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Asset Price Bubbles

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Abstract

This article reviews the theoretical literature on asset price bubbles, with an emphasis on the martingale theory of bubbles. The key questions studied are as follows: First, under what conditions can asset price bubbles exist in an economy? Second, if bubbles exist, what are the implications for the pricing of derivatives on the bubble-laden asset? Third, if bubbles can exist, how can they be empirically determined? Answers are provided for three frictionless and competitive economies with increasingly restrictive structures. The least restrictive economy just assumes no arbitrage. The next satisfies no arbitrage and no dominance. The third assumes the existence of an equilibrium.

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1. INTRODUCTION

Asset price bubbles have fascinated economists for centuries. One of the earliest alleged price bubbles was the Dutch tulip mania of 1634–37 (Garber 1989, 1990), followed by the Mississippi bubble of 1719–20 (Garber 1990) and the related South Sea bubble of 1720 (Garber 1990, Temin & Voth 2004). More recently, alleged stock price bubbles include the run-up in equity prices prior to the 1929 US stock price crash (White 1990, De Long & Shleifer 1991, Rappoport & White 1993, Donaldson & Kamstra 1996) and the NASDAQ dot-com price bubble of 1998–2000 (Ofek & Richardson 2003; Brunnermeier & Nagel 2004; Cuñado, Gil-Alana & Perez de Gracia 2005; Pastor & Veronesi 2006; Battalio & Schultz 2006). Of course, there was also the alleged US housing price bubble prior to 2007 (Goodman & Thibodeau 2008, Mikhed & Zemcik 2009, Clark & Coggin 2011).

The purpose of this article is to review the theoretical literature related to asset price bubbles. The key questions studied are as follows: First, under what conditions can asset price bubbles exist in an economy? Second, if bubbles exist, what are the implications for the pricing of derivatives on the bubble-laden asset? Third, if bubbles can exist, how can they be empirically tested? Answers are provided to these questions for three frictionless and competitive economies with increasingly restrictive structures. The least restrictive economy just assumes no arbitrage. The next satisfies no arbitrage and no dominance (ND). The third assumes the existence of an equilibrium.

The major focus of this article is on the martingale theory of asset price bubbles based on the insights of Loewenstein & Willard (2000a,b); Cox & Hobson (2005); Heston, Loewenstein & Willard (2007); and Jarrow, Protter & Shimbo (2007, 2010). For an excellent review of the martingale theory of bubbles' mathematics, see Protter (2013). This article briefly reviews the classical theoretical literature on asset pricing models, with an eye toward its relation to the martingale theory. A nice review of the classical literature was done by Camerer (1989). We leave an updated review of the empirical literature to subsequent research.

In this review, Section 2 addresses the definition of an asset price bubble. Given this definition, Section 3 characterizes the properties of bubbles in a no-arbitrage economy. Sections 4 and 5 study the properties of price bubbles in more restricted economies, imposing the additional assumptions of ND and economic equilibrium, respectively. Section 6 discusses empirical testing, and Section 7 provides closing arguments.

2. DEFINITION OF A BUBBLE

An asset's market price exhibits a bubble when the market price exceeds the asset's fundamental value. The market price for an asset is unambiguous, but the meaning of an asset's fundamental value is not. This section discusses the definition of an asset's fundamental value.

2.1. The Economy

We consider a continuous or discrete time model on the time interval $[0, T]$, where T can be a fixed finite time or ∞. For simplicity, we give the notation for a continuous time model and discuss the discrete time model's differences via remarks.

Let $(\Omega, \mathcal{F}, \mathbb{F}, P)$ be a filtered complete probability space characterizing the randomness in the economy. We assume that the filtration $\mathbb{F} = (\mathcal{F}_t)_{t\geq 0}$ satisfies the usual hypotheses (Protter 2001). Markets are competitive and frictionless. Competitive means that traders act as price takers, believing their trades have no quantity impact on market price. Frictionless means that there are no transaction costs and no trading constraints, e.g., short sale constraints or margin requirements, and that shares are not infinitely divisible.

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A risky asset and a money market account that is locally riskless are traded in the economy. Locally riskless means that over the next time step of smallest granularity in the model, the return on the asset is known with probability 1. For simplicity we consider only one risky asset, but the logic easily applies to an arbitrary but finite collection of risky assets.

We denote the value at time t of a money market account as

$$B_t = \exp\left(\int_0^t r_u du\right), \tag{1}$$

where $(r_t)_{t\geq 0}$ is a nonnegative adapted process representing the default-free spot rate of interest. Let $\tau \leq T$ be a stopping time that represents the maturity (or life) of the risky asset. Let $D = (D_t)_{0\leq t<\tau}$ be a càdlàg (left continuous with right limits) semimartingale process[1] adapted to $\mathbb{F}$ that represents the cumulative cash flow process from holding the risky asset. Let $X \in \mathcal{F}_\tau$ be the terminal payoff or liquidation value of the asset at time τ. We assume that $X, D \geq 0$.

2.2. The Market Price

Let the market price of the risky asset be given by the nonnegative càdlàg semimartingale $S = (S_t)_{0\leq t\leq\tau}$ representing the price ex-cash flow (because S is càdlàg) where, by the previous assumption, $S_\tau \equiv X$ at the liquidation date. For convenience, after the liquidation date τ, we define the risky asset price to be the liquidation value invested in the money market account, i.e., $S_t \equiv B_t \frac{X}{B_\tau}$. We also note that after the liquidation date, there are no more cash flows to the risky asset, i.e., $\int_\tau^T dD_t \equiv 0$.

