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Design and Development of a Web–Based Application for Solving Linear Physical Systems using Gauss-Seidel Iterative Method

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Abstract: ***This work focused on the design and development of a web-based application for numerical solution of physical systems using Gauss-Seidel iteration method. The work was limited to a system of three linear equations which is about the systems of equations mostly encountered when solving practical problems involving many physical systems. JavaScript, a web compatible program was used to develop the application. The application was tested using three systems of linear equations and it was seen to achieve, universality, accessibility, accuracy and speed. The application will serve as a tool that will help eliminate errors; since iterative processes are prone to errors and end up wasting time when performed with pen and paper . The work envisages computational Physics as a means of bringing back the attention of student to world of work and purposeful learning. This application will be an addition to database of the few existing web-based applications in the teaching and learning of Physics.***

Keywords: Computational, Gauss-Seidel, Numerical, Iteration, Web-based.

1. Introduction

The greatest contribution of cyber age technology is the development of computer and its use in all works of life. The use of computer in teaching learning process has brought many stages of its evolution. A host of research studies have been conducted to explore the effectiveness of computer Aided Learning (CAL) in various fields of study and at different levels of education.

Computer-Aided Learning is the process by which written and visual information is presented in a logical sequence to learner through the use of a computer. The student learns by reading the text material presented or by observing the graphic information displayed. CAL can be characterized as interactive and individualized learning as it usually involves a dialogue between one student and a computer program and the student can learn at his own pace and time frame. [5] in [9].

Around the world most people have come to believe that we should embrace the approaches which place a greater emphasis on student centered learning processes in which students are responsible for learning, rather than the approaches based on teacher-centered instruction. Only by using such methods it is possible to bring forth the creative thinking, intelligence, and individual skills of students [2,10]

When these methods of teaching are applied to the teaching-learning process, great wealth can be derived in the achievement of students. As the relationship between teaching, research and computer evolution, new tools are required not only to treat numerical problems, but also to solve various diverse problems from algorithm implementations, computer modeling, simulation and visualization to website design and presentation [1].

Systems of linear equations arise in a large number of areas both directly in modeling physical situations and indirectly in the numerical solutions of the other mathematical models. These applications occur in virtually all areas of the physical, biological and social sciences [7].

Some of these applications are smeared in Physics and Engineering examples include mechanical system of forces, electrical networks under Kirchhoff's rules, fluid flow and many others. Solving such systems of linear equations usually requires a method that will provide the much needed accuracy and speed which is imperative since there exist many methods of solving such systems. The direct methods of solving linear equations are known to have their difficulties. Turner (1989), envisaged that the problem with Gauss elimination approach lies in control of the accumulation of rounding errors [7].

Iterative, or *indirect methods*, start with an initial guess of the solution $\mathbf{x}$ and then repeatedly improve the solution until the change in $\mathbf{x}$ becomes negligible. Since the required number of iterations can be very large, the indirect methods are, in general, slower than their direct counterparts. However, iterative methods do have the following advantages that make them attractive for certain problems:

1) It is feasible to store only the nonzero elements of the coefficient matrix. This makes it possible to deal with very large matrices that are sparse, but not necessarily banded. In many problems, there is no need to store the coefficient matrix at all.
2) Iterative procedures are self-correcting, meaning that round off errors (or even arithmetic mistakes) in one iterative cycle are corrected in subsequent cycles [8].

The most preferred method of approaching such problems is using numerical iterative methods. Since these methods overcomes the rounding errors associated with direct methods. Several iterative methods exist ranging from Jacobi, Gauss-Seidel, successive over relaxing method, gradient methods and so on. An iterative method for solving a linear system constructs an iteration series that under some conditions converges to the exact solution of the system.

Since iterative methods may involve many iterates or loops the question of how fast an iteration process converges and how accurate the results are. Iterative methods have the advantage of simplicity of operation and ease of implementation on computers, and they are relatively insensitive to propagation of errors; they would be used in preference to direct methods for solving linear systems involving several hundred variables particularly, if many of the coefficients were zero.

Performing these iterative processes with pen and paper is prone to a lot of errors and this could be daunting. Most students of Physics and Engineering today find it boring to sit and perform up to five to seven iterates thereby neglecting this aspect of mathematics wherever it appears in their problems. Therefore a more accurate and fast computer application can be employed to solve these problems. This will also draw the attention of students back to numerical processes which yield better results.

Many computer programming languages (MATLAB, SCILAB, Visual Basic, C++, C#,…) can be used to program iterative processes. Using a web-based application that is compatible with all browsers and user platforms is of great importance since accessibility is one target considering the developing nations. From the foregoing, JavaScript language comes to mind as the most suitable language that is web friendly with all browsers and susceptible to most user platforms (Laptops, Desktops, mobile phones, and any other platform that can be browsed with).

