Recursion & Backtracking

(1) Introduction

- Print Increasing
- Print Decreasing
- Print Increasing Decreasing
- Power-linear
- Power-logarithmic
- Print Zigzag
- Tower Of Hanoi

(2) Recursion & Arrays

- Display Array
- Display Array In Reverse
- Max Of An Array
- First Index
- Last Index
- All Indices Of Array

(3) Recursion & ArrayList {Get}

- Get Subsequence
- Get Kpc
- Get Stair Paths
- Get Maze Paths
- Get Maze Path With Jumps

(4) Recursion on the way up (print)

- Print Subsequence
- Print Kpc
- Print Stair Paths
- Print Maze Paths
- Print Maze Paths With Jumps
- Print Encodings
- Print Permutations

(5) Backtracking

- Flood Fill
- Target Sum Subsets
- N Queens
- Knights Tour

Principle of mathematical Induction (PMI)

$$\sum_{i=1}^{n} i = \frac{n*(n+1)}{2}$$

① Trivial Case

$$\sum_{i=1}^{0} i = 0$$

$$\sum_{i=1}^{1} i = 1$$

② ★ Assume $\sum_{i=1}^{k} i = \frac{k*(k+1)}{2}$

③ To prove $\sum_{i=1}^{k+1} i = (1+2+\cdots k)+(k+1) = \frac{k*(k+1)}{2} + (k+1)$

$$= \frac{(k+1)*(k+2)}{2}$$

if $n = k+1$ LHS = RHS

Recursion {High-level thinking}

n<5

1 2 3 4 (5)

(1) Recursive function :→ Expectation

void printInc(int n) :→ 1, 2, 3, - ... n

(2) Recursive fn :→ Smaller values :→ faith (call)

void printInc(n-1) :→ 1, 2, 3, - ... n-1

1 2 3 4

(3) Meeting Expectation with faith

System.out.println(n);

(4) Base Case

↳ To stop recursion

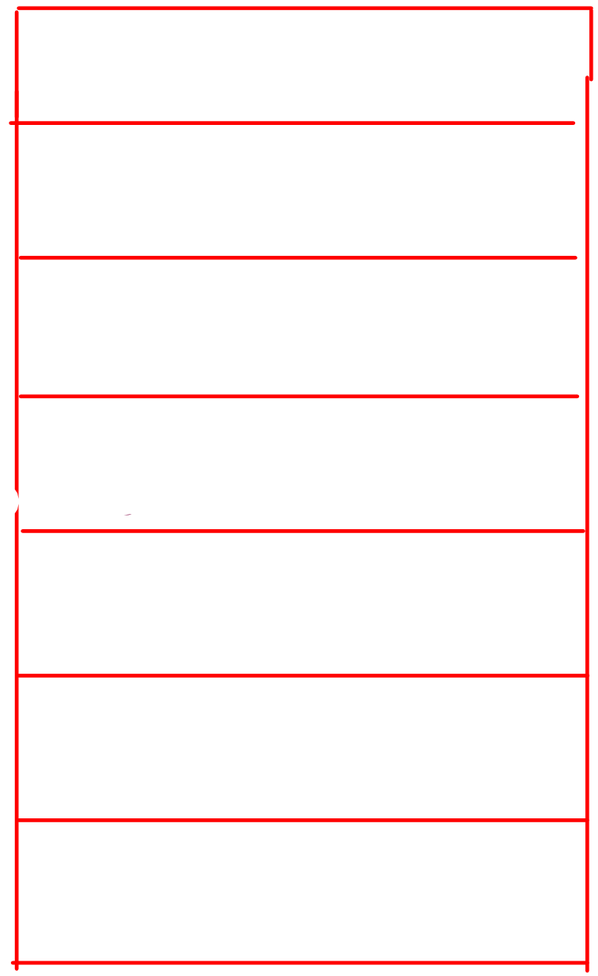
① → Stack overflow / memory limit exceeded

how- level thinking

function call stack

```
void printInc(int n) {
Base case → (3) if(n==0) return;
Faith → (1) printInc(n-1);
Meeting Expectation in faith → (2) System.out.println(n);
}

main() {
    printInc(5);
}
```



```java
public static void main(String[] args) throws Exception {
    Scanner scn = new Scanner(System.in);
    int n = scn.nextInt();
    printIncreasing(n);
}

public static void printIncreasing(int n){
    if(n == 0){
        // Base Case
        return;
    }

    // 1. Faith : 1, 2, .. n-1
    printIncreasing(n - 1);

    // 2. Meet Expectation with Faith
    System.out.println(n);
}
```

Tail Recursion

↓

work is being done while returning

} → post work

Q2) Print Decreasing

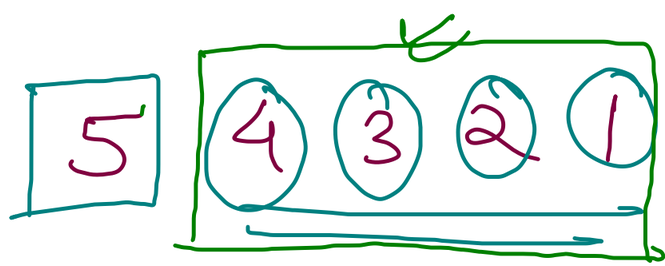
(1) printDecreasing(int n) :→ Expectation

n, n-1, n-2, ---- 1

(2) printDecreasing(n-1) :→ faith

n-1, n-2, n-3, --- 1

(3) System.out.println(n) :→ Meeting Expectation should be before the call.



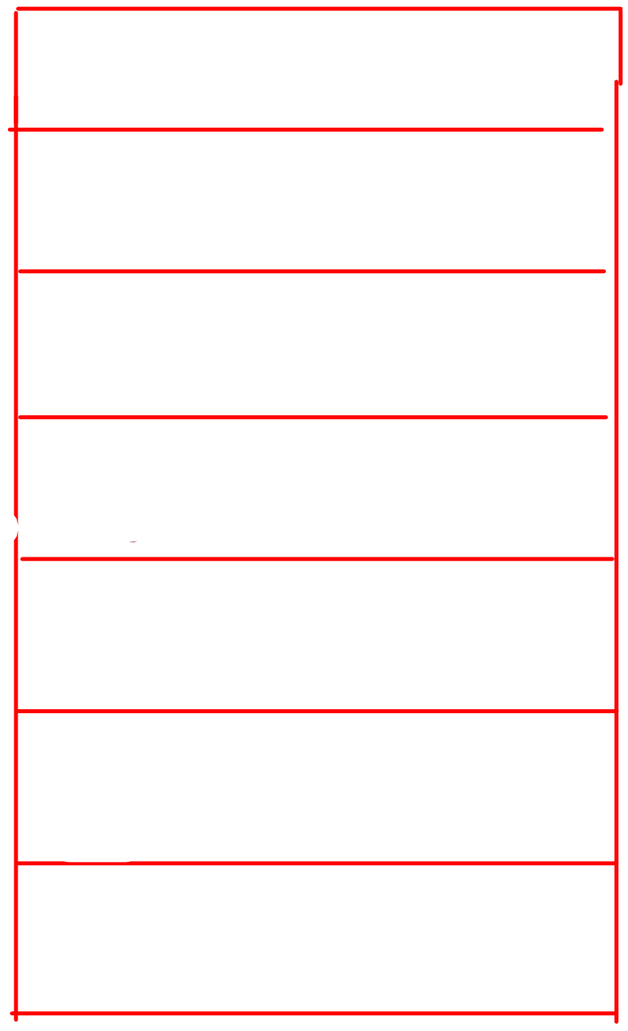
n=5 5 4 3 2 1

```
void printDec(int n){
    if(n==0) return;
    System.out.println(n);
    printDec(n-1);
}
```

