BUBBLES DETECTION FOR INTER-WAR EUROPEAN HYPERINFLATION: A THRESHOLD COINTEGRATION APPROACH

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Abstract

This paper undertakes an empirical investigation into the existence of inflationary bubbles during the inter-war European hyperinflation for Germany, Hungary, Poland and Russia. Our Monte Carlo simulations show that the residual-based threshold cointegration methodology of Caner and Hansen (2001) is better able to detect periodically collapsing bubbles. Moreover, this methodology possesses greater power against nonlinear stationary alternatives in a finite sample than several commonly used cointegration tests that do not allow for multiple regime shifts. The empirical results of the threshold cointegration tests provide evidence of stationary, regime-switching processes in money demand dynamics, but suggest that there are no inflationary bubbles in any of the countries. (JEL C13, C32, E30)

Introduction

Price bubbles are explosive asset pricing processes generated by self-fulfilling expectations independent of market fundamentals. The empirical investigation of price bubbles in high-inflation economies is important because their existence has far-reaching implications for anti-inflation policies. In particular, the crucial choice of an appropriate policy to reduce the inflation rate may depend on the underlying causes of the inflation. If, for example, inflationary bubbles are not present in the observed price series, all that is needed are restrictive monetary and fiscal policies to control for market fundamentals. If, however, inflation is being driven by a bubble phenomenon, it is necessary to take action against the expectation mechanism in order to circumvent the speculative bubble path (Funke et al. 1994). In some instances, this may require the government of a high-inflation country to undertake a radical change in policy.

Diba and Grossman (1988a) were the first to suggest the use of cointegration econometric methods for bubble testing. They argued that if an explosive bubble exists, it should not be possible to make the residuals of a regression of the asset price on underlying fundamentals stationary after any order of differencing. In other words, asset prices and fundamentals are not cointegrated in the presence of a price bubble.

There are, however, two major empirical problems in applying the standard linear cointegration methodology to the study of bubbles. The first is related to the difficulty of detecting periodically collapsing bubbles. Unlike the deterministic bubble that explodes infinitely without end, the periodically collapsing bubble, as proposed by Evans (1991), can repeatedly burst and re-explode, which results in a nonlinear data generation process. In spite of having an explosive root in its conditional expectation, the nonlinearity of the periodically collapsing bubble makes the asset price appear to follow a linear autoregressive process than an explosive one. As a result, periodically collapsing bubbles are not detectable using linear tests. In response to this problem, Bohl (2003) argues that the nonlinear behavior resulting from periodic price run-ups followed by

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crashes can be captured by a threshold process. Therefore, he proposes that the observation of threshold nonlinearity in asset prices be considered as evidence of periodically collapsing bubbles.

Besides speculative bubbles, however, nonlinear behavior in asset prices may also be caused by changes in fundamentals (Norden and Schaller, 1996). This gives rise to a second empirical problem, which is an important issue because governments normally introduce monetary reforms in the midst of periods of hyperinflation and if such reforms are anticipated by economic agents, expected changes in market fundamentals may occur (Flood, et al. 1986; Hamilton and Whiteman, 1985; Hamilton, 1986; Flood and Hodrick, 1990). Under such a circumstance, the residuals of the regression model under study would display nonlinear behavior when there is a potential regime shift in the fundamentals. Since the linear cointegration tests would not have sufficient power to separate nonstationarity from nonlinearity, the stationary switching processes of fundamentals may be mistaken for explosive bubbles. In the bubble-testing literature, the second problem is usually circumvented by truncating the final months of a hyperinflation during which the expectations of monetary reform began to take effect (for instance, Flood and Garber, 1980b; Casella, 1989; Engsted, 1993, 1994, 1996; Durlauf and Hooker, 1994; Hooker, 2000; and Chan, et al. 2003). Nevertheless, this approach is flawed because the existence of inflationary bubbles during the truncated periods may be precluded, *a priori*, without theoretical justification.

Against the above background, the threshold autoregressive (TAR) unit root method developed by Caner and Hansen (2001) is adopted for investigating the existence of inflationary bubbles since this approach allows for the joint considerations of nonstationarity and nonlinearity. In order to verify this point, Monte Carlo simulations are undertaken to examine whether the TAR approach is better at detecting periodically collapsing bubbles than the standard cointegration tests. In addition, the TAR method is applied to data from the 1920s hyperinflations in Germany, Hungary, Poland and Russia, and the results are compared to other cointegration tests that do not allow for multiple regime shifts in the variables. In contrast to earlier studies, the observations from which our data are sampled extend through periods of monetary reform and hence the hyperinflationary episodes are analyzed in their entirety. These TAR tests do not provide evidence that would be consistent with the existence of price bubbles during the inter-war European hyperinflation episodes.

Cagan's Model and the Bubble

According to Cagan (1956), the expected rate of change in the price of goods is the only important cost of holding real money balances during a hyperinflation. Mathematically, the linear form of the Cagan money demand model under rational expectations is:

$$M_t - p_t = \alpha + \beta E_t(\Delta p_{t+1}) + u_t \tag{1}$$

where M_t is the natural logarithm of the money stock at time t; p_t is the natural logarithm of the price level at time t; $M_t - p_t$ denotes real money balances; Δp_{t+1} is the inflation rate over the period from time t to time t+1; E_t is the conditional expectations operator; α is a constant; β is the semi-elasticity of real money balances with respect to the expected inflation rate; and u_t denotes money demand disturbances. Theoretically, the value of β should be negative because economic agents will substitute real goods for money when the value of money is expected to fall dramatically.

