Bubble Detection in Financial Markets
A Survey of Theoretical Bubble Models and Empirical Bubble Detection Tests

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Abstract

Asset price bubbles have fascinated economists for decades. In consequence, the literature on bubbles and their detection is abundant, with many researchers taking very opposite positions on the topic, however. This survey gives a structured overview of the two branches of research that have received the most attention in bubble research. First, we describe the theoretical models that have been developed to model bubble phenomena. These can be divided into rational bubble models and behavioral bubble models. Second, we provide a structured overview of empirical methods for the detection of rational bubbles. We focus in particular on recently developed bubble detection methods, namely recursive unit root tests, fractional integration tests, and regime-switching tests. These tests are predominantly advanced stationarity- and cointegration-based tests and as such are not based on the fundamental factors of the assets but rather focus on the time series of the asset prices. As a result, they avoid testing a joint hypothesis of the presence of rational bubbles and the validity of the model used to determine the asset's value. Furthermore, they are capable of detecting multiple, periodically collapsing bubbles.

While a consensus both on the appropriate theoretical bubble model as well as on the most applicable empirical bubble detection test has not been reached, especially empirical research has made significant progress. Different bubble detection tests now increasingly find overlapping evidence of rational bubbles when used to analyze the same time series. Nonetheless, many results are still inconclusive and bubbles remain an interesting avenue for further research.

Keywords: bubbles, bubble detection, econometric tests, theoretical models
JEL: C52, G10, G12

1 Introduction

Persistent and substantial divergences between an asset's price and its fundamental value, commonly referred to as *asset price bubbles*, *speculative bubbles*, or simply *bubbles*, have fascinated economists for decades whilst at the same time causing much controversy. In consequence, vast

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amounts of theoretical and empirical literature have been produced on the subject. Although the bubble detection research as a whole and the literature on empirical bubble detection methods in particular have made great strides in the past few years, many questions remain unsolved and bubble phenomena are still a very controversial issue.

Indeed, there is a range of alternative explanations for exuberance in asset prices. Economists have attempted time and again to develop extensions of the standard present-value model of stock prices, which in its most basic form defines the stock price as the present value of future dividends, that more accurately reflect prices. The insight from these attempts is straightforward. The more restrictive the model's assumptions on the fundamentals, the likelier it seems that a bubble is detected. As Gürkaynak (2008) points out, it is more likely that bubbles can be ruled out as a possible explanation, when some of the assumptions are relaxed. The question on which alternative structure of fundamentals to impose has not yet been fully resolved. In consequence, the closely related issue of whether bubbles exist in asset markets remains one of the fundamental debates in economics and finance.

A major issue in the discussions on bubble detection is that bubbles are often seen as market irrationality and thus pose a challenge to the efficient market hypothesis. As a consequence, many economists dismiss bubbles as loose talk and unscientific thinking. For example, Eugene F. Fama, the father of the efficient market theory, does not believe that asset prices exhibit bubbles (Fama 2014). O'Hara (2008), a supporter of the bubble theory, on the other hand, warns that when economists value the structure of their models more highly than the substance they convey, this can result in the scientific community loosing connection with the real world. Many researchers agree with this more moderate stance on asset price bubbles and thus acknowledge their existence. Nonetheless, they point out the importance of finding reasonable economic explanations for such apparently inexplicable phenomena and make an effort to classify speculative events as bubbles only as a means of last resort (Garber 1990). Recent theoretical work on bubble detection shows that the dynamics of asset prices may well contain a bubble component and that the explosive price behavior caused by the bubble can still be consistent with rational behavior among investors (Al-Anaswah and Wilfling 2011). These theoretical developments have fostered wider acceptance of the bubble theory in the scientific community.

Due to the devastating effects bubbles can have on economies, also policy makers increasingly stress the importance of understanding why and when asset prices can deviate from fundamental values. History has shown time and again that bubbles can lead to misallocation of resources in economies and that their burst can negatively impact real economic activity. The global financial crises of 2007-2008 has once again highlighted the importance of being able to detect bubbles and revealed the consequences that await us, when we fail to do so. It has demonstrated afresh that a bursting bubble can cause the collapse of major financial institutions, bring nations to the brink of bankruptcy, and lead to outright financial and economic crises. In our environment of increasingly interconnected economies the risk of potential contagion between markets is especially high.

This paper is structured as follows. Section 2 provides a short overview of the different research branches that have emerged in the bubble literature. Section 3 covers the theoretical bubble models that have been proposed up to this date. Then, section 4 surveys the econometric methods developed over the past decades for the detection of rational bubbles. It describes the procedures, their strengths and weaknesses, and potential modifications other authors have suggested. Additionally, it provides an overview of the actual empirical applications the different tests have been used in and summarizes the results obtained. Section 5 concludes.

2 Research Branches on Asset Price Bubbles

The literature on bubbles is plentiful and as a result, it is not always easy to have a full picture of where the literature stands. Generally, research on asset price bubbles can be roughly divided in to four strands.

Historical A wide range of papers focus on bubbles from a historical perspective. In economic literature numerous bubble episodes have been identified over the past centuries. Asset price bubbles have been documented as early as the 17^{th} century, where the prices of rare tulip bulbs skyrocketed during the *Dutch Tulip Mania* (1634-1637). The price exuberance came to a sudden halt when bulb prices plummeted in February 1637. The 18^{th} century saw the *Mississippi Bubble* (1719-1720) during which share prices of the Mississippi Company, a company that held a monopoly on the development of France's vast territory in Mississippi (North America), were hugely inflated. In parallel, the market price of the South Sea Company, a British trading company that held a trading monopoly with the Spanish colonies in the West Indies and South America, experienced a similar trajectory which later became known as the *South Sea Bubble* (1720). In the 20^{th} century stock and real estate prices soared in the United States (U.S.) during an episode known as the *Roaring Twenties* (1920s). The exuberance ended abruptly on "Black Tuesday" (October 29, 1929) when stock prices on the New York Stock Exchange plummeted. Japan experience a similar surge in equity and real estate prices during the *Japanese Bubble* (1986-1991) that plunged the Japanese economy into stagnation when it crashed.

The academic literature has also identified episodes of exuberance in more recent history. During the *Dot-com* or *Internet Bubble* (1994-2000) the prices of internet stocks were highly inflated. The most recently documented bubble has come to be known as the *U.S. Housing Bubble* (2001-2006). The burst of this speculative bubble led to the global financial crisis of 2007-2008. For a more detailed account of the most famous bubble episodes see Kindleberger and Aliber (2005), for instance.

Theoretical There is also a large and growing number of papers on theoretical models for bubbles. A theoretical understanding of the bubble concept is essential for developing and employing suitable empirical methods. Therefore, an indepth survey of the literature on theoretical bubble models will be provided in section 3.

Empirical Empirical bubble literature is primarily concerned with mechanisms for econometrically detecting bubbles and measuring their extent. It strives to uncover the complex forces behind the rapid price increases and the subsequent collapses. A significant body of literature on empirical bubble detection was developed in the 1980s and early 1990s, however, this strand of literature has made particularly great strides over the past few years. Empirical bubble detection mechanisms are especially valuable for regulators, since they enable them to understand, monitor, and control systemic risk in the financial system. Additionally, methods for detecting bubbles may help them consider suitable policy measures.

Econometric bubble detection mechanisms are discussed in detail in section 4. There, we lay a special emphasis on the econometric bubble detection tests that have been developed recently and have not yet been evaluated in previous literature surveys. We also include previous empirical bubble detection methods but refer to the literature survey by Gürkaynak (2008) for a more detailed account of these. We show that empirically identifying bubble episodes is still a notoriously challenging task. Recent research on empirical bubble detection has focused predominantly on stationarity- and cointegration based tests, nonetheless, there is still no consensus on which methodology should ultimately be used and what conclusions can be drawn from the variety of different methods.

Experimental The fourth strand of bubble literature focuses on experimental bubble analysis, which has gained popularity in recent years. For a recent overview of bubbles and crashed in experimental markets see Palan (2013).

Experimental research has the advantage that researchers have control over the market environment and can isolate the different mechanisms that drive bubbles. It is therefor particularly suited for exploring the causes and the psychology of investor behavior. Experimental bubble research was founded by Smith et al. (1988). It generally supports the bubble hypothesis. Experimental studies find that assets are often traded in high volumes at prices substantially above their fundamental values. However, short-sale constraints and the level of investor experience tend to reduce the appearance of bubbles.

