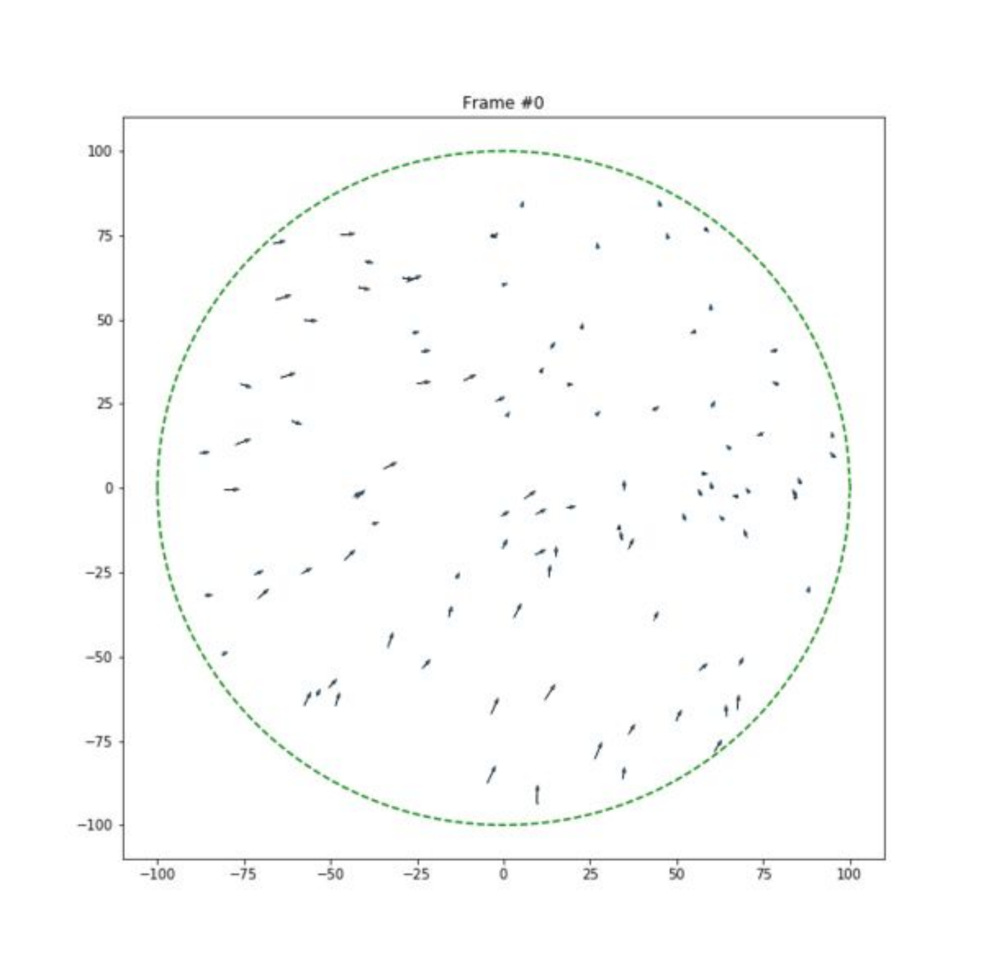


Forces are inverse; h = 0.001.