Let G be the cumulative gains process associated with the market price of the risky asset:

$$G_t = S_t + B_t \int_0^t \frac{1}{B_u} dD_u. \tag{2}$$

Note that all the asset's cash flows are also invested in the money market account, and because $B, S, X, D \geq 0$, we have $G \geq 0$.

Remark 1 (Discrete time). For a discrete time model, the integral becomes a summation over the cash flow payment dates.

2.3. No Free Lunch with Vanishing Risk

To define the asset's fundamental value, we need to assume some additional structure on the economy. The minimal structure, consistent with the existence of an economic equilibrium, is to assume that the market has no arbitrage opportunities.

In this regard, we assume that the economy satisfies no free lunch with vanishing risk (NFLVR), a technical extension of the standard definition of no arbitrage. This extension excludes both (*a*) zero investment self-financing trading strategies that have nonnegative liquidation values that are strictly positive with positive probability, and (*b*) limiting arbitrage opportunities, i.e., the limits of sequences of zero investment self-financing trading strategies that have a small probability of a loss. Condition *a* in this definition is just the standard definition of no arbitrage. Trading strategies are holdings in the risky asset and money market account (for all times and states) that

[1]A stochastic process X is a semimartingale if it has a decomposition $X_t = X_0 + M_t + A_t$, where (*a*) $M_0 = A_0 = 0$; (*b*) A is adapted, càdlàg, and of finite variation on compacts; and (*c*) M is a local martingale (hence càdlàg) (Protter 2001, chapter 2). The definition of a local martingale is provided later in this article.

are predictable (depend on only current and past information) and that are admissible (the value of the trading strategy is bounded below), the purpose of which is to exclude doubling strategies from the economy (for further discussion, see Jarrow & Protter 2008).

By the first fundamental theorem of asset pricing (Delbaen & Schachermayer 1994, 1998), NFLVR holds if and only if there exists a probability measure Q equivalent to P such that $\frac{G_t}{B_t}$ is a local martingale.[2] A local martingale is an adapted and càdlàg stochastic process X_t where there exists a sequence of stopping times (τ_n) such that $\lim_{n\to\infty} \tau_n = \infty$ and $X_{\min\{t,\tau_n\}}$ is a martingale for all n.

A local martingale is a generalization of a martingale that extends the martingale's fair game property to a game that has a random termination time, which depends on information generated when playing the game. This information is essential when considering extended arbitrage opportunities. Given that $\frac{G_t}{B_t}$ is a local martingale, there are only three possibilities: $\frac{G_t}{B_t}$ is a uniformly integrable martingale, a martingale, or a strict local martingale.[3] We return to these observations below. Q is called an equivalent local martingale measure.

Remark 2 (Discrete time). For a discrete time model (as shown in Kopp 1984, p. 46), all local martingales are martingales, and the qualifier "local" can be omitted. In a discrete time and finite horizon economy, NFLVR can also be replaced with the standard definition of no arbitrage (Dalang, Morton & Willinger 1990).

Given that the economy can be incomplete, by the second fundamental theorem of asset pricing (Jarrow & Protter 2008) we know that the equivalent local martingale measure (ELMM) Q need not be unique. If it is nonunique, then there exists a continuum of possible ELMMs. To define the fundamental value, we need to identify the unique ELMM from the collection of all possible ELMMs reflected in the market price.

To obtain a unique ELMM, we assume that the model considered is embedded in a larger economy that is complete. This larger economy uniquely determines the ELMM. For example, if enough static trading in call options exists, then the ELMM can be uniquely identified via the traded call option prices (in this regard, see Jacod & Protter 2010, Schweizer & Wissel 2008). For the remainder of this article, we assume that the market selected ELMM Q has been determined in this manner. We call this market selected ELMM the ELMM chosen by the market.

2.4. The Fundamental Value

Given the ELMM Q chosen by the market, we can now define the risky asset's fundamental value. The risky asset's time t fundamental value, denoted by FV_t, is defined by the expression

$$FV_t = E_Q\left(\frac{X \cdot 1_{\{\tau<\infty\}}}{B_\tau} + \int_t^\tau \frac{1}{B_u} dD_u \,\middle|\, \mathcal{F}_t\right) B_t, \tag{3}$$

where $E_Q(\cdot)$ denotes expectation under the ELMM Q.

The risky asset's fundamental value is equal to the price a trader would pay if after purchase, they had to hold the asset in their portfolio forever. Indeed, Expression 3 is the expected discounted cash flows received from holding the asset, where the expectation operator using the ELMM Q implicitly adjusts for risk.

The asset market price bubble is then

$$\beta_t = S_t - FV_t. \tag{4}$$

[2] In general, $\frac{W_t}{B_t}$ is a σ-martingale, but because it is nonnegative (bounded below), it is a local martingale.

[3] A strict local martingale is a local martingale that is not a martingale.

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A bubble reflects the notion that the resale value of the asset is higher than the price paid if it were to be held forever.[4]

There is an alternative definition of fundamental value that can be formulated in this setting using the notion of price operators defined on the space of random variables generated by the cash flows from the risky asset's and the money market account's price processes. This definition uses the notion of a charge, a finitely additive measure, and its relation to the price operator (see Gilles 1988, Gilles & LeRoy 1992, Jarrow & Madan 2000). As shown by Jarrow, Protter & Shimbo (2010), this alternative definition is equivalent to that based on local martingale measures (as discussed above), and consequently we omit the details of this alternative definition.