Experimental bubble research is useful in assessing the validity of the results obtained by theoretical and empirical research. As Brunnermeier (2008) points out, several empirical bubble tests rely heavily on backward induction. Their results are to be critically questioned, since experimental evidence demonstrates that individuals often disregard the backward induction principle. Experimental results also call into question the validity of some of the theoretical reasoning behind the rational bubble models covered in subsection 3.1. On the other hand, behavioral bubbles models, which are another subcategory of theoretical bubbles models discussed in subsection 3.2, find support in experimental studies.

3 Theoretical Bubble Models

It is important to understand the propagating mechanisms that can lead to explosive characteristics in asset prices and economists have developed a range of different theoretical models for this purpose. This section is designed to provide a concise overview of the most dominant theories that have been developed over the past decades. Many authors before us have attempted to provide structured overviews of these models. For a recent literature survey see Scherbina and Schlusche (2014).

We distinguish two broad categories of standard approaches to modeling bubbles, as can be seen from figure 1. First, we focus on rational bubble models which attempt to explain exuberance in financial prices in settings where all investors are assumed to be fully rational. These models constitute the oldest and most developed class of potential bubble explanations and will be covered in subsection 3.1. Second, section 3.2 describes behavioral bubble models. These are sometime also referred to a irrational bubble models because they abandon rational investor behavior to a varying degree. In such models rational traders interact with non-rational (i.e., behavioral) traders and this may cause the emergence of a bubble.

It is important to point out that the terminology used to distinguish the different model subcategories is not always consistent. For this reason, we make a special effort to include the different names the various models may have in academic literature in every subsection. These alternative terms are also included in figure 1.

3.1 Rational Bubble Models

The first theoretical bubble models developed were *rational bubble models*. They attempt to explain explosive behavior in economic and financial variables in a setting in which all agents are assumed to be rational with regards to their expectations of future earnings and in which markets are predominantly efficient. The classic rational bubble has a longstanding tradition in theoretical literature, with seminal papers by Samuelson (1958), Diamond (1965), Blanchard and Watson (1982), Tirole (1982, 1985) and Froot and Obstfeld (1991), to name a few.

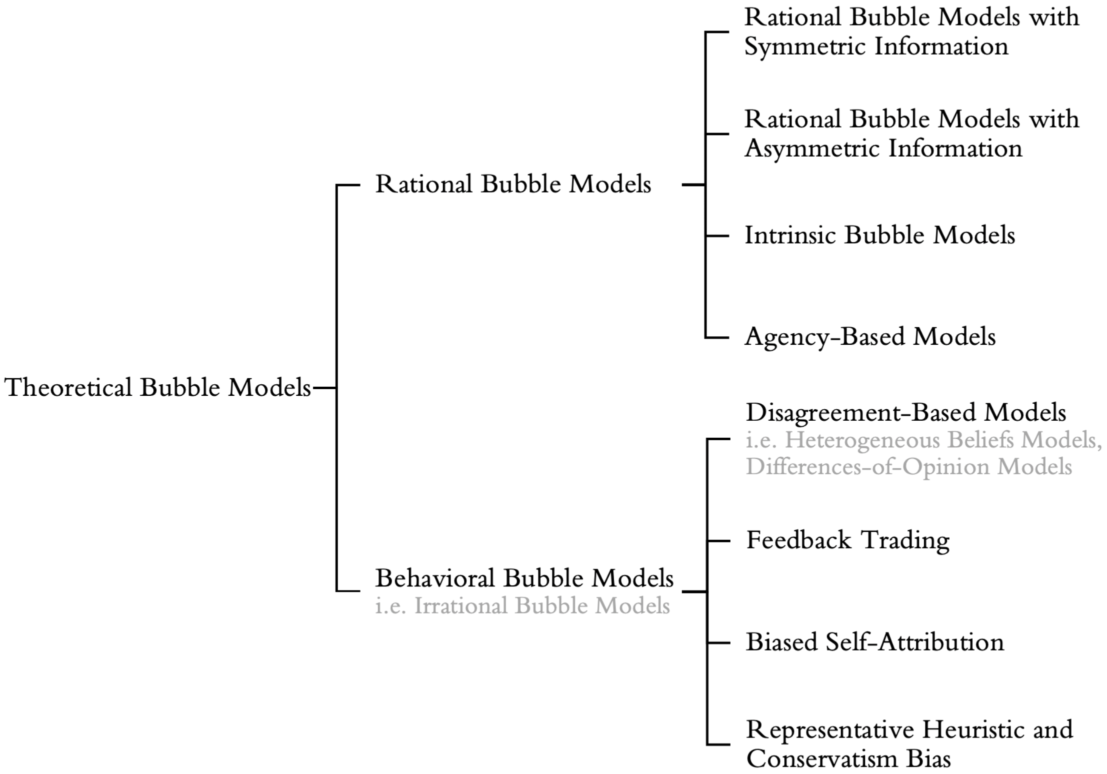

Figure 1: Theoretical Bubble Models

The bubble literature generally divides rational bubble models into a range of subcategories. How this is done is not always consistent. Following Brunnermeier (2008), we first distinguish two categories based on whether investors have symmetrical or asymmetrical information. The two subcategories are described in subsections 3.1.1 and 3.1.2, respectively. Furthermore, we also include intrinsic bubble models into the category of rational bubble models. They constitute a special case of rational bubble and are covered in subsection 3.1.3. Finally, also agency-based attempts to explain bubbles in asset prices fall into the rational bubble model category. These developments in the rational bubble literature are discussed in subsection 3.1.4.

3.1.1 Rational Bubble Models with Symmetric Information

Rational bubble models generally build on the *present-value model* of equity prices, also known as the *market fundamentals model* or simply the *standard model*. In this model, an asset's value is determined solely by the value of the discounted fundamentals. In its most basic version for stock prices, the current stock price is equal to the present value of the expected future dividends.

The standard present-value model builds on four main assumptions (Gürkaynak 2008). First, all investors are rational and risk neutral. This implies that the model does not incorporate risk premia. Second, the discount rate used to determine the present value of the future payments is constant over all periods, and third, the dividend generating process is not expected to change. The fourth assumption relates to the information available to all investors. As the name suggests, in *rational bubble models with symmetric information* all investors possess the same information.

Numerous studies (see, for example, LeRoy and Porter (1981) and Shiller (1981)) have demonstrated that such simple present-value models are unable to explain fluctuations in stock prices.

For this reason, *rational bubbles models* have been developed. Rational bubble models are a special case of the general present-value model, that include a second component to explain the deviation between discounted fundamentals and observable asset prices. In addition to the fundamentals component of the standard model, the rational bubble model also contains a bubble component. This rational bubble component is not a mispricing effect but a basic component of the asset price and it is uncorrelated with the market fundamentals (Froot and Obstfeld 1991). A rational investor is willing to to pay more for the stock than he knows is justified by the value of the discounted dividend stream because he believes that he will be sufficiently compensated for the extra payment induced by the bubble term through future price increases. Pricing is still rational and arbitrage opportunities can be ruled out.

Pioneers on rational bubble models with symmetric information include Blanchard (1979), Blanchard and Watson (1982), Flood and Garber (1980), and Tirole (1985). Their models build on the so-called "no bubble condition" which states that in the present-value model of equity prices the value of the bubble must be zero at all times. The problematic aspect of this approach is that the fundamental component cannot be directly observed so assumptions have to be made to determine it. One potential approach is to assume that dividends follow a random walk with drift while the bubble process is characterized by an explosive autoregressive process (Diba and Grossman 1988). This implies that whenever a bubble is not present, the price also follows a random walk with drift. Put differently, the fundamental component in the rational bubble model converges, while the bubble component is non-stationary. As we will see in section 4, some empirical methods attempt to exploit this fact in order to detect rational bubbles.

Rational bubble models under symmetric information suffer from a range of problems. For example, while they allow bubbles to exist in the price of an infinitely lived asset, this is only possible if the bubble's growth rate is equal to the discount rate. Furthermore, rational bubble models do not allow a bubble to exisit in the price of an finitely lived asset.[1] Finally, since all agents have rational expectations and share the same information, rational bubble models are not able to explain any volume and simple theoretical arguments can be used to rule out bubbles under certain conditions (Barberis et al. 2018; Brunnermeier 2008).