① ② ③ ④

① ⇒ Preorder

head Recursion
work is done before calling functions



5 4 3 2 1

Q) Print Increasing Decreasing

High-Level Thinking

① Expectation

PDI(int n) :→ n, n-1, n-2, -- -1, 1, 2, --- n

n=4 :→ 4 3 2 1 1 2 3 4

② Faith

PDI(n-1) :→ n-1, n-2, n-3, --- 1, 1, 2, 3, --- n-2, n-1

n=3 :→ 3 2 1 1 2 3

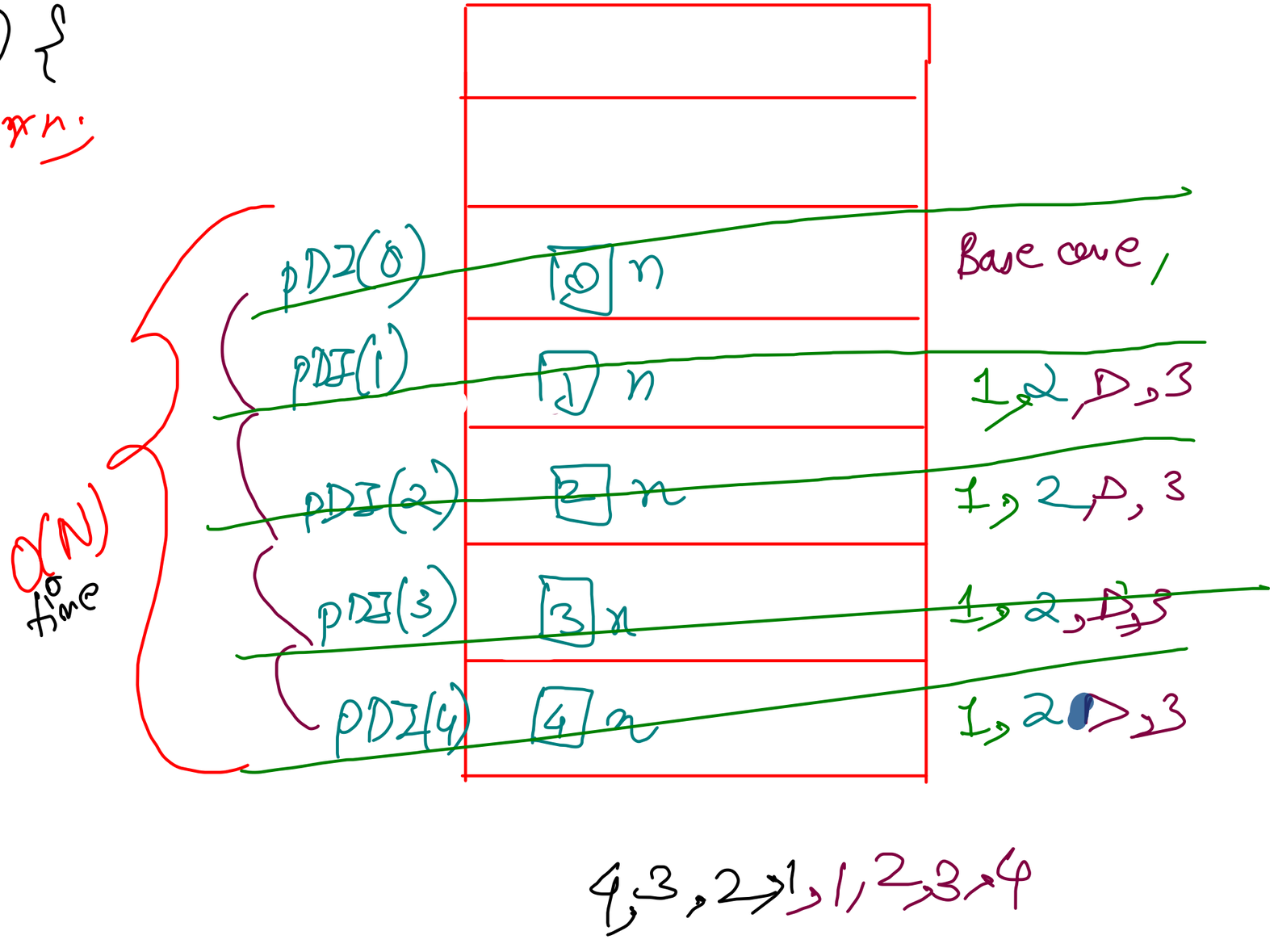
③ Meeting Expectation

Sys0(n) → Preorder
Sys0(n) → Postorder

N = 4 4,3,2,1,1,2,3,4

```
void pDI( int n) {
  ①  if(n==0) return;
Preorder{ ②  Syso(n);
failure{  ③  pDI(n-1);
Postorder{④  Syso(n);
}

main() {
    pDI(n);
}
```



4,3,2,1,1,2,3,4

Factorial

$5! = 5 * 4 * 3 * 2 * 1$

High level thinking

1. Expectation

int fact(int n) :→ $n * (n-1) * (n-2) * \cdots 1$

2. Faith

int fact(n-1) :→ $(n-1) * (n-2) * \cdots 1$ → factnm1

3. Meeting Expectation :→

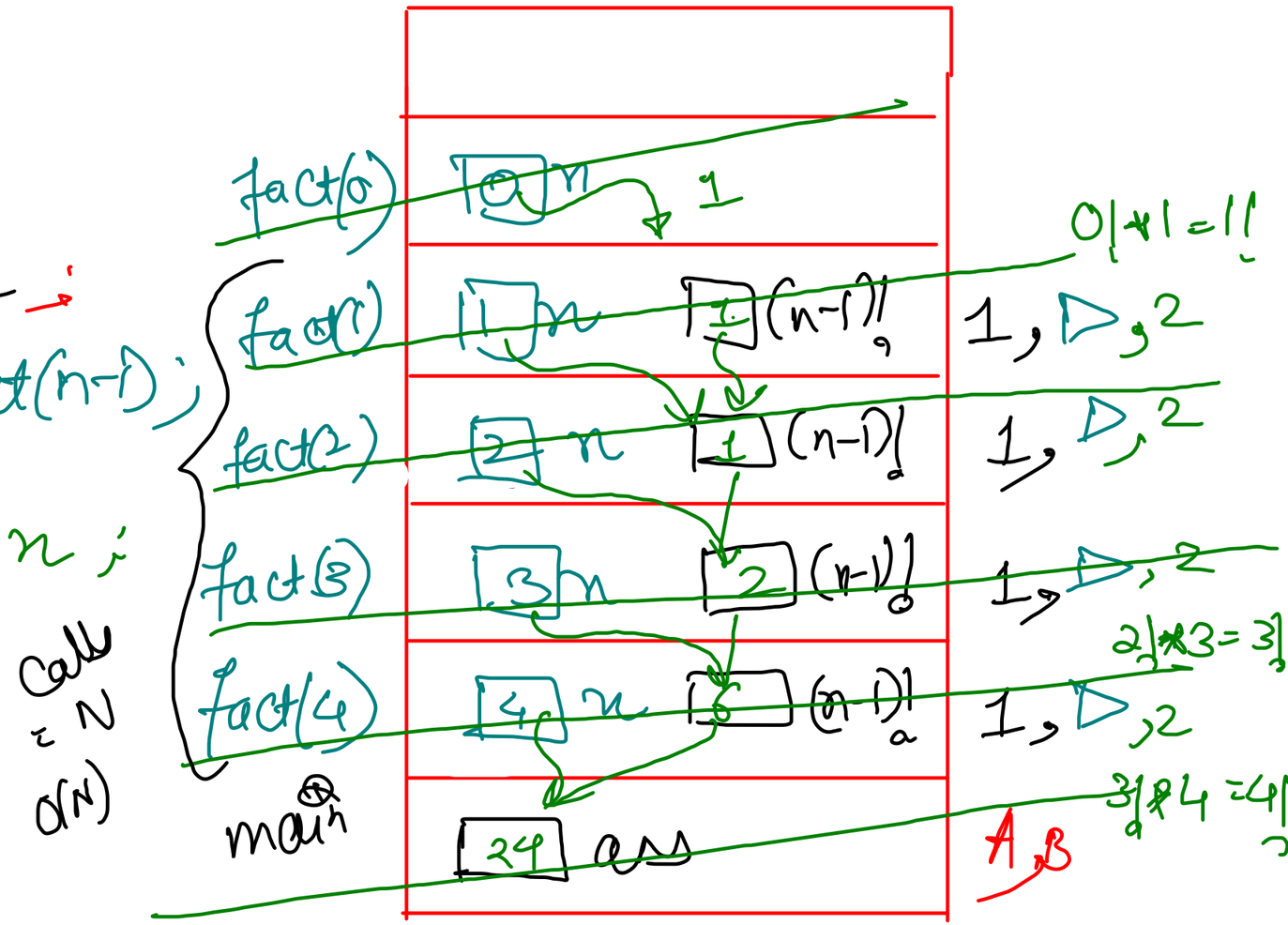
```
{ return (fact nm1) * n; }
```

How-Level thinking

N = 4 4×3×2×1

```
int fact (int n) {
Base case (0)  if (n==0) return 1;
faith {   (1)  int factnm1 = fact(n-1);
Meeting {
Expectation (2)  return factnm1 * n;
}
```

```
main() {
(A) int ans = fact(n);
(B)   Syso (ans);
}
```



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Power - Linear $\Rightarrow O(N)$

