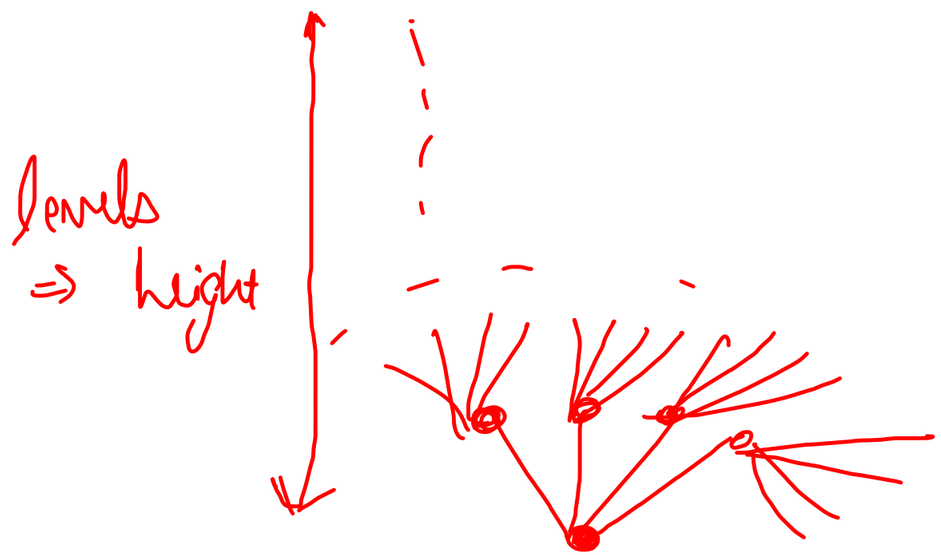
Recursion

1. Expectation
2. Faith
3. Meeting expectation with faith
4. Base Case

#Time Complexity

$(calls)^{height}$ + preorder * height + postorder * height



① Print Increasing

1.1 fun(n) : 1 --- n

1.2 fun(n-1) : 1 --- (n-1)

1.3 M.E ⇒ Postorder : Sys0(n);

n=0 : return;

② Print decreasing

1.1 fun(n) : n --- 1

1.2 fun(n-1) : (n-1) -- 1

1.3 M.E ⇒ Preorder : Sys0(n);

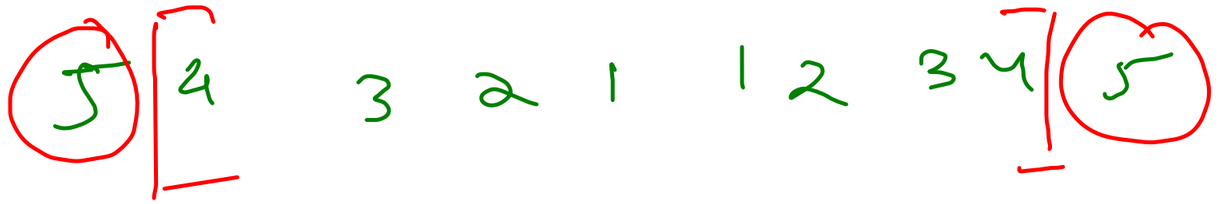
n=0 : return;

{ TC ⇒ $(1)^n$ + k*n = O(n) }

C++ Main

③ Print Increasing Decreasing

fun(5) :



fun(4):

4 3 2 1 1 2 3 4

M.E :→

Pre :
Syso(n)

fun(n-1)

Post
Syso(n)

TC ⇒ $\left(\frac{1}{}\right)^{n}$ + k * n + k * n

= 2 * n + 1 ⇒ O(n)

Power - linear

power(x, n)

$x^{n-1} = x * x * \dots$ (n-1) times

return $x * x^{n-1}$;

power(x, n)

$x * x * x * \dots$ n times

Power - logarithmic

$x^{n/2}$ = power(x, n/2)

if (n%2 == 0)
return $x^{n/2} * x^{n/2}$;

if (n%2 == 1)
return $x^{n/2} * x^{n/2} * x$;

Base Case : $n = 0$: return 1;