3.1.2 Rational Bubble Models with Asymmetric Information

In *rational bubble models with asymmetrical information* the assumption of the present-value model that all investors have the same information is relaxed. This means that investors are asymmetrically informed, but at the same time they still share a common prior distribution (Brunnermeier 2008). Asymmetric knowledge can mean that it may not be common knowledge whether or not there is a bubble in the asset price. Alternatively, it can also mean that not all investors know that the other investors know there is a bubble. Due to this lack of higher-order mutual knowledge bubbles can also be present in the price of a finitely lived assets under certain conditions.

One condition that can induce bubbles in such a setting are short-sale restrictions. Allen et al. (1993) allow for the existence of a bubble in their model when common knowledge is absent and short-sale constraints bind. All agents know that the asset is overvalued but they do not know that other agents know this as well. Agents are willing to hold an overvalued asset because they hope to resell it at a higher price to another agent who may value the asset highly in certain states due to his particular information structure. Kindleberger and Aliber (2005) refer to this

[1] See Scherbina and Schlusche (2014) for an explanation on the reasoning behind this.

belief as the “greater fool theory”, because the investor is convinced he can sell the asset to an even greater fool.

3.1.3 Intrinsic Bubble Models

Intrinsic bubbles constitute a special class of rational bubble model that was first introduced by Froot and Obstfeld (1991). Unlike in the bubble models mentioned in the subsections above, in intrinsic bubble models the bubble component is a deterministic function of the fundamentals and not a function of time. This bubble parametrization has a few advantages. First, the bubble can overreact to changes in fundamentals. Second, it does not have to explode in relation to the fundamental value, and third, it may even disappear entirely.

In intrinsic bubble models, since the bubble is tied to the level of dividends, dividends are modeled explicitly. Due to this, the bubble process depends completely on the level of the dividends and does not take off on its own. When there is no bubble, prices are a linear function of dividends and the price-dividend ratio is a constant. However, since the stock price is correlated with the fundamentals, it is more sensitive to dividend innovations than is justified by the linear pricing equation. Non-linearity in the relationship between stock prices and dividends is interpreted as an intrinsic bubble and the differences in the behavior of the price-dividend ratio when a bubble is present compared to when it is not, is exploited to form a bubble test. Unfortunately, a non-linear relationship between stock prices and dividends can only be interpreted as a sign of bubbles because the model is assumed to be linear. If the “true” model is actually non-linear, this is no longer the case.

3.1.4 Agency-Based Models

A newer category of rational bubble models are *agency-based models*. These theoretical models ascribe bubbles to the numerous incentive problems that many key economic agents such as money managers, institutions, and information intermediaries are confronted with. Agency-based models can be classified into three broad categories. The first type of agency-based model explains the emergence of bubbles based on herding. Herding is a phenomenon in finance in which the reaction of market participants is influenced more by the behavior of other market participants than by market fundamentals. Research has demonstrated that money managers herd for a range of reasons, which include reputational concerns, limited time and resources, and relative performance-based compensation. The second type of agency-based models ascribe bubbles to the limited liability many economic agents are subject to. Limited liability implies that money managers are entitled to keep all the gains on the upside while at the same time facing only limited downside losses. This imbalance can lead to trading behavior that can foster bubbles in an asset’s price. The third category of agency-based models explain the bubble phenomenon based on misaligned incentives of information intermediaries. Perverse incentives may cause agents to ride a bubble rather than alert the public of it. For a more in-depth review of the literature on agency-based models see Scherbina and Schlusche (2014).

3.2 Behavioral Bubble Models

Empirical evidence has shown time and again that rational bubble models persistently fail to explain observable asset price levels. Therefore, more recent theoretical models of bubbles, so-called *behavioral bubble models*, try to better fit the data by relaxing the assumption of the present value model that all investors are perfectly rational. By incorporating insights from psychology, they attempt to offer better explanations for bubble phenomena and other return anomalies. Bubbles associated with behavioral models are often referred to as irrational bubbles. Therefore, behavioral bubble models are sometime termed *irrational bubble models*.

We distinguish four subcategories of behavioral bubble models.[2] Disagreement-based bubble models are characterized by the fact that different investors have different beliefs on the fundamental value of an asset. For this reasons these models are also known as heterogeneous beliefs models. They are covered in subsection 3.2.1. As mentioned above, behavioral bubble models are characterized by the fact that rational, sophisticated traders interact with irrational traders. When these irrational market participants trade, they are influenced by psychological biases. Models based on feedback trading, biased self-attribution, and a combination of the representative heuristic and the conservatism bias are examples of models that rely on psychological biases to explain bubble episodes. These three behavior bubble model subcategories are dealt with in subsections 3.2.2, 3.2.3, and 3.2.4, respectively.

3.2.1 Disagreement-Based Models

Disagreement-based models, which are sometime also referred to as *heterogeneous beliefs models* or *differences-of-opinion models*, were founded by Miller (1977) and Harrison and Kreps (1978).[3] In these models investors hold heterogeneous beliefs on asset valuations. This means that, unlike in the models described above, traders have non-common prior beliefs, or psychological biases. They agree to disagree about the fundamental value of the asset and as a result anomalous asset market phenomena such as asset price bubbles can occur (Brunnermeier 2008).

In a theoretical model known as the *resale option theory*, heterogeneous beliefs combined with the presence of market frictions such as short-sale constraints induce speculative investor behavior and asset price bubbles. This theory is called resale option theory because investors believe that they will be able to resell an asset to other more optimistic buyers for a speculative profit in the future. The optimists push up the asset price. At the same time, pessimists cannot counterbalance the price increase due to the short-sale restrictions they face (Miller 1977). This imbalance can foster the occurence of bubbles. Other market frictions such as noise trader risk are also possible. For a complete overview of models with varying types of market frictions see Griffin et al. (2011).

As Scheinkman and Xiong (2003) points out, disagreement-based models are effective in explaining the large trading volume and high price volatility often associated with asset price bubbles. Furthermore, they can explain the fact that bubbles are more likely to arise in prices of assets that are difficult to value. This is due to the fact that opinions play an important role in forming valuations. A bubble will burst when either the uncertainty about the asset value or the corresponding market restrictions, such as the potential constraints on short selling mentioned above, disappear. The disadvantage of disagreement-based models is that while they can explain positive price bubbles, they are unable to account for negative bubbles.

3.2.2 Feedback Trading

Behavioral bubble models based on *feedback trading* are based on the following reasoning: An asset experiences an initial price increase in response to good news. This price increase is observed by a group of traders, called feedback traders, who buy the asset in response to the

[2] For more details on the behavioral model category we refer to the literature survey by Scherbina and Schlusche (2014).

[3] For a detailed overview of disagreement-based models see Hong and Stein (2007).

past price movement rather than based on the asset's current valuation. This pushes the price up. The further price increase attracts additional feedback traders, whose demand for the asset pushes prices even higher. At some point, this behavior leads to prices that exceed the fundamentals. In short, feedback traders amplify past price movements instead of focusing on fundamentals.

Models based on feedback trading can be traced back to De Long et al. (1990) who abandon perfect rationality and allow for irrational pricing. They state that speculative investors may initiate or contribute to price movements based on the expectation that positive-feedback traders will purchase the securities later at even higher prices. Shiller (2000) points out that psychological biases may play a role in the formation of such feedback mechanisms. He terms this kind of theoretical model *feedback loop theory*. Xiong (2013) provides a more detailed overview of models based on feedback trading. He points out that the feedback loop theory requires substantial distortions in aggregate beliefs of investors in order to generate a bubble and that the bubble will burst once the feedback trading loop is broken.

Feedback trading models have distinct advantages over disagreement-based models. Unlike the latter, they can explain both positive and negative bubbles. Furthermore, they can account for high trading volumes.

3.2.3 Biased Self-Attribution

The third subcategory of behavioral bubble models we distinguish are based on *biased self-attribution* which is a cognitive bias that has been thoroughly documented in psychology. It causes people to put more weight on signals that confirm their own beliefs than justified while at the same time disregarding contradictory signals as noise. Theoretical models that explain bubble phenomena through biased self-attribution were founded by Daniel et al. (1998). In their model, the bubble bursts when the positive sentiment about the price exuberance is reversed. Like models based on feedback trading, biased self-attribution theories can explain positive as well as negative bubbles. Again, bubbles are more likely to arise in prices of hard-to-value assets. This is due to the fact that judgment is important in forming the underlying valuations.

