Homework Assignment #1

Question #1

a) $E[f(x)] = 10 \times 0.1 + 5 \times 0.2 + \frac{10}{7} \times 0.7$

$= 3$ //

b) $E\left(\frac{1}{p(x)}\right)$: $E(f(x)] = \sum_{x \in X} p(x) \cdot f(x)$

$E\left[\frac{1}{p(x)}\right] = \sum_{x \in X} p(x) \cdot \frac{1}{p(x)}$

$= 0.1 \cdot \frac{1}{0.1} + 0.2 \cdot \frac{1}{0.2} + 0.7 \cdot \frac{1}{0.7}$

$= 3$

c) $E\left[\frac{1}{p(x)}\right]$: Since $E[f(x)] = \sum_{x \in X} p(x) \cdot f(x)$

$E\left[\frac{1}{p(x)}\right] = \sum_{x \in X} p(x) \cdot \frac{1}{p(x)}$

$= \sum_{x \in X} 1 = n$

$\therefore$ it will be the number of elements present.

d) $E[f(x)^2]$ & $E[f(x)]^2$

$E[f(x)^2] = (0.1 \times 10^2) + (0.2 \times 5^2) + (0.7 \times 10/7^2)$

$= 115/7$

$E[f(x)]^2 = [0.1 \times 10 + 0.2 \times 5 + 0.7 \times 10/7]^2$

$= 3^2$

$= 9$

Question #2

A: $p(H) = 0.75$ B: $p(H) = 0.5$ C: $p(H) = 0.25$

A: $p(T) = 0.25$ B: $p(T) = 0.5$ C: $p(T) = 0.75$

a) $x,\ E[x] =$

$z(H) = \{0.75,\ 0.5,\ 0.25\}$

$= 0.75 \times 1 + 0.5 \times 1 + 0.25 \times 1$

$= 1.5$ //

b) $p(D = 3 \text{ heads } \& \ 2 \text{ tails} \mid \text{coin} = C) = p(D = 3H \ \& \ 2T)$

Bayes: $p(y \mid x) = \dfrac{p(x \mid y) \cdot p(y)}{p(x)}$

$\dfrac{15}{512}$

$= \dfrac{p(\text{coin } C \mid 3H2T) \cdot p(3H2T)}{P(\text{coin } C)}$

$= \dfrac{[10 \cdot 0.25^3 \cdot 0.75^2][85/384]}{[1/3]}$

Coin A $P_1 = 0.75^3 \cdot 0.25^2 \cdot 10$

Coin B $P_2 = 0.5^5 \cdot 10$

Coin C $P_3 = 0.25^3 \cdot 0.75^2 \cdot 10$

$\dfrac{\text{Sum for } P(3H2T)}{3} = 85/384$

$= \dfrac{3825}{65536}$ // $= 0.058$ //

Question #3

$$p(x) = \begin{cases} \frac{1}{10} & \text{if } 0 \leq x \leq 10 \\ 0 & \text{else} \end{cases}$$

$$\text{cost} = x^3 = p(x)$$

$$E[x] = \int_x x\,p(x)\,dx$$

$$= \int_0^{10} \frac{1}{10} x^3\,dx$$

$$= \frac{1}{10} \int_0^{10} x^3\,dx$$

$$= \frac{1}{10} \left[\left[\frac{1}{4} x^4 \right]_0^{10} \right]$$

$$= \frac{1}{10} [2500]$$

$$= 250$$

Question # 4

b) $M_1 = 0.422$

$M_2 = 0.278$

$M_3 = 0.060$

$M_4 = -0.351$

$M_5 = -0.424$

$$\bar{V} = \frac{1}{n-1} \sum_{i=1}^{n} (M_i - \bar{M})^2$$

$$\bar{M} = -0.015/5 = -0.003$$

$$= \frac{1}{4} \sum_{1}^{4} (M_i - \bar{M})^2$$

$$= \frac{1}{4}\left[(0.422+.003)^2 + (0.278+.003)^2 + (0.060+.003)^2 + (-0.351+.003)^2 + (0.424+.003)^2\right]$$