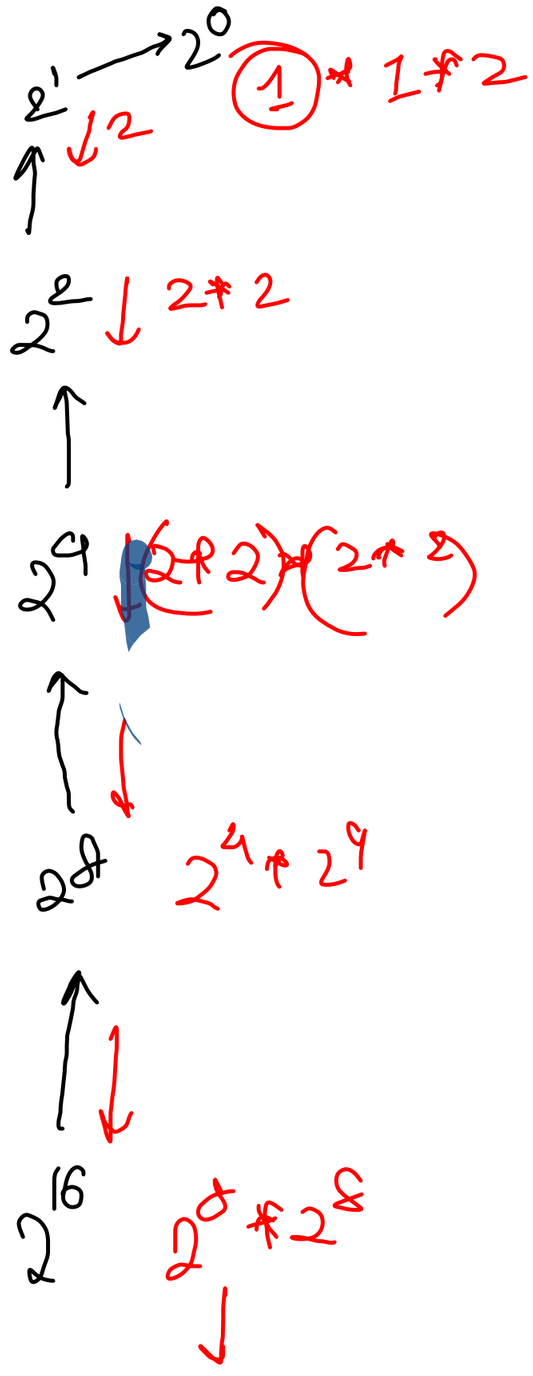
TC ⇒ $(1)^n + k * n \Rightarrow O(n)$

TC ⇒ $(1)^{\log_2 n} + k * \log_2 n$

⇒ $O(\log n) \simeq O(1)$

Power - log

log n height




```java
public static int power(int x, int n){
    // Base Case x^0 = 1
    if(n == 0) return 1;

    // 1. Faith : x ^ n/2
    int xpnb2 = power(x, n/2);

    // 2. x^n = x^n/2 * x^n/2
    int xpn = xpnb2 * xpnb2;

    // 3. If n is odd
    if(n % 2 == 1) xpn = xpn * x;

    // 4. Return
    return xpn;
}
```

Power - 2

```
if(n%2==0)
  return power(x, n/2) * power(x, n/2)
else
  retn pow(x, n/2) * pow(x, n/2) * x;
```