By defining the rational expectation forecasting errors as $\eta_{t+1} = \Delta p_{t+1} - E_t(\Delta p_{t+1})$, which are assumed to be serially uncorrelated, the Cagan model (1) can be expressed alternatively as:

$$M_t - p_t = \alpha + \beta \Delta p_{t+1} + \xi_t \tag{2}$$

where $\xi_t = u_t - \beta\eta_{t+1}$. Using Eq. (1) the price level, p_t, is expressed as:

$$p_t = \frac{\alpha}{\beta-1} - \frac{M_t}{\beta-1} + \frac{\beta}{\beta-1}E_t(p_{t+1}) + \frac{u_t}{\beta-1} \tag{3}$$

Recursively substituting forward for the expected next-period price, using the law of iterated expectations, and imposing the transversality condition, $\lim_{i\to\infty}\left(\frac{\beta}{\beta-1}\right)^{i+1} E_t(p_{t+1+i}) = 0$, yields the following fundamental price solution, p_t^f:

$$p_t^f = -\alpha + \frac{1}{1-\beta}\sum_{i=0}^{\infty}\left(\frac{\beta}{\beta-1}\right)^i E_t(M_{t+i} - u_{t+i}) \tag{4}$$

Eq. (4) shows that the fundamental price is determined by the discounted values of the current and expected money supply as well as money demand disturbances. Since the transversality condition has been imposed upon (4), the fundamental price represents only a particular solution to (3). Following Engsted (1993), equation (4) can be re-arranged as follows:

$$(M_t - p_t^f) = \alpha + \beta M_t + (\beta-1)\sum_{i=1}^{\infty}\left(\frac{\beta}{\beta-1}\right)^i E_t(\Delta^2 M_{t+i}) + (\frac{1}{1-\beta})\sum_{i=0}^{\infty}\left(\frac{\beta}{\beta-1}\right)^i E_t(u_{t+i}). \tag{5}$$

If the transversality condition fails to hold, the general solution to (3) is equal to $p_t^f + B_t$, where B_t is a homogenous or bubble solution represented by any arbitrary process that satisfies:

$$B_t = \frac{\beta}{\beta-1}E_t(B_{t+1}), \text{ where } \beta < 0 \text{ and } \left(\frac{\beta-1}{\beta}\right) > 1 \tag{6}$$

Evans (1991) formulates a class of periodically collapsing bubbles that present formidable problems for the cointegration analysis. Their behavior is captured by the following equations:

$$B_{t+1} = \left(\frac{\beta-1}{\beta}\right) B_t\ \omega_{t+1}, \text{ for } B_t \le \kappa; \tag{7a}$$

$$B_{t+1} = [\delta_o + \theta_{t+1}\Pi^{-1}\left(\frac{\beta-1}{\beta}\right)(B_t - \delta_o\left(\frac{\beta}{\beta-1}\right))]\omega_{t+1}, \text{ for } B_t > \kappa, \tag{7b}$$

where κ and $\delta_o > 0$, $0 \leqq \Pi \leqq 1$, ω_{t+1} is an exogenous identically independently distributed (i.i.d.) positive random variable with $E_t(\omega_{t+1}) = 1$,[1] θ_t is an exogenous i.i.d. random Bernoulli process that takes the value of 1 with probability Π and the value of 0 with probability $(1-\Pi)$, where the value of Π represents the probability of bubble continuation and $(1-\Pi)$ the probability of bubble collapse.

As long as $B_t \leq \kappa$, the bubble will grow at the mean rate $(\beta-1)/\beta$. However, when $B_t > \kappa$ the bubble will fall into another regime in which it grows at the mean rate $[(\beta-1)/\beta]\Pi^{-1}$ with a probability of bursting equal to $1-\Pi$. When the bubble actually collapses, it falls to a mean value of δ_0 and starts to explode again. Hence, B_t is nonstationary with a complex nonlinear process.

Methodology

The testing procedure for inflationary bubble detection used in this paper is taken from Engsted (1993, 2003), which extends the arguments of Campbell and Shiller (1987). The procedure is implemented by conducting two cointegration analyses in sequence: (i) real money balances, $M_t - p_t$, and inflation, Δp_{t+1}, in Eq.(2), and (ii) real money balances and money growth, ΔM_t, in Eq.(5). Eq.(2) shows that as long as the sequence of money demand disturbances, u_t, are stationary,[2] the linear combination between $M_t - p_t$ and Δp_{t+1} will be stationary regardless of whether there are price bubbles or not. Specifically, in the case where price bubbles do not exist, $M_t - p_t$ will cointegrate with Δp_{t+1} in the manner defined by Engle and Granger (1987).[3] On the other hand, as argued by Engsted (2003), if price bubbles exist, both variables $M_t - p_t$ and Δp_{t+1} will share the same explosive component with the bubbles. Therefore, regressing $M_t - p_t$ onto Δp_{t+1} will still produce stationary residuals with finite variance as long as the Cagan model remains valid. Based on these observations, it is possible to infer that u_t is nonstationary when the first test fails to establish a cointegration relationship between $M_t - p_t$ and Δp_{t+1} in Eq. (2).

If the first cointegration test supports the evidence of a stationary u_t in the residuals, then it is possible to proceed to detect the existence of inflationary bubbles by examining the cointegrated relationship in Eq.(5). In case where u_t is stationary, the variables $M_t - p_t$ and ΔM_t in Eq.(5) will be cointegrated when no inflationary bubbles are present.[4] If a bubble exists, the right-hand side of Eq. (5) must be augmented by $-B_t$, and consequently $M_t - p_t$ is not cointegrated with ΔM_t. In sum, three possible outcomes can be inferred from the cointegration analyses of Eqs.(2) and (5). First, $M_t - p_t$ is cointegrated with Δp_{t+1} in Eq.(2) and ΔM_t in Eq.(5) respectively, it implies that u_t is stationary and the possibility of bubbles is precluded. Second, if $M_t - p_t$ is

[1] The strictly positive values of δ_0 and ω_{t+1} ensure that the periodically collapsing bubble is strictly positive and never vanishes. Following the argument of Diba and Grossman (1988b), when a stochastic bubble collapses to zero, it cannot start again.

[2] It should be noted that the rational expectation error, γ_{t+1}, in Eq.(2) is white noise by definition.

[3] In the absence of bubbles, the nominal variables of price and money supply would normally behave as I(2) series during hyperinflation (Taylor,1991, Engsted, 1993, 1994, 1996, 1998 and Haldrup, 1998). Consequently, it implies that $M_t - p_t$, Δp_{t+1} and ΔM_t are I(1), and $\Delta^2 M_t$ is I(0).

[4] It implies that the expected future values of u_{t+i} and $\Delta^2 M_{t+i}$, $\forall i > 0$, are assumed to be I(0).

cointegrated with Δp_{t+1} but not with ΔM_t, it indicates that u_t is stationary, and bubbles are likely to be present because the non-bubble transversality condition is violated. Finally, if $M_t - p_t$ is not cointegrated with Δp_{t+1} in Eq.(2) and ΔM_t in Eq.(5) respectively, it means that u_t is not stationary, which implies that the Cagan model is misspecified and the presence of bubbles cannot be examined. To tackle this problem, the Cagan model would have to be re-specified so as to include any nonstationary fundamental variables that were omitted at the outset.