$a^b = a * a * a * \cdots$ b times

$3^5 = 3 * 3 * 3 * 3 * 3$

(1) Expectation

constant, variable

power(x, n) :→ $\{ 3*3*3*3*3 \}$

$x * x * x * \cdots$ n times

(2) Faith

power(x, $n-1$) :→ $\{ 3*3*3*3 \}$

$x * x * \cdots$ $(n-1)$ times

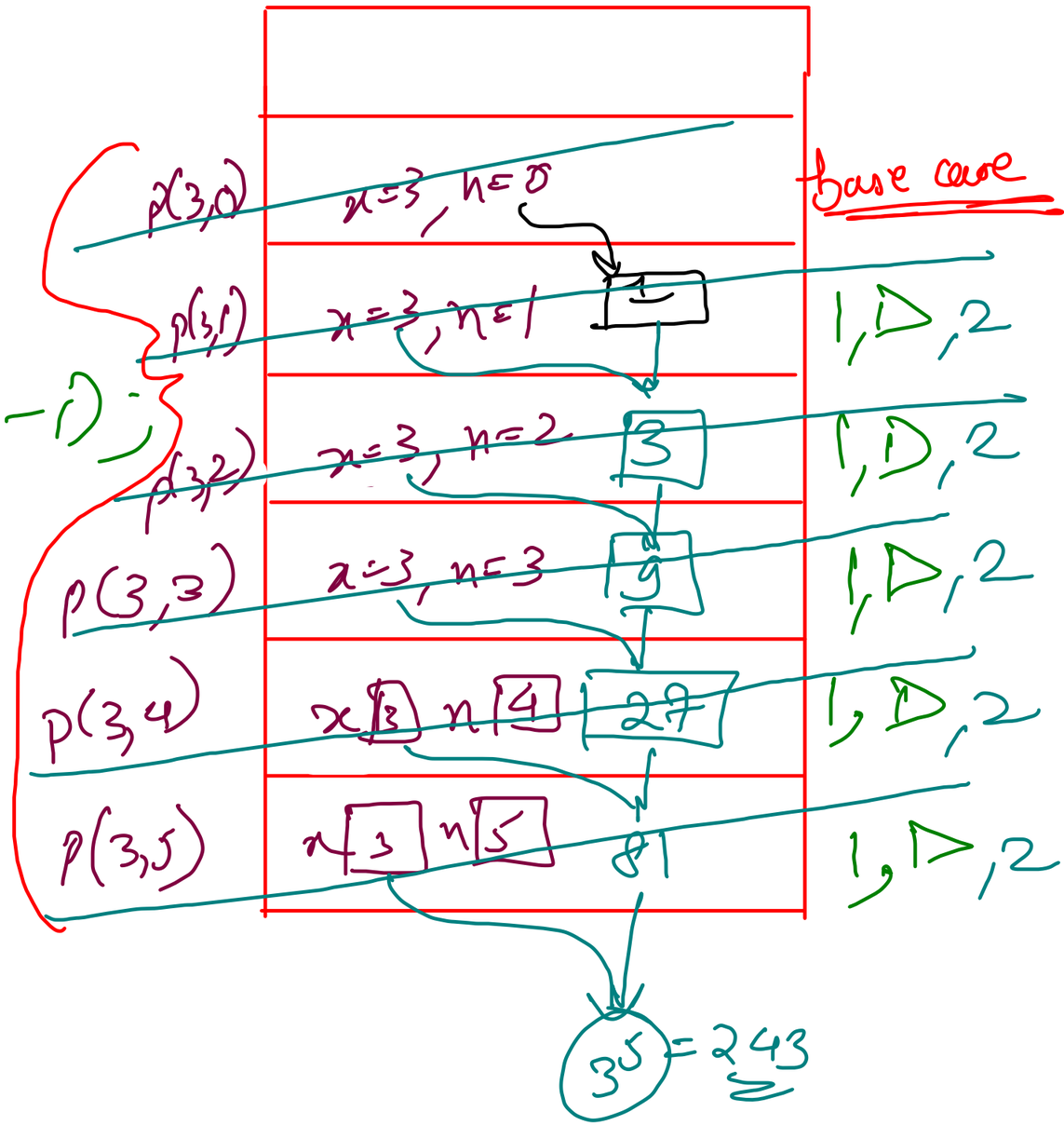
(3) Meeting Expectation :→

return pxnm1 * x;

$3^4 * 3 = 3^5$

How - Level Thinking

```
int power(int x, int n) {
    ① if(n == 0) return 1;
    ② int pxnm1 = power(x, n-1);
    ③ return pxnm1 * x;
}
```



```java
public static void main(String[] args) throws Exception {
    // write your code here
    Scanner scn = new Scanner(System.in);
    int x = scn.nextInt();
    int n = scn.nextInt();
    int ans = power(x, n);
    System.out.println(ans);
}

public static int power(int x, int n){
    if(n == 0) return 1; // x ^ 0 = 1
    int pxnm1 = power(x, n - 1); // faith
    return pxnm1 * x; // meeting expectation
}
```

Power - logarithmic

$a^b = a * a * a * \cdots$ b times

$3^5 = 3 * 3 * 3 * 3 * 3$

$x^n = x^{n/2} * x^{n/2}$ (n = even)

$3^8 = 3^4 * 3^4$

$x^n = x^{n/2} * x^{n/2} * x$ (n = odd)

$3^9 = 3^4 * 3^4 * 3$

① Expectation

power(x, n) :→ { 3 * 3 * 3 * 3 * 3 }

x (constant), n (variable)

$x * x * x * \cdots$ n times

② Path

power(x, n/2) :→

pxnby2

$x * x * \cdots$ n/2 times

③ Meeting Expectation :→

return pxnby2 * pxnby2 ;

*x if(n%2==1)

Low-level Thinking

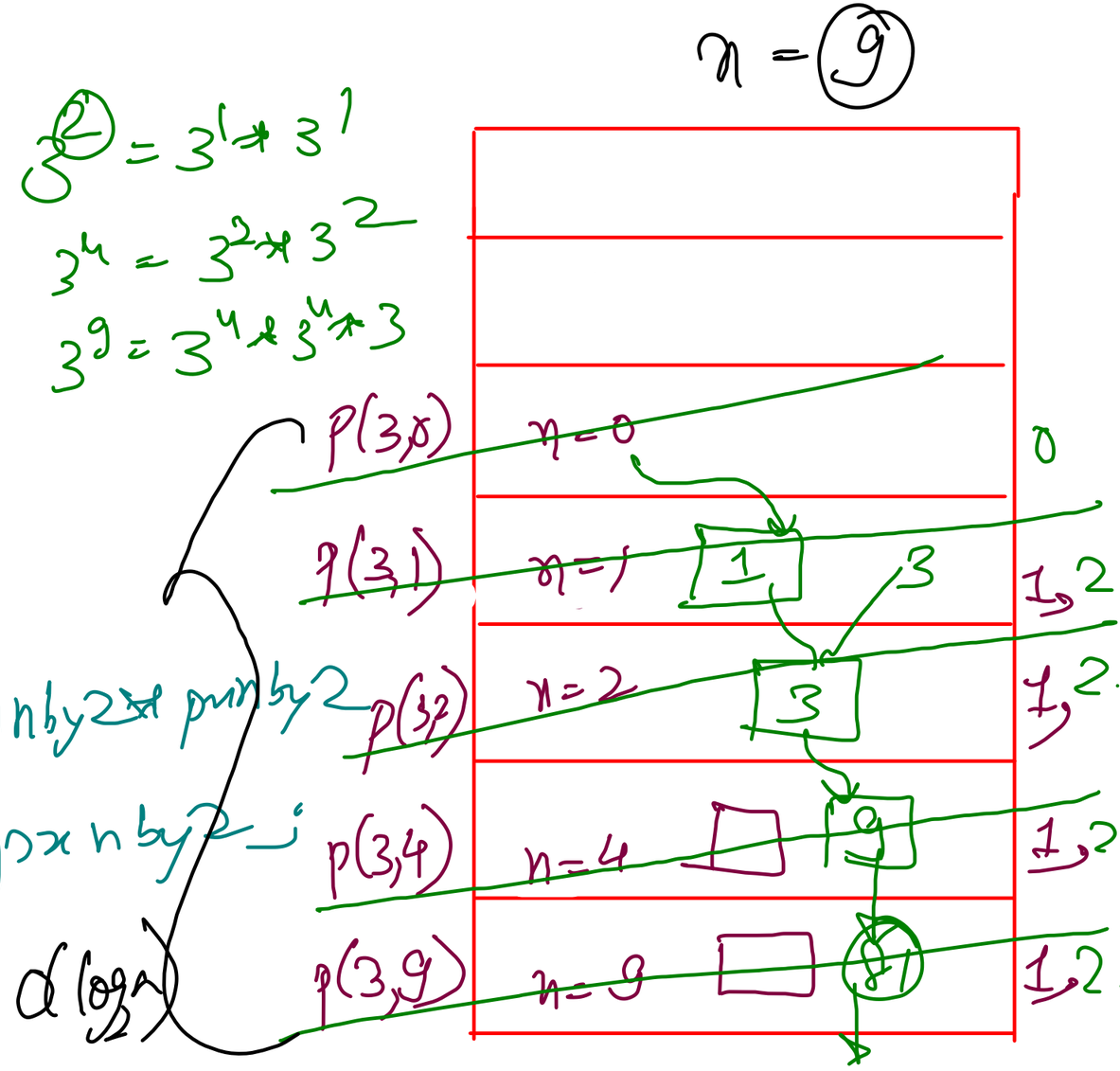
```
int power(int x, int n) {
    if(n==0) return 1;
    int pxnby2 = power(x, n/2);
    if(n%2==1) return x * pxnby2 * pxnby2;
    else return pxnby2 * pxnby2;
}
```

① ② ③ ④ ⑤ ⑥ ⑦ ⑧ ⑨ ⑩ ⑪ ⑫ ⑬ ⑭ ⑮ ⑯ ⑰ ⑱ ⑲ ⑳

$3^9 = 3^4 * 3^4 * 3$

$3^4 = 3^2 * 3^2$

$3^2 = 3^1 * 3^1$



```java
public static void main(String[] args) throws Exception {
    Scanner scn = new Scanner(System.in);
    int x = scn.nextInt();
    int n = scn.nextInt();
    int ans = power(x, n);
    System.out.println(ans);
}

public static int power(int x, int n){
    if(n == 0) return 1;

    int pxnby2 = power(x, n/2);

    if(n % 2 == 1) return pxnby2 * pxnby2 * x;
    else return pxnby2 * pxnby2;
}
```