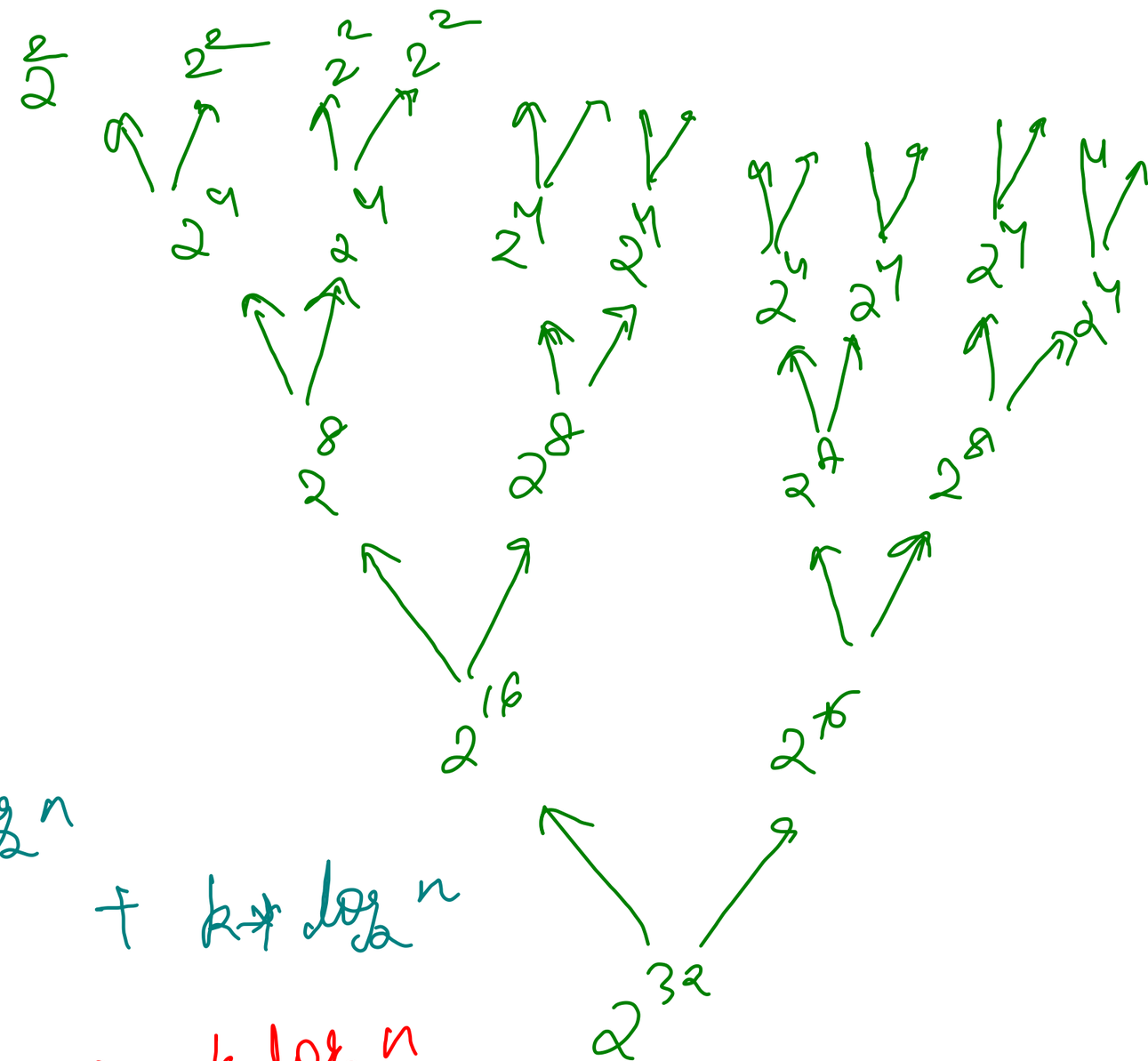
height $\Rightarrow \log_2 n$

calls $\Rightarrow 2^{\log_2 n}$

$= (2)^{\log_2 n} + k * \log_2 n$

$= n + k \log_2 n$

$\Rightarrow O(n)$



long

① 10^{18} % mod

② → (a+b) % m

Because AB

int max value

$10^9 + 7$

long ⇒ $\approx 10^{18}$

int ⇒ $2^{31} - 1 \approx 10^9$

(a+b) % m

(a*b) % m

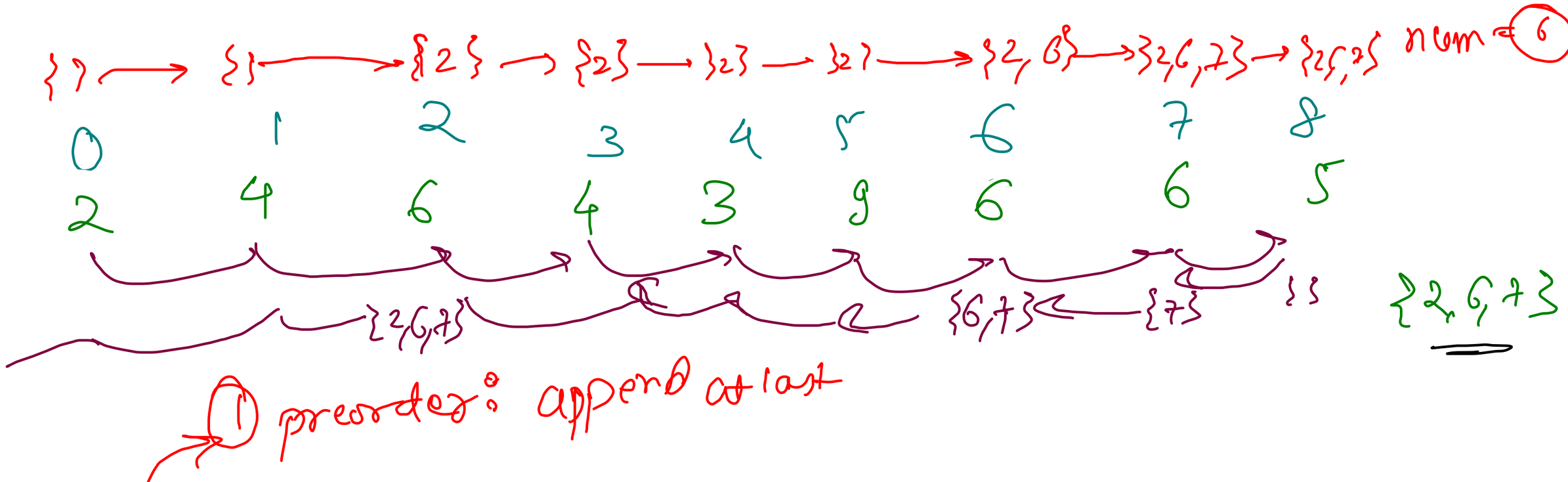