2.5. SUMMARY

The definition for an asset price bubble focuses on the motives underlying the investor's demands for buying the asset to resell versus buying the asset to consume its cash flows (hold it forever). It captures the idea that the market price can exceed the fundamental value because of a bigger fool belief. An investor (here, the fool) may buy the asset at a price that exceeds it fundamental value, believing that it can be resold at a higher price in the future to someone else (the bigger fool). In this definition, the key is to characterize conditions under which asset price bubbles can exist under increasingly restrictive structures imposed on the economy. The increasingly restrictive structures are NFLVR, ND, and equilibrium. We discuss each of these increasingly restrictive economies in turn.

3. NO FREE LUNCH WITH VANISHING RISK

The first market studied is one where, as above, only NFLVR is assumed to hold. We maintain the competitive and frictionless market assumptions unless otherwise noted.

3.1. Properties of Bubbles

This section characterizes the types of bubbles that can exist in the economy given NFLVR.

Theorem 3 (Nonnegative bubbles).

$$\beta_t \geq 0.$$

Proof. Because the discounted cumulative gains process $\frac{G_t}{B_t}$ is nonnegative, it is a Q supermartingale. Taking conditional expectations, this implies

$$S_t \geq E_Q\left(\frac{X \cdot 1_{\{\tau<\infty\}}}{B_\tau} + \int_t^\tau \frac{1}{B_u} dD_u \middle| \mathcal{F}_t\right) B_t. \tag{5}$$

The definition of a bubble completes the proof. □

This theorem shows that asset price bubbles can only be nonnegative. This follows from the observation that the minimal value obtained from buying any risky asset is its fundamental value, thereby providing a lower bound on the asset's market price.

[4]A recent paper by Shiryaev, Zhitlukhin & Ziemba (2015) has an empirically motivated definition of a price bubble. They define a bubble to exist in an asset's price process if there is a positive probability of a downward jump in the asset's drift and a change in the asset's volatility at some stopping time θ over a fixed future time horizon $[1, \ldots, T]$, where the jump must occur by time $T+1$ with probability 1. For a similar approach to the definition and detection of price bubbles, see Andersen & Sornette (2004).

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Theorem 4 (Bubble types). If there exists a nontrivial bubble $\beta \not\equiv 0$, then we have three possibilities based on the properties of the risky asset's liquidation time:

1. If $P(\tau = \infty) > 0$, then $\frac{\beta}{B}$ is a local Q-martingale (which could be a uniformly integrable Q-martingale).
2. If $P(\tau < \infty)$ equals 1 and is unbounded, then $\frac{\beta}{B}$ is a local Q-martingale (which could be a Q-martingale) but not a uniformly integrable Q-martingale.
3. If τ is a bounded stopping time, then $\frac{\beta}{B}$ is a strict local Q-martingale.

This theorem is proved by Jarrow, Protter & Shimbo (2007). It implies that there are three types of bubbles. The implications of this theorem for the risky asset's cumulative gains process are as follows:

1. Type 1 bubbles occur when the risky asset has a payoff at $\tau = \infty$. In this case, the fundamental value is $FV_t = E_Q(\int_t^\infty \frac{1}{B_u} dD_u | \mathcal{F}_t) B_t$, so that the bubble captures the entire market value of the payoff at ∞, i.e., $\beta_t = S_t - E_Q(\int_t^\infty \frac{1}{B_u} dD_u | \mathcal{F}_t) B_t$.
2. Type 2 bubbles occur when the risky asset's cumulative gains process $\frac{G}{B}$ is not a uniformly integrable Q-martingale, i.e., S_t does not equal its fundamental value $FV_t = E_Q(\frac{X \cdot 1_{\{\tau < \infty\}}}{B_\tau} + \int_t^\tau \frac{1}{B_u} dD_u | \mathcal{F}_t) B_t$. In this case, the cumulative gain's process could be a Q-martingale.
3. Type 3 bubbles happen when the cumulative gains process $\frac{G}{B}$ is a strict local Q-martingale, rather than a Q-martingale or a uniformly integrable Q-martingale.

Remark 5 (Discrete time). In discrete time models, only type 1 and 2 bubbles can exist. This is because in a discrete time model, all local martingales are martingales (see Kopp 1984, p. 46; Kabanov 2008).

We have the following properties of type 2 and 3 bubbles:

Theorem 6 (Type 2 and 3 bubbles). Suppose $P(\tau < \infty) = 1$. Then

1. $\beta_\tau = 0$;
2. if $\beta_t = 0$, then $\beta_u = 0$ for all $u \geq t$; and
3. if no cash flows, then $S_u = E_Q(\frac{S_t}{B_t} | \mathcal{F}_u) B_u + \beta_u - E_Q(\frac{\beta_t}{B_t} | \mathcal{F}_u) B_u$ for any $u \leq t \leq \tau$.

This theorem is proved by Jarrow, Protter & Shimbo (2007). Condition 1 states that bubbles must burst on or before the asset's liquidation date τ. This is because at that date, the market price and the fundamental price are identical. Condition 2 states that if the bubble ever bursts before the asset's liquidation date, then it can never start again. Alternatively stated, in the context of this model, either bubbles must exist at the start of the model, or they never will exist. Also, if they exist and burst, then they cannot start again. Requiring bubbles to exist at the start of the model is a weakness of the theory. However, this aspect of the model can be relaxed in an incomplete market via shifts in the market chosen equivalent local martingale measure (see Jarrow, Protter & Shimbo 2010; Biagini, Follmer & Nedelcu 2014). Shifts in the market chosen ELMM capture random structural shifts in the underlying economy corresponding to changing beliefs, preferences, endowments, institutional structures, market clearing mechanisms, or political/regulatory considerations.