$n = 2^x \Rightarrow$ no of calls $= x$

$\log_2 n = \log_2(2^x)$

$\log_2 n = x * \log_2 2$

$\log_2 n = x \Rightarrow$ no of call $\Rightarrow O(\log_2 n)$

n

2^{20}

$O(N) \Rightarrow 1048576$

$O(\log_2 N) \Rightarrow O(\log_2 2^{20})$

$= O(20)$

constant

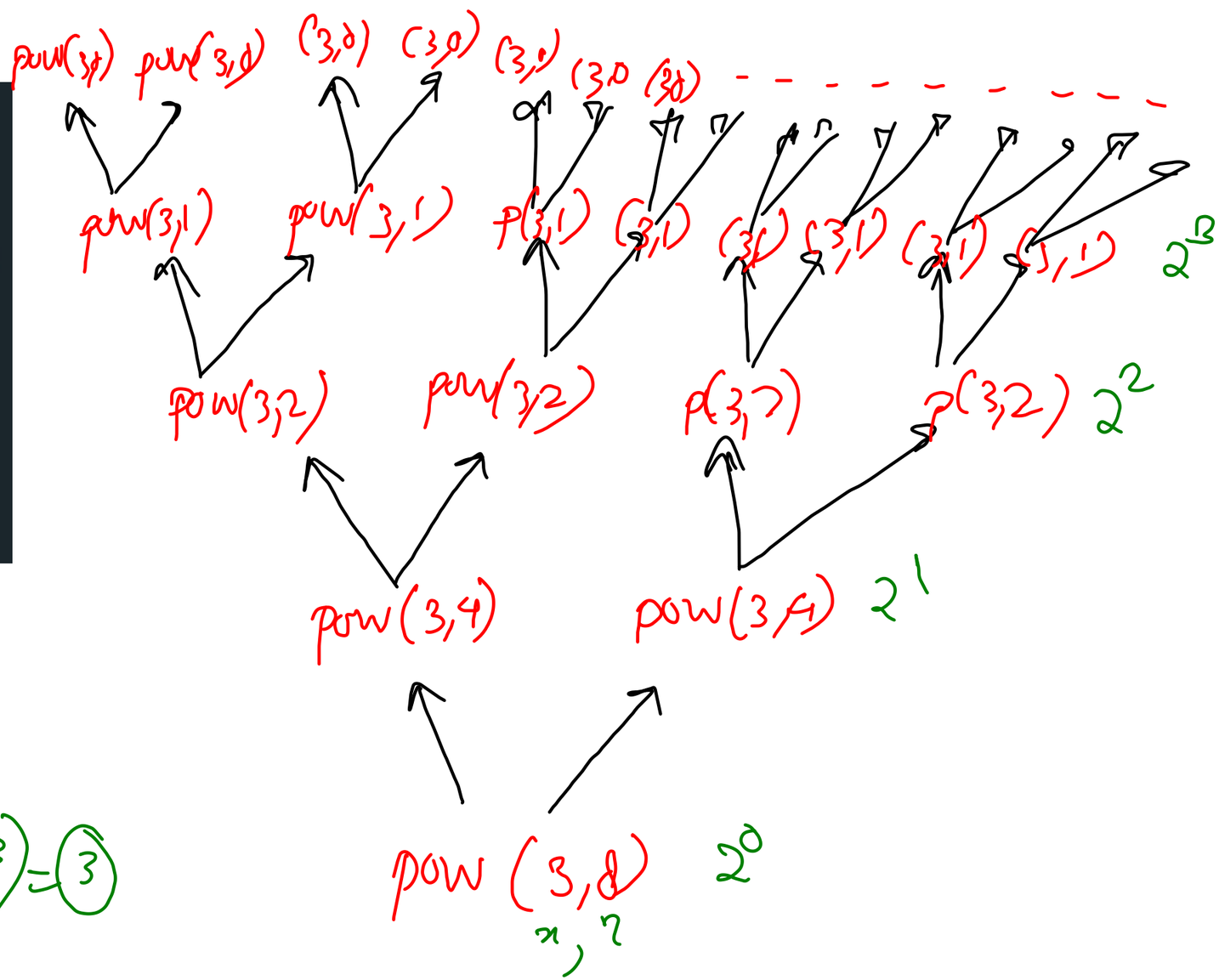
Today's Questions

1. Power - 3rd method
2. Print ZigZag
3. Display Array
4. Display Array - Reverse {Home work}
5. Max of Array
6. First Index of Array
7. Last Index of Array

Power → III rd method

```java
public static int power(int x, int n) {
  if(n == 0){
    return 1;
  }
  int xpn = power(x, n/2) * power(x, n/2);

  if(n % 2 == 1){
    xpn = xpn * x;
  }

  return xpn;
}
```

L R



$2^0+2^1+2^2+2^3 = \frac{2^4-1}{2-1}$

$\approx 2^4$

$\approx 2^{3+1}$

$\approx 2^{\log_2 n+1}$

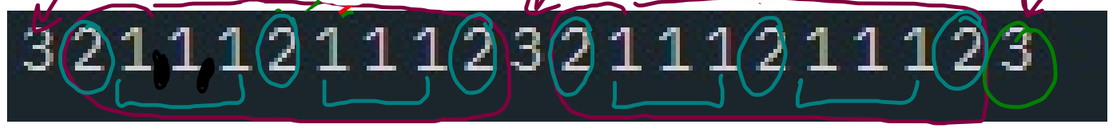
$\approx n$

$n = 8$

$O(\log_2 n) = O(\log_2 2^3) = 3$

(9) Print Zigzag

(1) Expectation :- printZigzag(int n)



3

2

(2) Faith :- printZigzag(n-1)

2 1 1 1 2 1 1 1 2

(3) Meet Expectation :-

```
Syso(n)
printZigzag(n-1)
Syso(n)
printZigzag(n-1)
Syso(n)
```

printZigzag(1)

1 1 1

printZigzag(0)

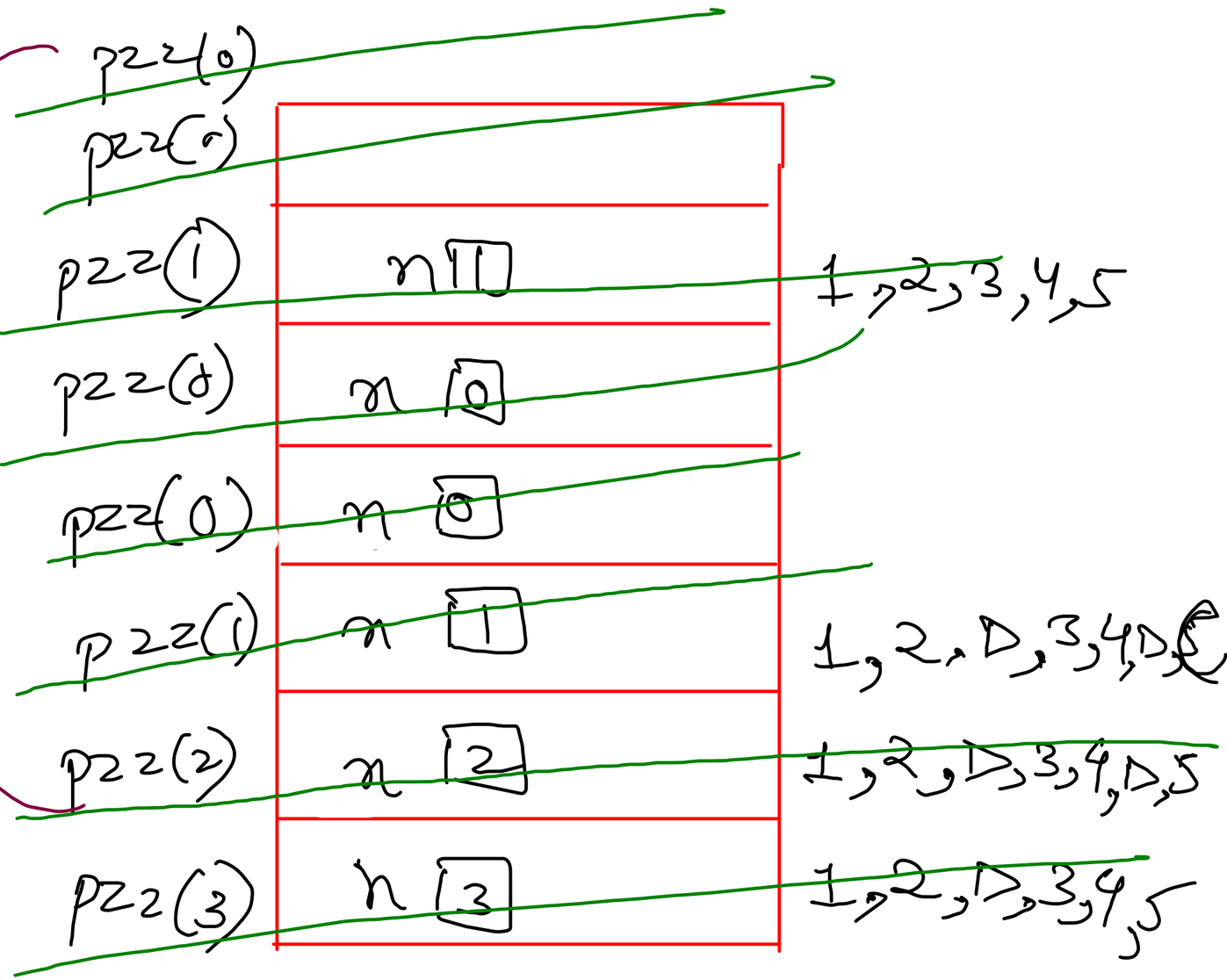
return;