Modulus formulas

1. $(a+b) \% m = (a \% m + b \% m) \% m$

2. $(a-b) \% m = ((a \% m - b \% m) \% m + m) \% m$

3. $(a * b) \% m = (a \% m) * (b \% m) \% m$

4. Division $\Rightarrow$ **Modulo Inverse**

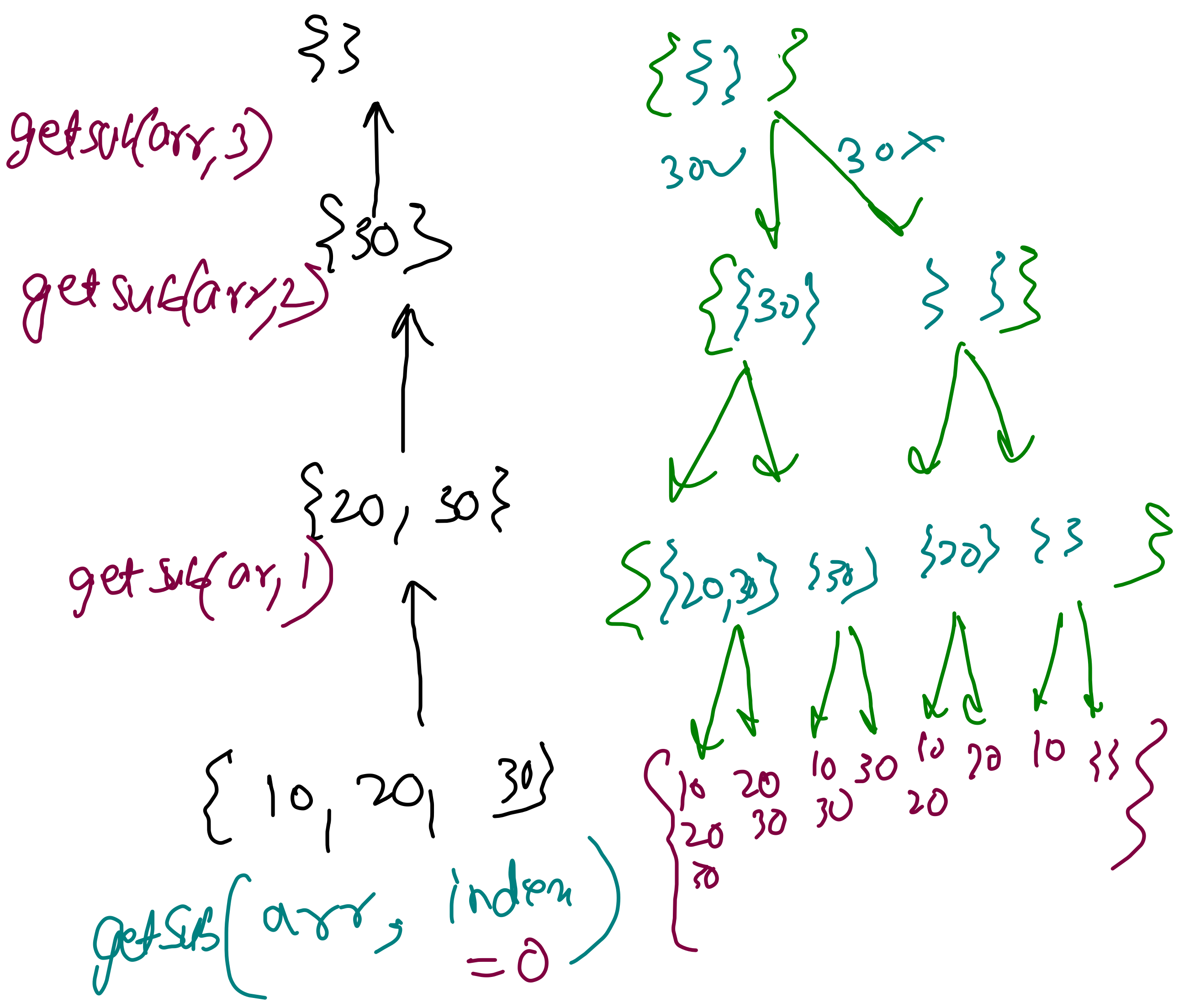


① preorder: append at last

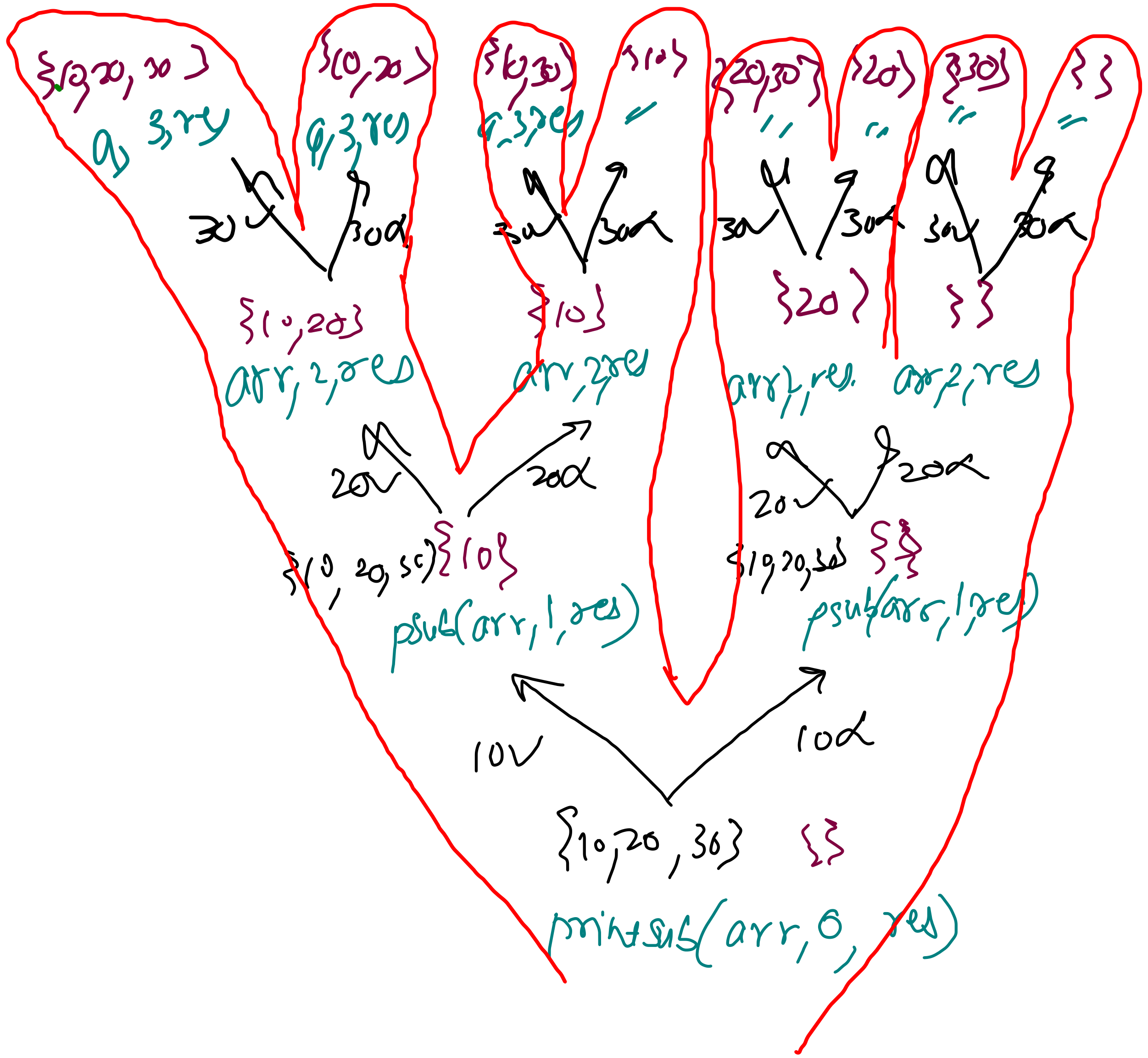
fun(arr, 0, res, num)

② postorder: append at first

Get Subsequence



Print Subsequence



$$TC \Rightarrow \left(2^{n} + k*n + k*n\right) \Rightarrow O(2^{n})$$

Today's Target

1. Permutations
   - → Box on level
   - → Item on level
2. Combinations
   - → Box on level
   - → Item on level
3. String Permutations {Unique}
   - → Box on level
   - → Item on level
4. Palindromic Permutations