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Finally, Condition 3 relates the asset price at time u to the conditional expectation of its price at time t plus the asset's price bubble. This condition is used below in the valuation of various derivatives.

For some risky assets, bubbles can never exist, as the following theorem shows.

Theorem 7 (Bounded payoffs). Suppose $P(\tau < \infty) = 1$. If $\frac{X}{B_\tau}$ and $\int_0^\tau \frac{1}{B_u} dD_u$ are bounded, then there are no type 3 bubbles.

Proof. Under the assumptions of the theorem, $\frac{G_t}{B_t}$ must be a nonnegative bounded local martingale, and hence a martingale (Klenke 2008, p. 488). Because type 3 bubbles imply that $\frac{G}{B}$ is a strict local martingale, this completes the proof. □

This theorem shows that if any risky asset has bounded payoffs, then it can have no type 3 bubbles. Examples of such a risky asset include default-free zero-coupon bonds of finite maturities as long as interest rates are nonnegative. Various other derivatives also satisfy this condition, as discussed below.

Example 8 (Asset price process with a bubble in a complete market). We now give an example, in a complete market, of an asset's price process that exhibits a bubble. Assume the risky asset has no cash flows. Let the asset's market price process be given by the following stochastic differential equation driven by a Brownian motion Z:

$$dS_t = \sigma(S_s)dZ_s + \mu(S_s)ds; \quad S_0 = x. \tag{6}$$

Here, the asset's price volatility $\sigma(S_t)$ is stochastic and depends on the level of the asset's price.

This evolution implies a complete market. Under suitable integrability conditions, there exists an equivalent probability measure Q such that

$$\frac{S_t}{B_t} = S_0 + \int_0^t \sigma(S_s)d\tilde{Z}_s, \tag{7}$$

where $\tilde{Z}$ is a Brownian motion under Q. Hence, given the necessary integrability conditions, this evolution is consistent with NFLVR.

In this situation, it is known (Delbaen & Shirakawa 2002, Mijatovic & Urusov 2012) that the process (S/B) is a strict local martingale if and only if

$$\int_\epsilon^\infty \frac{x}{\sigma(x)^2} dx < \infty, \tag{8}$$

for some $\epsilon > 0$. Hence, a bubble exists for this asset price process if and only if Expression 8 holds.

This is an example of a price process in a complete market that is either a type 2 or a type 3 bubble, depending on the asset's underlying liquidation time τ, which is implicit and not explicitly given in this example.

Example 9 (Asset price process with a bubble in an incomplete market). Here we give an example, in an incomplete market, of an asset's price process that exhibits a bubble. This example is due to Sin (1998). Assume there are no cash flows on the underlying asset and let (S_t, v_t) satisfy

$$\begin{aligned}\frac{dS_t}{S_t} &= r_t dt + v_t^\alpha(\sigma_1 dZ_{1t} + \sigma_2 dZ_{2t}),\\ \frac{dv_t}{v_t} &= \rho(b - v_t)dt + a_1 dZ_{1t} + a_2 dZ_{2t}\end{aligned}$$

under the ELMM Q, where (Z_{t1}, Z_{t2}) is a two-dimensional independent Brownian motion process, $S_0 = x$, $v_0 = 1$, and $\alpha > 0$, $\rho \geq 0$, $b > 0$, a_1, σ_1, a_2, σ_2 are constants. This evolution implies an

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incomplete market. Then $\frac{S_t}{B_t}$ is a strict local martingale under Q if and only if $a_1\sigma_1 + a_2\sigma_2 > 0$. Hence, a bubble exists for this asset price process if and only if $a_1\sigma_1 + a_2\sigma_2 > 0$.

This is an example of a price process in an incomplete market that is either a type 2 or a type 3 bubble, depending on the asset's underlying liquidation time τ, which is implicit and not explicitly given in this example. For additional examples of continuous sample path processes with stochastic volatilities exhibiting bubbles, see Andersen & Piterbarg (2007) and Lions & Musiela (2007). For examples of discontinuous sample path processes exhibiting bubbles, see Keller-Ressel (2014) and Protter (2015).

For the remainder of this article, we only study risky assets that have no embedded price component analogous to that of fiat money, i.e., that contain a strictly positive liquidation value X at $\tau = \infty$. To exclude these type 1 bubbles, we add the following assumption:

Assumption (No type 1 bubbles).

$$P(\tau < \infty) = 1. \tag{9}$$

Remark 10 (Discrete time). In discrete time models, under this assumption, only type 2 bubbles can exist and only in infinite horizon models. This remark elucidates why the classical empirical literature on bubbles (discussed below), which consists of discrete time models, focuses on infinite horizons.

3.2. Derivatives

Do asset price bubbles affect the standard arbitrage-free pricing methodology for derivatives? As shown below, the answer is almost always yes, but there are some exceptions. For the remainder of this section, we assume that the risky asset's liquidation time τ is bounded, and that there are no cash flows on the risky asset. We assume that European call and put options trade on the risky asset with strike price K and maturity date $U \leq T$. The call's payoff at expiration is $C_U = \max\{S_U - K, 0\}$, and the put's payoff at expiration is $P_U = \max\{K - S_U, 0\}$. Let C_t and P_t be the call's and the put's market prices at time $t \leq U$.