The cointegration test is conducted by applying the two-regime TAR unit root method developed by Caner and Hansen (2001) to the OLS estimated residuals of regressions (2) and (5) sequentially.[5] The TAR regression can be represented by the following equation:

$$\Delta y_t = \Theta_1' x_{t-1} I(z_{t-1} < \lambda) + \Theta_2' x_{t-1} I(z_{t-1} \geq \lambda) + e_t, \quad t = 1 \ldots T, \tag{8a}$$

where y_t, in our case, is the OLS residual or disequilibrium error of either regression (2) or (5); $I(.)$ is an indicator function; e_t is an identically and independently distributed (i.i.d.) innovation term; T is the number of usable observations; λ is a value of threshold; $x_{t-1} = (y_{t-1}, 1, t, \Delta y_{t-1}, \ldots, \Delta y_{t-k})'$ is a vector of regressors; $\Theta_i = (\rho_i, c_i, b_i, \Psi_{i,t-1}, \ldots, \Psi_{i,t-k})'$ represents a vector of parameters of the TAR model associated with the corresponding regressors of x_{t-1}; and z_{t-1} is the observable threshold variable that must be predetermined, strictly stationary and ergodic. At an empirical level, there are two possible ways to specify z_{t-1}, either as $y_{t-1} - y_{t-1-m}$ or $y_{t-m} - y_{t-m-1}$, which are known as long difference and lagged difference types of threshold, respectively, where m is a delay number. The long difference or lagged difference type of threshold is used to ensure the stationarity of z_{t-1}. The TAR model of (8a) can account for a general nonlinear phenomenon because all TAR parameters in Θ_i are allowed to shift.

The estimation of the TAR model is first implemented by estimating the parameters of (8a) using the OLS method at each possible value of λ:

$$\Delta y_t = \hat{\Theta}_1'(\lambda) x_{t-1} I(z_{t-1} < \lambda) + \hat{\Theta}_2'(\lambda) x_{t-1} I(z_{t-1} \geq \lambda) + \hat{e}_t(\lambda) \tag{8b}$$

Using $\hat{\sigma}^2(\lambda)$ to denote the estimated residual variance of Eq. (8b), Chan (1993) argues that the estimate of the threshold value, $\hat{\lambda}$, obtained by minimizing $\hat{\sigma}^2(\lambda)$ of (8b), is superconsistent. Denoting the values of other parameters in (8b) that are estimated at $\hat{\lambda}$ as $\hat{\Theta}_1 = \hat{\Theta}_1(\hat{\lambda})$, $\hat{\Theta}_2 = \hat{\Theta}_2(\hat{\lambda})$, and $\hat{\sigma}^2 = \hat{\sigma}^2(\hat{\lambda})$ yields the fitted TAR model as follows:

$$\Delta y_t = \hat{\Theta}_1'(\hat{\lambda}) x_{t-1} I(z_{t-1} < \hat{\lambda}) + \hat{\Theta}_2'(\hat{\lambda}) x_{t-1} I(z_{t-1} \geq \hat{\lambda}) + \hat{e}_t(\hat{\lambda}) \tag{8c}$$

Let $\hat{\sigma}^2(\hat{\lambda})$ be the residual variance estimated at the value $\lambda = \hat{\lambda}$. The estimated values of ρ_1 in $\hat{\Theta}_1(\hat{\lambda})$, ρ_2 in $\hat{\Theta}_2(\hat{\lambda})$ and $\hat{\sigma}^2(\hat{\lambda})$ will be used to construct the inferential statistics for the threshold nonlinearity and cointegration analysis. Also, the value of the delay number, m, in 8(c)

[5] As pointed out by Balke and Fomby (1997), the OLS estimates of a cointegration vector remain superconsistent under regular conditions even though the disequilibrium error exhibits threshold nonlinearity.

is estimated by minimizing the residual variance at $\hat{\lambda}$.

The existence of threshold effects on the residuals, y_t, of regression (2) or (5) can be tested under the joint null hypothesis of no threshold effect, that is, $\Theta_1(\hat{\lambda}) = \Theta_2(\hat{\lambda})$, for all parameters in (8c). The Wald statistic, W_T, for examining these joint restrictions is given by:

$$W_T = T\left(\frac{\sigma_0^2}{\hat{\sigma}^2(\hat{\lambda})} - 1\right) \tag{9}$$

where σ_0^2 is the residual variance from the OLS estimation of the null linear ADF regression. Rejection of the W_T statistic indicates that the residual displays threshold nonlinearity. The asymptotic distribution of the W_T statistic depends on whether the residual of the equation under study is nonstationary. Consequently, Caner and Hansen (2001) suggest the use of unconstrained and constrained bootstrap methods to approximate the distribution of the W_T statistic. The unconstrained method does not impose any unit root restrictions on the estimated parameters in (8c), whereas the constrained counterpart requires the imposition of a unit root restriction. Moreover, individual Wald test statistics, denoted by W(1), W(*t*) and $W(y_{t-1})$, are used to examine whether the coefficients of the intercept, linear time trend and y_{t-1}, respectively, that is, c_i, b_i and ρ_i, are equal between the two regimes.