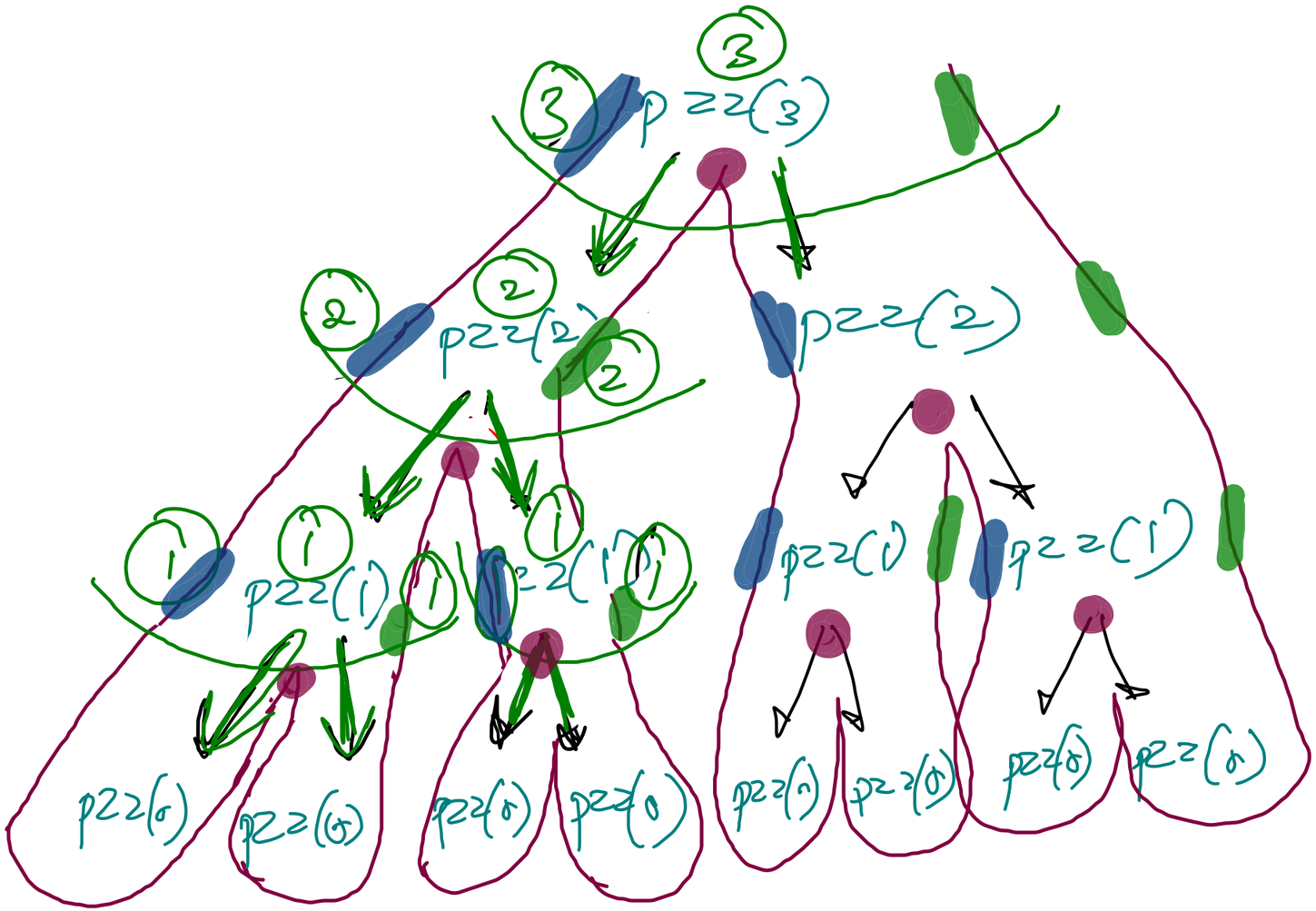
```java
public static void pzz(int n){
    if(n == 0) return;

    System.out.print(n + " ");
    pzz(n - 1);
    System.out.print(n + " ");
    pzz(n - 1);
    System.out.print(n + " ");
}
```

x2

3, 2, 1, 1, 1, 2, 1, 1, 1, 2, 3

2, 1, 1, 1, 2, 1, 1, 1, 2, 3






```java
public static void pzz(int n){
    if(n == 0) return;

    System.out.print(n + " ");
    pzz(n - 1);
    System.out.print(n + " ");
    pzz(n - 1);
    System.out.print(n + " ");
}
```

Depth first Search

⇓

Recursive tree/
callertree

Recursion

$$(calls)^{height} + \{preorder + inorder + postorder\} * height$$

eg Print zigzag

$$2^{n} + \{k + k + k\} * n = 2^{n} + 3k * n$$

$$O(2^n)$$

Recursion & Arrays

(Q) Display Array

```
public static void displayArr(int[] arr, int idx){

}
```

① Expectation

{ 10, 20, 30, 40, 50 }

displayArr(arr, 0)

② Faith

display (arr, 1)

{ 20, 30, 40, 50 }

③ Meeting Expectation

↳ Preorder

syso(arr[idx])

display (arr, idx+1)

④ Base case

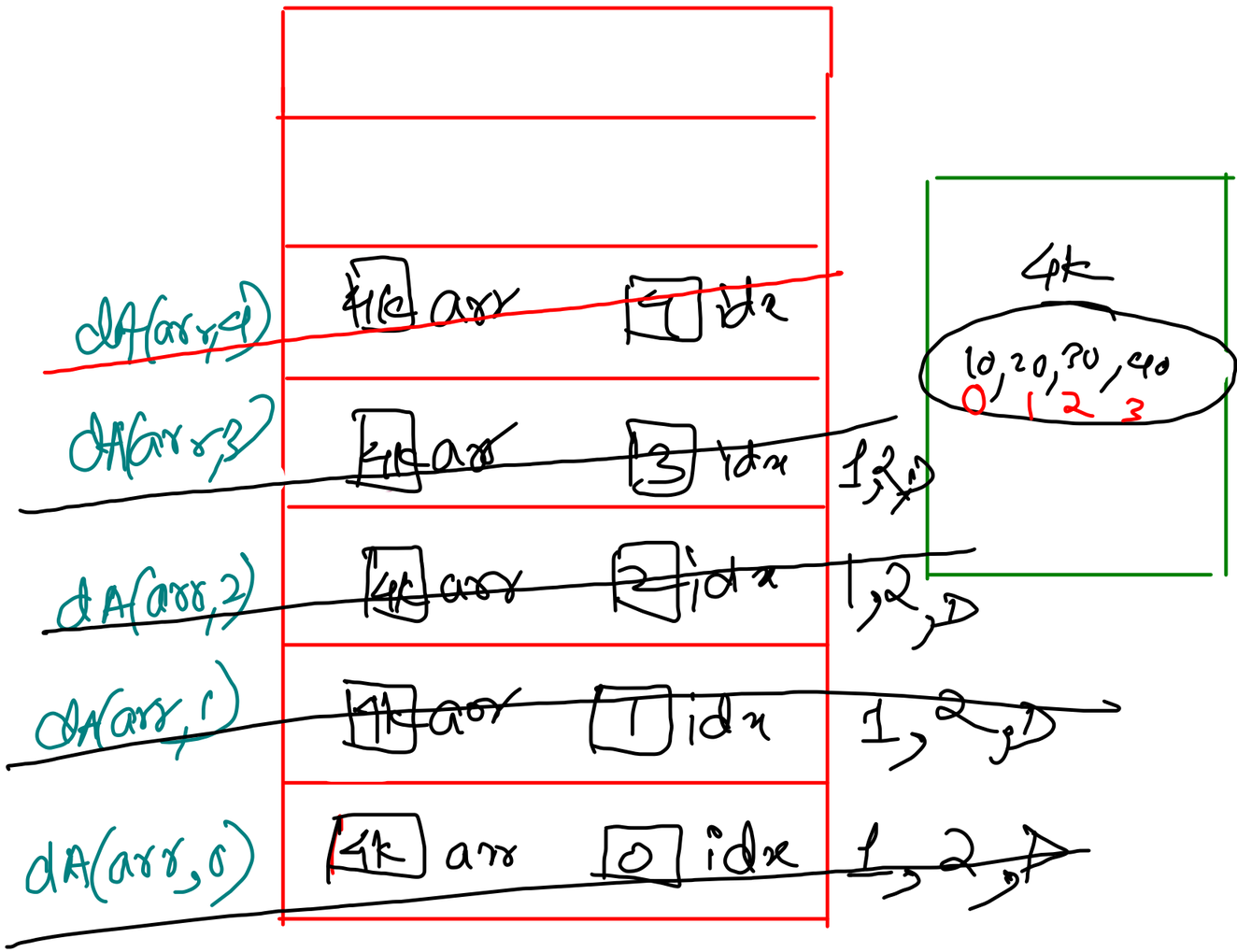
if (idx == arr.length)

return;

```java
public static void displayArr(int[] arr, int idx){
    if(idx == arr.length) return;

    // preorder -> Meeting Expectation
    System.out.println(arr[idx]);

    // faith
    displayArr(arr, idx + 1);
}
```