15-20 → follow up } → 30 Q

Homework :-

- Factor combinations
- Dictionary Order
  - → small
  - → large

Combinatn

^nC_r

$$\frac{n!}{(n-r)!\,r!}$$

i i — —

4C2

identical items

Permutation

Selectn + Arrangement

^nP_r

$$\frac{n!}{(n-r)!}$$

non-identical items

12 — — 21 — —

1 — 2 — 2 — 1 —

1 — — 2 2 — — 1

— 12 — — 21 —

— 1 — 2 — 2 —

— — 12 — — 21

$^4P_2 = {}^4C_2 * 2!$

Combinations → Box on level

box = 3

box = 2

box = 1

box = 0

{1, 2}

n = 4

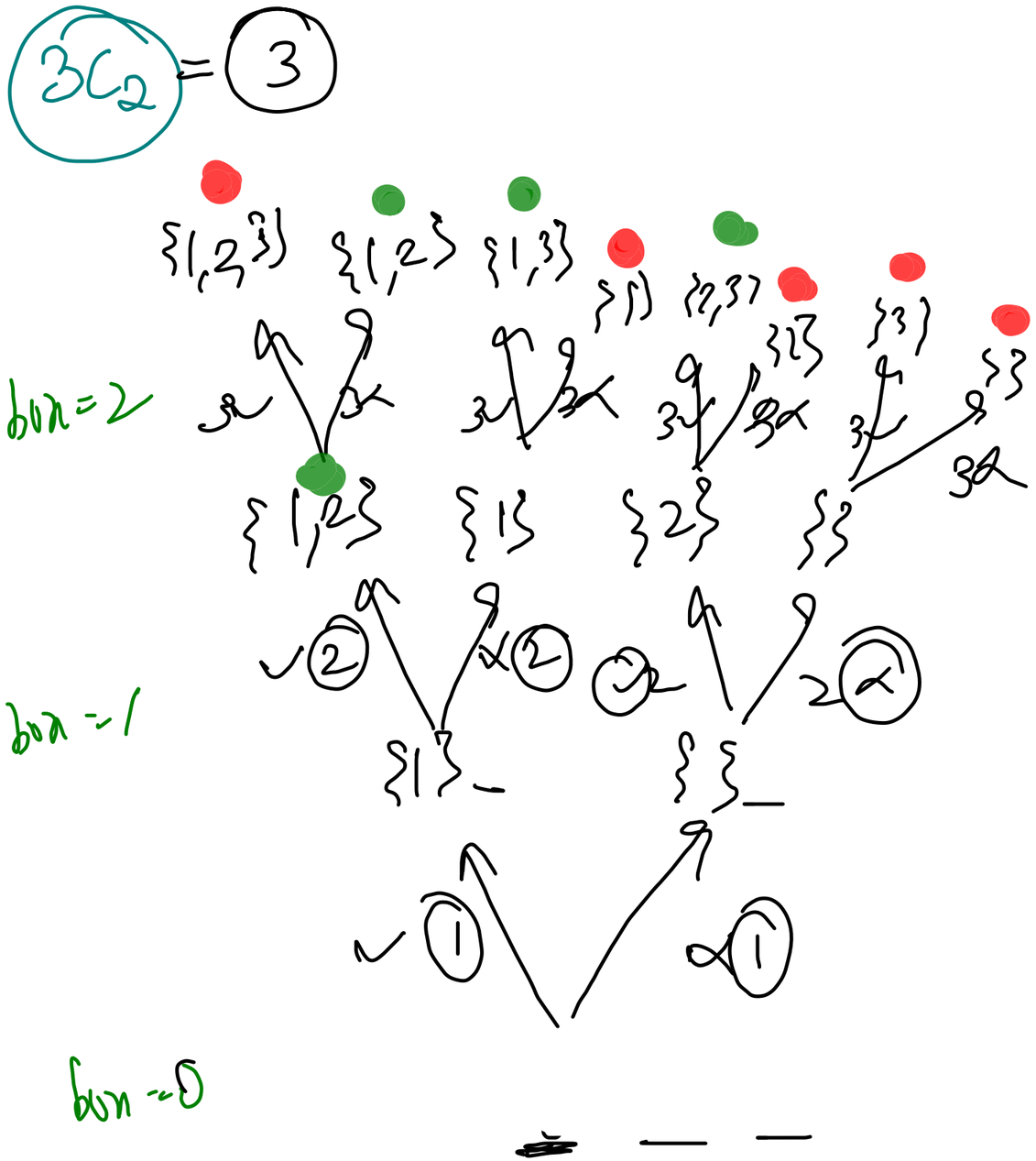
r = 2

$$2^n = {}^nC_0 + {}^nC_1 + {}^nC_2 + \cdots {}^nC_n$$

$$2^4 = {}^4C_0 + {}^4C_1 + {}^4C_2 + {}^4C_3 + {}^4C_4$$

$$2^4 = 1 + 4 + 6 + 4 + 1$$

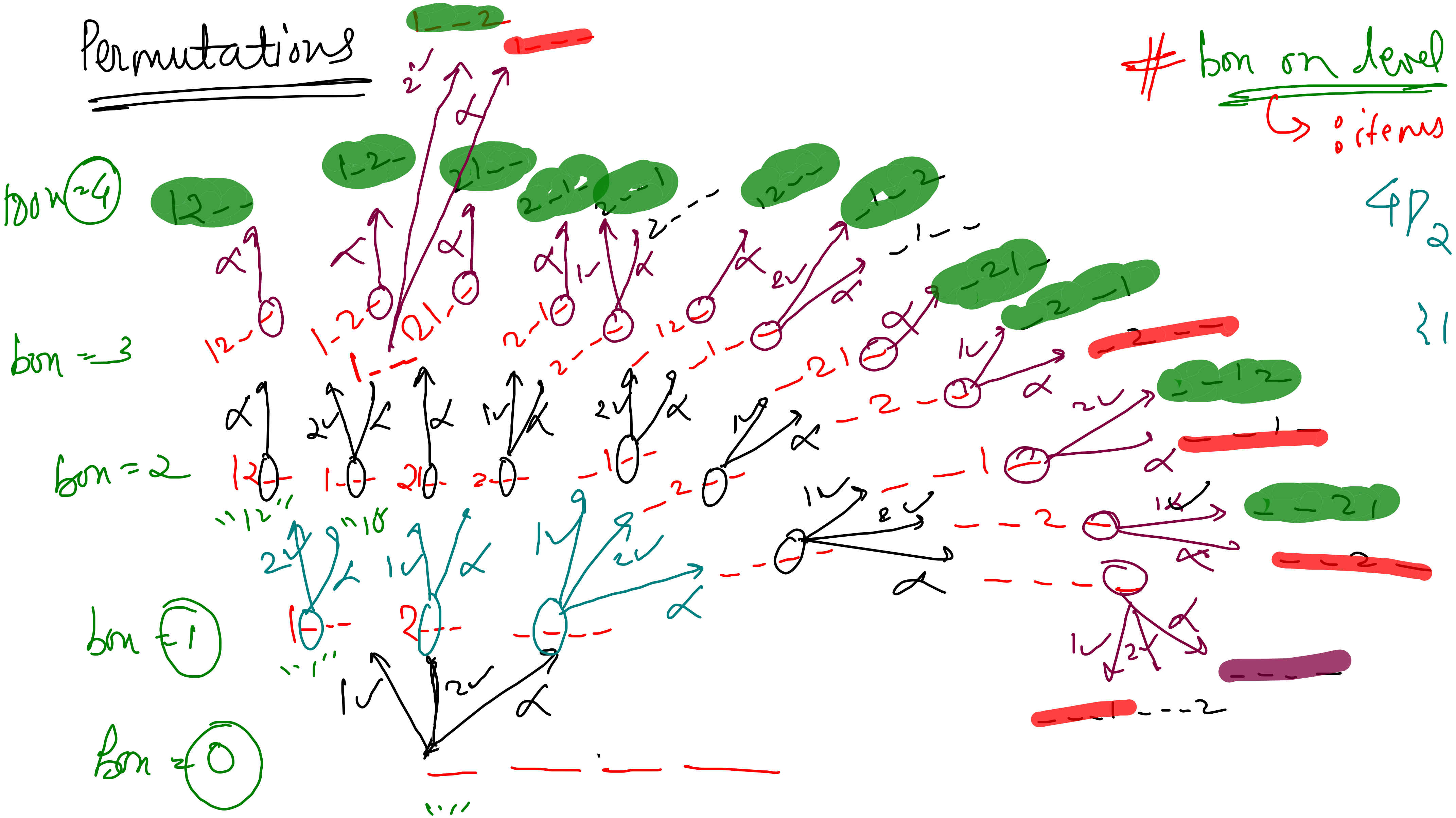
TC ⇒ $(2)^n + \cdots + \cdots$




```java
public void combine(List<List<Integer>> combinations,
    List<Integer> combination, int currentBox, int n, int k){
    if(currentBox == n){
        if(combination.size() == k){
            // deep copy
            List<Integer> temp = new ArrayList<>(combination);
            combinations.add(temp);
        }
        return;
    }

    // options -> current Box -> item should be placed or not

    // yes
    combination.add(currentBox + 1);
    combine(combinations, combination, currentBox + 1, n, k);
    combination.remove(combination.size() - 1);

    // no
    combine(combinations, combination, currentBox + 1, n, k);
}
```

