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# A test for rational bubbles in the NASDAQ stock index: A fractionally integrated approach

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**Abstract**

In this paper we test for the presence of rational bubbles in the NASDAQ stock market index over the period 1994:06–2003:11 by means of a methodology based on fractional processes. The results suggest that the existence of bubbles depends on the sampling frequency used in the analysis. We cannot reject the unit root hypothesis when using monthly data on price–dividend ratios, which according to the present value model suggests the existence of rational bubbles. However, we reject this hypothesis in favor of fractional alternatives when using daily and weekly data. This might be explained by the temporal aggregation and/or the sample sizes used in the application.


*JEL classification:* G12; E40; C32
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## 1. Introduction

During the second half of the 1990s, the US economy experienced a remarkable performance in productivity growth. From 1995:04 to 2000:04, the productivity

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growth per worker was 2.4% per year (with a GDP growth rate of 3.5% per year), compared with an increase of only 1.4% per year from 1972:02 to 1995:04 (and 2.9% real GDP growth). This improvement in productivity growth was accompanied by an unprecedented boom in equipment investment. Furthermore, the recent improvement in the US productivity growth may be related with the information technology revolution (see, e.g., Oliner and Sichel, 2000; Jorgenson, 2001; Nordhaus, 2002; etc.). [1] The expansion in the late 1990s was also accompanied by a massive boom in the stock market and the spectacular rise of the NASDAQ index. [2] The NASDAQ composite index containing mostly technology shares, soared from 373 in December 1990 to 1052 in December 1995, surpassing 2000 in December 1998, and finally peaking at 4069 in December 1999, while the Standard and Poor's 500 (S&P 500) stock index varied from 330 in December 1990 to 1469 in December 1999. The continuous rise in stock prices over the 1990s in the US has led some leading economists, such as Greenspan (1996), Shiller (2001) and Stiglitz (2003), to suggest the existence of a bubble in the US stock market. Thus, Stiglitz (2003) considers the stock market bubble the seed of the destruction of the US economy in the 1990s. The term bubble in the stock market suggests a possible deviation of the stock price from the fundamental price, proxied by the present value of all its future cash flows or dividends.

Many papers have studied the presence of bubbles in stock markets (e.g., Campbell and Shiller, 1987, 1988; Diba and Grossman, 1988; Froot and Obstfeld, 1991; Craine, 1993; Timmermann, 1995; Crowder and Wohar, 1998; Lamont, 1998; Bohl, 2003; Nasseh and Strauss, 2003, among others). Most of these papers test the existence of stock market bubbles using traditional unit-root tests to the price–dividend ratio with long span data of the S&P 500 composite index. Campbell and Shiller (1987) test for rational bubbles using annual data for the S&P 500 from 1871 to 1986, obtaining persistent deviations of stock prices from the present-value model, and thus, rational bubbles, when testing for cointegration between stock prices and dividends. Froot and Obstfeld (1991) and Craine (1993) reach the same conclusion testing for a unit root in the price–dividend ratio using annual data of the S&P 500 for 1900–1988 and 1876–1988 respectively. However, Diba and Grossman (1988) find that stock prices do not contain explosive rational bubbles when analyzing data of S&P 500 index for 1871–1986. [3] Furthermore, according to a more recent literature, the evidence of bubbles may be different depending on whether or not the IT bubble observed since the mid 90s is included. For example, Bohl (2003) obtains different results when he analyzes the stock market behavior excluding or including the

[1] The concept of the "new economy" was coined in the business press in the mid 1990s to mean an economy which is able to benefit from the globalization of business and the revolution in information and communication technology.

[2] In addition, the internet companies experienced a dramatic rise in the late 1990s and a spectacular fall in 2001 (see, for example, Ofek and Richardson, 2003).

[3] Most of the studies which have analyzed the presence of stock market bubbles do not include the recent IT bubble observed since the middle of the 1990s, since their sample periods end in the 1980s. The major long-run stock price fluctuations or possible bubbles analyzed in most of these studies correspond to the bull markets of 1921–1929 or 1949–1966 or to the bear markets of 1929–1933 or 1973–1975.

IT bubble observed since 1995: while his results from the subsample 1871–1995 cannot be interpreted in favor of the existence of bubbles in the US stock market, the findings from the 1871–2001 sample period indicate their presence. Moreover, Nasseh and Strauss (2003) analyze the presence of bubbles for the period 1979:03–1999:02 and find evidence of a breakdown since the mid 1990s in the relation between stock prices and dividends.

In line with the above papers, we also test for rational bubbles. The main differences of the present work with the former ones are the following. First, we test for bubbles in the NASDAQ composite index and not in the mostly analyzed S&P 500. Second, instead of using classic approaches based on $I(1)/I(0)$ integration and cointegration techniques, we use a methodology based on fractional processes, that is, we study the order of integration of the stock prices and dividends (in logs) in the NASDAQ index, and the difference between the two variables, but using a methodology based on fractional integration. In doing so, we avoid the strong dichotomy produced by the $I(0)/I(1)$ specifications, and consider a wider variety of $I(d)$ models, with $d$ not necessarily constrained to be 0 or 1. We use a version of the tests of Robinson (1994a) along with a semiparametric method (Robinson, 1995a) in order to examine the univariate properties of stock prices, dividends and stock prices–dividends ratio. Finally, the empirical analysis is carried out using different sampling frequencies; that is, we use daily, weekly and monthly data of the NASDAQ composite index and dividends in order to test for bubbles.

The paper is organized as follows. The following section provides a brief discussion of the present value model under rational expectations, and introduces the notion of rational bubbles in stock markets. Section 3 presents the testing procedure of Robinson (1994a) and the semiparametric method. In Section 4, the rational bubbles hypothesis is tested on daily, weekly and monthly prices and dividends from the NASDAQ stock market, covering the period 1994:06–2003:11. Section 5 summarizes the empirical results, while Section 6 contains some concluding comments.

## 2. A model of rational bubbles

We define (see, for example, Campbell et al., 1997) the simple return on a stock as

$$R_{t+1} = \frac{P_{t+1} - P_t + D_{t+1}}{P_t} = \frac{P_{t+1} + D_{t+1}}{P_t} - 1, \tag{1}$$

where $R_{t+1}$ denotes the return on the stock held from time $t$ to $t+1$, and $D_{t+1}$ is the dividend in period $t+1$. The subscript $t+1$ denotes the fact that the return only becomes known in period $t+1$. Taking the mathematical expectation on Eq. (1), based on information available at time $t$, and rearranging terms, we obtain

$$P_t = E_t\left[\frac{P_{t+1} + D_{t+1}}{1 + R_{t+1}}\right]. \tag{2}$$

Solving Eq. (2) forward $k$ periods yields the semireduced form

$$P_t = E_t\left[\sum_{i=1}^{k}\left(\frac{1}{1+R_{t+i}}\right)^i D_{t+i}\right] + E_t\left[\left(\frac{1}{1+R_{t+k}}\right)^k P_{t+k}\right]. \tag{3}$$

In order to obtain a unique solution to (3), we need to assume that the expected discounted value of the stock in the indefinite future converges to zero:

$$\lim_{k\to\infty} E_t\left[\left(\frac{1}{1+R_{t+k}}\right)^k P_{t+k}\right] = 0. \tag{4}$$

The convergence assumption allows us to obtain the fundamental value of the stock as the sum of the expected discounted dividends:

$$F_t = E_t\left[\sum_{i=1}^{\infty}\left(\frac{1}{1+R_{t+i}}\right)^i D_{t+i}\right]. \tag{5}$$

Ruling out the convergence assumption – Eq. (4) – leads to an infinite number of solutions, any one of which can be written in the form

$$P_t = F_t + B_t, \tag{6}$$

where

$$B_t = E_t\left[\frac{B_{t+1}}{1+R_{t+1}}\right]. \tag{7}$$

The second term in the right-hand side in Eq. (6) (i.e. (7)) appears in prices only because it is expected to be present in the next period with a value $B_tE_t(1 + R_{t+1})$. The term $B_t$ is called a "rational bubble", in the sense that it is entirely consistent with rational expectations and the time path of expected returns.