Because the put's payoff is bounded, applying Theorem 7 gives the following result:

Theorem 11 (Put valuation). Assume there are no cash flows on the risky asset. Then

$$P_t = E_Q\left(\frac{\max\{K - S_U, 0\}}{B_U}\middle| F_t\right) B_t. \tag{10}$$

This theorem states that the put's market price has no bubble, and that it is a uniformly integrable martingale under Q. Hence, we can now answer the question posed at the start of this section. In the presence of asset price bubbles, the standard option pricing methodologies can still be employed to value and hedge put prices. However, the same is not true for call options. Call option prices can have bubbles because the call's payoff is unbounded if the underlying asset's market price is unbounded, which is the usual case.[5] Examples of call options with bubbles can be found in Heston, Loewenstein & Willard (2007). In this setting, European put–call parity need not hold as well (Cox & Hobson 2005). For related results, see Kardaras, Kreher & Nikeghbali (2015).

[5] For example, this is true for the geometric Brownian motion that underlies the Black–Scholes–Merton option pricing model.

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Remark 12 (Strategic trading). Bubbles can exist in frictionless markets when the price taking assumption is relaxed and there are large investors who trade strategically, knowing that their trades have a quantity impact on the price. Such a quantity impact on the price may be due to asymmetric information considerations (informed traders in rational markets) or portfolio rebalancing effects caused by sufficiently large trades in markets with symmetric information.

In such economies, due to the strategic manipulation of the market price by a large trader, the market price can exceed that which would otherwise exist in the ideal economy, generating asset price bubbles. An example of such an asset price bubble is generated by a pump and dump strategy, where the large trader buys the underlying risky asset to generate upward price momentum, then sells into this momentum to collect trading profits. Corners and short squeezes are other manipulative trading strategies that generate price bubbles. These price bubbles were first studied in the literature by Jarrow (1992). Jarrow (1994), Frey & Stremme (1997), and Frey (1998), among others, studied the impact that such market manipulation price bubbles have on the pricing of derivatives. Price bubbles generated by strategic trading can also exist in markets where there are many traders, each of whose trades have a quantity impact on price (Jarrow, Protter & Roch 2012).

Remark 13 (Short sale constraints). In competitive markets with trading constraints, NFLVR needs to be modified because not all admissible self-financing trading strategies can be employed. In this context, a weakened notion of NFLVR needs to be defined that respects the existence of the trading constraints. In such economies, NFLVRs can exist. Jarrow, Protter & Pulido (2015) study an economy where there are short sale constraints on risky assets. They show that short sale constraints facilitate the existence of price bubbles, even with the trading of futures contracts on the underlying risky assets.

4. NO DOMINANCE

As shown above, bubbles violate many of the classical option pricing theorems, and in particular, put–call parity. These violations occur because of the absence of sufficient structure on the economy within the NFLVR framework. Interestingly, however, in actual markets, put–call parity is almost never empirically violated (for example, see Klemkosky & Resnick 1980, Kamara & Miller 1995, Ofek & Richardson 2003), suggesting that more structure than only NFLVR is needed to understand price bubbles in realistic economies.

Because bubbles are market-wide phenomena, thought to be generated by the collective actions of all traders (the bigger fool theory), it seems appropriate that when considering their existence, one needs to impose some additional structure on the economy beyond NFLVR. Consistent with this intent, but less restrictive than imposing an economic equilibrium, Jarrow, Protter & Shimbo (2007, 2010) added Merton's (1973) ND condition. ND is a restriction imposed by the collective action of all traders.

This section introduces the additional assumption of ND and explores its implications for asset price bubbles. We maintain the competitive and frictionless market assumptions.

4.1. Definition

An asset is dominated if (*a*) there exists a trading strategy whose cash flows and liquidation value are always greater than or equal to the cash flows and liquidation value of the risky asset and strictly greater with positive probability, and (*b*) the cost of constructing the trading strategy is less than the market price of the asset. An economy satisfies ND if no traded asset is dominated.

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A dominated asset need not imply the existence of an arbitrage opportunity. This is due to the admissibility condition in the definition of a trading strategy, which precludes shorting an asset whose price is unbounded above. NFLVR and ND are distinct conditions, although both imply the standard no-arbitrage condition.

ND is a condition related to supply equalling demand. To understand why, note that NFLVR is a mispricing opportunity that any single trader can exploit in unlimited quantities. In contrast, ND compares two investment positions, where a trading strategy dominates holding the asset directly. The trader, if they desire the payoffs from the asset in their optimal portfolio, will never hold the asset. Because all traders see the same dominance situation, no one in the market will hold the asset. Hence, supply for the asset (if positive) will exceed demand.

We note that in a finite horizon economy, in discrete or continuous time, it has been shown that NFLVR and ND are satisfied if and only if there exists an equivalent martingale measure, as distinct from a local martingale measure (Jarrow & Larsson 2012).

4.2. Properties of Bubbles

In a complete market, one can show the following restriction is implied by ND:

Theorem 14 (Complete markets). Assume NFLVR and ND. Then in a complete market, there are no type 2 or type 3 bubbles.