{1,2} , {1,3} , {2,3}

Permutations



box on level
↳ : items were on options + α

$4P_2$

$\{1, 2\}$

$4P_0 + 4P_1 + 4P_2$

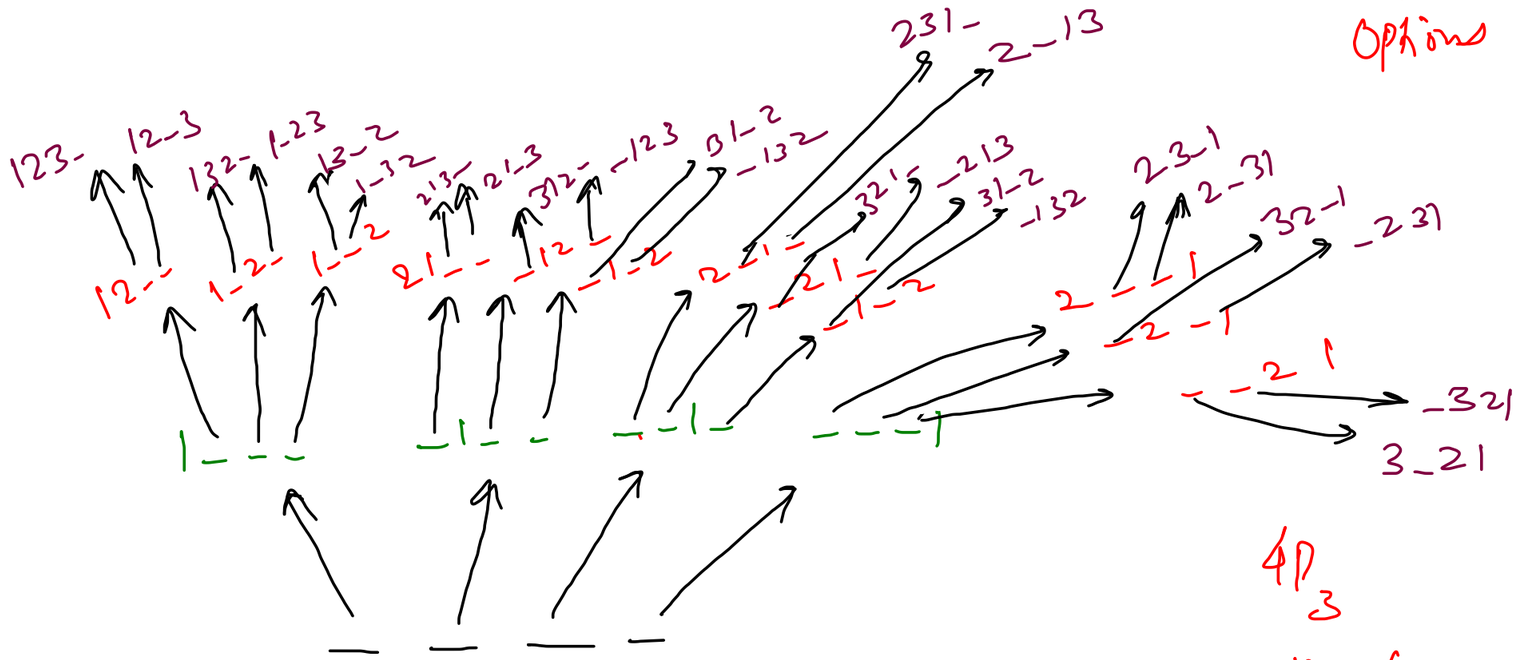
Permutation - 2

items on level : boxes are Options

item = 3

item = 2

item = 1



$^4P_3$

n = 4

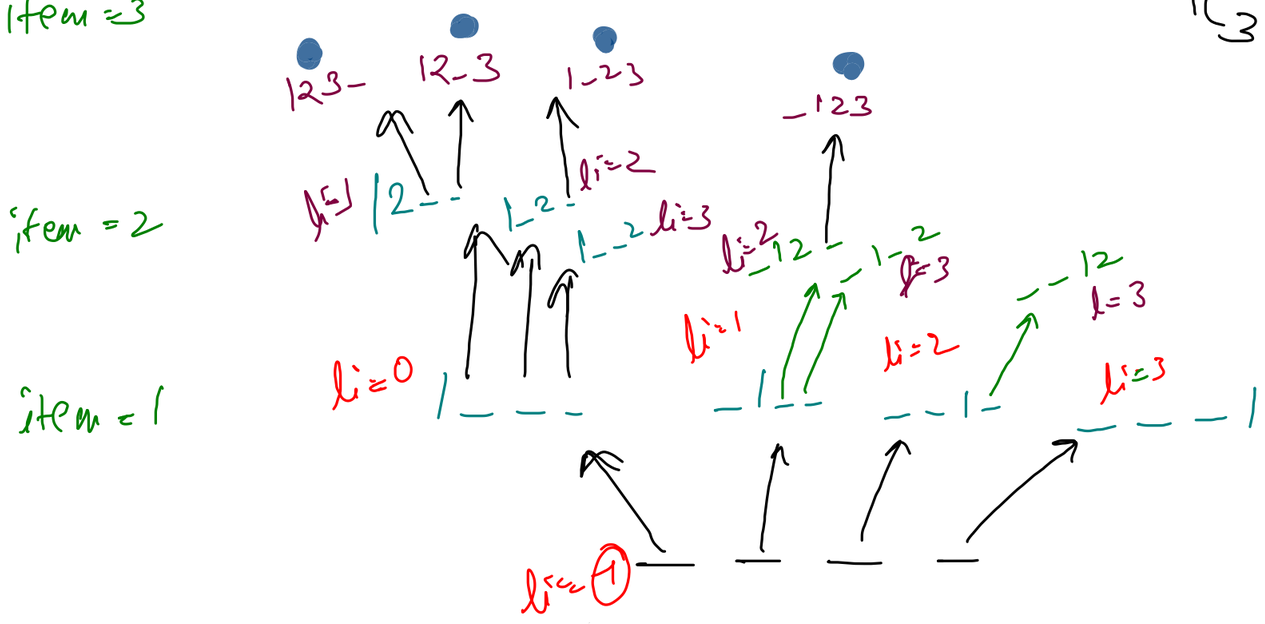
b = 3

```java
public static void permutations(int[] boxes, int ci, int ti){
  // write your code here
  if(ci == ti){
      for(int val: boxes) System.out.print(val);
      System.out.println();
  }

  // Item -> Chose box
  for(int i=0; i<boxes.length; i++){
      if(boxes[i] == 0){
          boxes[i] = ci + 1;
          permutations(boxes, ci + 1, ti);
          boxes[i] = 0;
      }
  }
}
```