In this study, the Gauss-Seidel iterative method was programmed with JavaScript codes to achieve, a web-based solution of systems of linear equations involving physical systems. This work is significant because it will help to eke out curiosity in students and bring back their attention to Physics since the application is accessible all platforms.

2. Literature Survey

2.1. Gauss-Seidel Method

The Gauss-Seidel Method of solving linear systems is a technical improvement over the Jacobi method. This is utilized in the solution of a square system of n linear equations with unknown x

$Ax = b$ 1

It is defined by the iteration

$L_* x^{(k+1)} = b - U x^{(k)}$ 2

Where $x^{(k)}$ is the kth approximation of x. $x^{(k+1)}$ is the next iteration of y. L_* is the lower triangle component of the matrix A. the upper triangle is U. hence the matrix A can be written as

$A = L_*$ +U 3 [6]

We have that:

$$x_i^{(k+1)} = \frac{1}{a_{jj}}\left(b_i - \sum_{j<i} a_{ij}\, x_i^{(k+1)} - \sum_{j>i} a_{ij}\, x_j^{(k)}\right),$$

$where\ i,j = 1,2,\dots,n$

[6].

The convergence properties of the Gauss-Seidel method are dependent on the matrix A. the process converges if A is symmetric positive definite and A is strictly or irreducibly diagonally dominant. The Gauss-Seidel method sometimes converges even if these conditions are not satisfied.

2.2 Web-based Applications

There exist many stand-alone programs in different languages that can perform iterative processes but a few exist that will operate on all platforms on the web. A web application or web app is defined as any software that runs in a web browser [4]. It is created in a browser-supported programming language (such as the combination of JavaScript, HTML and CSS) and relies on a web browser to render application. It can also be referred to as any program that is accessed over a network connection using HTTP, rather than existing within a device's memory.

JavaScript is a scripting language designed primarily for adding interactivity to web pages and creating web applications. JavaScript is widely supported; it is available in the following browsers: Netscape Navigator (beginning with version 2.0), Microsoft Internet Explorer (beginning with version 3.0), Firefox, Safari, Opera, and Google Chrome. JavaScript is the language used in writing the iterative program in this research work, this is because JavaScript is one of the most adaptive, flexible, versatile and effective of languages used on the web.

3. Methodology

3.1. Design Method

This Web-based application is a dynamic application, programmed in HTML environment using JavaScript. Adobe DreamWeaver CS3 was used as a compiler for the program. In order to achieve the full functionalities of a flexible and enticing web pages, cascading style sheets (CSS) were used.

3.2 Algorithm

The step-by- step solution to the problem is shown below:

We assume that a_{jj} =1 for j =1,…,n. This can be achieved if we can rearrange the equations so that no diagonal coefficient is zero; then we divide each equation by the corresponding diagonal coefficient. We have

$A = I + L + U\ (a_{jj} = 1)$ 3.1

Where I is the $n \times n$ unit matrix.

$Ax = (I + L + U)x = b$ 3.2

Taking Lx and Ux to the right we have

$x = b - Lx - Ux$ 3.3

From (3.3) we took new approximations and the main diagonal old ones. Thus;

$x^{(k+1)} = b - Lx^{(k+1)} - Ux^{k}$ 3.4

Where $x^{(k+1)}$ and x^{k} are the new and old approximations respectively. At this point our approximation a_{jj}=1 is no longer valid since it is taking care of automatically by the factor $1/a_{jj}$.

The algorithm computes solution for x of the system Ax=b given initial approximation $x^{(0)}$, where $A = [a_{jk}]$ is an $n \times n$ matrix with $a_{jj} \neq 0, j = 1, \dots, n$

INPUT: A, b initial approximation $x^{(0)}$, tolerance $\epsilon > 0$, maximum number of iterations N.

$For\ k = 0, \dots, N-1, do:$

$For\ j = 1, \dots n, do:$

$$x_j^{(k+1)} = \frac{1}{a_{jj}}\left(b_j - \sum_{i=1}^{j-1} a_{ji}\, x_i^{(k+1)} - \sum_{i=j+1}^{n} a_{ji}\, x_i^{(k)}\right)$$

End

$if\ max\left|x_j^{(k+1)} - x_j^{k}\right| < \epsilon$

$then\ OUTPUT\ x^{(k+1)}$ Stop

End

OUTPUT: no satisfying solution. The tolerance condition failed.