3.2.4 Representativeness Heuristic and Conservatism Bias

In line with a previous literature survey by Scherbina and Schlusche (2014), the fourth category of behavioral models we point to explains bubbles using the *representativeness heuristic* and *conservatism bias*. Representativeness heuristic and conservatism bias are cognitive biases extensively documented in psychology literature that are used when making judgments about the probability of an event under uncertainty. Representativeness is defined as "the degree to which [an event] (i) is similar in essential characteristics to its parent population, and (ii) reflects the salient features of the process by which it is generated" (Kahneman and Tversky 1972). The representativeness heuristic states that investors rely on representativeness to make decisions. However, this can lead to wrong judgments because of the flawed reasoning that something is more likely simply because it is more representative. In the financial context the use of the representativeness heuristic implies that investors overreact to "strong" or attention-grabbing news. They do this by putting too much weight on such signals relative to their base probabilities. Conservatism bias, on the other hand, is the tendency to revise one's belief insufficiently when presented with new evidence. In the financial context this means that investors underreact to routine and non-attention grabbing signals (Scherbina and Schlusche 2014).

Models based on these two biases were founded by Barberis et al. (1998). They can explain positive as well as negative bubbles, and furthermore, they are able to show why prices experience an initial phase of underreaction followed by an overreaction that results in a bubble. Additionally, they explain why the bubble will burst once the positive sentiment about the bubble is reversed.

Behavioral bubble models try to base their assumptions on evidence documented in psychology and, as mentioned in section 2, they find more and more support in experimental studies. However, most empirical research on bubble detection up to now has focused on rational bubbles. Much work remains to be done in empirically proving or refuting behavioral bubble theories.

4 Empirical Tests for Rational Bubbles

Theoretical bubble models have received considerable attention in academic literature. While a profound theoretical understanding of the bubble phenomenon and its causes is without doubt very important, it is, however, by no means sufficient. Academics and policy makers alike highlight the importance of sound empirical methods for detecting bubbles in asset prices. As Giglio et al. (2016) stress, bubbles should not be treated as a purely theoretical problem. Rather, the debate over the existence of bubbles is an inherently empirical question.

The goal of empirical bubble literature is to develop mechanisms to identify the origination, termination, and extent of explosive behavior in asset prices based on explicit quantitative measures. These mechanisms must be able to empirically separate the contribution of rational bubbles and market fundamentals to exuberance detected in the data. The exuberance detected should only be attributed to bubbles when all other phenomena that effect asset prices have be ruled out as possible explanations. Even though the literature on empirical bubble detection has made great strides over the past decades, it is still a notoriously challenging task to design appropriate tests for identifying bubble episodes. The reason for this lies in the fact that the determinants of the fundamental value are generally not observable and it is therefore challenging to determine an asset's fundamental value.

Empirical evaluation of bubble phenomena requires some econometric technology. Gürkaynak (2008) provides a thorough survey of the literature on econometric tests for rational bubbles in the context of the present value of dividends model. He points out that while a large number of studies have attempted to empirically separate the contribution of rational bubbles and market fundamentals to asset price movements, there is still no general agreement on which bubble detection methodology should ultimately be used. Furthermore, there is no consensus on whether bubbles are actually present in the different time series that have been analyzed. He states: "For each paper that finds evidence of bubbles, there is another one that fits the data equally well without allowing for a bubble."

In this paper we provide a more recent survey of econometric methods for detecting rational bubbles. We provide a brief overview of the empirical tests that have been covered in previous literature reviews but focus predominantly on the methods that have been developed since the Gürkaynak (ibid.) survey. We focus on methods for detecting rational bubbles, i.e. methods for testing the validity of the rational bubble model, because these models have received the most attention in empirical bubble research. Furthermore, as Giglio et al. (2016) emphasizes, the theoretical plausibility of the classic rational bubble as an equilibrium phenomenon has been reaffirmed by the theoretical literature. In addition to the overview of the methods that have been proposed, we also include the time series and sample periods that are analyzed in the applications of the novel bubble detection tests and the conclusions the different papers come to. This sets our survey apart from that of Gürkaynak (2008), who provides a methodological overview but does not include a survey of applications.

As can be seen from figure 2, we divide the literature on empirical bubble detection into two broad categories. In subsection 4.1, we briefly describe the methods that have been dealt with in previous literature surveys. These early econometric detection methods include variance bound tests, West (1987) two-step tests, and standard stationarity- and cointegration-based tests. Subsection 4.2 deals with econometric bubble detection methods that have been developed more recently. All tests in this category are advanced stationarity- and cointegration-based tests. As we will show, this field of research seems very promising in terms of accurate bubble detection.

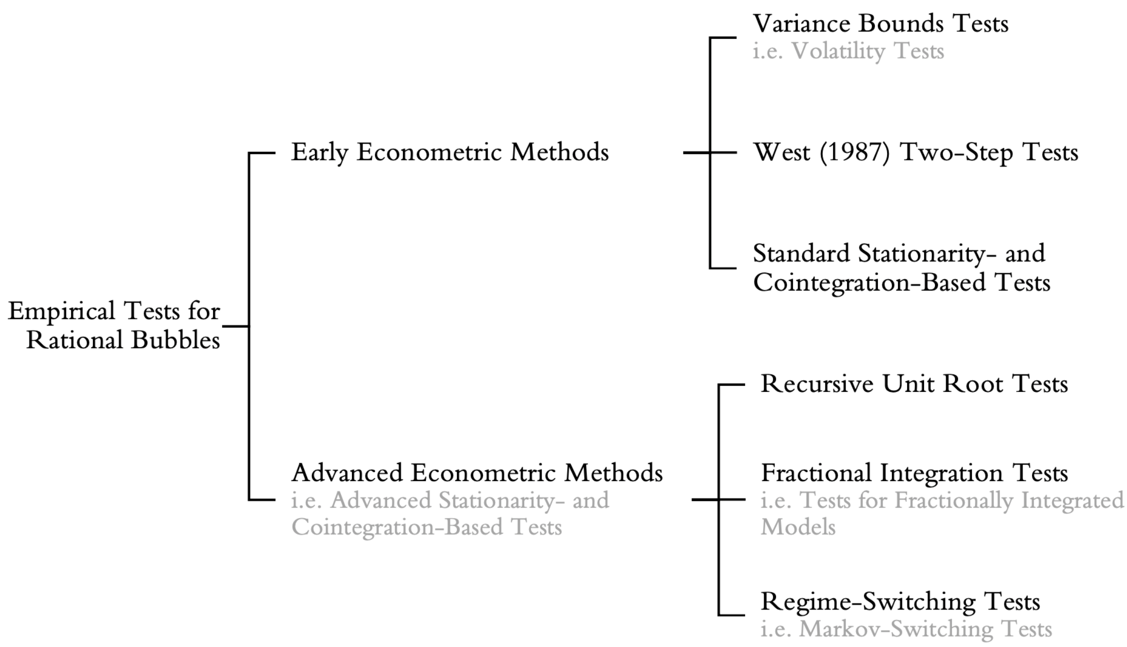

Figure 2: Empirical Tests for Rational Bubbles

4.1 Early Econometric Methods

Early research on empirical detection of rational bubbles dates back to the 1980s and early 1990s. Following Frömmel and Kruse (2012) we include three model types into the category of early econometric studies on rational bubbles, namely variance bounds tests, West (1987) two-step tests, and standard stationarity- and cointegration-based tests. These will be briefly covered in subsection 4.1.1, 4.1.2 and 4.1.3, respectively. For a indepth survey of these methods see Gürkaynak (2008).

4.1.1 Variance Bounds Tests

Variance bounds tests, sometimes also referred to as *volatility tests*, are essentially tests of the present-value model. Put simply, they try to uncover whether there is something apart from fundamentals that can explain the apparently inexplicably high volatility that is observed in stock prices. Research showing that stock prices exhibit a level of volatility that cannot be explained by fundamental factors such as new information about future dividends is abundent, dating back to Shiller (1981) and LeRoy and Porter (1981). Shiller (1981) and Grossman and Shiller (1981) developed variance bounds test to empirically explain this observation. However, as Gürkaynak (2008) points out, these authors do not explicitly design their tests as bubble detection tests or even refer to bubbles as a potential explanation for the excessive volatility they observe. Only in later applications was the excessive volatility interpreted as evidence of stock market bubbles (Brunnermeier 2008).