{preorder}

10, 20, 30, 40



⑥ Maximum In an Array

{ 20, -30, 50, 60, 0, 10, 40 }

① Expectation

int maxOfArray(arr, 0) :→ max arr[0 : l-1]
⇒ 60

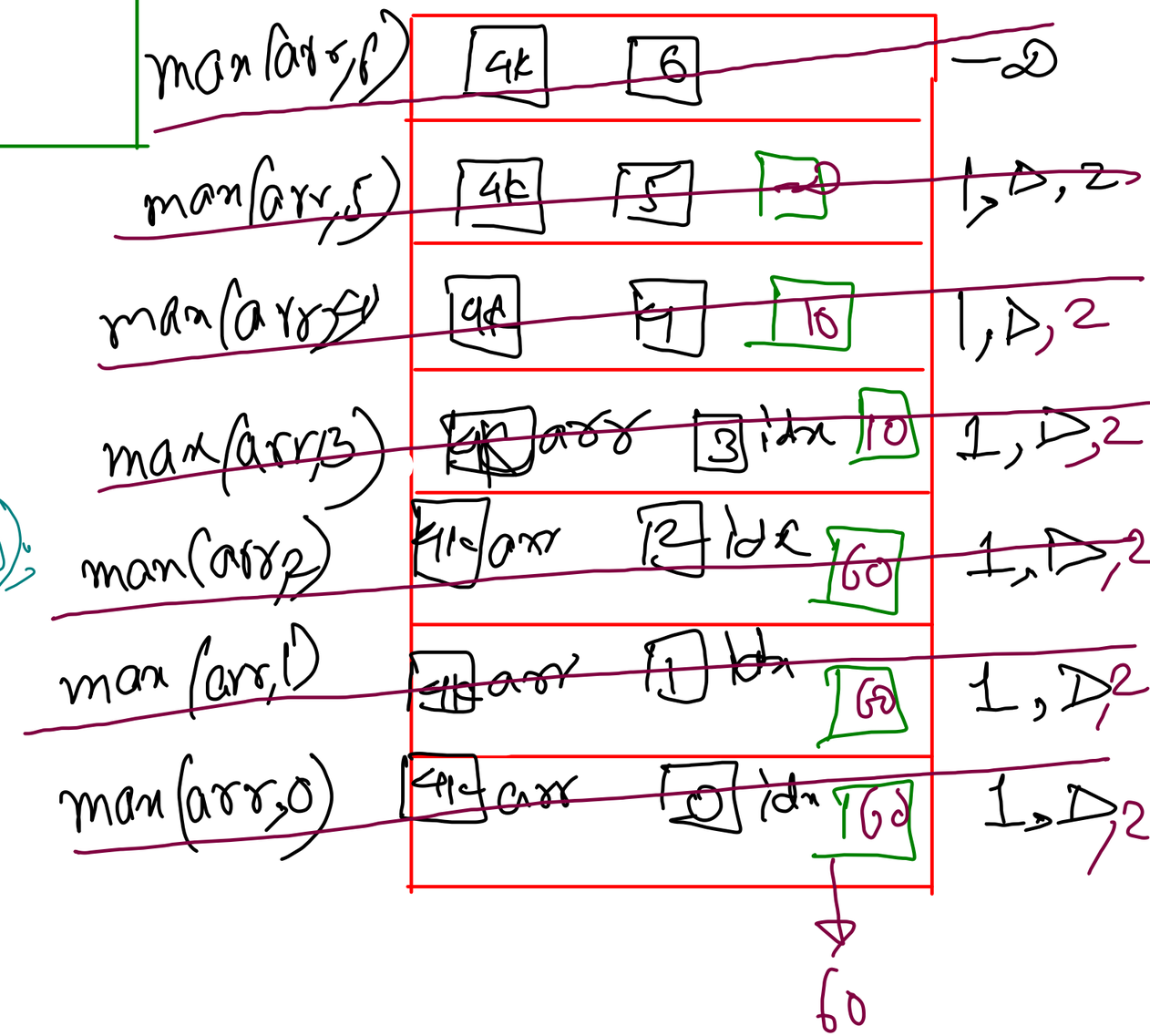
② Faith int maxOfArr(arr, 1) :→ max arr[1 : l-1]

③ Meeting Expectation :→ return max(arr[idx], maxCall)

heap

4k { 20, -30, 50, 60, 0, 10 }
0 1 2 3 4 5

```
int maxOfArr(int[] arr, int idx){
    ① if(idx == arr.length) return -∞;
    ② int maxTemp = max(arr, idx+1);
    ③ return Math.max(maxTemp, arr[idx]);
}
```



Addition

$x + ? = x$ (? = 0)

Subtraction

$x - ? = x$ (? = 0)

Multiplication

$x * ? = x$ (? = 1)

Minimum

$min(x, +\infty) = x$

$+\infty$ → Integer. MAX_VALUE

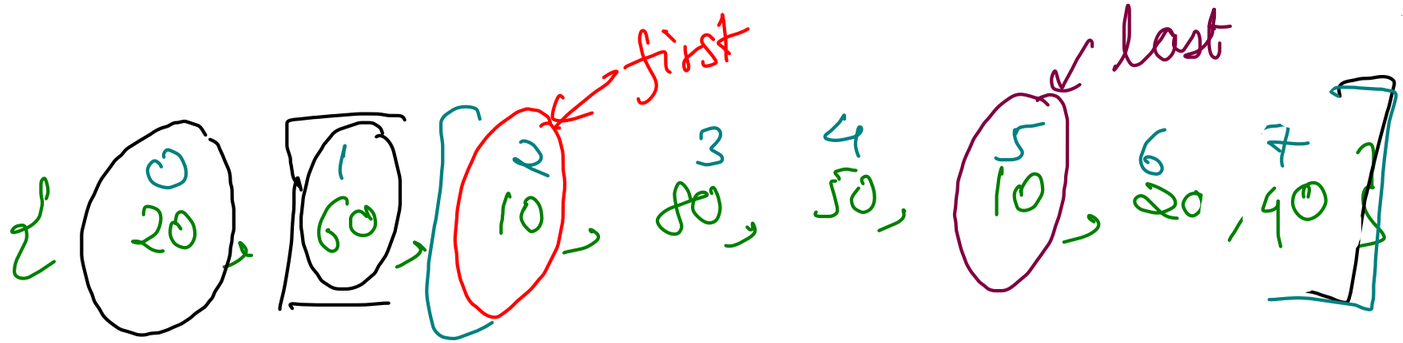
$+2^{31} - 1$

Maximum

$min(x, -\infty) = x$

$-\infty$ → Integer. MIN_VALUE

-2^{31}



First Index

fI (arr, 0) :→ 2

faith

temp = fI(arr, idx+1)

Meeting Exp

if(arr[idx] == target) T → return idx;
F → return temp;

L→R

lastIndex

lI (arr, a.l-1)

faith

temp = lI(arr, idx-1)

R→L

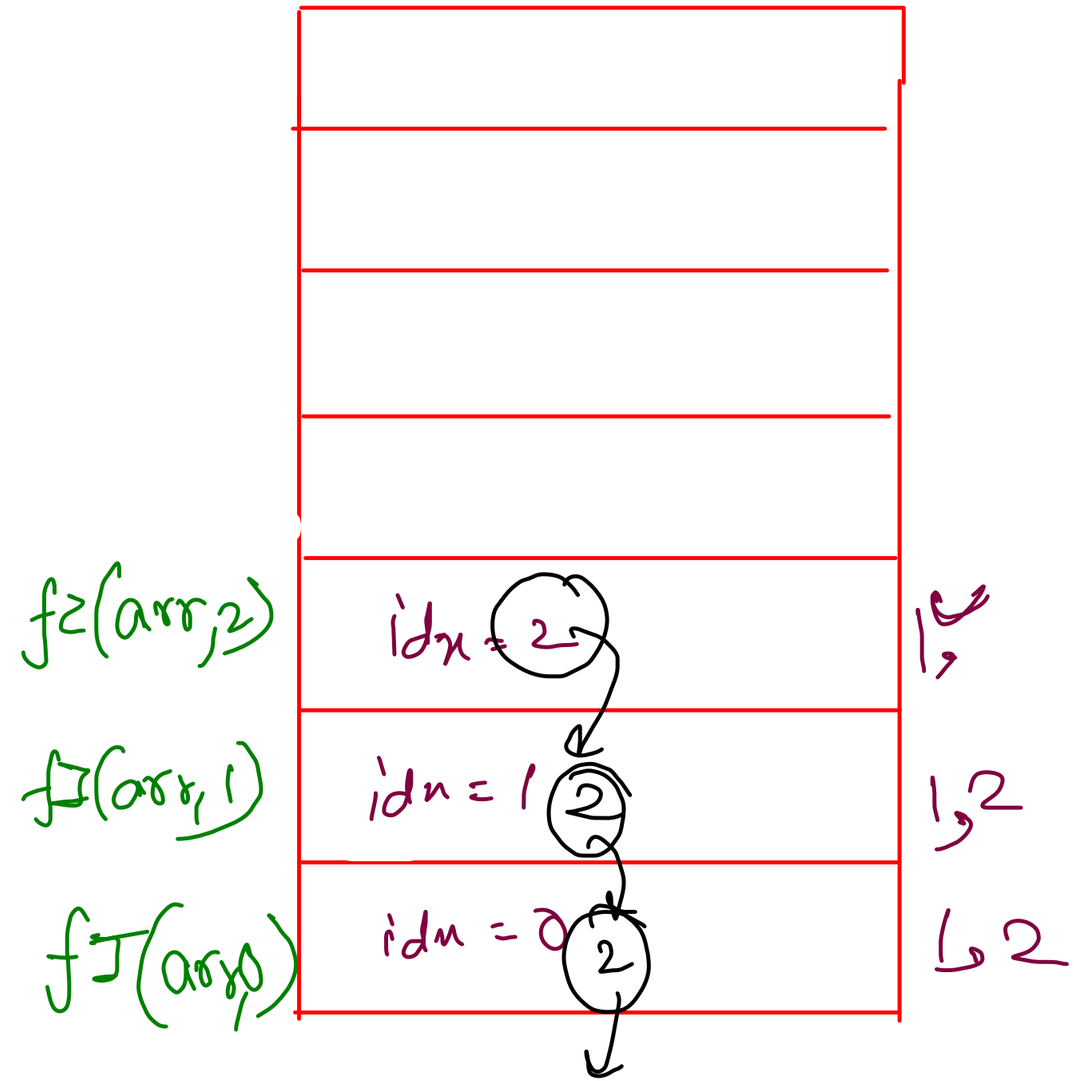
Meeting Exp

if(arr[idx] == target)
T → return idx;
return temp;

```java
public static int firstIndex(int[] arr, int idx, int x){
    if(idx == arr.length) return -1;

    if(arr[idx] == x){
        // meeting expectation
        return idx;
    } else {
        // faith
        return firstIndex(arr, idx + 1, x);
    }
}
```