As pointed out by Caner and Hansen (2001), the parameters ρ_1 in $\hat{\Theta}_1(\hat{\lambda})$ and ρ_2 in $\hat{\Theta}_2(\hat{\lambda})$ of (8c) control the stationarity of the process for y_t. When both parameters are equal to zero, this implies the existence of nonstationary roots in y_t over both regimes; in other words, the regression equation under study is not cointegrated. To run this test empirically, two Wald statistics developed by Caner and Hansen (2001) are used. First, the two-sided Wald statistic, R_{2T}, which tests the null hypothesis, $\rho_1 = \rho_2 = 0$, against the two-sided alternative that $\rho_1 \neq 0$ or $\rho_2 \neq 0$ or both, is implemented. Mathematically, the two-sided Wald statistic can be expressed as:

$$R_{2T} = t_1^2 + t_2^2, \tag{10a}$$

where t_1 and t_2 are the standard Dickey-Fuller (DF) t-ratio statistics for the estimates of ρ_1 in $\Theta_1(\hat{\lambda})$ and ρ_2 in $\Theta_2(\hat{\lambda})$, respectively, from the fitted TAR model of (8c). Second, since the stationarity test usually focuses upon the negative values of ρ_1 and ρ_2, a one-sided Wald statistic, R_{1T}, is introduced for testing the null hypothesis, $\rho_1 = \rho_2 = 0$, against the one-sided alternative that $\rho_1 < 0$ or $\rho_2 < 0$ or both, where R_{1T} can be written as:

$$R_{1T} = t_1^2 I(\rho_1 < 0) + t_2^2 I(\rho_2 < 0). \tag{10b}$$

The presence of stationary roots in the residuals over both of the regimes can be checked by testing the null hypothesis, $\rho_1 = \rho_2 = 0$, against the alternative hypothesis of $\rho_1 < 0$ and $\rho_2 < 0$. The alternative hypothesis is supported if the R_T (R_{2T} and R_{1T}) Wald statistics reject their respective null hypotheses and also if both the point estimates of ρ_1 and ρ_2 satisfy the sufficient condition of stationarity (Chan and Tong, 1985), namely, $-2 < (\rho_1, \rho_2) < 0$.[6] The asymptotic distributions of the R_T Wald statistics are non-standard and dependent upon the existence of threshold effects and as suggested by Caner and Hansen (2001), the unidentified bootstrap, which assumes no threshold effects on parameters of the estimated TAR model, is used in the bootstrap approximation. However, the identified bootstrap method that assumes threshold effects on the TAR parameters, suffers greater size distortion than the unidentified one in the simulation study of Caner and Hansen (2001), and for this reason it is not used for conducting the bootstrap approximation.

By means of Monte Carlo experimentations, we can compare the performance of the TAR method with that of the standard cointegrating augmented Dickey-Fuller (ADF) and Z statistics, as well as the Gregory and Hansen (1996) ADF and Z statistics. While the standard ADF test corrects for serial correlation by adding lagged terms in the ADF regression (Said and Dickey, 1984), the standard Z_a and Z_t statistics of Phillips and Ouliaris (1990) employ a semi-parametric method to asymptotically eliminate the bias caused by the weakly dependent and heterogeneously distributed innovations of the DF regression. The finite-sample critical values of the standard ADF and Z statistics are obtained from Phillips and Ouliaris (1990). Standard ADF and Z statistics do not allow for any structural changes in the residuals of OLS regressions; however, another class of ADF and Z statistics suggested by Gregory and Hansen (1996), denoted ADF*, Z_t^* and Z_a^*, is designed to allow for a single break of unknown timing in the cointegration vector. The ADF*, Z_t^* and Z_a^* are chosen to be the smallest values of the standard ADF, Z_a and Z_t statistics, respectively, across all values of $\tau \in T$, which means $ADF^* = \inf_{\tau \in T} ADF(\tau)$, $Z_t^* = \inf_{\tau \in T} Z_t(\tau)$ and $Z_a^* = \inf_{\tau \in T} Z_a(\tau)$. The single structural change is modeled in this study as a change in the slope coefficient in the regime-shift cointegrating regression model.[7] The critical values of the ADF*, Z_t^* and Z_a^* test statistics are obtained by Monte Carlo simulations.

Monte Carlo Experiments

The Monte Carlo study attempts to examine the relative finite-sample performance of the cointegration tests chosen to identify the existence of periodically collapsing bubbles. To do this, a series, P_{1t}, is constructed that contains a periodically collapsing bubble using the following equations:

$$P_{1t} = 0.5 F_t + B_t , \tag{11a}$$

[6] The individual DF t-ratio statistics, t_1 and t_2, may be used to test for the presence of non-stationarity in each regime. The simulation study conducted by Caner and Hansen (2001), however, shows that the t_1 statistic is found to have substantially lower power than both the t_2 statistic and the R_T Wald statistics when the simulated series have stationary roots over two regimes and $|\rho_1|$ is much closer to zero than $|\rho_2|$. There are similar problems in Enders and Siklos' (2001) threshold model. To circumvent this problem, Enders and Siklos (2001) suggest using an F statistic for conducting cointegration analysis when the point estimates of ρ_1 and ρ_2 satisfy the sufficient conditions for stationarity proposed by Chan and Tong (1985).

[7] The structural change can also be modeled as a single break in intercept in a level-shift model as well as in a level-shift in a trend model. However, our subsequent Monte Carlo findings and empirical results of the three cointegrating regression models are quite similar. Thus, to save space, we only report the results of the regime-shift model in this paper. The empirical results of the other two models are available upon request.

$$F_t = F_{t-1} + v_t, \text{ where } F_0 = 0 \text{ and } v_t \sim N(0,1), \tag{11b}$$

where F_t represents an I(1) fundamental process, the periodically collapsing bubble B_t is generated according to Eqs.(7a) and (7b) using the same parameter values as those of Evans (1991), viz. $\kappa = 1$, $\delta_o = 0.5$, $B_0 = \delta_o$, $(\beta - 1)/\beta = 1.05$. To match the average sample size of the actual data series used in the subsequent empirical work, the sample length is set at 45.

The existence of B_t, which is a nonstationary process by construction, implies the non-cointegration of P_{1t} and F_t (Diba and Grossman, 1988a). In addition, the residuals from the regression of P_{1t} on F_t should reflect the pattern of periodic collapses in the bubble, which the standard linear cointegration tests may wrongly identify as mean-reversing processes, thereby biasing the tests towards incorrect rejection of non-cointegration (Taylor and Peel, 1998). Bohl (2003) argues that the econometric techniques with threshold effects can be used to capture the asymmetric characteristics of periodically collapsing bubbles as illustrated by applying the TAR model of (8a) to the residuals of regression of (11a). When the threshold variable, z_{t-1}, lies above the threshold, λ, it is associated with a period of bubble explosion in one regime followed by a sudden collapse to the threshold in the next regime. In other words, the second regime is interpreted as the collapse of bubble. On the other hand, the path of z_{t-1} does not show bubble eruptions followed by a collapse when its value is below λ. These asymmetric movements of periodically collapsing bubbles can be captured by the threshold mechanism in the TAR cointegration approach. Because of this, the TAR cointegration tests may be less likely to interpret the nonlinear explosive bubbles as a stationary, mean-reverting process, thereby leading to fewer incorrect rejections of the bubble hypothesis than standard tests.[8]