3.3 Program Design and Structure

The application is a two page application that allows the user to enter all the coefficients of the system of linear equation in form of matrix A.

$$\begin{pmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{pmatrix}\begin{pmatrix} x_1 \\ x_2 \\ x_3 \end{pmatrix} = \begin{pmatrix} b_1 \\ b_2 \\ b_2 \end{pmatrix}$$

Where x_1, x_2 and x_3 are the expected values that solves the system. We have represented a system of three linear equations to depict most of the cases obtained while solving physical problems. The designed view of the program is shown in fig1. below.

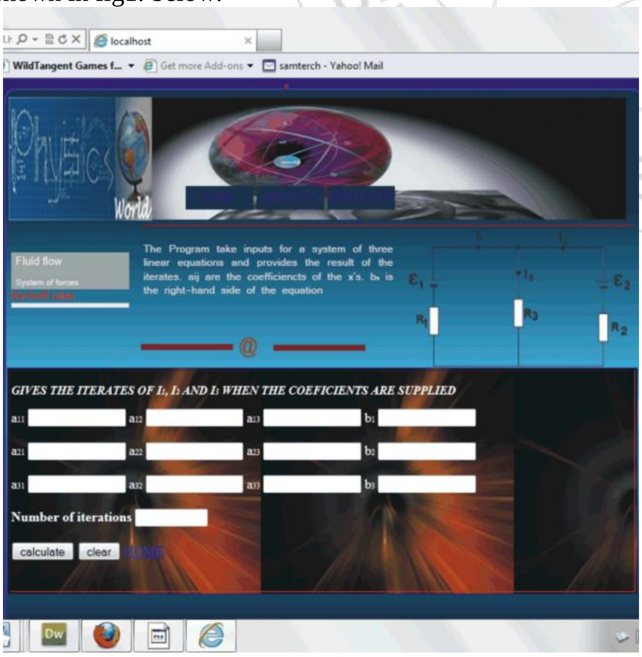

Figure 1: Designed web-based application for Gauss-Sidel iteration

After the coefficients of the system a_{11}, a_{12}, a_{13}, $b_1 \dots, a_{jj}$ is inputted and the calculate button clicked. The result is displayed on a new page.

4. Results and Discussion

Here, examples on systems of linear equations were taken and tested in the application to see the results of the iterates.

(a)Suppose the system below

$5x_1 + 2x_2 - x_3 = 6$

$x_1 + 6x_2 - 3x_3 = 4$

$2x_1 + x_2 + 4x_3 = 7$

Inputting the values of the coefficients in the program we have

Table 1: Results of iterates for example 1

i_1	i_2	i_3
0.0000	0.0000	0.0000
1.2000	0.4667	1.0333
1.2200	0.9800	0.8950
0.9870	0.9497	1.0191
1.0239	1.0056	0.9866
0.9951	0.9941	1.0039
1.0031	1.0014	0.9981
0.9991	0.9992	1.0007
1.0005	1.0003	0.9997
0.9998	0.9999	1.0001
1.0001	1.0000	1.0000

The results as displayed on the browser page is shown in fig2.

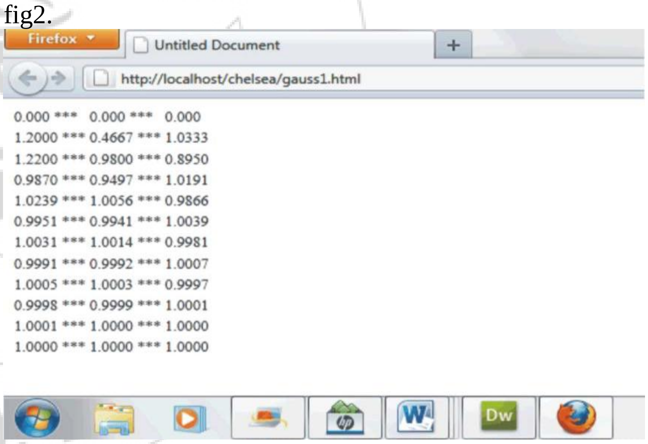

Figure 2: Results of example (a) on the browser window

In order to exhibit the relevance this kind of systems in Physics, we used examples in electrical network under Kirchhoff's rules; where the voltage rule states that: *In any closed loop in a network, the algebraic sum of the voltage drops (i.e. products of current and resistance) taken around the loop is equal to the resultant e.m.f. acting in that loop* [3]

(b) Consider the electrical network below

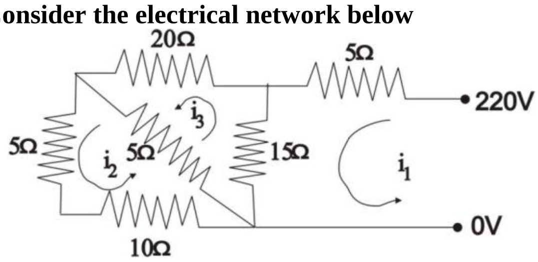

Figure 3: Electrical network

The system of equations developed for the three loops using Kirchhoff's rule stated above, is shown below:

$$20i_1 + 0i_2 - 15i_3 = 220$$
$$0i_1 + \quad 20i_2 - 5i_3 = 0$$
$$-15i_1 - 5i_2 + 40i_3 = 0$$

Inputting the coefficients of the values of the currents in the web-based application we obtain the following iterates.