The presence of time-varying expected stock returns has led to a nonlinear relation between prices and returns. Campbell and Shiller (1988) suggest a log-linear approximation to (1) and they write it as

$$\begin{aligned} r_{t+1} &= \log(1+R_{t+1}) = \log(P_{t+1}+D_{t+1}) - \log(P_t) \\ &= p_{t+1} - p_t + \log[1+\exp(d_{t+1}-p_{t+1})], \end{aligned} \tag{8}$$

where lower case letters represent the natural logarithm of a variable. Eq. (8), a nonlinear function of the log dividend–price ratio, can be approximated around the mean by a first-order Taylor expansion:

$$r_{t+1} \approx \alpha + \lambda p_{t+1} + (1-\lambda)d_{t+1} - p_t, \tag{9}$$

where $\alpha$ and $\lambda$ are (scalar) parameters. Thus, (9) is a linear difference equation for the log stock price. Solving forward and imposing the no transversality condition, we obtain

$$p_t = \frac{\alpha}{1-\lambda} + \sum_{j=0}^{\infty}[(1-\lambda)d_{t+1+j} - r_{t+1+j}]. \tag{10}$$

Finally, taking the mathematical expectation of (10) based on information available at time $t$, and rearranging in terms of the log dividend–price ratio, we obtain

$$d_t - p_t = -\frac{\alpha}{1-\lambda} + E_t\left[\sum_{j=0}^{\infty} \lambda^j[(1-\lambda)d_{t+1+j} - r_{t+1+j}]\right]. \tag{11}$$

According to (11), if stock prices ($p_t$) and real dividends ($d_t$) follow integrated processes of order one, and no bubbles are present, the log stock price and the log dividends are cointegrated with the cointegrating vector $(1, -1)$ and the log dividend–price ratio ($d_t - p_t$) is a stationary process under no rational bubble restriction. On the contrary, the presence of a unit-root in the log dividend–price ratio is consistent with rational bubbles in stock markets.

In this context, the usual form of testing the absence of rational bubbles assumes that the individual series (log of real stock prices and log of real dividends) are both nonstationary $I(1)$. We start our analysis by examining this hypothesis. However, instead of using classic methods, based on autoregressive (AR) models, we use a parametric testing procedure developed by Robinson (1994a) and a semiparametric one (Robinson, 1995a) for testing $I(d)$ statistical models.

For the purpose of the present paper, we define an $I(0)$ process $\{u_t, t = 0, \pm 1, \ldots\}$ as a covariance stationary process, with spectral density function that is positive and finite at the zero frequency. In this context, we say that $\{x_t, t = 0, \pm 1, \ldots\}$ is $I(d)$ if

$$(1-L)^d x_t = u_t, \quad t = 1, 2, \ldots, \tag{12}$$

with $x_t = 0$ for $t \leqslant 0$. The polynomial in (12) can be expressed in terms of its binomial expansion such that

$$(1-L)^d = \sum_{j=0}^{\infty} \binom{d}{j} (-1)^j L^j = 1 - dL + \frac{d(d-1)}{2} L^2 - \cdots$$

for all real $d$. Clearly, the unit-root case corresponds to $d = 1$ in (12). If $d > 0$, $x_t$ is said to be long memory, so-named because of the strong association between observations widely separated in time. In the following section, we present two procedures for testing this type of model.

## 3. Testing for fractional integration

There exist many different ways of testing for unit-root models. Perhaps, the most common ones are the tests of Fuller (1976) and Dickey and Fuller (1979). They consider a process of the form

$$(1-\rho L)y_t = \mu + u_t, \tag{13}$$

which, under the null hypothesis

$$H_0 : \rho = 1, \tag{14}$$

becomes the random walk model if $u_t$ is white noise. The tests are based on the auxiliary regression:

$$(1 - L)y_t = \pi y_{t-1} + \mu + u_t, \tag{15}$$

where $\pi = \rho - 1$, and the test statistic is the "$t$-value" corresponding to $\pi$ in (15). Due to the nonstandard asymptotic distributional properties of the "$t$-values" under the null hypothesis: $H_0 : \pi = 0$, Dickey and Fuller (1979) provide the fractiles of simulated distributions, which give us the critical values to be applied when testing the null against the alternative: $H_a : \pi < 0$. The tests can be extended to allow for autocorrelated disturbances, and then the auxiliary regression (15) may be augmented by lagged values of $(1 - L)y_t$, and also with other deterministic regressors, like an intercept or a linear time trend, though this unfortunately changes the distribution of the test statistic. Another limitation of these tests is that they lose validity if the disturbances are not white noise or AR processes. Also, Phillips (1987) and Phillips and Perron (1988) consider tests that employ a nonparametric estimate of the spectral density of $u_t$ at the zero frequency, for example, a weighted autocovariance estimate.

Robinson (1994a) proposes a Lagrange multiplier (LM) test of the null hypothesis

$$H_0 : d = d_0, \tag{16}$$

in a model given by (12), where $d_0$ can be any real number and where $u_t$ is $I(0)$. The $x_t$ in (12) can be the time series we observe, though it may also be the errors in a regression model of the form

$$y_t = \beta' z_t + x_t, \tag{17}$$

where $\beta = (\beta_1, \ldots, \beta_k)'$ is a $(k \times 1)$ vector of unknown parameters, and $z_t$ is a $(k \times 1)$ vector of deterministic regressors that may include, for example, an intercept (e.g., $z_t \equiv 1$), or an intercept and a linear time trend. The functional form of the test statistic, denoted by $\widehat{R}$, is described in Appendix A. Robinson (1994a) shows that under certain regularity conditions [4]

$$\widehat{R} = \hat{r}^2 \to_d \chi_1^2 \quad \text{as } T \to \infty. \tag{18}$$

Thus, we are in a classical large-sample testing situation and the conditions on $u_t$ in (12) are far more general than Gaussianity, with a moment condition only of order 2 required. Because $\widehat{R}$ involves a ratio of quadratic forms, its exact null distribution could have been calculated under Gaussianity via Imhof's algorithm. However, a simple test is approximately valid under much wider distributional assumptions. An approximate one-sided $100\alpha\%$-level test of $H_0$ (16) against the alternative: $H_a : d > d_0$ ($d < d_0$) will reject $H_0$ if $\hat{r} > z_\alpha$ ($\hat{r} < -z_\alpha$), where the probability that a standard normal variate exceeds $z_\alpha$ is $\alpha$. Moreover, Robinson (1994a) shows that the above test is efficient in the Pitman sense, i.e., that against local alternatives of form: $H_a : d = d_0 + \delta T^{-1/2}$, with $\delta \neq 0$, the limit distribution of $\hat{r}$ is normal with variance 1

[4] These conditions are very mild, regarding technical assumptions which are satisfied by the model in (17) and (12).

and mean that cannot (when $u_t$ is Gaussian) be exceeded in absolute value by that of any rival regular statistic.

There exist other procedures for estimating and testing the fractionally differenced parameter, some of them also based on the likelihood function. We believe that as in other standard large-sample testing situations, Wald and LR test statistics against fractional alternatives will have the same null and local limit theory as the LM tests of Robinson (1994a). Sowell (1992) employed essentially such a Wald testing procedure but it requires an efficient estimate of $d$, and while such estimates can be obtained, the LM procedure of Robinson (1994a) seems computationally more attractive.

In the following section we will also implement a semiparametric procedure for estimating the fractional differencing parameter. It is semiparametric in the sense that we just consider a process like (12) with no specification for the $I(0)$ disturbances. We will use a Whittle estimator in the frequency domain proposed by Robinson (1995a).