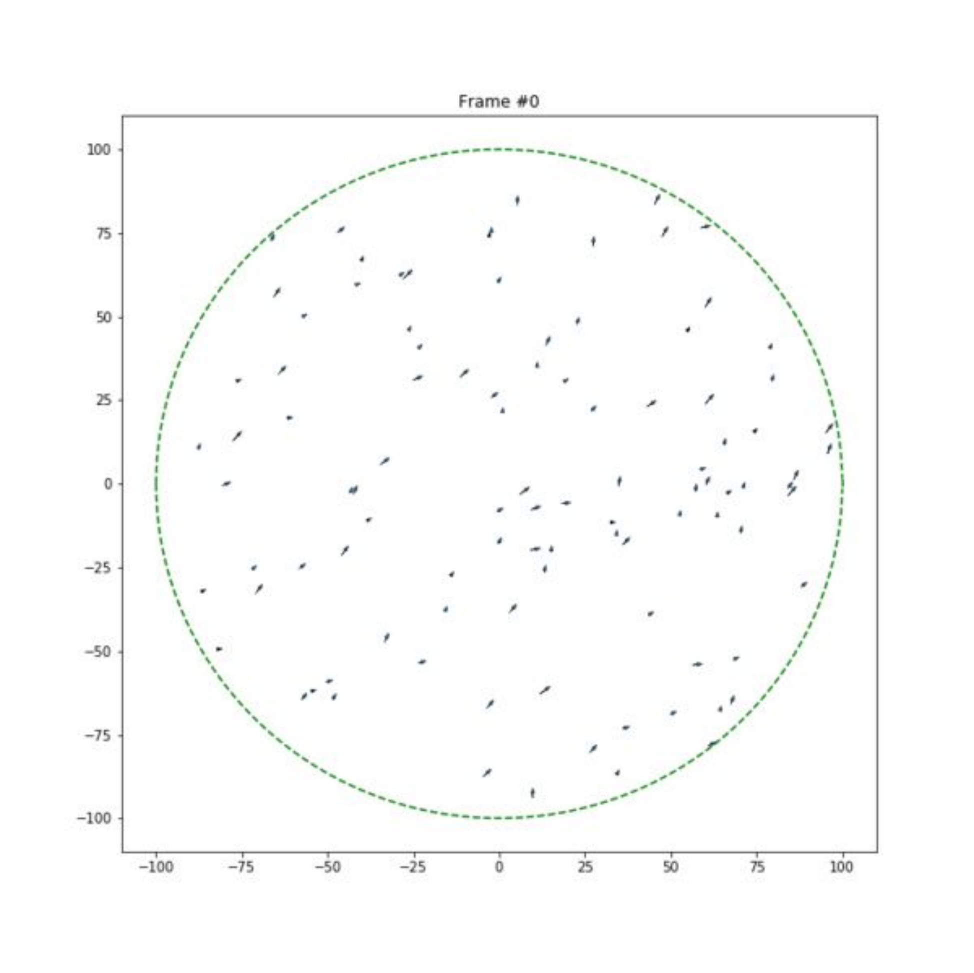


A = 0; B = 0; N = 100; epsilon = 0.0001; alpha = 1; h = 0.01; with double reflection

Forces are direct



Forces are inverse

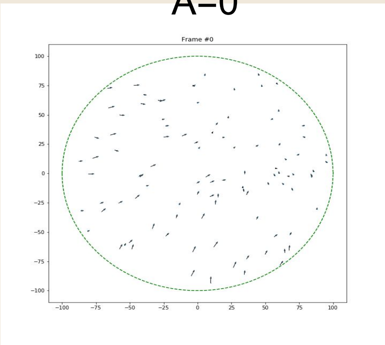


which kind of steady-state (equilibrium state) can exist in such kind of systems?

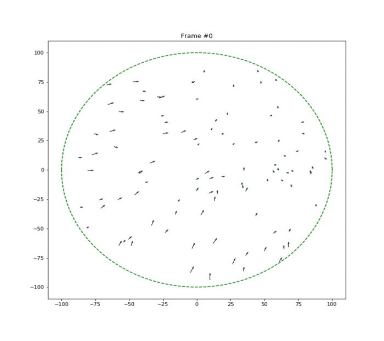
Will all the particles “run” into the state of black hole or self-organize into some structures?

Experiment with different A,B

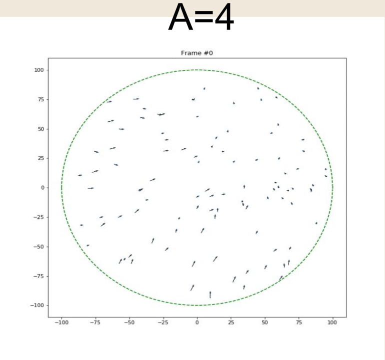
A=0



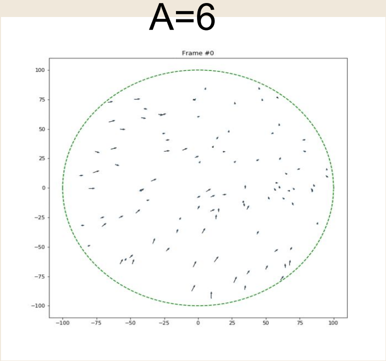
A=2



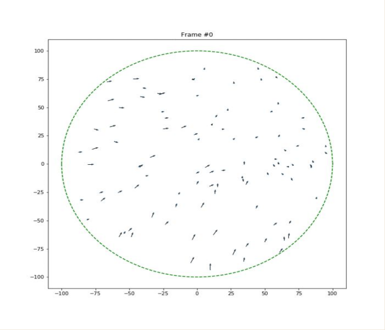
A=4



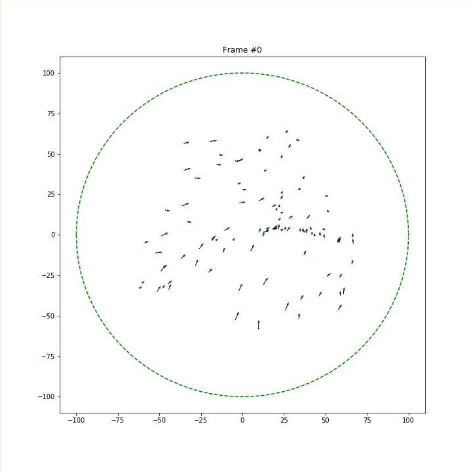
A=6



A=8



B=2



Experiment with different epsilon

Experiment with different alpha

Error Analysis