This theorem is proved by Jarrow, Protter & Shimbo (2007).[6] The logic of the proof is easy to understand. If a type 2 or type 3 bubble exists, then the market price of the asset is strictly greater than its fundamental value, which represents the cost of buying and holding the asset synthetically constructed. This violates ND, implying that there can be no such bubbles in the economy. The difficulty of the proof is in showing the asset's fundamental value can be constructed synthetically as an admissible trading strategy. Type 2 and type 3 bubbles can still exist in an incomplete market under ND.

4.3. Derivatives

For the remainder of this section, we assume that the risky asset's liquidation time τ is bounded and that there are no cash flows on the risky asset. We assume that European call and put options trade on the risky asset with strike price K and maturity date $U \leq T$. The call's payoff at expiration is $C_U = \max\{S_U - K, 0\}$, and the put's payoff at expiration is $P_U = \max\{K - S_U, 0\}$. Let C_t and P_t be the call's and the put's market prices at time $t \leq U$, respectively.

Under ND, we can prove the following additional results to those already collected under NFLVR alone. For the first result, we have to assume that a default-free zero-coupon bond paying a dollar at time U is traded. Denote its time t market price as $p(t, U)$.

Theorem 15 (Put–call parity). Assume there are no cash flows on the risky asset. Then

$$C_t = P_t + S_t - p(t, U)K. \tag{11}$$

This follows by the original argument used by Merton (1973). If it is violated, then the portfolio represented by either the left or right side of the equality dominates the portfolio on the other

[6]In a finite horizon economy, this follows from the second fundamental theorem, which implies a unique local martingale measure in a complete market, and the application of the theorem noted above from Jarrow & Larsson (2012).

side of the equality, contradicting ND. We note that the risky asset price S in Expression 11 may exhibit an asset price bubble. Given the satisfaction of put–call parity, we can now characterize a European call option's market price in the presence of bubbles.

Theorem 16 (Call valuation). Assume there are no cash flows on the risky asset and that interest rates are nonnegative. Then

$$C_t = E_Q\left(\frac{\max\{S_U - K, 0\}}{B_U}\middle|\mathcal{F}_t\right)B_t + \beta_t - E_Q\left(\frac{\beta_U}{B_U}\middle|\mathcal{F}_t\right)B_t. \tag{12}$$

Proof. If interest rates are nonnegative, an application of Theorem 7 gives $p(t, U) = E_Q(\frac{1}{B_U}|\mathcal{F}_t)B_t$. We have $S_U - K = \max\{S_U - K, 0\} + \max\{K - S_U, 0\}$. Taking conditional expectations, substituting the equation in Condition 3 of Theorem 6, and using Expression 11 and algebra gives the result. □

This theorem shows that call options inherit the underlying asset's price bubble. More importantly, the call's market price will not equal its risk-neutral value if the underlying risky asset exhibits a price bubble. The standard option pricing methodology fails.

However, all is not lost. Expression 12 can be used to compute the market price of a European call option in a market where the underlying asset's price exhibits a bubble. In this case, one must first use the underlying asset's price process to estimate the size of the asset's price bubble via Condition 3 of Theorem 6, and then substitute into Expression 12 along with the standard risk-neutral valuation formula for the option's price.

Much more is known about the impact of asset price bubbles under NFLVR and ND for the market prices of various other derivatives. For example, American call and put prices can have no price bubbles. For puts, this follows because payoff is bounded above. For calls, this follows because the ability to exercise early enables the call holder to exploit the existence of a bubble in the underlying asset. Indeed, it can be shown that with bubbles, an American call option has a positive probability of early exercise, even if the underlying asset has no cash flows. This is in contradiction to the standard results in nonbubble economies, such as the Black–Scholes–Merton model (Jarrow, Protter & Shimbo 2010).

For the pricing of forward and futures contracts, some interesting differences arise in the presence of asset price bubbles. As might be expected, a forward contract's forward price inherits the bubble of the underling asset analogous to the manner in which European calls inherit the underlying asset's price bubble in Theorem 16. More surprising, perhaps, is that futures contracts can exhibit their own price bubbles, independent of those in the underlying asset, due to marking to market (daily settlement) (Jarrow & Protter 2009; Jarrow, Protter & Shimbo 2010).

The pricing of foreign currencies and foreign currency derivatives in the presence of price bubbles in the underlying foreign currency spot exchange rates is studied in Jarrow & Protter (2011). The new insights obtained in this regard are that: (*a*) exchange rate bubbles can be negative, in contrast to asset price bubbles, (*b*) exchange rate bubbles are caused by price level bubbles in either or both of the relevant countries currencies, and (*c*) price level bubbles decrease the expected inflation rate in the domestic economy.

5. EQUILIBRIUM

This section studies the impact that economic equilibrium has on the existence of asset price bubbles. Economies are considered that satisfy the competitive and frictionless market assumptions, as well as those that do not.

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5.1. Definition

This section defines an economic equilibrium. To avoid the introduction of additional notation, it is assumed that the reader is familiar with the notion of an Arrow–Debreu equilibrium as given, for example, by Duffie (2001).

We consider a continuous or discrete time model with finite or infinite horizon. The randomness in the economy is generated by a filtered probability space satisfying the usual conditions as employed in the previous sections.

The economy trades a collection of assets, including a money market account that is defined to be a locally riskless investment. The risky assets are in positive supply, whereas the money market account is in zero supply. The supply of the risky assets is assumed to be fixed across time. There is a single consumption good in which the asset prices are denominated.

In the economy is a collection of traders who are characterized by their beliefs and preferences and by an endowment of shares in the assets. Their objective is to choose holdings in the assets and consumption over time to maximize their preferences, subject to their budget constraints.