The estimate is sometimes called a quasi maximum likelihood estimate (QMLE), and it is basically a local "Whittle estimate" in the frequency domain, based on a band of frequencies that degenerates to zero. The estimate of $d$ is implicitly defined by

$$\hat{d} = \arg\min_{d} \left( \log C(d) - 2d \frac{1}{m} \sum_{j=1}^{m} \log \lambda_j \right), \tag{19}$$

$$\text{for } d \in (-1/2, 1/2); \quad C(d) = \frac{1}{m} \sum_{j=1}^{m} I(\lambda_j) \lambda_j^{2d}, \quad \lambda_j = \frac{2\pi j}{T}, \quad \frac{m}{T} \to 0,$$

where $m$ is a bandwidth parameter number. Under finiteness of the fourth moment and other mild conditions, Robinson (1995a) proves that

$$\sqrt{m}(\hat{d} - d^*) \to_d N(0, 1/4) \quad \text{as } T \to \infty,$$

where $d^*$ is the true value of $d$ and with the additional requirements that $m \to \infty$ slower than $T$ and $m < T/2$. [5] A multivariate extension of this estimation procedure can be found in Lobato (1999), but that procedure is invalid if the multivariate model is cointegrated.

There exist other semiparametric procedures for estimating the fractional differencing parameter, for example, the log-periodogram regression estimate (LPE), initially proposed by Geweke and Porter-Hudak (1983), and modified later by Künsch (1986) and Robinson (1995b), and the averaged periodogram estimate (APE) of Robinson (1994b). We have decided to use in this article the QMLE, mainly because of two reasons: The first one is computational, using the QMLE, we do not need to employ, apart from $m$, any other additional user-chosen numbers in the estimation

[5] The exact requirement is that $(1/m) + ((m^{1+2\alpha}(\log m)^2)/(T^{2\alpha})) \to 0$ as $T \to \infty$, where $\alpha$ is determined by the smoothness of the spectral density of the short run component. In case of a stationary and invertible ARMA, $\alpha$ may be set equal to 2 and the condition is $(1/m) + ((m^5(\log m)^2)/(T^4)) \to 0$ as $T \to \infty$.

(as is the case with the LPE and the APE). The second reason is that we do not have to assume Gaussianity in order to obtain an asymptotic normal distribution, the QMLE being more efficient than the LPE. [6]

## 4. Data and test results

In this section we examine whether the NASDAQ composite stock market index and its corresponding dividend yield satisfy the condition required for the absence of rational bubbles, that is, we test for mean reversion in the log price–dividend ratio, using monthly, weekly and daily data over the period 1994:06–2003:11. All these data were taken from Bloomberg. The data consist of daily closing values for the NASDAQ index and cover the 10-year period 1994:06–2003:11. [7] For weekly data, we have used the Wednesday prices and dividends, and when there was no trading on a given Wednesday, the trading day before Wednesday was used. The monthly data was constructed using the last trading day of each month. The first thing we do is to examine the existence of bubbles by means of traditional methods. In particular, we carry out the Augmented Dickey–Fuller (ADF) and Phillips–Perron (PP) tests to each of the variables. The results are presented in Table 1. As shown in the table, we cannot reject the unit-root null hypothesis in any of the cases, suggesting the existence of a rational bubble in the NASDAQ index. However, these results should be taken with caution. It is a well-known fact that these tests have very low power against AR alternatives if the roots are close to the unit root. Moreover, Diebold and Rudebusch (1991) and Hassler and Wolters (1994) show that classic unit-root tests have also very low power against fractionally based alternatives.

We perform the tests of Robinson (1994a) described in Section 3 to the two individual series. Denoting each of the time series by $y_t$, we employ throughout the model given by (12) and (17), $z_t = (1, t)'$, $t \geqslant 1$, $z_t = (0, 0)'$ otherwise. Thus, under the null hypothesis $H_0$ (16),

$$y_t = \beta_0 + \beta_1 t + x_t, \quad t = 1, 2, \ldots, \tag{20}$$

$$(1 - L)^{d_0} x_t = u_t, \quad t = 1, 2, \ldots, \tag{21}$$

and we treat separately the cases $\beta_0 = \beta_1 = 0$ a priori; $\beta_0$ unknown and $\beta_1 = 0$ a priori; and both $\beta_0$ and $\beta_1$ unknown, i.e., we consider respectively the cases of no regres-

[6] Velasco (2000) showed that Gaussianity is not necessary for the LPE either.

[7] In this paper and due to data availability problems, we only analyze the period 1994:06–2003:11, which includes the rapid increase in stock prices since the mid 1990s and their decrease in 2000. We believe that the results should not change very much since the IT bubble included in our sample is seen in the literature as one of the clearest examples of stock market bubbles, mainly when the NASDAQ stock index is analyzed. However, the lack of data for a longer time span does not make possible to analyze the existence of a structural break in the relation between stock prices and dividends before the IT bubble. Although the results are not directly comparable, the results found in this paper are consistent with those of Koustas and Serletis (2005) who also applies fractional integration techniques for the S&P 500 index for a longer period, 1871–2000.

Table 1
ADF and PP unit root tests

| | Augmented Dickey–Fuller | | | Phillips–Perron | | |
|---|---|---|---|---|---|---|
| | (i) | (ii) | (iii) | (i) | (ii) | (iii) |
| *Testing the order of integration of log prices* | | | | | | |
| Monthly data | 1.13 | −1.91 | −1.06 | 1.02 | −1.89 | −1.10 |
| Weekly data | 1.17 | −1.77 | −0.07 | 1.11 | −1.76 | −0.99 |
| Daily data | 1.12 | −1.77 | −0.95 | 1.13 | −1.15 | −0.95 |
| *Testing the order of integration of log dividends* | | | | | | |
| Monthly data | 0.34 | −1.20 | −0.70 | 0.33 | −1.25 | −0.70 |
| Weekly data | 0.33 | −1.20 | −0.21 | 0.35 | −1.25 | −0.60 |
| Daily data | 0.40 | −1.00 | −0.43 | 0.37 | −1.15 | −0.56 |
| *Testing the order of integration of log (prices/dividends)* | | | | | | |
| Monthly data | 0.04 | −1.89 | −0.24 | 0.02 | −1.89 | −0.05 |
| Weekly data | 0.07 | −1.77 | −0.07 | 0.03 | −1.78 | −0.10 |
| Daily data | 0.05 | −1.77 | −0.07 | 0.04 | −1.76 | −0.03 |

(i) With no regressors; (ii) with an intercept; (iii) with an intercept and a linear time trend.
The number of the lags included was determined using Akaike Information Criteria (AIC).
When these tests are applied to the first differences of these variables, in all the cases we reject the unit-root hypothesis.

sors in the undifferenced regression (20), an intercept, and an intercept and a linear time trend. We model the $I(0)$ process $u_t$ to be both white noise and to have parametric autocorrelation.

We start with the assumption that $u_t$ in (21) is white noise. Thus when $d = 1$, for example, $y_t$ behaves, for $t > 1$, like a random walk when $\beta_1 = 0$, and a random walk with drift when $\beta_1 \neq 0$. However, we report test statistics not merely for the null $d_0 = 1$ in (16) but for $d_0 = 0, (0.25), 2$, thus including also a test for stationarity ($d_0 = 0.5$) and for $I(2)$ ($d_0 = 2$), as well as other fractionally integrated possibilities.

The test statistic reported across Tables 2–4 is the one given by $\hat{r}$ in Appendix A. Thus, for a given $d_0$, significantly positive values of $\hat{r}$ are consistent with orders of integration higher than $d_0$, whereas significantly negative ones imply orders of integration smaller than that hypothesized under the null. A noticeable feature observed in the table is that if the disturbances are white noise, the values of $\hat{r}$ monotonically decrease with $d_0$. This is something to be expected since they are one-sided statistics. Thus, for example, we would wish that if $H_0$ (16) is rejected with $d = 0.75$ in favor of alternatives of form $d > 0.75$, an even more significant result in this direction should be obtained when $d = 0.50$ or 0.25 are tested.