When used for bubble detection, the null hypothesis of variance bounds tests states that an asset's price contains no bubble. This implies that the price should be solely driven by market fundamentals. To test the null hypothesis, the variance of the observed price series is compared to the variance of the ex post rational price. For the stock market, the observed price series is based on the present value of the expected dividends while the ex post rational price is calculated as the present value of the actual subsequent real dividends. The variance of the latter logically acts as an upper bound on the variance of the first. Put differently, the market prices should not be more volatile than the ex post rational price series. The reasons for this is that the observed prices are based on expected dividends and do not have the variation introduced by future forecast errors which the ex post price includes.

Numerous studies have shown that this requirement is violated dramatically by real-world data. Shiller (1981) applies test to the real prices and real dividends of the S&P 500 index (1871-1979; annual data) and the Dow Jones Industrial Average (1928-1979; annual data). He shows that for both data sets the price volatility of the observed time series exceeds the bound imposed by the variance of the ex post rational price. Grossman and Shiller (1981) also analyze the S&P 500 stock price index (1889-1979; annual data) and again demonstrate that the variance bound is violated. Since both papers do not use the analysis for bubble detection purposes, the authors draw no explicit conclusions on whether their results can be interpreted as a sign of a bubble in the time series. However, they do state that the present-value model is inadequate to explain observable prices.

Blanchard and Watson (1982) are the first to introduce variance bounds tests into the context of empirical bubble detection. They analyze the same data set as Shiller (1981) and again show that the variance bound is violated. They interpret the violation as an indicator for the presence of bubbles. However, they point out that the violation may also be due to other phenomena such as irrationality. Also Tirole (1985) suggests that the variance bound may be violated due to the presence of bubbles. Although he does not perform an empirical analysis of his own, he states that the presence of bubbles may well be responsible for price fluctuations in assets like gold or stocks.

Variance bounds tests have a major disadvantage. Since they are model-dependent tests of bubbles, they jointly test for the presence of a bubble in the data and the validity of the model used to derive the asset's price. This means that a rejection of the null hypothesis of no bubbles cannot unambiguously be attributed to a bubble in the data. A rejection can also be caused by the failure of any other assumption of the present-value model. Also a combination of bubbles and model misspecifications could be a valid explanation. This problem is referred to as the *joint hypothesis problem*. However, as Gürkaynak (2008) points out, while variance bounds tests may fail to unambiguously attribute excess volatility to asset price bubbles, they do show that there is something more volatile to asset prices than is assumed in the standard present value model. Recall from section 3.1.1 that one standard assumption of the present-value model is that the required expected returns are constant over time. If this assumption is relaxed, time-varying required expected returns may be an alternative explanation for price exuberance. In his paper, Cochrane (1992) suggests exactly this. He points out that variation in the required expected returns may be a suitable explanation for excessive volatility.

4.1.2 West (1987) Two-Step Tests

West (1987) proposes a test to circumvent the joint hypothesis problem. He sequentially employs two separate tests, the first of which tests the model hypothesis while the second one tests the no-bubble hypothesis. This *two-step test* exploits the fact that the parameters that are required to calculate the expected discounted value of dividends can be estimated in two different ways. One calculation method is not affected by a potential bubble in the data while the second method is. He uses a Hausman misspecification test to determine whether the estimates derived from the two calculation methods are the same. Unlike variance bounds tests, West (1987) two-step tests are explicitly designed to test the no-bubble null hypothesis against the alternative hypothesis that bubbles are present. When no bubble is present, the stock price can be explained by the standard present-value model and the two estimates of the linear relationship should be equal. Under the alternative hypothesis of a rational bubble, the equality of the estimates no longer holds. Due to fact that the stepwise procedure tests the model hypothesis separately from the no-bubble hypothesis, the results can be clearly attributed either to a bubble or to a misspecification in the model. West (1987) applies his method to the S&P 500 index (1871-1980; annual data) and the Dow Jones index (1928-1987; annual data). He rejects the null hypothesis of no bubble in both data sets.

4.1.3 Standard Stationarity- and Cointegration-Based Tests

The third subcategory of early econometric bubble detection techniques we distinguish are *standard stationarity- and cointegration-based tests*. These tests are derived from the relationship between an asset's price and its fundamental value. When used for bubble detection purposes in stock markets, these tests often analyze the relation between share prices and dividends. This is done either by testing the divided-price ratio or dividend yield for stationarity, by testing to see whether stock prices are stationary when they are differenced the number of times required to make dividends stationary, or by testing for cointegration between prices and dividends (Gürkaynak 2008).

One method to determine whether a time series variable is stationary or not is via *unit root tests*. Their null hypothesis is that the time series possesses a unit root and is non-stationary. Standard unit root tests developed early on in the bubble detection literature test for the alternative hypothesis of stationarity which is located on the left side of the probability distribution of the test statistic. As we will see in subsection 4.2, newer stationarity test often focus on the right side.

Various standard unit root tests have been proposed in the bubble detection literature. Campbell and Shiller (1987) perform a test to see whether an asset's price and its fundamentals are cointegrated. They apply their cointegration-based test to the S&P 500 stock price index (1871-1986; annual data) and a U.S. Treasury 20-year yield series (1959-1983; monthly data). They find a bubble in both series. In their seminal paper, Diba and Grossman (1988) employ stationarity tests to test for evidence against explosive rational bubbles in stock prices. They use a model that assumes a constant discount rate, but at the same time allows unobservable variables to influence market fundamentals. The reasoning behind their test is as follows: If a rational bubble does not exist and if the first differences of the dividends and the first differences of the unobservable variables are stationary, the first differences of the stock prices should also be stationary. Similarly, if a rational bubble does not exist and if the levels of the unobservable variables and the first differences of dividends are stationary, then stock prices and dividends are cointegrated. The authors test the no-bubble hypothesis using a series of autocorrelation patterns, unit root tests, and cointegration tests. They analyze the S&P 500 stock price index (1871-1986; annual data) and fail to reject the null hypothesis of no bubbles. They conclude that explosive rational bubbles do not exists in the stock prices.

Froot and Obstfeld (1991) use cointegration tests to empirically test for intrinsic bubbles in the S&P 500 stock price index (1900-1988; annual data). To be precise, they run cointegration regressions of the price-dividend ratios on a constant and dividends and reveal a strong evidence for a non-linear relationship between stock prices and dividends. They conclude that this non-linearity may be due to a bubble if the relation between prices and dividends is indeed linear. However, they acknowledge the fact that alternative explanations may also be valid if the model is actually non-linear. Craine (1993) tests for rational bubbles in the S&P 500 (1872-1988; annual data) using augmented Dickey-Fuller unit root tests applied to the price-dividend ratio. The tests cannot reject the unit root null. The author therefore concludes that either the price-dividend ratio contains a rational bubble or the discount factor must be stochastic.

Numerous other authors have employed stationarity- and cointegration-based tests to detect rational bubbles in series such as the S&P 500. As can be seen from the papers mentioned above, the results are inconclusive. While some papers find evidence of rational bubbles in the data, others do not. This is due to the fact that detecting non-stationarity in time series and estimating cointegration relationships is by no means as straightforward a task as it may seem. There exist many different tests that all have different size/power properties and often do not agree on the result. Furthermore, failing to reject the stationarity or cointegration conditions that underpin the no-bubble hypothesis is not conclusive proof that bubbles are indeed absent from the data. In what is by now generally referred to as *Evans' critique*, Evans (1991) shows that standard unit root and cointegration tests, such as the ones mentioned above, have little power in detecting complex patterns, such as bubbles that collapse to a small non-zero value and then continue increasing. These *periodically collapsing bubbles* make the data look more like stationary processes rather than explosive processes. Standard unit root and cointegration tests cannot effectively distinguish between a stationary process and a periodically collapsing bubble process and are therefore inappropriate tools for bubble detection purposes. Evans (ibid.) reinforces his critique using a time series simulated from a nonlinear model that produces periodically collapsing bubbles. Taylor and Peel (1998) react to Evans' critique by proposing a new cointegration test for rational bubble detection that alleviates the problem of substantial size distortions that standard cointegration tests are subject to when periodically collapsing bubbles are present in the data. They apply the test to the S&P500 stock price index (1871-1987; annual data) and find no evidence of rational bubbles in the time series. Froot and Obstfeld (1991) draw a bleak conclusion on the effectiveness of standard stationary- and cointegration-based models. They state that it may be impossible to determine conclusively whether deviations from present-value prices are nonstationary or stationary and that econometricians should not feel too comfortable about how well they really understand stock prices.

Also the overview of early econometric tests provided in this paper shows that these bubble detection mechanisms provide unsatisfactory results at best. Not only do the different approaches lead to different conclusions when applied to similar or identical data sets, many also suffer from weaknesses such as low power and size distortions or face challenges such as the joint hypothesis problem or Evans' critique.