Simulation experiments can be implemented to test for the non-cointegration between P_{1t} and F_t in the presence of the simulated Evans' (1991) bubble, B_t. The number of rejection frequencies of the non-cointegration null hypothesis is found by applying the cointegration test statistics to the residuals obtained from the OLS regression of P_{1t} on F_t, and then calculating the number of Monte Carlo replications that have rejected the non-cointegration null at the 5% significance level. Table 1 reports the simulation results of all the chosen cointegration tests, in which we find that the rejection frequencies of the non-cointegration null hypothesis using the TAR R_T tests are much smaller than those using other cointegration tests in most cases. For example, over the entire range of bubble survival probability, Π, the incorrect rejection rate of the TAR R_T tests does not exceed 14.5%; the rejection rate of the ADF* test is 48% and the standard ADF test is 78%. Hence, by incorporating the threshold effects in the cointegration framework, the TAR R_T tests are generally much less biased towards incorrect rejection of non-cointegration than cointegration tests that do not allow for threshold processes.[9]

[8] Many researchers employ sophisticated tests of nonstationarity to detect periodically collapsing bubbles, although such tests have been subject to criticisms (Charemza and Deadman, 1995). For example, Funke, et al. (1994) and Hall, et al. (1999) employed Markov-switching techniques to capture the exploding dynamics of periodically collapsing bubbles. Taylor and Peel (1998) proposed a robust cointegration test, which is subject to a smaller size distortion when periodically collapsing bubbles actually exist.

[9] As shown in Table 1, when the value of Π is 0.8 or greater, which implies that the artificial bubble processes are nearly deterministic, the empirical sizes of the TAR R_T statistics are somewhat larger than those of the ADF or Z statistics. In spite of this, Flood and Garber (1980b) argued that as deterministic bubbles explode without end, they might not exist.

Table 1: Rejection Frequencies (%) of Non-Cointegration Null from the Regression of P_{1t} on F_t.

	TAR Cointegration Test		Standard Linear Cointegration Tests			Gregory-Hansen Cointegration Tests		
Π	R_{1T}	R_{2T}	ADF	Z_a	Z_t	ADF*	Z_a^*	Z_t^*
0.20	12.17	11.50	49.90	48.80	52.60	38.70	39.90	38.70
0.40	11.87	11.63	71.80	72.50	73.70	48.75	57.19	57.19
0.50	13.60	13.30	78.10	75.80	79.50	37.43	51.29	49.29
0.60	14.43	14.14	72.40	55.60	76.20	26.60	37.45	34.04
0.625	14.14	13.71	65.10	45.10	69.90	24.62	34.15	31.69
0.65	12.29	11.86	56.00	36.80	62.00	23.67	32.66	28.48
0.70	12.70	12.60	37.00	21.70	42.60	19.90	26.90	24.10
0.75	10.50	10.40	21.60	10.40	25.70	17.50	22.10	19.20
0.80	7.80	7.90	10.10	4.90	13.60	14.90	20.20	17.60
0.825	5.86	5.57	6.60	3.40	9.50	13.60	17.70	14.80
0.85	4.80	4.70	3.80	2.20	5.40	12.50	15.80	13.30
0.90	4.00	4.00	2.10	1.50	2.50	10.40	13.90	11.50
0.95	4.30	4.20	1.20	1.20	1.50	7.30	12.30	8.40

Notes: (1) The Monte Carlo study is based on 1000 replications for each case, with the finite sample of 45. (2) The simulated critical values for ADF* and Z_t^* are -5.46, and for Z_a^* are -34.71. The simulation is based on 20,000 replications for a sample size of 45.

Another detection issue arises when econometricians apply cointegration tests that do not allow for multiple regime shifts to hyperinflation data. For example, when monetary reforms were anticipated during a hyperinflation, the agents' expectations about the underlying fundamentals would change, as explained by Flood and Garber (1980a) and LaHaye (1985). Due to their lack of power to discriminate stationary nonlinearity from nonstationarity, the standard cointegration tests may mistake nonlinearity for nonstationarity, which in turn weakens their ability to test for bubbles. To examine the power of the cointegration tests in the presence of a regime shift in the data, Monte Carlo simulations are implemented, in which the data generating process for the artificial series is formulated as follows:

$$P_{2t} = 0.5\,F_t + y_t^s, \tag{12a}$$

$$y_t^s = y_{t-1}^s + \Theta_1{}' x_{t-1} I(z_{t-1} < \lambda) + \Theta_2{}' x_{t-1} I(z_{t-1} \geq \lambda) + e_t \text{, where}$$

$$y_0^s = 0 \text{ and } e_t \sim N(0,1);\ \Theta_i{}' = (\rho_i, c_i)',\ i = 1, 2,\ x_{t-1} = (y_{t-1}^s, 1)',\ z_{t-1} = y_{t-1}^s - y_{t-2}^s. \tag{12b}$$

where F_t is an I(1) process defined in the same manner as Eq.(11b), and y_t^s represents the simulated residual series of the regression (12a). As suggested by Saikkonen and Ripatti (2000), the threshold nonlinearity can be used to model the regime shifts in the conditional expectations of the fundamental process. In addition, the regime-switching process of fundamentals would normally appear in the residuals of the regression model under study (Blackburn and Sola, 1996). Therefore, y_t^s is generated to model the expected regime shifts in the fundamental process in the Monte Carlo study. The use of y_t^s is to simulate the situation where the Cagan model (5) contains no bubbles but the sequences of expected $\Delta^2 M_{t+i}$ and u_{t+i} display nonlinear stationary dynamics. The regime-switching effects of y_t^s in our Monte Carlo study are constructed by setting the threshold parameter $c_1 = 0$ and varying c_2 among {0, 1, 1.5}, choosing ρ_2 among {-0.1, -0.3, -0.5, -0.7} when $\rho_1 = -0.1$, and selecting ρ_2 among {-0.05, -0.3, -0.5, -0.7} when $\rho_1 = -0.3$. Both the values of c_i and ρ_i are allowed to shift in order to consider more general regime-switching dynamics of fundamentals. Note that the threshold effect is absent when $c_1 = c_2 = 0$ and also $\rho_1 = \rho_2$. Because the absolute values of all of these ρ_i's are smaller than unity, the simulated residual, y_t^s, is stationary. The Monte Carlo study of Caner and Hansen (2001) is employed in setting the threshold value, λ, equal to zero and the threshold variable, z_{t-1}, equal to Δy_{t-1}^s.