Table 2 reports the results for the two individual series assuming that $u_t$ is white noise. Starting with the stock prices, we see that we cannot reject the unit-root hypothesis ($d = 1$) for stock prices when using monthly and weekly data (for the three cases of no regressors, an intercept and an intercept and a linear time trend) and daily data (in the case of no regressors). However, this hypothesis is rejected in favor of $d < 1$ when we use daily data and include an intercept and/or a linear time trend. The last column of the table reports the 95%-confidence intervals of those

Table 2
Order of integration of log-prices and log-dividends using white noise

| | | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|
| *Testing the order of integration of log-prices, using white noise disturbances* | | | | | | | | | | |
| Monthly data | | | | | | | | | | |
| $z_t/d_0$ | 0.00 | 0.25 | 0.50 | 0.75 | 1.00 | 1.25 | 1.50 | 1.75 | 2.00 | Conf. interval |
| (i) | 25.02 | 13.70 | 9.53 | 3.96 | **−0.26** | −2.55 | −3.73 | −4.41 | −4.83 | [0.88; 1.13] |
| (ii) | 25.02 | 21.51 | 15.72 | 8.05 | **1.27** | −2.15 | −3.62 | −4.32 | −4.74 | [0.99; 1.19] |
| (iii) | 25.39 | 23.11 | 17.61 | 8.68 | **1.28** | −2.20 | −3.64 | −4.33 | −4.73 | [0.99; 1.19] |
| Weekly data | | | | | | | | | | |
| $z_t/d_0$ | 0.00 | 0.25 | 0.50 | 0.75 | 1.00 | 1.25 | 1.50 | 1.75 | 2.00 | Conf. interval |
| (i) | 74.38 | 45.81 | 29.90 | 10.63 | **−0.27** | −4.90 | −7.11 | −8.36 | −9.16 | [0.94; 1.05] |
| (ii) | 74.38 | 65.96 | 46.98 | 17.20 | **0.13** | −5.57 | −7.81 | −8.95 | −9.64 | [0.97; 1.05] |
| (iii) | 74.89 | 69.37 | 50.68 | 18.06 | **0.12** | −5.58 | −7.81 | −8.95 | −9.65 | [0.97; 1.05] |
| Daily data | | | | | | | | | | |
| $z_t/d_0$ | 0.00 | 0.25 | 0.50 | 0.75 | 1.00 | 1.25 | 1.50 | 1.75 | 2.00 | Conf. interval |
| (i) | 217.05 | 147.29 | 90.52 | 26.87 | **−0.34** | −10.26 | −14.90 | −17.54 | −19.23 | [0.97; 1.02] |
| (ii) | 217.04 | 195.95 | 126.84 | 28.15 | −2.86 | −11.47 | −15.37 | −17.67 | −19.19 | [0.94; 0.97] |
| (iii) | 218.46 | 203.61 | 133.89 | 29.16 | −2.86 | −11.47 | −15.37 | −17.67 | −19.19 | [0.94; 0.97] |
| *Testing the order of integration of log-dividends, using white noise disturbances* | | | | | | | | | | |
| Monthly data | | | | | | | | | | |
| $z_t/d_0$ | 0.00 | 0.25 | 0.50 | 0.75 | 1.00 | 1.25 | 1.50 | 1.75 | 2.00 | Conf. interval |
| (i) | 18.71 | 17.03 | 10.95 | 4.55 | **0.008** | −2.46 | −2.73 | −4.44 | −4.88 | [0.90; 1.14] |
| (ii) | 18.71 | 15.68 | 10.47 | 4.55 | **0.11** | −2.35 | −3.55 | −4.16 | −4.52 | [0.91; 1.15] |
| (iii) | 17.31 | 14.96 | 10.32 | 4.56 | **0.l2** | −2.34 | −3.54 | −4.16 | −4.52 | [0.91; 1.15] |
| Weekly data | | | | | | | | | | |
| $z_t/d_0$ | 0.00 | 0.25 | 0.50 | 0.75 | 1.00 | 1.25 | 1.50 | 1.75 | 2.00 | Conf. interval |
| (i) | 62.15 | 53.82 | 33.21 | 11.87 | **0.22** | −4.70 | −7.00 | −8.27 | −9.08 | [0.96; 1.07] |
| (ii) | 62.15 | 53.47 | 35.51 | 15.06 | 2.44 | −3.35 | −6.13 | −7.69 | −8.67 | [1.03; 1.15] |
| (iii) | 58.55 | 51.46 | 35.15 | 15.06 | 2.44 | −3.34 | −6.13 | −7.68 | −8.66 | [1.03; 1.15] |
| Daily data | | | | | | | | | | |
| $z_t/d_0$ | 0.00 | 0.25 | 0.50 | 0.75 | 1.00 | 1.25 | 1.50 | 1.75 | 2.00 | Conf. interval |
| (i) | 191.33 | 166.09 | 97.72 | 28.63 | **0.02** | −10.20 | −14.91 | −17.57 | −19.27 | [0.98; 1.02] |
| (ii) | 191.35 | 168.93 | 105.21 | 30.15 | **−1.21** | −11.31 | −15.63 | −18.04 | −19.59 | [0.97; 1.00] |
| (iii) | 183.83 | 164.76 | 104.83 | 30.13 | **−1.21** | −11.31 | −15.63 | −18.03 | −19.59 | [0.97; 1.00] |

(i) With no regressors; (ii) with an intercept; (iii) with an intercept and a linear time trend. In **bold**: The nonrejection values of the null hypothesis at the 5% significance level.

values of $d_0$ where $H_0$ cannot be rejected, using a grid of $d_0$-values of 0.01. [8] We see that using monthly and weekly data, the intervals include the unit root in all the cases, while using daily data, the upper limit of the intervals are strictly lower than one when an intercept and/or a linear time trend are included in the model.

As far as the dividends are concerned, the results suggest that $H_0$ cannot be rejected with $d = 1$ in any of the cases except when using weekly data with an intercept and/or a linear time trend, where the lower limit of the intervals are strictly higher

[8] In other words, we test sequentially $H_0$ (16) for $d_0 = 0, (0.001), 2$ and choose those values where $H_0$ cannot be rejected.

Table 3
Order of integration of log-prices and log-dividends with Bloomfield

*Testing the order of integration of log-prices with Bloomfield (S = 1)*

Monthly data

| $z_t/d_0$ | 0.00 | 0.25 | 0.50 | 0.75 | 1.00 | 1.25 | 1.50 | 1.75 | 2.00 | Conf. interval |
|---|---|---|---|---|---|---|---|---|---|---|
| (i) | 44.93 | 24.61 | 17.12 | 7.11 | **−0.46** | −4.58 | −6.70 | −7.91 | −8.68 | [0.92; 1.05] |
| (ii) | 44.93 | 38.63 | 28.24 | 14.46 | 2.29 | −3.87 | −6.50 | −7.77 | −8.50 | [1.02; 1.13] |
| (iii) | 45.59 | 41.51 | 31.63 | 15.59 | 2.30 | −3.96 | −6.53 | −7.77 | −8.50 | [1.02; 1.12] |

Weekly data

| $z_t/d_0$ | 0.00 | 0.25 | 0.50 | 0.75 | 1.00 | 1.25 | 1.50 | 1.75 | 2.00 | Conf. interval |
|---|---|---|---|---|---|---|---|---|---|---|
| (i) | 43.68 | 22.45 | 15.02 | 5.93 | **−0.18** | −3.30 | −4.79 | −5.74 | −6.37 | [0.91; 1.08] |
| (ii) | 43.68 | 35.74 | 25.39 | 12.26 | 2.32 | −2.14 | −4.14 | −5.25 | −5.96 | [1.04; 1.22] |
| (iii) | 46.30 | 40.02 | 28.93 | 13.51 | 2.68 | −2.19 | −4.13 | −5.23 | −5.96 | [1.04; 1.21] |

Daily data

| $z_t/d_0$ | 0.00 | 0.25 | 0.50 | 0.75 | 1.00 | 1.25 | 1.50 | 1.75 | 2.00 | Conf. interval |
|---|---|---|---|---|---|---|---|---|---|---|
| (i) | 141.13 | 80.13 | 48.42 | 15.58 | **−0.47** | −6.64 | −10.10 | −12.00 | −13.28 | [0.95; 1.03] |
| (ii) | 140.94 | 116.64 | 69.65 | 17.92 | −2.18 | −8.36 | −11.14 | −12.67 | −13.74 | [0.93; 0.98] |
| (iii) | 143.79 | 122.30 | 74.53 | 19.32 | −2.18 | −8.37 | −11.14 | −12.67 | −13.74 | [0.93; 0.98] |