4.2 Advanced Stationarity- and Cointegration-Based Tests

Around the turn of the millennium, interest in the detection of rational bubbles waned, partly due to the weaknesses of the early testing procedures as well as the inconclusive results they delivered. However, the global financial crisis of 2007-2008 has rekindled the interest among empirical economists in asset price bubbles and their potential global consequences, and at the same time provided a rich environment for further empirical research (Phillips and Yu 2011).

The advanced econometric tests that have been developed over the past two decades generally fall into the category of stationarity- and cointegration-based tests. They strive to overcome the weaknesses associated with early bubble detection tests, such as the fact that multiple breaks can seriously diminish the discriminatory power of standard stationarity and cointegration tests as pointed out by Evans (1991). One important aspect of these advanced tests is the specification of their null and alternative hypotheses. As we stated in subsection 4.1.3, standard unit root tests generally test for the null hypothesis of a unit root (i.e., non-stationarity) against the stationarity alternative. This implies that the alternative is located on the left-side of the probability distribution. More recent tests, on the other hand, often test for non-stationary behavior of the price time series against a mildly explosive alternative, which is located on the right side of the probability distribution of the test statistic. Other methods employ a fractional integration technique and another category of methods, tests for a change from an I(0) to an I(1) process (Frömmel and Kruse 2012).

Some of the newer papers also include methods that can be used as warning systems for future bubbles. These methods seek to help market participants avoid the risks from the eventual collapses of bubbles. Obviously, designing early warning systems is as challenging a task as is the development of reliable bubble detection mechanisms. As Phillips et al. (2015) point out, warning systems must have a high positive detection rate that ensures early and effective policy implementation to be effective. At the same time they need to have a low false detection rate to avoid unnecessary policy measures. Since early warning mechanisms constitute a challenging topic of their own, they are not covered in this paper.

Advanced stationarity- and cointegration-based tests can be subdivided into three broad categories. The first category are recursive unit root tests, which are covered in subsection 4.2.1. Subsequently, subsection 4.2.2 deals with tests for fractionally integrated models. Finally, subsection 4.2.3 covers regime-switching tests.

4.2.1 Recursive Unit Root Tests

A recent advancement in econometric bubble detection has been the development of *recursive unit root tests*, pioneered by Phillips et al. (2011) and refined by Phillips et al. (2015). These unit root tests are recursive in the sense that they are applied to subsamples of data on a period-by-period basis rather than to the full sample. Bubbles are detected by locating subsamples with distinct explosive behavior. Due to their recursive nature, these tests are able to distinguish periodically collapsing bubbles from pure unit root processes, giving them a distinct advantage over standard stationarity and cointegration tests. Both authors mentioned above also include methods for the identification of the starting and ending points of bubble episodes. However, these date-stamping procedures are not covered in this paper.

Like previous unit root tests, the new recursive tests build on the assumption that stock prices follow a random walk and therefore contain a unit root. During times when stock prices deviate substantially from their fundamental values, it is assumed that this assumption no longer holds. Instead, when there is a bubble in the time series, the data may be characterized by mildly explosive behavior (Bohl et al. 2013). This mildly explosive behavior can be modeled by an autoregressive process with a root that exceeds but is still close to unity.

The Phillips et al. (2011) (PWY) Method

Recursive unit root test were founded by Phillips et al. (2011).[4] They define financial exuberance in terms of explosive autoregressive behavior, in line with the explanation above, and assess the empirical evidence of price exuberance with the help of forward recursive regression tests. To be

[4] We will refer to their method as the Phillips/Wu/Yu (PWY) method hereinafter.

precise, they attempt to detect rational bubbles by applying a *right-tailed sup augmented Dickey-Fuller (sup ADF) test*.[5] The forward recursive regressions of the sup ADF test are performed by using subsets of the total sample that are extended by one observation at each pass. The starting point stays fixed to the first observation in the full sample. The test is right-tailed in the sense that it tests for a unit root (i.e., non-stationarity) against the alternative of an explosive root which is located on the right side of the probability distribution. If the explosive root holds, i.e., if the alternative hypothesis of mildly explosive behavior is not rejected, this is interpreted as a sign of the existence of a rational bubble in the asset price. Phillips et al. (2011) apply their technique to the NASDAQ composite price index and the NASDAQ composite dividend yields (1973-2005; monthly data) and successfully document periods of mildly explosive behavior.

A number of other studies have evaluated the applicability of the PWY method and applied it in empirical settings. For example, Phillips and Yu (2011) modify the PWY method in terms of the initialization of the procedure. Instead of fixing the initial condition to the first observation of the full sample as proposed by Phillips et al. (2011), the authors select the initial observation based on an information criterion. They apply their modified test to a U.S. house price index (1990-2009; monthly data), the crude oil price (1999-2009; monthly data), and the spread between Baa and Aaa bond rates (2006-2009; daily data). They reveal bubble characteristics in all three time series. Additionally, they show how the bubble migrates first from the housing market, on to the commodity market, and finally to the bond market.

Homm and Breitung (2012) compare the PWY method in terms of testing power to other test procedures that originate from the literature on tests for a change in persistence. Using Monte Carlo simulations they show that the PWY method works satisfactorily as a detection mechanism for structural breaks. However, they show that the alternative test procedures have higher finite sample power. In addition to the NASDAQ data sets used in Phillips et al. (2011), they investigate a range of stock market indices (S&P 500 (1980-2000), FTSE 100 (1985-1999), Nikkei 225 (1957-1990), Hang Seng (1980-2007), Shanghai (1991-2007)), two commodity time series (Brent crude oil (1985-2008), gold (1985-2007)), and housing indices (U.S. (1987-2006), Spain (1987-2007), U.K. (1991-2007), Japan (1957-1990)). When they apply the PWY method to monthly data, the test detects bubbles in all house price series, in the S&P 500, the Nikkei 225, and the gold price. For the remaining series, the PWY test does not reject the no-bubble hypothesis.

Gutierrez (2013) suggests a nonparametric bootstrap methodology to compute the empirical distribution and the critical values of the PWY method. Bootstrapping helps compute the critical values of the test statistics in finite samples more accurately than those based on asymptotic theory. He applies the modified method to future prices of four commodities (wheat, corn, soybean (1985-2011; daily data); rough rice (1988-2011; daily data)) and finds evidence of explosive behavior in all series but soybeans. Bettendorf and Chen (2013) use the PWY method to analyze the Sterling-U.S. dollar nominal exchange rate (1972-2012; monthly data). They find strong evidence for explosive behavior in the time series. However, they conclude that their findings should not be interpreted as an indication of bubbles in the data, but instead show that the explosive behavior might be driven by the relative prices of traded goods. Bohl et al. (2013) employ the sup ADF test of Phillips et al. (2011) to analyze the prices of German renewable energy stock indices (ÖkoDAX, DAXsubsector Renewable Energies (2004-2011; weekly and monthly data)). The supremum of the recursive ADF test statistic is highly significant.

[5] A sup ADF test relies on the repeated estimation of the conventional ADF model on a forward expanding sample sequence. The test is obtained based on the supremum value of a sequence of forward recursive ADF unit root test statistics.

The authors interpret this as an indication that rational bubbles are indeed present in the time series.[6]

More recently, Harvey et al. (2016) evaluate of the performance of the PWY method to detect explosive autoregressive behavior when the volatility of the innovation process is subject to non-stationarity. They criticize the assumption of the PWY method that the unconditional variance of the innovation process is stationary under both the unit root null and the explosive alternative. They base their criticism on empirical evidence indicating that there may actually be potential structural breaks in the unconditional volatility (i.e., the unconditional variance in some sub-samples can be larger than in others). They conclude that the PWY test can be badly oversized for plausible models of non-stationary volatility. This can mean that the PWY test spuriously rejects the no-bubble hypothesis, indicating the presence of a bubble when one does not actually exist. They propose wild bootstrap implementations of the method and show them to be effective in controlling finite sample size under non-stationary volatility. The apply their implementations to several commodity price series (crude oil: Brent, West Texas Intermediate (WTI); precious metals: gold, silver, platinum; non-ferrous metals: aluminum, copper (2000-2013; weekly and monthly data)) and find strong evidence of a speculative bubble in all time series but aluminum. However, they point out that when employing their wild bootstrap implementations, there is considerably less clear evidence for the presence of rational bubbles than when the original PWY method is used.