{ 20 50 10 80 50, 10, 20, 40 }

```java
public static int lastIndex(int[] arr, int idx, int x){
    if(idx == -1) return -1; // search unsuccessful

    if(arr[idx] == x){
        return idx;
    } else {
        return lastIndex(arr, idx - 1, x);
    }
}
```



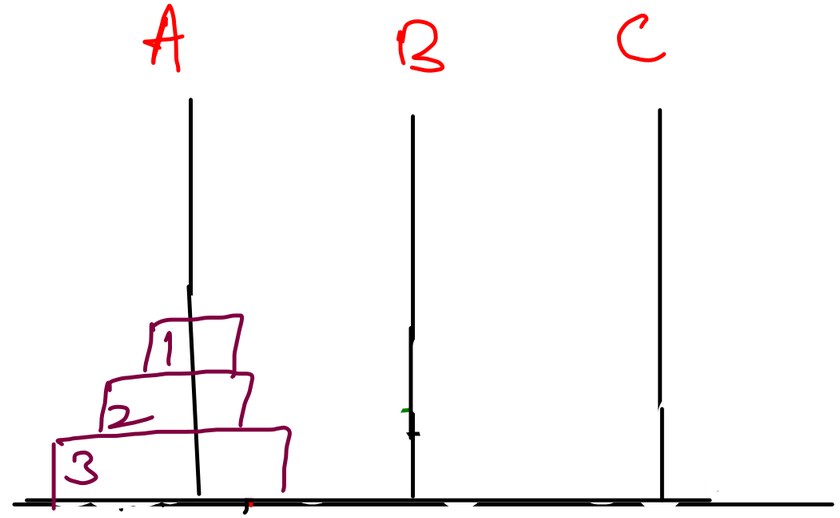
Today's Questions

1. Tower of Hanoi
2. All indices of Array

+ Discussion on DSA based projects

Tower of Hanoi

1. There are 3 towers. Tower 1 has n disks, where n is a positive number. Tower 2 and 3 are empty.
2. The disks are increasingly placed in terms of size such that the smallest disk is on top and largest disk is at bottom.
3. You are required to
   3.1. Print the instructions to move the disks.
   3.2. from tower 1 to tower 2 using tower 3
   3.3. following the rules
      3.3.1 move 1 disk at a time.
      3.3.2 never place a smaller disk under a larger disk.
      3.3.3 you can only move a disk at the top.



Constraints:

(1) only one disk can be moved at a time

(2) Larger disk cannot be placed on top of smaller disk

(1) Expectation

(Source) (dest) (auxilliary)
from to using

tower of hanoi (n , A , B , C) → instructing to move 3 disks from A to B using C

(2) Faith

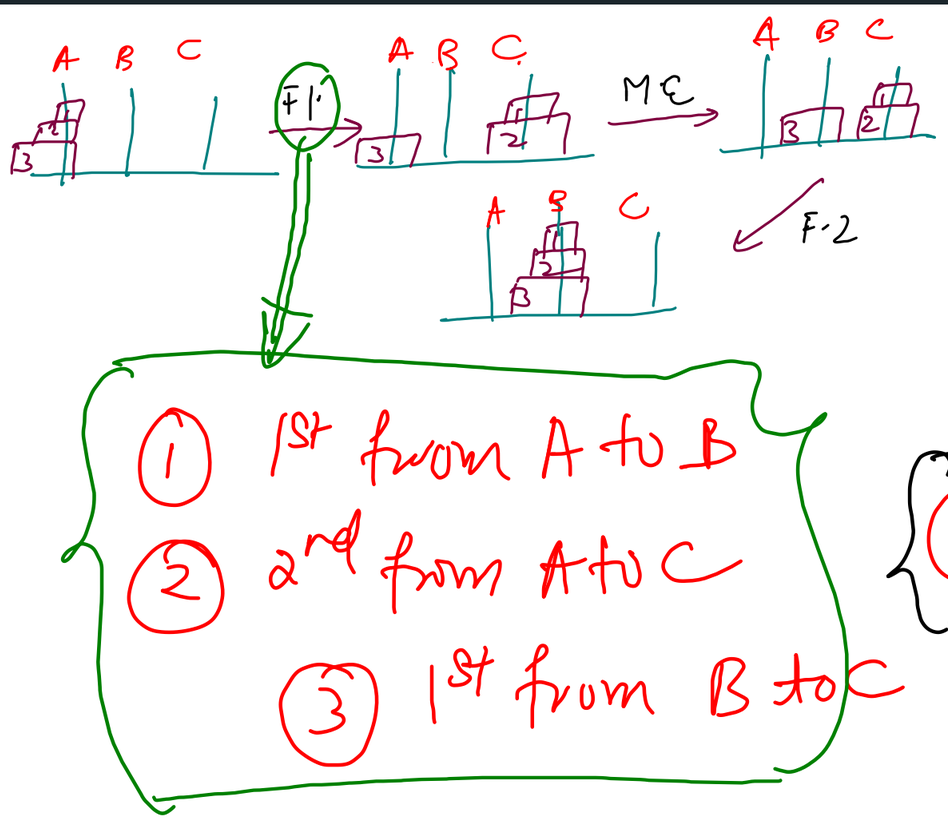
(2.1) tower of hanoi (n-1, A, C, B) → move (n-1) disks from A to C using B

Meeting Expectation

(2.2) Move nth disk from A to B

(2.3) tower of hanoi (n-1, C, B, A) → move (n-1) disks from C to B using A.

```java
public static void toh(int n, int srcTower, int destTower, int auxTower){
    if(n == 0) return;

    toh(n - 1, srcTower, auxTower, destTower);
    // Faith -> N - 1 disks from source to auxiliary

    System.out.println(n + "[" + srcTower + "->" + destTower + "]");
    // Move nth disk from source to destination

    toh(n - 1, auxTower, destTower, srcTower);
    // Faith -> N - 1 disks from auxiliary to destination
}
```