*Testing the order of integration of log-dividends with Bloomfield (S = 1)*

Monthly data

| $z_t/d_0$ | 0.00 | 0.25 | 0.50 | 0.75 | 1.00 | 1.25 | 1.50 | 1.75 | 2.00 | Conf. interval |
|---|---|---|---|---|---|---|---|---|---|---|
| (i) | 33.60 | 30.58 | 19.67 | 8.18 | **0.02** | −4.41 | −6.71 | −7.98 | −8.76 | [0.94; 1.07] |
| (ii) | 33.60 | 28.15 | 18.81 | 8.18 | **0.20** | −4.22 | −6.37 | −7.47 | −8.12 | [0.95; 1.08] |
| (iii) | 31.09 | 26.87 | 18.54 | 8.19 | **0.21** | −4.21 | −6.36 | −7.47 | −8.12 | [0.95; 1.08] |

Weekly data

| $z_t/d_0$ | 0.00 | 0.25 | 0.50 | 0.75 | 1.00 | 1.25 | 1.50 | 1.75 | 2.00 | Conf. interval |
|---|---|---|---|---|---|---|---|---|---|---|
| (i) | 35.53 | 28.81 | 16.96 | 6.69 | **0.25** | −3.06 | −4.80 | −5.83 | −6.61 | [0.94; 1.10] |
| (ii) | 35.53 | 27.71 | 16.86 | 6.59 | **0.43** | −2.68 | −4.45 | −5.40 | −6.20 | [1.04; 1.22] |
| (iii) | 31.69 | 25.85 | 16.53 | 6.59 | **0.43** | −2.67 | −4.44 | −5.38 | −6.16 | [1.04; 1.21] |

Daily data

| $z_t/d_0$ | 0.00 | 0.25 | 0.50 | 0.75 | 1.00 | 1.25 | 1.50 | 1.75 | 2.00 | Conf. interval |
|---|---|---|---|---|---|---|---|---|---|---|
| (i) | 119.03 | 96.73 | 53.93 | 16.71 | **0.02** | −6.57 | −9.73 | −11.70 | −13.01 | [0.97; 1.04] |
| (ii) | 119.00 | 98.91 | 57.79 | 19.64 | **0.77** | −6.83 | −10.45 | −12.22 | −13.59 | [0.99; 1.05] |
| (iii) | 112.11 | 95.97 | 57.73 | 19.63 | **0.77** | −6.82 | −10.45 | −12.22 | −13.59 | [0.99; 1.05] |

(i) With no regressors; (ii) with an intercept; (iii) with an intercept and a linear time trend. In **bold**: The nonrejection values of the null hypothesis at the 5% significance level.

than one. Thus, we can conclude that, there is no evidence of mean reversion in the NASDAQ stock prices and dividends, and most of the cases support the hypothesis of a unit-root, which is consistent with the results reported in Table 1 based on traditional methods.

The significance of the above results, however, might be in large part due to the unaccounted for $I(0)$ autocorrelation in $u_t$. Thus, we also fitted other models, taking into account a weakly autocorrelated structure on the disturbances. First, we imposed AR processes and, though not reported in this paper, the results showed a lack of monotonicity in the value of $\hat{r}$ with respect to $d_0$. This lack of monotonicity could be explained in terms of model misspecification as is argued, for example, in

Table 4
Order of integration of log (prices/dividends) using white noise and Bloomfield

| | | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|
| *Testing the order of integration of log (prices/dividends) using white noise disturbances* | | | | | | | | | | |
| Monthly data | | | | | | | | | | |
| $z_t/d_0$ | 0.00 | 0.25 | 0.50 | 0.75 | 1.00 | 1.25 | 1.50 | 1.75 | 2.00 | Conf. interval |
| (i) | 25.50 | 20.95 | 12.97 | 5.51 | **−0.03** | −2.62 | −3.69 | −4.26 | −4.63 | [0.91; 1.12] |
| (ii) | 25.50 | 22.47 | 16.34 | 7.34 | **0.63** | −2.44 | −3.69 | −4.31 | −4.70 | [0.95; 1.16] |
| (iii) | 24.83 | 22.56 | 16.71 | 7.60 | **0.63** | −2.46 | −3.71 | −4.32 | −4.71 | [0.96; 1.15] |
| Weekly data | | | | | | | | | | |
| $z_t/d_0$ | 0.00 | 0.25 | 0.50 | 0.75 | 1.00 | 1.25 | 1.50 | 1.75 | 2.00 | Conf. interval |
| (i) | 70.94 | 32.26 | 1.99 | −6.57 | −8.72 | −9.48 | −9.89 | −10.16 | −10.35 | [0.51; 0.56] |
| (ii) | 70.94 | 44.36 | 4.15 | −6.73 | −8.82 | −9.53 | −9.92 | −10.18 | −10.37 | [0.53; 0.58] |
| (iii) | 66.67 | 38.49 | 3.85 | −6.67 | −8.82 | −9.53 | −9.92 | −10.18 | −10.36 | [0.53; 0.58] |
| Daily data | | | | | | | | | | |
| $z_t/d_0$ | 0.00 | 0.25 | 0.50 | 0.75 | 1.00 | 1.25 | 1.50 | 1.75 | 2.00 | Conf. interval |
| (i) | 214.28 | 111.68 | 4.97 | −14.56 | −18.35 | −19.97 | −21.00 | −21.74 | −22.30 | [0.53; 0.54] |
| (ii) | 214.28 | 143.90 | 7.73 | −15.00 | −18.52 | −20.05 | −21.05 | −21.77 | −22.32 | [0.53; 0.54] |
| (iii) | 208.62 | 129.46 | 7.65 | −14.94 | −18.52 | −20.05 | −21.05 | −21.77 | − 22.32 | [0.54; 0.55] |
| *Testing the order of integration of log (prices/dividends) with Bloomfield (S = 1)* | | | | | | | | | | |
| Monthly data | | | | | | | | | | |
| $z_t/d_0$ | 0.00 | 0.25 | 0.50 | 0.75 | 1.00 | 1.25 | 1.50 | 1.75 | 2.00 | Conf. interval |
| (i) | 45.79 | 37.62 | 23.30 | 9.90 | **−0.06** | −4.70 | −6.63 | −7.64 | −8.32 | [0.95; 1.05] |
| (ii) | 45.79 | 40.36 | 29.35 | 13.19 | **1.14** | −4.38 | −6.63 | −7.74 | −8.44 | [0.99; 1.09] |
| (iii) | 44.59 | 40.51 | 30.00 | 13.64 | **1.12** | −4.42 | −6.65 | −7.76 | −8.45 | [0.99; 1.09] |
| Weekly data | | | | | | | | | | |
| $z_t/d_0$ | 0.00 | 0.25 | 0.50 | 0.75 | 1.00 | 1.25 | 1.50 | 1.75 | 2.00 | Conf. interval |
| (i) | 7.85 | **0.49** | −3.06 | −4.81 | −5.87 | −6.50 | −6.85 | −7.16 | −7.38 | [0.20; 0.37] |
| (ii) | 7.85 | **0.40** | −3.34 | −4.93 | −5.93 | −6.39 | −6.86 | −7.17 | −7.39 | [0.20; 0.36] |
| (iii) | 5.52 | **−0.24** | −3.36 | −4.92 | −5.93 | −6.39 | −6.86 | −7.17 | −7.39 | [0.16; 0.35] |
| Daily data | | | | | | | | | | |
| $z_t/d_0$ | 0.00 | 0.25 | 0.50 | 0.75 | 1.00 | 1.25 | 1.50 | 1.75 | 2.00 | Conf. interval |
| (i) | 31.09 | −2.49 | −9.95 | −12.06 | −13.24 | −14.19 | −14.83 | −15.48 | −15.95 | [0.20; 0.23] |
| (ii) | 31.09 | −3.59 | −9.99 | −12.16 | −13.27 | −14.21 | −14.84 | −15.49 | −15.95 | [0.18; 0.21] |
| (iii) | 17.65 | −4.80 | −9.99 | −12.16 | −13.27 | −14.21 | −14.84 | −15.49 | −15.95 | [0.14; 0.17] |

(i) With no regressors; (ii) with an intercept; (iii) with an intercept and a linear time trend. In **bold**: The nonrejection values of the null hypothesis at the 5% significance level.