$$= \frac{0.5619}{4} = \boxed{0.140}$$ //

c) $M_1 = -0.073$

$M_2 = -0.021$

$M_3 = 0.071$

$M_4 = 0.047$

$M_5 = -0.045$

$$\bar{V} = \frac{1}{n} \sum_{i=1}^{n} (M_i - \bar{M})^2$$

$$\bar{M} = \frac{-0.021}{5} = -0.0042$$

$$= \frac{1}{5} \sum_{1}^{5} (M_i - \bar{M})^2$$

$$= \frac{1}{5}\left[(-0.073+.0042)^2 + (-0.021+0.0042)^2 + (0.071+0.0042)^2 + (0.047+0.0042)^2 + (-0.045+0.0042)^2\right] = \frac{0.015}{5} = \boxed{0.003}$$ //

* varience decreased as number of samples increased *

a) $\sigma^2 = 10$ * Gaussian

true mean = unknown

$n = 30$

$M = 0.214$

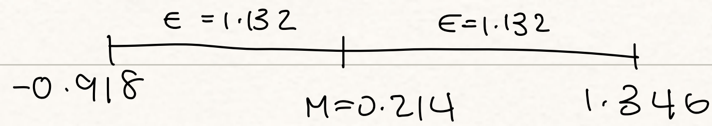
95 confidence

$\delta = 0.05$

$$\epsilon = \frac{1.96\,\sigma}{\sqrt{n}}$$

$$= \frac{1.96\sqrt{10}}{\sqrt{30}}$$

$$= 1.132 \;//$$



$CI = (-0.918\,,\ 1.346)$

e) $\sigma^2 = 10$

Not Gaussian

$n = 30$

$\bar{M} = 0.214$

95% Confidence

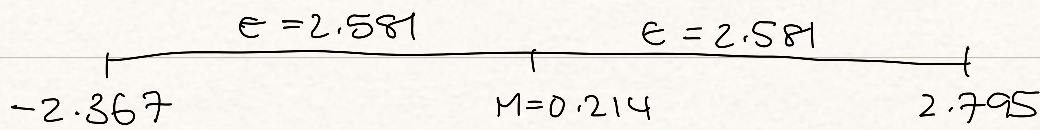
$\delta = 0.05$

Chebyshev:

$$\epsilon = \frac{4.47\,\sigma}{\sqrt{n}}$$

$$= \frac{4.47\sqrt{10}}{\sqrt{30}}$$

$$\epsilon = 2.581 \;//$$



$CI = (-2.367\,,\ 2.795)$

♡ ♡

Question # 5

a) $E[\bar{v}] = \sigma^2$

$\bar{v} = \frac{1}{n-1} \sum_{i=1}^{n} (x_i - \bar{x})^2$ } equal

$\frac{1}{n-1} \sum_{i=1}^{n} (x_i - \bar{x})^2 = \sigma^2$

multiply by $(n-1)$

$\underbrace{\sum_{i=1}^{n} (x_i - \bar{x})^2} = \sigma^2 (n-1)$

Since equal, Substitute

$\bar{V}_b = \frac{1}{n} \sum_{i=1}^{n} (x_i - \bar{x})^2$

$\bar{V}_b = \frac{1}{n} (n-1) \sigma^2$

$E[\bar{V}_b] = (1 - \frac{1}{n}) \sigma^2$

b) $Var[\bar{x}] = \frac{1}{n} \sigma^2$ with var σ^2

$Var[\bar{v}] = \frac{2(n-1)}{n^2} \sigma^4$

Chebyshev: $\rightarrow Pr(|Y - E[Y]| < \epsilon) > 1 - V/\epsilon^2$

$n = 10$

Put in for $n = 10$

$Var[\bar{x}] = \frac{1}{10} \sigma^2 \rightarrow \frac{10}{100} \sigma^2$ } 10

$Var[\bar{v}] = \frac{18}{100} \sigma^4$ } $18\sigma^2$

$\epsilon = \sqrt{\frac{\sigma^2}{\delta n}}$ will have a tighter confidence estimate around the sample varience $\bar{v}$! This is bcz the epsiolon will be smaller thus adding & subtracting a less value

to the mean leading to a tighter bound.
varience is more variable to sample size and the lower the variability the tighter the confidenc interval