The PWY method has two distinct advantages. Not only is the sup ADF test simple to use in practical applications, it has also been shown to work satisfactorily in comparison to other recursive procedures for structural breaks (Homm and Breitung 2012). However, as Phillips et al. (2015) show, it only serves as an effective bubble detection method when there is a single period of exuberance in the data. The test is less consistent and suffers from reduced power when there are periodically collapsing bubbles in the time series. This means that the method is not effective in analyzing long time series or rapidly changing market data where multiple periods of exuberance and collapse are suspected.

The Phillips et al. (2015) (PSY) Method

The PWY method was further extended by Phillips et al. (ibid.) to account for the case of multiple bubble episodes in the data. This method has become the prototype for the recent literature on recursive testing for bubbles.[7]

The PSY method is a generalized version of the Phillips et al. (2011) sup augmented Dickey–Fuller (sup ADF) test for bubbles. Like the normal sup ADF test, the *generalized sup ADF (GSADF) test* relies on recursive right-tailed ADF tests. However, instead of fixing the initial condition of the technique to the first observation in the sample, the new procedure uses flexible window widths in the recursive regressions. This modification significantly improves the test's discriminatory power when multiple bubbles occur in the data. It thereby overcomes the weakness of the PWY method and is far more suitable for dealing with multiple periods of exuberance and collapse. This in turn makes it more applicable in the empirical study of long historical time series. Using simulations, the authors compare the performance of their own method to the performance of the PWY method as well as a recursive CUSUM test suggested in Homm and Breitung (2012). They show that the PSY method significantly outperforms both methods when multiple bubbles occur in the data, thereby overcoming a weakness in the earlier applications of

[6] Bohl et al. (2013) also use a second powerful variant of the commonly known ADF test, namely the Markov regime-switching ADF test, to test for rational bubbles. For more information on this test see subsection 4.2.3.

[7] We will refer to the method as the Phillips/Shi/Yu (PSY) method hereinafter.

unit root test for economic bubbles. Phillips et al. (2015) apply their method to S&P 500 stock market data (1871-2010; monthly data). Their method successfully identifies the well-known historical bubble episodes.

Bettendorf and Chen (2013) apply a working paper version of the PSY method to the nominal Sterling-dollar exchange rate in addition to using the PWY method, as we have mentioned above. The data set used in the empirical application of the PSY test as well as the results they obtain, coincide with those of the PWY method. Etienne et al. (2014) apply a working paper version of the PSY method to a range of agricultural futures contracts (grain: corn, soybeans, soybean oil, wheat; livestock: feeder cattle, live cattle, lean hogs; soft commodities: cocoa, coffee, cotton, sugar (1970-2011; daily data)). They find evidence of multiple periods of price explosiveness in all markets but point out that negative bubbles contribute significantly to explosive episodes in prices. Etienne et al. (2015) apply the same working paper version of PSY method to identify explosive episodes in corn, soybeans, and wheat futures markets (2004-2013; daily data). They find that only approximately two percent of their sample experienced explosive prices and conclude that grain futures markets are driven primarily by fundamental factors.

Also Pavlidis et al. (2017) use the PSY method for detecting bubbles that periodically collapse. However, rather than apply the test to the stock price and a set of observed market fundamentals, they apply the sup ADF test to the difference between the future spot price and the forward price. They exploit the fact that the probabilistic structure of periodically collapsing bubbles creates a gap between future spot and forward (futures) asset prices in small samples. When there is a bubble in the market, both the forward exchange rate and the future spot rate incorporate a bubble. However, they include it with different weights. The forward rate becomes a downward biased predictor of the future spot rate. The difference between the forward rate and the future spot rate depends on the bubble process and, as such, is explosive. As long as the forecast errors for fundamentals are not explosive, the presence of explosive dynamics in the difference between the two rates can be attributed to the existence of bubbles. Using Monte Carlo simulations they show that the PSY method exhibits good power properties in their setting. Additionally, Pavlidis et al. (ibid.) propose a method that relies on estimating rolling Fama regressions. This method tests the unbiasedness hypothesis and can also be used to detect bubbles. Pavlidis et al. (ibid.) provide two empirical applictions of their methods. First, they analyze German mark-U.S. dollar spot and forward exchange rates (1921-1923; weekly data). Both the PSY method as well as the analysis based on rolling Fama regressions reject the no-bubble null hypothesis and the authors conclude that a speculative bubble process exists in the foreign exchange market. In their second empirical application, the authors use S&P 500 stock market data (1982-2015; weekly data). The PSY method suggests that episodes of exuberance also exist in the U.S. stock market. The rolling Fama regressions, on the other hand, fail to reject the null hypothesis for this data set. Pavlidis et al. (ibid.) attribute this failure to the fact that the power of the test declines with the maturity horizon. In conclusion, Pavlidis et al. (ibid.) show that their adaption of the PSY method is effective in detecting bubble periods. Additionally, it has a one big advantage. Since it only requires data on spot and forward (futures) prices, which, contrary to market fundamentals, is readily available at high frequencies and over long time periods of time, it is a highly applicable bubble detection methodology.

In summary, recursive unit root tests have a two main advantages. First, since they are not based on fundamental factors but focus on the time series of the stock prices, they avoid testing a joint hypothesis of the presence of bubbles and the validity of the model used to determine the stocks' fundamental value. Second, they are effective in detecting periodically collapsing bubbles.

4.2.2 Fractional Integration Tests

Another subcategory of advanced stationarity- and cointegration-based tests are *fractional integration tests*, or *tests for fractionally integrated models.* Early research on stationarity- and cointegration-based tests rely on integer degrees of integration. However, a newer line of research tests for fractional integration in asset prices instead. Unlike stationary and unit-root processes, fractionally integrated processes are persistent (i.e., posses long memory) but are also mean reverting. Because fractionally integrated processes additionally allow significant deviations from equilibrium in the short run, some authors deem them more appropriate for modeling low frequency behavior of stock prices, dividends, and their corresponding equilibrium relationship (Koustas and Serletis 2005).

Koustas and Serletis (ibid.) utilize autoregressive fractionally integrated moving average (ARFIMA) processes to detect rational exuberance in stock prices. They apply their fractional integration technique to the S&P 500 composite stock market index and dividend yields (1871-2000; annual data). They are able to obtain robust rejections of the unit-root and stationarity null hypotheses in the log dividend yield and therefore come to the conclusion that the log dividend yield is a fractionally integrated process. They take this as evidence for long memory. Since a fractionally integrated dividend yield is assumed to be inconsistent with rational bubbles in stock market prices, they interpret this as an indication that there is no rational bubble in the data. Also Cuñado et al. (2005) use the fractional integration technique for bubble detection purposes. They use a version of the tests of Robinson (1994) which they apply to the NASDAQ composite stock market index and its corresponding dividend yield (1994-2003; daily, weekly, and monthly data). Their results bring to light that the sampling frequency used in the analysis has a strong influence on whether the test detects the presence of bubbles or not. Using monthly data, the authors are unable to reject the unit-root null hypothesis and therefore conclude that a rational bubble exists in the data. However, when using daily and weekly data, the test does reject the null hypothesis in favor of the alternative hypothesis of fractional integration, which speaks against the existence of bubbles in the time series.

Frömmel and Kruse (2012) empirically analyze the S&P 500 dividend-price ratio (1871-2009; monthly data) by combing two types of tests. First, they apply a test for changing persistence under fractional integration proposed by Sibbertsen and Kruse (2009). The test finds a significant break in the memory of the time series. Second, they apply a unit root test against long memory proposed by Demetrescu et al. (2008) to the two subsamples created by the break-testing procedure. The unit root test suggests that the pre-break data is characterized by long memory (i.e., fractional integration) while the post-break data contains a unit root. The authors interpret these results as an indication of a rational bubble in the data. The paper reconciles previous results on fractional integration, where fractional integration is seen as an indication that there is no bubble in the data, with results from other studies that find evidence of a unit root that is consistent with rational bubbles.