Gil-Alana and Robinson (1997): Frequently, misspecification inflates both numerator and denominator of $\hat{r}$ to varying degrees, and thus affects $\hat{r}$ in a complicated way. However, it may also be due to the fact that the AR coefficients are Yule–Walker estimates and, though they are smaller than one in absolute value, they can be arbitrarily close to 1. A problem then may occur in that they may be capturing the order of integration of the series by means, for example, of a coefficient of 0.99 in case of using AR(1) disturbances. In order to solve this problem, we use other less conventional forms of $I(0)$ processes. One that seems especially relevant and convenient in the context of the present tests is that proposed by Bloomfield (1973), in which the spectral density function is given by

$$f(\lambda;\tau) = \frac{\sigma^2}{2\pi}\exp\left(2\sum_{s=1}^{S}\tau_s\cos(\lambda s)\right), \tag{22}$$

where $S$ refers to the number of parameters required to describe the short run components. Bloomfield (1973) showed that the logarithm of an estimated spectral density function is often found to be a fairly well-behaved function and can thus be approximated by a truncated Fourier series. He showed that (22) approximates well to the spectral density of an ARMA($p,q$) process, where $p$ and $q$ are small values, which usually happens in economics. Like the stationary AR($p$) model, the Bloomfield (1973) model has exponentially decaying autocorrelations and thus we can use a model like this for $u_t$ in (21). Formulae for Newton-type iteration for estimating the $\tau_s$ are very simple (involving no matrix inversion), updating formulae when $S$ is increased are also simple, and we can replace $\widehat{A}$ in Appendix A by the population quantity:

$$\sum_{l=S+1}^{\infty} l^{-2} = \frac{\pi^2}{6} - \sum_{l=1}^{S} l^{-2},$$

which indeed is constant with respect to the $\tau_s$ (unlike what happens in the AR case). Table 3 displays the results based on Bloomfield (with $S = 1$) disturbances. Other values for $S$ were also tried and the results were very similar to those reported here.

If we permit autocorrelation through the model of Bloomfield (1973), the results also suggest that $H_0$ cannot be rejected with $d_0 = 1$ for stock prices when using monthly and weekly data and also for daily data (in the case of no regressors) and for dividends in all the cases. Thus, we can conclude the analysis of these two tables by saying that both stock prices and dividends contain unit roots, and though fractional hypotheses can also be plausible in some cases, the unit root cannot be rejected.

Given the nonstationary $I(1)$ character of the two individual series, we next examine the price–dividend ratio (in logs). Fig. 1 displays plots of the series for the three sampling frequencies, daily, weekly and monthly, along with their corresponding correlograms and periodograms. The correlograms show a very slow decay, while the periodograms display in the three cases a large value at the smallest frequency, implying that the three series may have a component of long memory behavior at the long run or zero frequency. Thus, a simple visual inspection at this figure suggests that the three series are not $I(0)$ stationary. Fig. 2 is similar to Fig. 1 but for the differenced data. [9] Here, the correlograms and periodograms show, especially for daily and weekly data, that the series may now be overdifferenced with respect to the zero frequency. [10]

[9] The plots of the daily and weekly data show the existence of outliers. They were also taken into account and the results did not substantially differ from those reported in the paper.

[10] The periodogram is an estimate of the spectral density function $f(\lambda)$. If a series is overdifferenced, $f(0) = 0$, and thus, we should expect a similar pattern in the periodogram.

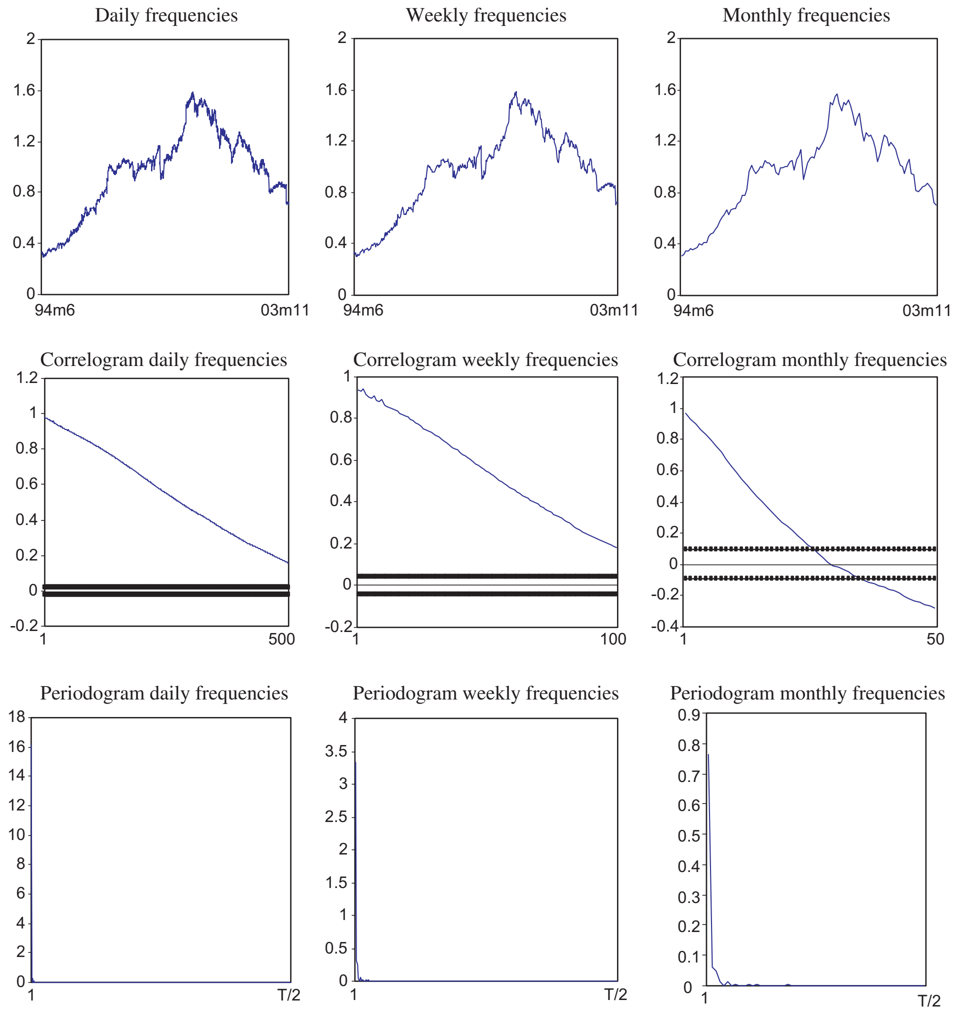


Fig. 1. Price/dividend ratios (in logs) with their corresponding correlograms and periodograms. * The large sample standard error under the null hypothesis of no autocorrelation is $1/\sqrt{T}$.

Table 4 displays the results of the same statistic as in Tables 2 and 3 for the logged price–dividend ratios. Starting with the case of white noise disturbances, we see that the results are very different depending on the periodicity of the data. For example, we cannot reject the unit-root hypothesis for this variable when using monthly data, a result that seems to support the existence of a rational bubble in the NASDAQ. However, we reject it in favor of values of $d$ lower than unity when using weekly and daily data. In these two cases, the upper limit of the intervals is strictly lower than one. In fact, the results indicate that although this variable is nonstationary ($d > 0.5$), it is mean reverting ($d < 1$).