By construction, the residual, y_t^s, is stationary and nonlinear, which implies cointegration of P_{2t} and F_t. Since the standard cointegration tests may not be able to distinguish nonstationarity from nonlinearity, one may unduly accept the false null of non-cointegration between P_{2t} and F_t. The present Monte Carlo experiments allow one to discern whether the finite-sample power of the TAR cointegration tests can out-perform other tests that do not incorporate multiple regime shifts.[10] Table 2 reports the correct rejection rates of the cointegration tests for the non-cointegration null hypothesis. The results show that the TAR R_T statistics exhibit the greatest power to reject the false non-cointegration null hypothesis across all the parameterizations of ρ_1, ρ_2 and c_2. For instance, when $c_2 = 1$, $\rho_1 = -0.3$, and ρ_2 changes from –0.05 to –0.7, the simulation shows that about 63.90% to 92.90% of Monte Carlo replications correctly reject the false non-cointegration null when the TAR R_{2T} statistic is used, but the replications that produce correct rejections of the null hypothesis range from just 0.33% to 39.75% using the standard ADF test, and from a mere 0.08% to 6.22% using the ADF* test. It is also important to note that the superiority of the TAR R_T statistics over other tests in the simulation study remains unchanged even when there are no identified threshold effects, i.e. $c_1 = c_2 = 0$ and $\rho_1 = \rho_2 = -0.1$ or $= -0.3$.The standard ADF and Z tests as well as the ADF* and Z* tests are subject to noticeable power loss, especially when there are regime shifts in mean values, i.e. $c_1 \neq c_2$, and also when the residuals in two regimes are near unit root processes, i.e. ρ_1 and ρ_2 are close to zero. The findings are similar to those of Pippenger and Goering (1993), which show that the standard DF statistics suffer from substantial power loss under stationary threshold processes. Even the ADF* and Z* tests that allow for a single break in the data cannot perform well when multiple regime-shifting processes are present. From this comparison, one can conclude that the TAR R_T tests are more powerful than other tests in correctly rejecting the false non-cointegration null hypothesis when the residuals of a cointegrating regression exhibit threshold stationarity.

[10] Similarly, Taylor and Peel (1998) requires the cointegration tests for bubble detection to have power to discriminate between explosive bubbles and mean-reverting dynamics due to fads in stock market sentiment that are observationally similar to the explosive bubbles.

Table 2. Power of the Cointegration Tests against the Alternative of Threshold Stationarity.

$c_1 = 0$	$c_2 = 0,\ \rho_1 = -0.1$				$c_2 = 1,\ \rho_1 = -0.1$				$c_2 = 1.5,\ \rho_1 = -0.1$			
$\rho_2 =$	-0.10	-0.30	-0.50	-0.70	-0.10	-0.30	-0.50	-0.70	-0.10	-0.30	-0.50	-0.70
R_{1T}	31.10	48.20	72.80	89.10	30.60	53.60	75.90	90.80	29.50	55.30	79.30	92.30
R_{2T}	30.20	46.20	70.00	88.30	29.50	51.60	73.50	89.80	27.90	53.60	77.00	91.30
ADF	8.85	15.48	26.47	42.30	0.56	4.81	14.02	29.58	0.22	2.33	9.77	24.30
Z_a	4.85	9.60	18.75	33.39	0.18	2.74	9.73	22.93	0.05	1.06	6.55	18.48
Z_t	9.92	17.94	30.35	47.44	0.83	6.40	17.72	35.09	0.34	3.56	12.84	29.67
ADF*	2.50	4.31	7.47	12.71	0.13	0.85	2.85	6.60	0.07	0.37	1.64	4.74
Z_a*	3.91	7.04	12.42	20.88	0.21	1.74	5.61	12.64	0.09	0.76	3.63	9.58
Z_t*	3.89	6.87	11.42	18.52	0.28	1.64	4.77	10.66	0.12	0.75	3.04	7.80
$c_1 = 0$	$c_2 = 0,\ \rho_1 = -0.3$				$c_2 = 1,\ \rho_1 = -0.3$				$c_2 = 1.5,\ \rho_1 = -0.3$			
$\rho_2 =$	-0.10	-0.30	-0.50	-0.70	-0.10	-0.30	-0.50	-0.70	-0.10	-0.30	-0.50	-0.70
R_{1T}	49.30	58.40	71.60	86.50	65.00	72.50	85.50	93.50	63.70	82.50	90.50	95.80
R_{2T}	47.20	55.40	69.70	84.70	63.90	70.30	84.20	92.90	62.70	81.50	89.60	95.30
ADF	12.10	30.75	51.37	72.01	0.33	3.14	15.45	39.75	0.25	0.77	6.89	25.87
Z_a	7.48	21.84	40.39	62.20	0.04	1.58	10.70	31.98	0.03	0.21	3.72	18.51
Z_t	14.11	34.53	55.79	76.18	0.44	4.75	20.68	47.78	0.27	1.23	9.82	33.01
ADF*	3.72	9.08	16.97	29.30	0.08	0.22	1.59	6.22	0.08	0.04	0.33	2.31
Z_a*	6.02	14.79	26.33	42.72	0.07	0.89	4.68	15.50	0.02	0.11	1.64	7.78
Z_t*	5.70	13.55	24.08	38.66	0.12	0.54	3.14	11.21	0.07	0.11	0.89	4.86

Notes: (1) The Monte Carlo study is based on 1,000 replications for each case with the finite sample of 45. (2) The true process of y_t^s displays threshold stationarity. The use of y_t^s in the data generating process is to simulate the situation where P_{2t} contains no bubbles while regime shifts in the fundamentals are allowed to appear in the residual of the regression model.