4.2.3 Regime-Switching Tests

The third category of advanced stationarity- and cointegration-based tests we distinguish are *regime-switching tests* or *Markov-switching tests.* Like all stationarity and cointegration tests, they make use of the fact that rational bubbles impose fairly strong restrictions on the time series properties of the asset price and use this to identify bubbles in data (Balke and Wohar 2009). Regime-switching try to capture discrete shifts in the generating process of time series. They were founded by Hamilton (1989) and introduced into the bubble literature by Hall et al. (1999). To be precise, Hall et al. (ibid.) propose a generalization of the Dickey-Fuller test procedure that makes use of the class of Markov-switching models. They treat each component of the bubble process as a separate Markov-regime with constant transition probabilities between the regimes. These so-called *Markov regime-switching augmented Dickey-Fuller unit-root (Markov-switching ADF) tests* should be able to distinguish between a moderately evolving regime and an explosive and subsequently collapsing regime, if a rational bubble exists. Thus, they are useful tools for modeling stock-market fluctuations that allow the bubble to switch between two states.

Recent Regime-Switching Analyses

Several authors have proposed different versions of regime-switching tests where the bubble switches between two or more states and applied them in different empirical analyses. We provide an overview of recent tests and applications. For reviews on early regime-switching tests see Gürkaynak (2008) and Al-Anaswah and Wilfling (2011).

Schaller and van Norden (2002) perform a switching regression analysis with elaborate functional forms of the bubble process to test for bubbles. They allow the collapse probability to be linked to the size of the bubble in the expanding bubble regime. They analyze the U.S. stock market (1926-1989; monthly data) based on data drawn from the Center for Research on Security Prices database. Their results show that there is something in the data, but that it cannot decisively be said that this is evidence for bubbles. Balke and Wohar (2009) also provide a regime-switching analysis. Rather than test whether a bubble exists or not, they attempt to decompose the log price-dividend ratio into a market fundamentals component and a bubble component using Bayesian Markov chain Monte Carlo methods. The market fundamentals component depends on expectations of future dividend growth and required returns while the bubble component is assumed to follow a Markov switching model that allows for the possibility of exploding and collapsing regimes. They allow the bubble component to temporarily expand or burst and collapse by specifying a Markov regime-switching model for the bubble component. The authors investigate how important the bubble component in the S&P 500 stock price index (1952-2005; quarterly data) is relative to market fundamentals. Their results show that different priors concerning market fundamentals and bubble components affect the inference made about the relative contribution of market fundamentals and rational bubbles to stock price fluctuations. When the econometrician allows for a persistent factor in dividend growth and/or required returns, the posterior distribution of the bubble's contribution to stock price movements is negligible. When prior beliefs rule out the presence of persistent changes in market fundamentals, on the other hand, the bubble component plays a significant role in explaining stock price movements.

Newer literature on regime-switching models also includes work by Al-Anaswah and Wilfling (2011), who use a state-space model with Markov-switching to detect speculative bubbles in stock price data. They express a present-value stock price model in state-space form which they estimate using the Kalman filter. Their Markov regimes represent two distinct states. In the first state the bubble survives, while in the second state the bubble collapses. They identify bursting stock price bubbles by statistically separating these regimes. In their empirical analysis, they analyze both artificially generated bubble data as well as five real-world data sets. The real-world data sets are monthly stock market data for the U.S. (1871-2004), Brazil (1994-2005), Indonesia (1990-2005), Malaysia (1986-2005), and Japan (1973-2005). The authors conclude that their procedure is successful in detecting price bubbles in all data sets and that Markov-switching in the data-generating process appears to be a statistically significant phenomenon.

Lammerding et al. (2013) investigate whether the market for crude oil (WTI (1983-2011; daily data), Brent (1989-2011; daily data)) is driven by speculative bubbles. They attempt to separate the oil price into a fundamental component and a bubble component by expressing a standard present-value oil price model in state-space form. Additionally, they introduce two Markov regimes into the state-space representation in order to distinguish between two distinct phases in the bubble process; one in which the oil price bubble is a stable process and one in which the bubble explodes. They estimate the entire Markov-switching state-space specification using an econometrically robust Bayesian Markov-Chain-Monte-Carlo (MCMC) methodology. They argue in favor of speculative bubbles in both time series. Bohl et al. (2013) test for explosive price behavior in renewable energy stocks in the mid-2000s. Besides employing the sup ADF test as mentioned in subsection 4.2.1, they employ another powerful variant of the commonly known ADF test, namely a Markov regime-switching ADF test. As with the sup ADF test, also the Markov-switching ADF test detects the presence of speculative bubbles in the data.

Tests for Regime-Switching Fundamentals

In the regime-switching models mentioned above, the bubble is allowed to switch between two or more states, while the fundamentals do not change. There are also *tests of regime-switching fundamentals*, where the fundamentals switch between two or more states. These test are ideal for detecting intrinsic bubbles. Recall from subsection 3.1.3 that intrinsic bubbles are a special type of rational bubble where the bubble component is a deterministic function of the fundamentals and not a function of time.

A model of regime-switching fundamentals was first developed by Driffill and Sola (1998). They use a framework to compare a regime-switching model of dividends, where the dividends are modeled as a Markov process with two states, and intrinsic bubbles as explanations of stock price behavior. They find that when they allow for regime switches in the fundamentals, the explanatory contribution of intrinsic bubbles is low. This once again highlights that the interpretation of a possible bubble component is very ambiguous. While their analysis shows that there seems to be a certain non-linearity in the data, it is not clear whether this non-linearity should be attributed to regime-switching fundamentals or intrinsic bubbles. More research remains to be conducted in order to get a better understanding of the origins of these non-linearities.

In conclusion, we can state that also regime-switching tests have the advantage that they avoid the joint hypothesis problem, since they are not based on fundamental factors, but rather focus on the time series of the stock prices themselves. This makes them superior to many early empirical bubble tests. However, many issues also regarding this class of models remain unsolved.

5 Discussion and Conclusion

Economists have been fascinated with bubbles in asset prices for decades. Not only have numerous authors attempted to provide an historical overview of the periods of financial exuberance and collapse that have occurred in the past, academics have also made numerous attempts to theoretically model and empirically test for bubbles in asset prices. Still, bubbles are not a generally accepted phenomenon and many researchers insist that asset prices do not exhibit price bubbles (Fama 2014). It has also been pointed out time and again that bubbles should only be used to explain asset prices as a means of last resort. Economists should always attempt to find reasonable economic explanations to apparently inexplicable price behavior before terming such an occurrence a bubble.

Over the years, numerous different theoretical models have been proposed in an attempt to explain asset prices. Before the turn of the millenium, Driffill and Sola (1998) came to the dire conclusion that it seems to be "a matter of taste and personal preference that makes the econometrician choose between bubble and fundamentals-based explanations of stock price behaviour." A decade later, Gürkaynak (2008) draws a similarly gloomy picture regarding the state of development of econometric bubble detection methods. He states that the methods developed are generally not able to detect bubbles in asset prices with a satisfactory degree of certainty. He criticizes that economists still find it difficult to differentiate between bubbles and fundamentals-based explanations and that empirical bubble detection tests still face many econometrics challenges.

Regarding theoretical bubble models, the state of research has taken a great stride forward with the development of the first behavioral bubble models. Many of these models provide more adequate explanations for bubble phenomena and other return anomalies than the more traditional rational bubble models because they incorporate insights from psychology into their reasoning. Experimental research seems to corroborate the validity of these models. Therefore, behavioral bubble models definitely constitute a promising direction of future bubble research.

Over the past few years, considerable advancement has also been made in the field of empirical bubble detection mechanisms. Research has focused predominantly on advanced stationarity- and cointegration-based methods to pinpoint phases of exuberance in financial markets. These test have the advantage that, since they are not based on fundamental factors but rather focus on the time series of the asset prices themselves, they avoid testing a joint hypothesis of the presence of rational bubbles and the validity of the model used. One advanced stationarity and cointegration method that has greatly advanced the applicability of bubble detection methods in time series with periodically collapsing bubbles, is the recursive unit root test proposed by Phillips et al. (2015) (PSY). Pavlidis et al. (2017) apply the PSY method to the difference between the future spot price and the forward price. This approach significantly increased the applicability of the PSY test because data on spot and forward prices is readily available at high frequencies and over long time periods for a wide range of assets, much in contrast to data on fundamentals. Other recent developments in empirical bubble detection include fractional integration methods and regime-switching tests, which both also constitute promising fields of research.

In summary, it can be said that different empirical bubble detection tests increasingly find overlapping evidence of rational bubbles in the various time series they are applied to. However, the empirical results on bubbles are still not entirely conclusive. In many cases, while one method fails to find bubbles in the data, another will conclude that there are bubbles in the exactly the same data set. Research may have focused to some extent but it is still developing in different directions within that new area of focus. More research must be carried out to attempt to guide the bubble detection literature towards some kind of consensus on methodology and conclusion.

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