0, A, C, B

0, B, A, C

1, B, C, A

0, C, B, A

0, A, C, B

1, A, B, C

2, A, C, B

3, A, B, C

1, ▷, 2, 3

1, ▷, 2, 3

1, ▷, 2, 3, ▷

1, ▷, 2, 3, ▷

0, C, B, A

0, A, C, B

1, A, B, C

0, B, A, C

0, C, B, A

1, C, A, B

2, C, B, A

1, ▷, 2, 3

1, ▷, 2, 3

1, ▷, 2, 3

① 1st from A to B

② 2nd from A to C

③ 1st from B to C

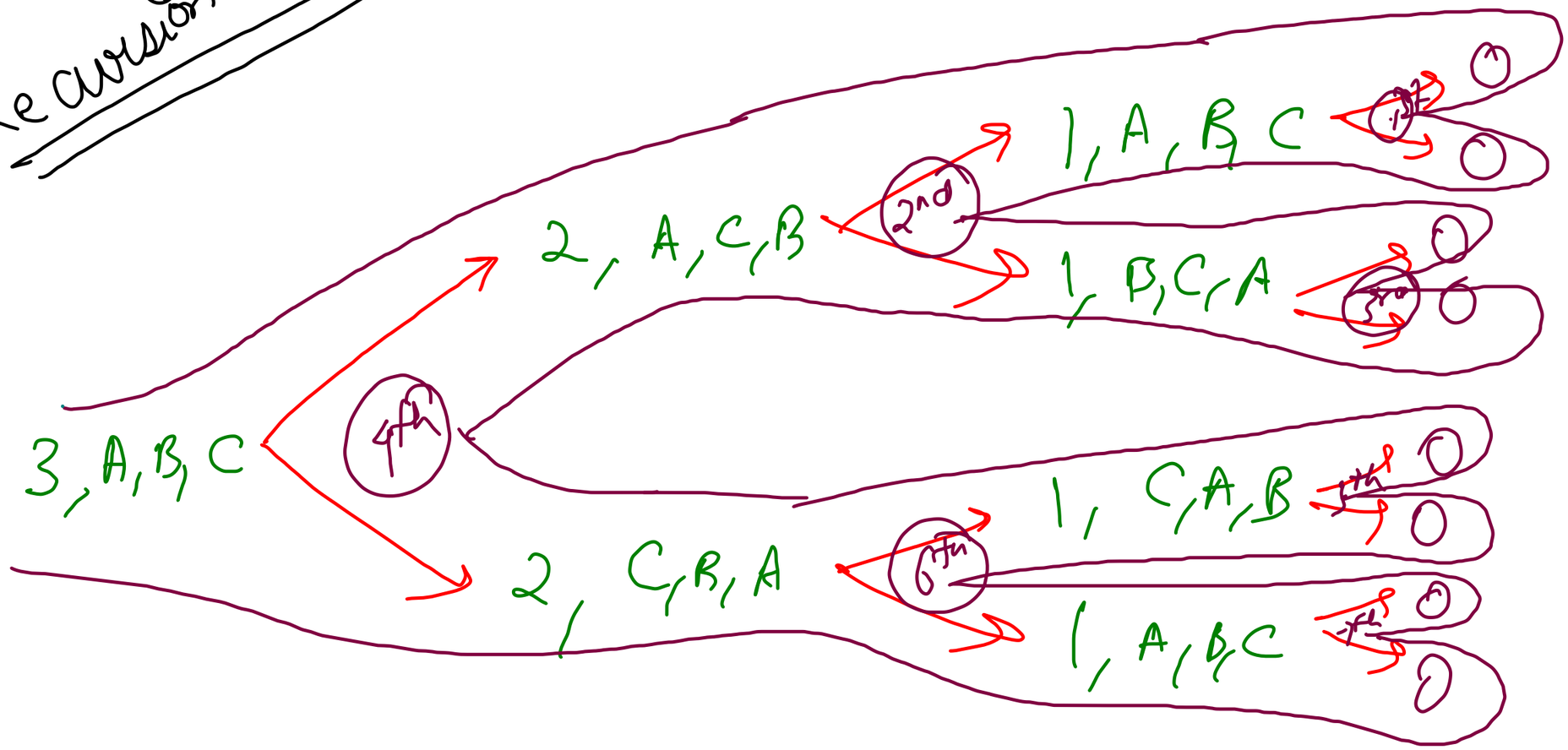
④ 3rd from A to B

⑤ 1st from C to A

⑥ 2nd from C to B

⑦ 1st from A to B

Recursion Tree



2 disks from A to C using B

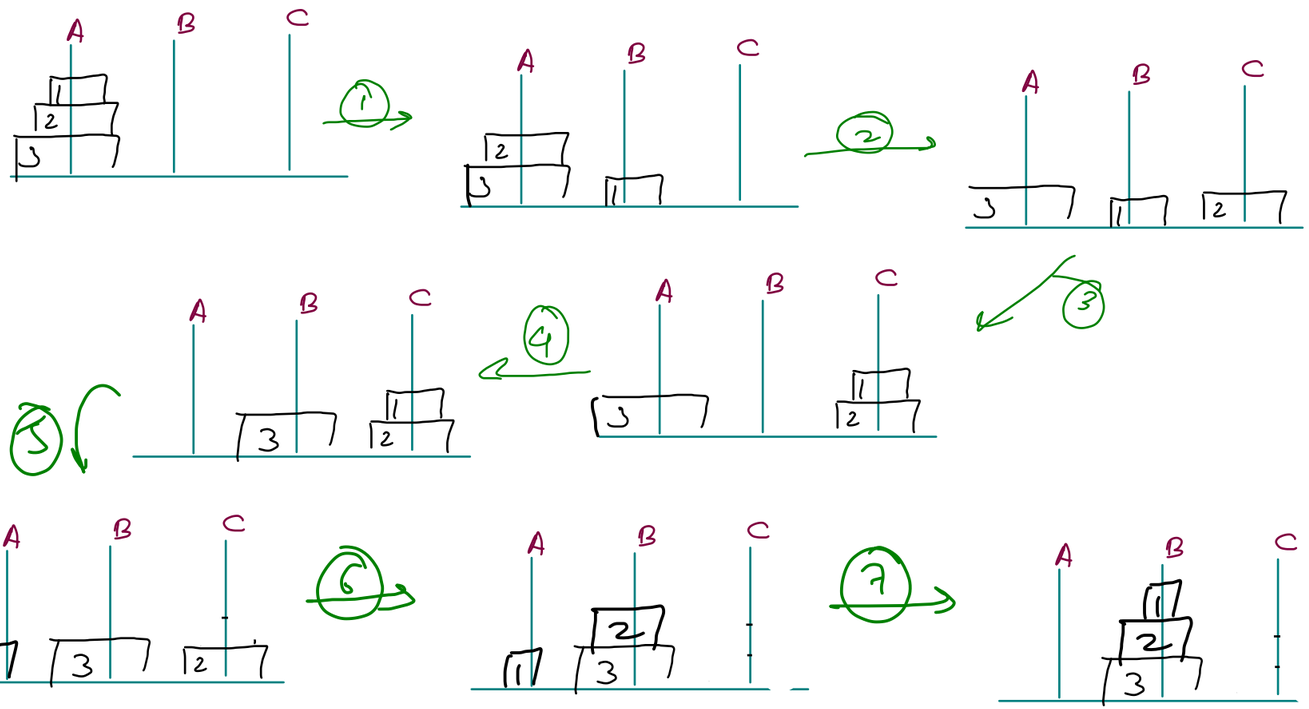
1. 1st from A to B
2. 2nd from A to C
3. 1st from B to C

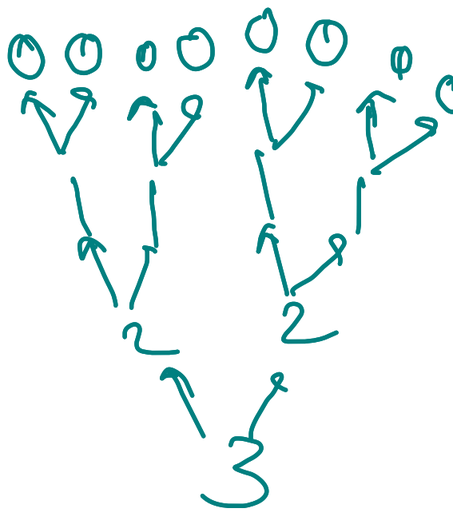
Meeting expectation

4. 3rd from A to B

2 disks from C to B

5. 1st from C to A
6. 2nd from C to B
7. 1st from A to B





DSA - based Projects {Unique* Project Ideas}

MERN

(1) Writing own library/header file for

→ B Tree, B+ Tree {SQL + DSA}

→ Segment Tree, Fenwick Tree, Number Theory Algorithms, DSU, Hashmap, Priority Queue/Heap, Graph Algorithms

{Competitive (Advanced DSA) + Generic Programming
⇓
Templates + OOPS}

(2) Games

Webdev ↳ CSS, JS

Canva

→ Snake & ladder :→ Graphs Algo {DFS} → {+ 2 player}

→ Sudoku :→ Backtracking {+ levels/Themes}

→ Crossword Puzzle :→ Backtracking {+ levels/themes}

→ Chess :→ 2D matrix {+ AI :→ Bot}

→ Jump Game :→ Greedy + Dynamic Programming {Climb Stairs}

{Frogs/Cartoon} → Mario Game

(3) Whatsapp/Telegram Clone

→ LRU Cache Algo } ⇒ Ranking/ ordering of chats

→ Trie (prefix tree) Data Structure } ⇒ Searching Contacts/Messages

→ CRUD operations { Firebase database }

→ Chatbots { API's }

→ Audio/Video calls → 1 to 1, 1 to many } Networking

④ Splitwise App Clone

↳ Minimizing Cash flow Algorithm → GFG

↳ CRUD applications { persistent data (database), + authentication }

mySQL
firestore

↳ System Design

↳ Priority Queue, Hashmap > Multiset Data Structures

My Project :- (Flutter) ⇒ LEN DEN

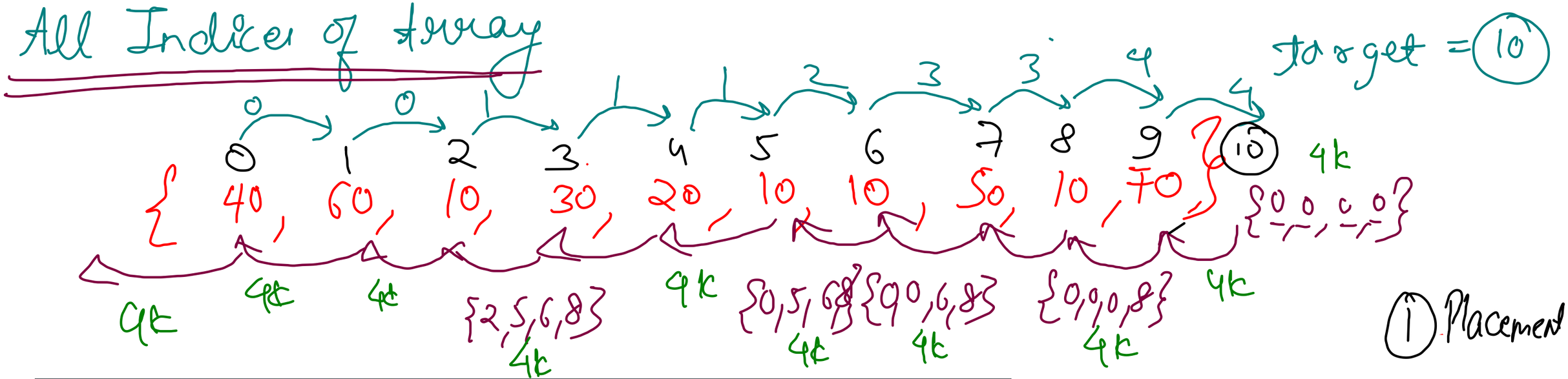
More Projects

- Sorting visualizers
- Path finding Algo visualizers

- Text Editor : {Stack Data Structure}

- File Zipping / URL Shortner (TinyUrl)
  - Huffman Encoding

All Indices of Array



① Placement

```java
public static int[] allIndices(int[] arr, int target, int idx, int count) {
    if(idx == arr.length){
        int[] base = new int[count];
        return base;
    }

    if(arr[idx] == target){

        int[] res = allIndices(arr, target, idx + 1, count + 1);
        res[count] = idx;
        return res;

    } else {

        int[] res = allIndices(arr, target, idx + 1, count);
        return res;

    }
}
```

① Total occ
size array
reduce

② Occurrences
from left
part

Codechef & codeforces

1. ~~Binary Tree~~
2. ~~S&O~~
3. BST
4. ~~Trie~~ Trie
5. Number Theory
6. DP
7. Graphs
8. DSU
9. ST, FT
10. Bit

leetcode Virtual Contest

1. BT
2. S&O
3. BST
4. DP
5. Graph
6. R&B
7. A&S
8. SES