Data and Empirical Results

With respect to the European inter-war hyperinflation, data on the German money supply (notes in circulation) and the cost of living index are collected from Tinbergen (1934). All the data for Hungary, Poland and Russia are taken from Young (1925). For Hungary, the data sets include the money stock (notes in circulation and deposits), retail and wholesale price indices, while the Polish data include the money supply (notes in circulation) and the wholesale price index only. The Russian data sets consist of the quantity of the money supply (paper money in circulation), and the retail and wholesale prices indices. The money supply data in Russia are recorded as first-of-month while other countries recorded as end–of-month. On the other hand, all the price data are monthly averages. In view of this, we follow Abel et al. (1979) to apply a geometric averaging method to change all the money supply data in order to conform to the format of the price data series.

Our sample data cover periods of monetary reform, which have usually been truncated in the literature on bubble testing. The sample period of the German data runs from December 1920 to December 1923 and so includes the monetary reforms implemented in October 1923. The Hungarian data cover the period from August 1921 to March 1925 where monetary reforms were launched in March 1924. The Hungarian price index represents retail prices from August 1921 to November 1923, and wholesale prices from December 1923 through March 1925. For Poland, the data cover March 1921 to March 1924, inclusive of the monetary reforms that began in January 1924. For Russia, the data spans from January 1920 to December 1924 wherein currency reforms started in December 1922 are included. From January 1920 to December 1922, the Russian price index numbers represent retail prices and from January 1923 to December 1924 wholesale prices. The dates of the monetary reforms are taken from Cagan (1956), Sargent (1982) and LaHaye (1985).

To identify bubbles for the four sample countries, the cointegration relationships of Eqs.(2) and (5) are examined. As a first step, the standard cointegrating ADF and Z statistics are employed to conduct the analysis.

Table 3: Cointegrating ADF, Z, ADF* and Z* Tests for Price Bubbles.

Country	α	β	ADF	Z_a	Z_t	ADF*	Z_a^*	Z_t^*
Cointegration test for the regression of $M_t - p_t$ on Δp_{t+1}								
Germany	8.3271	-0.6871	-2.1934	-18.5794	-2.0782	-5.7066[b]	-41.7062[a]	-5.7875[b]
Hungary	0.8043	-1.0091	-1.9475	-8.8528	-2.0928	-3.2581	-16.8531	-3.3222
Poland	0.9500	-1.0635	-3.5561	-17.6590	-3.6762	-3.7511	-21.3167	-3.7021
Russia	4.6679	-1.3484	-3.5202	-4.6238	-1.7547	-4.3593	-15.1740	-2.7136
Cointegration test for the regression of $M_t - p_t$ on ΔM_t								
Germany	8.3731	-0.9652	-3.4910	-20.8454	-3.5395	-5.0076[c]	-35.3824[b]	-5.4013[b]
Hungary	1.1051	-3.3883	-3.0770	-9.1919	-2.1644	-3.7454	-14.0172	-2.6687
Poland	1.1947	-2.003	-3.2723	-13.8429	-2.7591	-4.6920	-22.8866	-3.8103
Russia	4.6959	-1.4992	-2.0325	-8.2730	-1.9555	-4.0216	-16.3550	-2.8627

Notes: (1) The number of usable observations for Germany, Hungary, Poland and Russia are 37, 42, 37 and 58, respectively. (2) For conducting the standard cointegrating ADF and Z tests, the data are demeaned and detrended using the OLS method. The number of autoregressive lags in the ADF regressions is determined by the AIC criteria. (3) The significance of the ADF and Z tests are based upon the critical values from Phillips and Ouliaris (1990). (4) The significance of the ADF* and Z_t^* tests are based upon the simulated finite-sample critical values, which equal -4.9537, -5.3007 and -6.0294 at the 10%, 5% and 1% levels, respectively. (5) The significance of the Z_a^* test is based upon the simulated finite-sample critical values, which equal -31.9626, -34.7143 and -40.1184 at the 10%, 5% , and 1% significance levels, respectively. (6) Significance levels are denoted as follows: a(1%), b(5%), and c(10%).

As shown in Table 3, all the values of β of Eqs.(2) and (5) are found to have the correct negative sign. However, the ADF and Z tests cannot identify any cointegrated relationships between $M_t - p_t$ and Δp_{t+1} or between $M_t - p_t$ and ΔM_t for the four countries. Therefore, the standard ADF and Z tests reject the validity of the Cagan model for the sample countries. This result contradicts Engsted's (1993, 1994) findings, where data for the European hyperinflation are truncated before monetary reforms. Since the standard ADF and Z tests do not allow for any specifications of structural breaks in the series under test, the results may be biased towards the null of non-cointegration when structural breaks actually exist in the data. Therefore, the Gregory-Hansen ADF* and Z* tests, which allow for a single break in the cointegration vector at an unknown date, are used to re-estimate Eqs.(2) and (5). As shown in Table 3, the ADF* and Z* tests still cannot reject the null of non-cointegration in regressions (2) and (5) for Hungary, Poland and Russia; however, the results support a cointegrated relationship between $M_t - p_t$ and Δp_{t+1} as well as between $M_t - p_t$ and ΔM_t in the case of Germany. Given that Gregory and Hansen (1996) have shown that the ADF* and Z* tests have greater power to reject the false non-cointegration null hypothesis than the standard ADF test when the data contain only one regime shift, the result in the case of Germany may be attributed to the existence of a single break in the data.

To extend the analysis, we repeat the bubble testing procedure but this time use the residual-based threshold cointegration tests that take into account a threshold process with multiple regime shifts, rather than a single break, in the data series. The empirical results reported in Panel A of Table 4 show that all of the point estimates of ρ_1 and ρ_2 obtained from the regression of $M_t - p_t$ on Δp_{t+1} are negative and the R_T Wald statistics are all significantly different from zero. Hence, the TAR method supports cointegration relationships between $M_t - p_t$ and Δp_{t+1}, implying that the Cagan models are not rejected for the sample countries. Furthermore, the W_T Wald statistics for the residuals of the regression of $M_t - p_t$ and Δp_{t+1} are all greater than their corresponding unconstrained bootstrap critical values generated without the imposition of a non-cointegration restriction.[11] This indicates that the residual series in Eq.(2) exhibits threshold nonlinearity for all four countries due to the regime-shifting of one or more elements in $\hat{\Theta}_1(\hat{\lambda})$ and $\hat{\Theta}_2(\hat{\lambda})$. To proceed further, the threshold cointegration tests are applied to the regression of $M_t - p_t$ on ΔM_t in Eq.(5). As reported in Panel B of Table 4, the estimates of ρ_1 and ρ_2 are negative and the R_T Wald statistics are significant. This finding implies that $M_t - p_t$ is cointegrated with ΔM_t for all the sample countries. Moreover, the Wald statistics for the threshold effects on the residuals of the regression (5) are significant for all countries except Poland.