We first consider a competitive and frictionless market for the traded assets. The economy is said to be in equilibrium if the assets' price processes are such that given the price processes, all traders' demands for the asset shares maximize their preferences subject to their budget constraints, and supply equals demand for all times with probability 1.

We next consider other economies, where one out of (*a*) the preferences and/or beliefs, (*b*) the price taking assumption, or (*c*) the frictionless market assumption is changed. These other economies are intended to capture a market structure that approximates reality, the actual market. In these other economies, there is an analogous equilibrium price with the exception that the optimal demands are determined subject to one out of (*a*) different preferences and/or beliefs, (*b*) non–price taking behavior, or (*c*) constraints imposed by the relevant frictions.

Before discussing the literature, we note that for a competitive and frictionless economy with finite horizon, Jarrow & Larsson (2012) show that NFLVR and ND are both necessary conditions for the existence of an economic equilibrium. Consequently, for a competitive and frictionless market with finite horizon, all of the above results for asset price bubbles apply. In a related paper, Jarrow & Larsson (2014) show that NFLVR and ND are not necessary conditions for an economy with short sale constraints, implying that new insights are possible in these other economies.

5.2. Discrete Time

This section briefly reviews the literature on the existence of asset price bubbles in discrete time equilibrium models. An excellent review of the classical literature was done by Camerer (1989). Consequently, our discussion is brief, its purpose being to connect the classical literature to the martingale theory of bubbles, discussed above.

5.2.1. Competitive and frictionless economy. The classical literature only considers competitive and frictionless economies where there is no trading of derivatives. The nonexistence of asset price bubbles in an ideal economy equilibrium with finite horizon and discrete time was shown by Tirole (1982). For models with infinite horizon and discrete time, Santos & Woodford (1997) study sufficient conditions for the nonexistence of price bubbles. They also provide numerous different equilibrium economies where type 2 bubbles exist.

5.2.2. Other economies. In economies that are not competitive and frictionless, bubbles can arise for many reasons.

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5.2.2.1. Competitive markets. Bubbles can exist in competitive markets when traders behave myopically (Tirole 1982), when there are irrational traders (Harrison & Kreps 1978, De Long et al. 1990), in growing economies with infinite horizon and with rational traders (Tirole 1985, O'Connell & Zeldes 1988, Weil 1990), in economies where rational traders have differential beliefs or when arbitrageurs cannot synchronize trades (Abreu & Brunnermeier 2003), or when there are short sale/borrowing constraints (Santos & Woodford 1997; Scheinkman & Xiong 2003; Hong, Scheinkman & Xiong 2006; Scheinkman 2013).

5.2.2.2. Strategic trading. As noted previously, bubbles can exist in frictionless markets where the price taking assumption is relaxed and there are large investors who trade strategically, knowing their trades have a quantity impact on the price. Such bubbles were shown to exist in equilibrium by Allen & Gale (1992) and Allen & Gorton (1992). A recent survey of the market manipulation literature, both theoretical and empirical, was done by Putnins (2012).

5.3. Continuous Time

This section studies the existence of bubbles in continuous time models. Recall that in such an economy, there can only be type 2 and type 3 bubbles. This literature also only considers economies where there is no trading of derivatives.

5.3.1. Competitive and frictionless economy. There are two cases to consider: a complete and an incomplete market.[7] In a complete market that satisfies both NFLVR and ND, we know by Theorem 14 that type 2 and type 3 bubbles cannot exist. Hence, adding the restriction that the economy is in equilibrium adds no new results to the previous analysis based on NFLVR and ND alone. In an incomplete market, conditions characterizing the existence of asset price bubbles in equilibrium are an open research area.

5.3.2. Competitive with trading constraints. In an otherwise complete and competitive market, but with trading constraints, Hugonnier (2012) and Hugonnier & Prieto (2014) study models where type 3 bubbles exist in equilibrium. In these models, bubbles can exist in both the risky asset and the money market account. These models impose restrictive assumptions on the economy both to facilitate analytic solutions and to illustrate the existence of such bubbles. Extending and generalizing these models is an exciting area for subsequent research.

6. EMPIRICAL TESTING

This section briefly discusses the empirical methodology used to test for the existence of asset price bubbles of types 2 and 3. For this section, we maintain the competitive and frictionless market assumptions.

6.1. Discrete Time

The classical literature empirically testing for the existence of asset price bubbles is formulated in discrete time. As noted in Remark 10, an infinite horizon model is needed for the existence of asset price bubbles in a discrete time economy. Also, in such an economy, only type 2 bubbles can exist.

[7] Chen & Kohn (2011) study a partial equilibrium model where bubbles exist, extending the work of Harrison & Kreps (1978).

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To focus on type 2 bubbles, we assume that the risky asset's liquidation value is identically zero ($X \equiv 0$). This condition excludes type 1 bubbles. The null hypothesis that there are no type 2 asset price bubbles in the economy is given by the expression

$$\beta_t = S_t - \sum_{u=t}^{\infty} E_Q \left(\frac{\Delta D_u}{B_u} \middle| \mathcal{F}_t \right) B_t = 0. \tag{13}$$

To test this hypothesis, one must assume a particular model for $\{r, D, Q\}$. As such, empirical tests of Expression 13 involve a joint hypothesis with the model for $\{r, D, Q\}$.