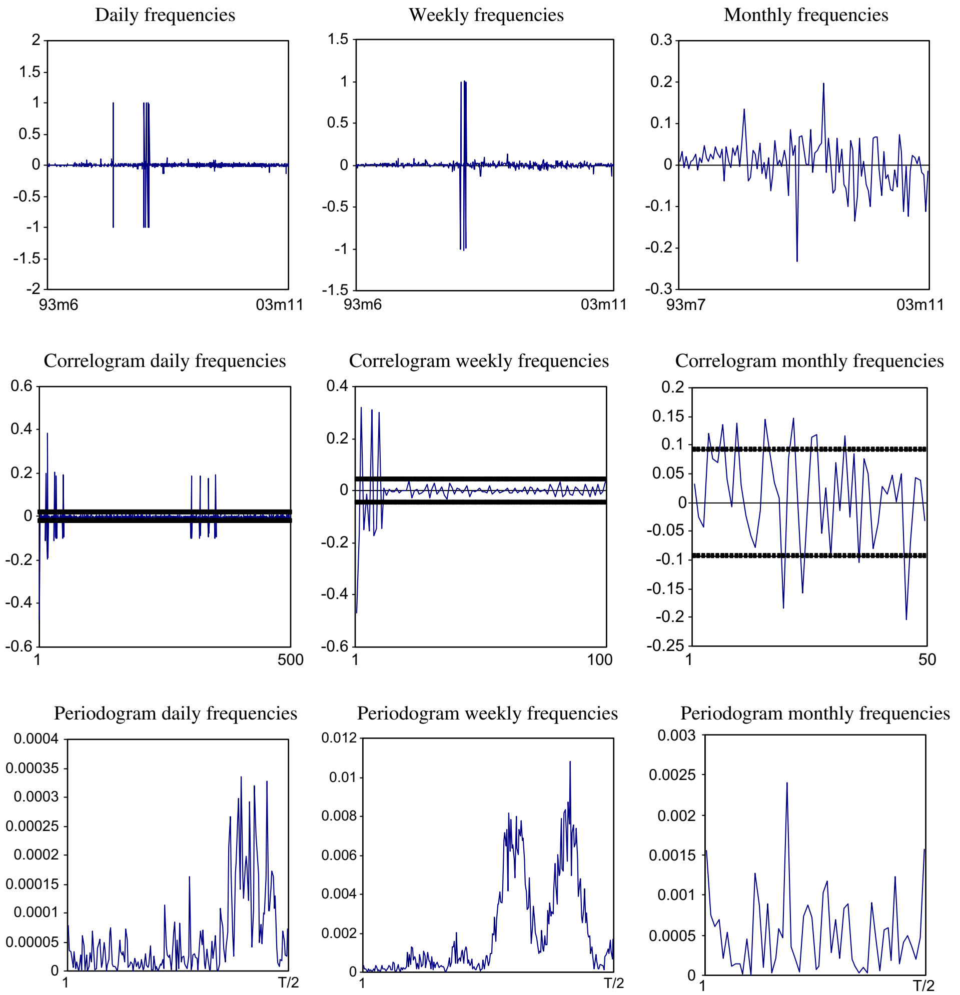


Fig. 2. First differenced series with their corresponding correlograms and periodograms. * The large sample standard error under the null hypothesis of no autocorrelation is $1/\sqrt{T}$.

If we permit autocorrelation through the model of Bloomfield (1973), the results also depend on the periodicity of the data. In the case of monthly data, we cannot reject the null hypothesis of a unit-root, which suggests the existence of a rational bubble in the NASDAQ index, while we reject this hypothesis in favor of an order of integration smaller than one when using weekly and daily data. In these two cases, we obtain a value of $d$ smaller than 0.5, suggesting thus that the price–dividend ratio will even be a stationary variable.

The results presented so far are very conclusive in favor of long memory behavior for the price dividend ratio in case of daily and weekly data. However, the proper

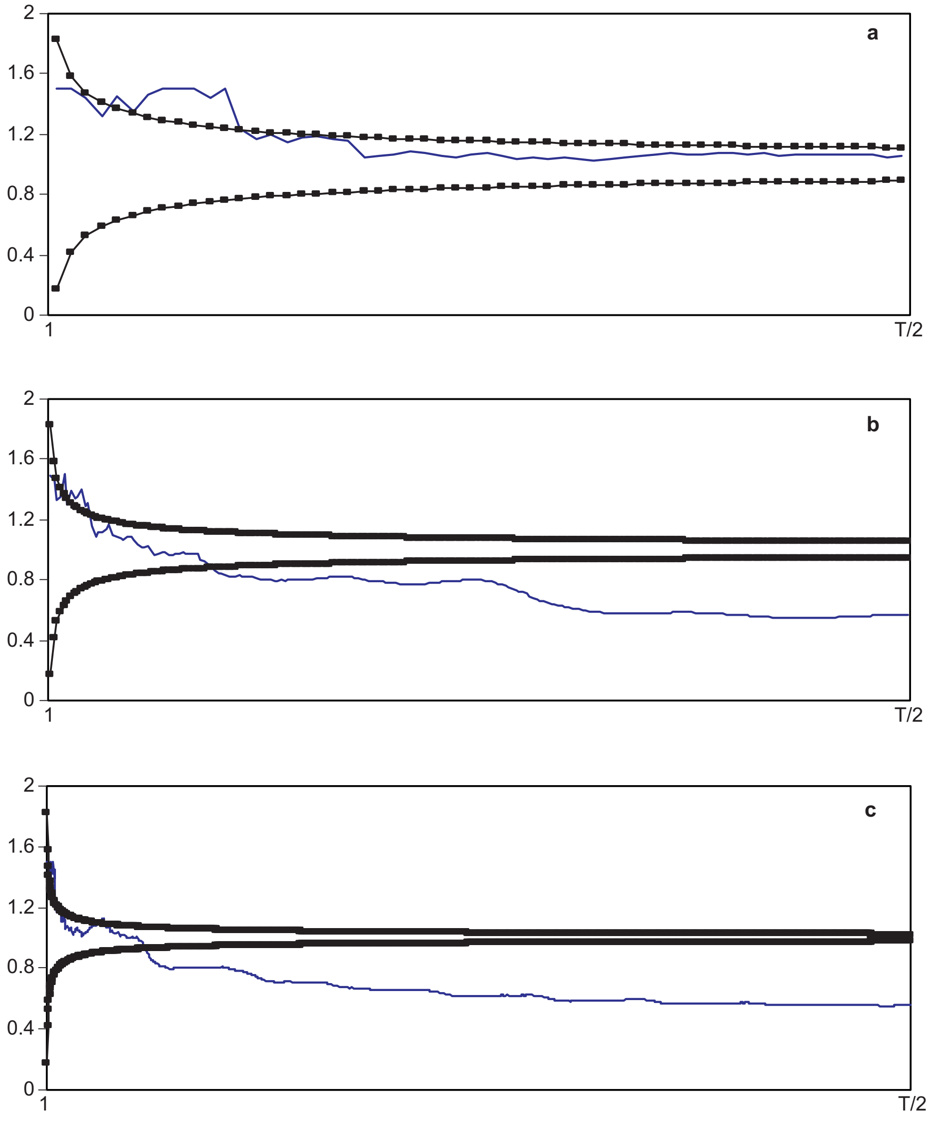


Fig. 3. (a) Quasi maximum likelihood estimates (QMLE, Robinson, 1995a) in MONTHLY data. (b) QMLE (Robinson, 1995a) in WEEKLY data. (c) QMLE (Robinson, 1995a) in DAILY data. The horizontal axis corresponds to the bandwidth parameter $m$, while the vertical one refers to the estimate of $\hat{d}$. The dotted lines corresponds to the 95%-confidence intervals of $I(1)$ stationarity.

order of integration of the series substantially varies depending on the structure of the disturbances. Thus, if they are white noise, the values are higher than 0.5, implying then nonstationary behavior. On the other hand, if we impose autocorrelation, the values are strictly smaller than 0.5, and thus being stationary. In the light of this conflicting evidence, we also display the results based on the semiparametric procedure described in Section 3.

Since it is not clear that the series are stationary, we carry out the analysis based on the first differenced data, then adding 1 to the estimated values of $d$ to obtain the proper order of integration. The results of $\hat{d}$ in (19) for the whole range of values of $m$ are displayed in Fig. 3. [11] We have also displayed in the figure the 95% confidence intervals corresponding to the $I(1)$ hypothesis. Starting with the monthly data, we observe that, apart from some small values of $m$, the estimates are within the $I(1)$ interval, which is consistent with the results obtained in Table 4. More interesting, if the frequency employed is weekly or daily, practically all values are below that interval, implying the existence of mean reversion. In both cases, the most stable behavior of the estimates takes place when $\hat{d}$ is around 0.5, which is precisely the boundary case between stationarity and nonstationarity. This may explain the discrepancies observed in Table 4 when using white noise or autocorrelated disturbances.