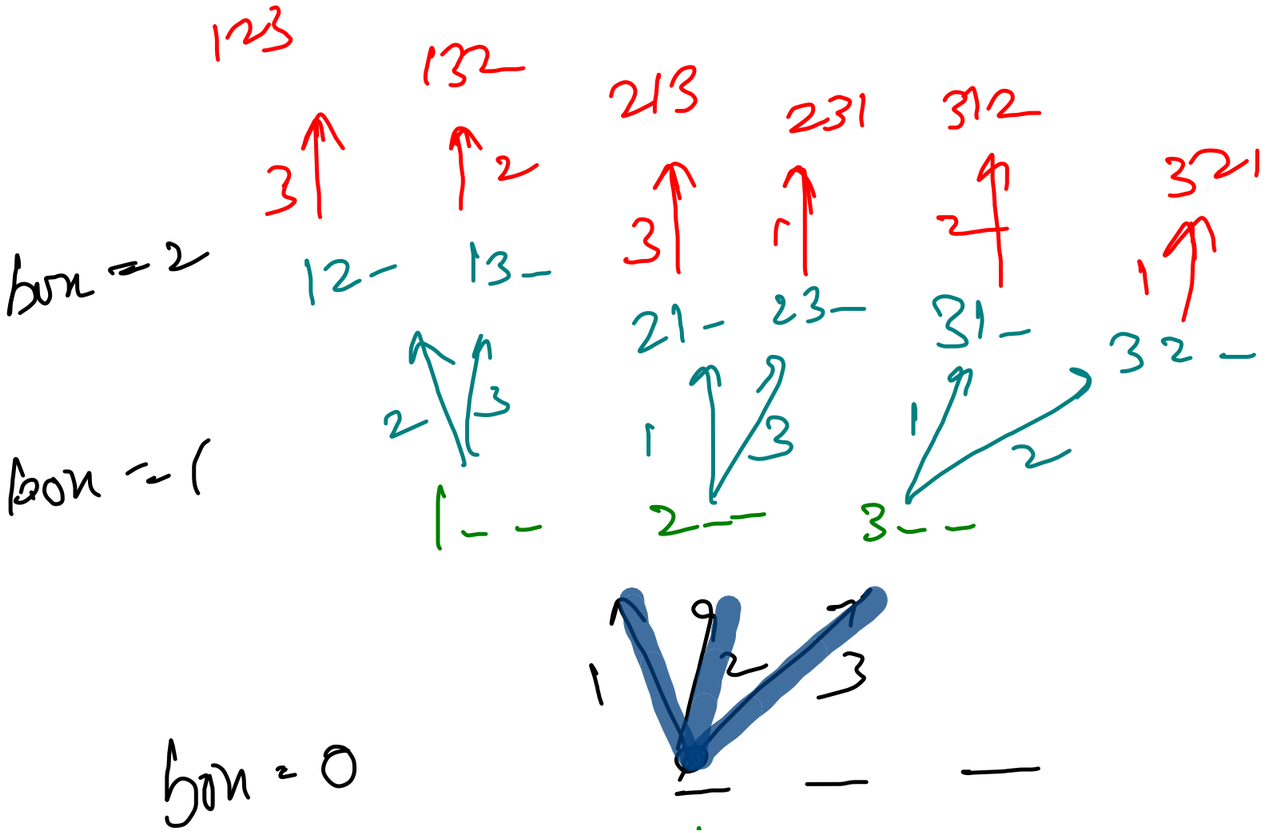
Combinations -2

{Items ⇒ level ; Option ⇒ box}

$^4C_3 = 4$




```java
public static void combunations(int[] boxes, int ci, int ti, int lastItemIdx){
  // write your code here
  if(ci == ti){
      for(int val: boxes){
          if(val == 0) System.out.print("-");
          else System.out.print("i");
      }

      System.out.println();
  }

  // Item -> Chose box
  for(int i=lastItemIdx + 1; i<boxes.length; i++){
      if(boxes[i] == 0){
          boxes[i] = ci + 1;
          combunations(boxes, ci + 1, ti, i);
          boxes[i] = 0;
      }
  }
}
```

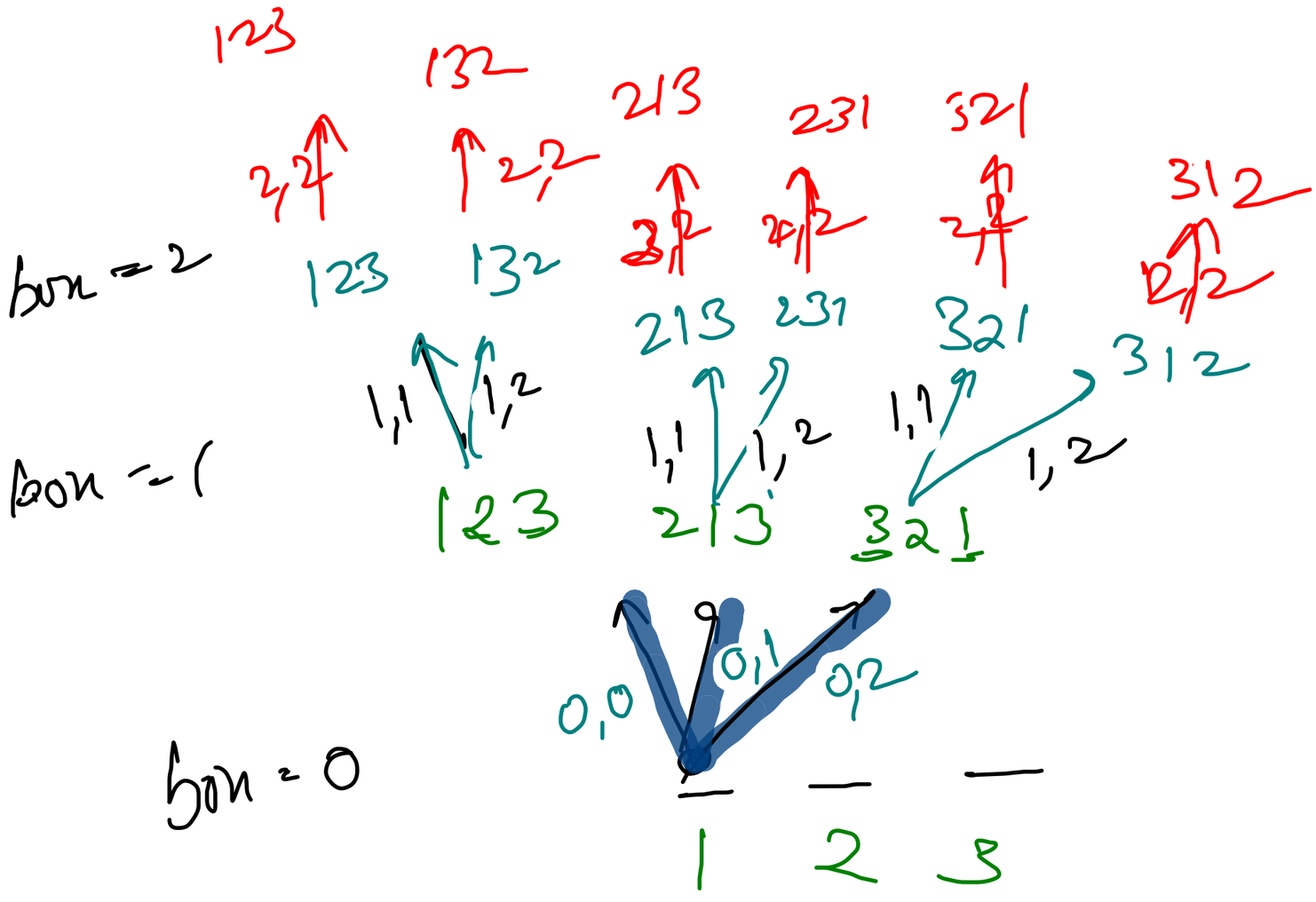


box = items

items

boxes = items

{1, 2, 3}




```java
public void permutations(int currentBox, int[] nums){
  if(currentBox == nums.length){
      List<Integer> copy = new ArrayList<>();
      for(int i=0; i<nums.length; i++){
          copy.add(nums[i]);
      }
      res.add(copy);
      return;
  }

  for(int i=currentBox; i<nums.length; i++){
      swap(nums, currentBox, i);
      permutations(currentBox + 1, nums);
      swap(nums, currentBox, i);
  }
```