In short, based upon the Engsted's (1993, 2003) procedure, the TAR model identifies cointegration relationships in both Eqs.(2) and (5), providing evidence contrary to explosive bubbles associated with periods of hyperinflation. Hence, even though the sample periods have been extended to cover the entire hyperinflation episodes, the empirical results that reject the bubble hypothesis do not contradict the work of Engsted (1993, 1994). Also, the rejection of the linearity null hypothesis in most cases favors the existence of stationary, regime-switching movements of money demand dynamics.[12]

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[11] As the non-cointegration null hypotheses are rejected in all cases, our conclusions of nonlinearity tests rely upon unconstrained bootstrap p-values. However, the empirical results based upon unconstrained and constrained bootstrap p-values are similar.

[12] Other than the switching processes of fundamentals, asymmetric intervention rules, money financing of perpetual government deficits and the presence of transaction costs may be additional economic justifications for threshold nonlinearity in a cointegrated system (Pippenger and Goering, 1993; Peel and Speight, 1994; and Balke and Fomby, 1997).

Table 4: Threshold Cointegration Test for Detecting Price Bubbles.

Country	$\hat{\lambda}$ [m,k]	R_{2T}	R_{1T}	ρ_1	ρ_2	W_T	W(1)	W(*t*)	W(y_{t-1})
Panel A: Threshold cointegration test statistics for the regression of $M_t - p_t$ on Δp_{t+1}									
Germany	-0.061[4,4]	85.8 (0.0092)^{a}	85.8 (0.0000)^{a}	-1.020	-1.690	130 (0.000)^{a} [0.109]	2.87 (0.837) [0.328]	18.20 (0.046)^{b} [0.039]^{b}	4.39 (0.002)^{a} [0.026]^{a}
Hungary	0.055[1,1]	22.0 (0.0631)^{c}	22.0 (0.0622)^{c}	-0.929	-0.271	25.3 (0.023)^{b} [0.033]^{b}	12.20 (0.054)^{c} [0.041]^{b}	19.10 (0.009)^{a} [0.011]^{b}	6.57 (0.116) [0.166]
Poland	-0.266[2,2]	21.8 (0.0502)^{b}	21.8 (0.0485)^{b}	-0.830	-0.654	22.4 (0.034)^{b} [0.034]^{b}	4.98 (0.349) [0.327]	10.80 (0.052)^{c} [0.061]^{c}	0.29 (0.752) [0.807]
Russia	-0.510[5,5]	28.6 (0.0380)^{b}	28.6 (0.0380)^{b}	-0.548	-0.245	21.7 (0.063)^{c} [0.120]	18.10 (0.023)^{b} [0.038]^{c}	13.40 (0.045)^{b} [0.086]^{c}	4.49 (0.189) [0.378]
Panel B: Threshold cointegration test statistics for the regression of $M_t - p_t$ on ΔM_t									
Germany	0.087[1,2]	32.7 (0.0484)^{b}	32.7 (0.0482)^{b}	-0.138	-1.280	59.4 (0.019)^{b} [0.026]^{b}	28.90 (0.037)^{b} [0.049]^{b}	31.90 (0.027)^{b} [0.021]^{b}	16.40 (0.049)^{b} [0.090]^{c}
Hungary	0.162[4,4]	28.3 (0.0357)^{b}	28.3 (0.0356)^{b}	-0.127	-0.984	26.2 (0.019)^{b} [0.036]^{b}	0.01 (0.986) [0.990]	0.98 (0.665) [0.710]	16.00 (0.004)^{a} [0.010]^{b}
Poland	-0.046[2,2]	24.9 (0.0456)^{b}	24.9 (0.0452)^{h}	-0.990	-0.769	12.2 (0.2940) [0.4360]	0.13 (0.909) [0.904]	8.08 (0.123) [0.184]	0.375 (0.733) [0.802]
Russia	-0.020[3,3]	29.7 (0.0170)^{b}	29.7 (0.0170)^{b}	-0.220	-0.360	28.5 (0.003)^{a} [0.021]^{b}	0.04 (0.950) [0.857]	10.80 (0.061)^{c} [0.114]	1.61 (0.481) [0.617]

Notes: (1) $\hat{\lambda}$ [m,k] denotes the point estimate of the threshold value with the estimated delay number (m) and the number of autoregressive lags in the TAR regression (k). The estimated values of $\hat{\lambda}$ and m are found by minimizing the residual variance of the TAR regression. The values of k are based upon the AIC criteria. (2) W(1), W(t) and W(y_{t-1}) stand for individual Wald statistics that are used to test for the null hypotheses that the TAR parameters for intercept, linear time trend, and y_{t-1} are equal between the two regimes, respectively. (3) The p-values of the TAR inferential statistics are generated by use of bootstrapping approximation. (4) Since both ρ_1 and ρ_2 are negative, R_{1T} and R_{2T} are equivalent in such a case. Therefore, their estimated values are the same. (5) (.) and [.] denote the unconstrained and constrained bootstrap p-values respectively for W_T, W(1), W(*t*) and W(y_{t-1}) statistics. Also, (.) below the R_T statistics denotes the unidentified bootstrap p-value. (6) Significance levels are denoted as follows: a(1%), b(5%), and c(10%). (7) The estimation results and the bootstrapping approximations are all produced by Hansen's program, which is available from his web page: http://www.ssc.wisc.edu/~bhansen

Conclusion

This paper attempts to solve two empirical problems in bubble testing with the application of the threshold cointegration methodology to the extended sample periods of the European inter-war hyperinflation. Based upon simulation experiments, the TAR methods are less biased towards rejecting non-cointegration in the presence of periodically collapsing bubbles. The empirical findings show that the null of non-cointegration using the TAR method is rejected in all cases, indicating the absence of bubbles. The stationary regime-switching process found in most cases may be caused by expected changes in the fundamentals of the Cagan model.

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