## 5. Summary of the empirical results

According to the present value model, the divergences of the price of a financial asset from its fundamental value should be temporary, and rational bubbles are persistent, systematic and increasing deviations of prices from their fundamental values. In this paper, we have tested for the existence of bubbles extending the results available in the previous literature in three directions. First, we analyze the presence of bubbles in the NASDAQ stock index, while most of the previous empirical analysis has focused on the S&P 500. Second, instead of using classic approaches based on $I(1)/I(0)$ integration and cointegration techniques we use a methodology based on fractional processes. Finally, we use different sampling frequencies, and analyze the robustness of our results using daily, weekly and monthly data of stock prices and dividends.

Using a version of the tests of Robinson (1994a), the results show that stock prices and dividends for the NASDAQ are both integrated of order 1 variables, which is in line with most of the empirical work on rational bubbles. Looking at the results based on the log ratio between the two variables, the results are mixed. When monthly data are used, the unit root null hypothesis cannot be rejected. However, when we use daily and weekly data, the order of integration appears to be higher than 0 but smaller than 1, suggesting that a certain degree of fractional cointegration exists between the two variables. Thus, we can reject the existence of rational bubbles since the log dividend yield presents mean reversion, with shocks affecting the series disappearing in the long run. The results show that mean reversion is obtained for

[11] Some attempts to calculate the optimal bandwidth numbers have been examined in Delgado and Robinson (1996). However, in the case of the Whittle estimator (QMLE), the use of optimal values has not been theoretically justified. Other authors, such as Lobato and Savin (1998) use an interval of values for $m$ but we have preferred to report the results for the whole range of values of $m$.

the weekly and daily data but not for the monthly case. Using a semiparametric procedure (QMLE, Robinson, 1995a) the estimates seem to be around 0.5 for the weekly and daily frequencies, and it is within the $I(1)$ interval for the monthly data.

Two arguments might explain the differences in the results when using monthly or daily (and weekly) data: the temporal aggregation problem and the sample size. The temporal aggregation problem was first mentioned in the contribution of Working (1960), although an extensive literature notes the relevance of temporal aggregation in many fields, such as the results of unit root tests (Schwert, 1989; Ng, 1995; Taylor, 2001, 2002). As pointed out by these authors, the use of low-frequency data may bias the results towards findings of slow convergence or random walk processes. For example, Taylor (2001) shows how if the actual adjustment horizon is of order of days, then monthly data cannot be expected to reveal it. In our context of stock market bubbles, we could say that if the stock prices adjustment to their fundamentals is of order of days or weeks, the use of monthly data to test for bubbles could be inappropriate and could bias the results towards finding unit roots in the price–dividend ratio, that is, towards finding bubbles. The sample size argument is consistent with the findings of Mandelbrot (1969), who argued that while many macroeconomic (or financial) series exhibit a persistent trend-cyclical behavior for a stretch of the data, when the same data is examined for a longer period, the persistent behavior tends to disappear. If this discrepancy with the results when using different sampling frequencies is due to low-powered instruments or to the temporal aggregation/sample size, or to a more profound factor calls for further research.

## 6. Concluding comments

In this paper we have used a version of the tests of Robinson (1994a) along with a semiparametric method (Robinson, 1995a) in order to examine the fractional univariate properties of stock prices, dividends and stock prices–dividends ratio (in logs) in the NASDAQ stock index.

A follow-up step in this article should be to examine the possibility of fractional cointegration. Pioneering work in this area are the papers of Cheung and Lai (1993), Baillie and Bollerslev (1994) and Dueker and Startz (1998). More recently, Gil-Alana (2003) proposes a very simple procedure for testing the null hypothesis of no cointegration against the alternative of fractional cointegration, and more elaborate techniques of fractional cointegration have been proposed by P.M. Robinson and his co-authors (e.g. Robinson and Marinucci, 2001; Robinson and Yajima, 2001; Robinson and Hualde, 2002, 2003). Extensions of the multivariate version of the tests of Robinson (1994a) which permit us to test fractional cointegration in a system-based model are being developed. There exist a reduced-rank procedure suggested by Robinson and Yajima (2002); however, it is not directly applicable here since that method assumes $I(d)$ stationarity ($d < 0.5$) for the individual series, while we consider $I(1)$ nonstationary processes. All these approaches can be performed to further examine the existence of bubbles in the NASDAQ stock market index.

The potential presence of structural breaks is another issue that may be examined in these series. Note that the tests of Robinson (1994a) described in the present paper permit us to include dummy variables to take into account structural breaks, with no effect on its standard null limit distribution. However, a proper study on this would require a detailed examination of the time and the type of break, which is out of the scope of the present work. Other issues such as the inclusion of explanatory variables or the existence of nonlinear structures will be addressed in future papers.

## Acknowledgements

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## Appendix A

The test statistic proposed by Robinson (1994a) for testing $H_0$ (16) in the model given by (17) and (12) is:

$$\widehat{R} = \hat{r}'\hat{r}, \quad \hat{r} = \left(\frac{T}{\hat{A}}\right)^{1/2} \frac{\hat{a}}{\hat{\sigma}^2},$$

where $T$ is the sample size and

$$\hat{a} = \frac{-2\pi}{T} \sum_{j=1}^{T-1} \psi(\lambda_j) g(\lambda_j; \hat{\tau})^{-1} I(\lambda_j); \quad \hat{\sigma}^2 = \sigma^2(\hat{\tau}) = \frac{2\pi}{T} \sum_{j=1}^{T-1} g(\lambda_j; \hat{\tau})^{-1} I(\lambda_j);$$

$$\hat{A} = \frac{2}{T} \left( \sum_{j=1}^{T-1} \psi(\lambda_j)^2 - \sum_{j=1}^{T-1} \psi(\lambda_j) \hat{\varepsilon}(\lambda_j)' \times \left( \sum_{j=1}^{T-1} \hat{\varepsilon}(\lambda_j) \hat{\varepsilon}(\lambda_j)' \right)^{-1} \times \sum_{j=1}^{T-1} \hat{\varepsilon}(\lambda_j) \psi(\lambda_j) \right);$$

$$\psi(\lambda_j) = \log \left| 2 \sin \frac{\lambda_j}{2} \right|; \quad \hat{\varepsilon}(\lambda_j) = \frac{\partial}{\partial \tau} \log g(\lambda_j; \hat{\tau}); \quad \lambda_j = \frac{2\pi j}{T}.$$

$\hat{\tau} = \arg\min_{\tau \in T^*} \sigma^2(\tau)$, where $T^*$ is a suitable compact subset of the $R^q$ Euclidean space. $I(\lambda_j)$ is the periodogram of $\hat{u}_t$, where

$$\hat{u}_t = (1 - L)^{d_0} y_t - \hat{\beta} w_t, \;\; w_t = (1 - L)^{d_0} z_t; \quad \hat{\beta} = \left( \sum_{t=1}^{T} w_t w_t' \right)^{-1} \sum_{t=1}^{T} w_t (1 - L)^{d_0} z_t,$$

and $g$ above is a known function coming from the spectral density of $u_t$:

$$f(\lambda_j; \tau) = \frac{\sigma^2}{2\pi} g(\lambda_j; \tau).$$

Note that these tests are purely parametric and, therefore, they require specific modeling assumptions to be made regarding the short memory specification of $u_t$. Thus, for example, if $u_t$ is white noise, $f = \sigma^2/2\pi$, and $g \equiv 1$, and if $u_t$ is AR(1) of the form: $u_t = \tau u_{t-1} + \varepsilon_t$, $g(\lambda_j;\tau) = |1 - \tau e^{i\lambda_j}|^{-2}$, with $\sigma^2 = V(\varepsilon_t)$, so that the AR coefficient corresponds to the parameter $\tau$.

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