As one might expect from such a vast literature, the evidence is inconclusive on the existence of bubbles because of this joint hypothesis. A brief summary of the different asset classes and time periods studied for the existence of asset price bubbles includes the following:

1. the Dutch tulip mania of 1634–37 (Garber 1989, 1990),
2. the Mississippi bubble of 1719–20 (Garber 1990),
3. the South Sea bubble of 1720 (Garber 1990, Temin & Voth 2004),
4. foreign currency exchange rates (Evans 1986, Meese 1986),
5. German hyperinflation in the early 1920s (Flood & Garber 1980),
6. US stock prices over the twentieth century (West 1987, 1988; Diba & Grossman 1988; Dezhbakhsh & Demirguc-Kunt 1990; Froot & Obstfeld 1991; McQueen & Thorley 1994; Koustas & Serletis 2005),
7. the 1929 US stock price crash (White 1990, De Long & Shleifer 1991, Rappoport & White 1993, Donaldson & Kamstra 1996),
8. land and stock prices in Japan 1980–1992 (Stone & Ziemba 1993),
9. the NASDAQ 1998–2000 internet stock price peak (Ofek & Richardson 2003; Brunnermeier & Nagel 2004; Cuñado, Gil-Alana & Perez de Gracia 2005; Pastor & Veronesi 2006; Battalio & Schultz 2006), and
10. US housing prices prior to 2007 (Case & Shiller 2003, Goodman & Thibodeau 2008, Mikhed & Zemcik 2009, Clark & Coggin 2011).

6.2. Continuous Time

For empirical tests of the continuous time model, we restrict our consideration to the risky asset's liquidation time τ being bounded. There are three different approaches that can be used to test for the existence of bubbles. We discuss each in turn.

6.2.1. Joint hypothesis with $\{r_t, D_t, X_t, \tau, Q\}$. The first approach is based on Expression 4, which we repeat here for convenience:

$$\beta_t = S_t - E_Q \left(\frac{X}{B_\tau} + \int_t^\tau \frac{1}{B_u} dD_u \middle| \mathcal{F}_t \right) B_t. \tag{14}$$

As with tests of bubbles in the discrete time model, to identify the bubble β_t using this expression, one needs a stochastic model for $\{r_t, D_t, X, \tau, Q\}$, generating a joint hypothesis. Conceptually, testing the satisfaction of this expression is similar to that in a discrete time model. It suffers from the same difficulties inherent in testing for bubbles in the discrete time model due to the joint hypothesis. Fortunately, for the continuous time model, there are two additional approaches that can be employed that partially avoid these difficulties.

6.2.2. Joint hypothesis with $\{S_t\}$. The second approach is based on specifying a stochastic process for S where the parameters of the asset's price law can be partitioned into two mutually exclusive and exhaustive sets, one where the process exhibits a bubble and one where it does not. Then standard statistics procedures can be used to estimate the parameters using historical time series data to determine whether the asset price process exhibits a bubble.

Expressions 8 and 9 provide two possible stochastic processes that can be used for this approach. The advantage of a joint hypothesis with $\{S\}$ versus $\{r_t, D_t, X, \tau, Q\}$ is that the hypothesis for $\{S\}$ can be independently tested, as historical time series for the asset's price are available. In contrast, it is difficult (if not impossible) to independently test the hypothesis for $\{r_t, D_t, X, \tau, Q\}$ because $\{X, \tau, Q\}$ are not observable for almost all asset price processes.

A statistical methodology for testing stock price bubbles based on Expression 8 was recently developed by Jarrow, Kchia & Protter (2011a). In that paper, they applied their methodology to various dot-com stocks during the alleged dot-com bubble period of 1998–2000, showing that some stocks contain bubbles but others do not. They have also applied their methodology to show that LinkedIn's IPO stock price in 2011 exhibited a price bubble (Jarrow, Kchia & Protter 2011b).

This approach, based on a joint hypothesis with $\{S_t\}$ for the empirical validation of asset price bubbles, is just in its infancy. Extensions and implementations of this approach represent a fruitful area for additional research.

6.2.3. Using derivatives $\{C_t, P_t\}$. The third approach to test for asset price bubbles requires the trading of call and put options on the risky assets suspected of having price bubbles. These tests are based on Expressions 10 and 12 for the European put and call values, respectively.

To apply this approach, historical time series of market prices for $\{S_t, D_t, C_t, P_t\}$ are needed. First, one hypothesizes a put option price model, i.e., one specifies $\{S_t, D_t, Q\}$. Then one validates or rejects the model by testing to see if the model price for the put, Expression 10, equals the market price. If accepted, then using the market prices for the call options, one compares the market prices to Expression 12 to determine if $\beta_t - E_Q(\frac{\beta_T}{B_T}|\mathcal{F}_t)B_t > 0$. If so, then the underlying stock exhibits a bubble.

In contrast to the first approach based on a model for $\{r_t, D_t, X, \tau, Q\}$, the advantage of this approach is that any model hypothesized for $\{S_t, D_t, Q\}$ can be first independently tested on put prices $\{P_t\}$ for its validity, before using the call prices $\{C_t\}$ to test for the existence of price bubbles. This approach for testing for asset price bubbles is currently unexplored in the literature.

7. CONCLUSION

This paper reviews the theoretical literature on asset price bubbles. Both the classical literature based on discrete time equilibrium models and the newer literature based on continuous time martingale theory are discussed. The martingale theory of bubbles can be used to both enrich our understanding of the classical literature and generate some new methodologies for empirically testing bubbles, most of which have not yet been explored.

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