Table 2: Results of iterates for example 2

i_1	i_2	i_3
0.0000	0.0000	0.0000
11.0000	0.0000	4.1250
14.0938	1.0313	5.4141
15.0606	1.3535	5.8169
15.3627	1.4542	5.9428
15.4571	1.4857	5.9821
15.4866	1.4955	5.9944
15.4958	1.4986	5.9982
15.4987	1.4995	5.9994
15.4995	1.4998	5.9998
15.4999	1.4999	5.9999
15.4999	1.5000	6.0000
15.5000	1.5000	6.0000

The result of the iterates as it appears on the browser window is shown in fig2. below.

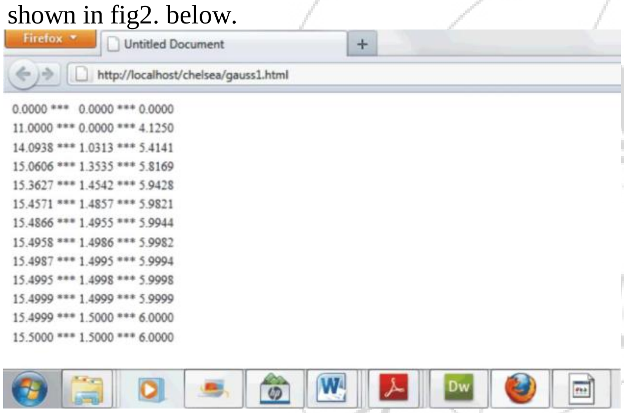

Figure 4: Results of example (b) on the browser window

(c)We also consider an example in electrical network using Kirchhoff's rules which is one of the practical examples in Physics below:

$$4i_1 + 2i_2 - i_3 = -6.3$$
$$0i_1 + \quad 3i_2 + 4i_3 = 14.8$$
$$i_1 - i_2 + 5i_3 = 13.5$$

Inputting the coefficient of currents i_1, i_2, and i_3; the result of the computed iteration is given below:

Table 3: Results of iterates for example 3

i_1	i_2	i_3
0.0000	0.0000	0.0000
-1.5750	4.9333	4.0017
-3.0412	-0.4023	3.2278
-0.5669	0.6296	2.9393
-1.1550	1.0143	3.1338
-1.2987	0.7549	3.1107
-1.1748	0.7857	3.0921
-1.1948	0.8105	3.1011
-1.2050	0.7985	3.1007
-1.1991	0.7991	3.0996
-1.1997	0.8005	3.1000
-1.2003	0.8000	3.1001
-1.2000	0.7999	3.1000
-1.1999	0.8000	3.1000
-1.2000	0.8000	3.1000

The result is also shown on another page of the browser in figure two below:

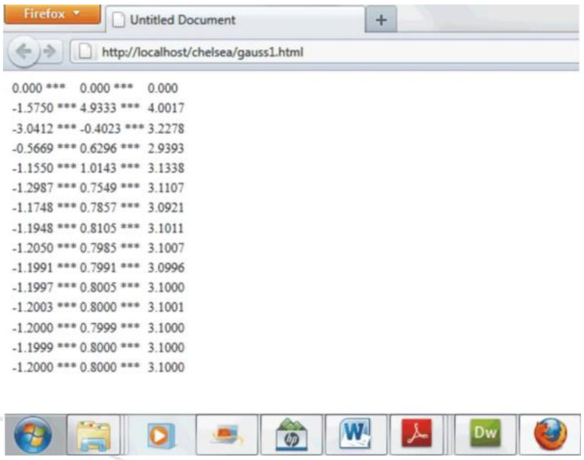

Figure 5: Results of example (c) on the browser window

The source code for the program is shown in the appendix.

5. Conclusion

In this research a web-based application for numerical solution of linear systems using Gauss-Seidel iterative method iteration has been developed and tested. The application performed effectively and is compatible with most browsers and is compliant with many platforms even with the mobile phones this will enhance accessibility even in regions of the world where computers and electricity supply is scarce.

This application will be an addition to database of existing applications in the teaching and learning of Physics. We therefore recommend that other web-based numerical applications should be designed in so many areas of physical Sciences to solve like problems.

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Appendix

```
//JavaScript Codes
<script type="text/javascript">
//program gets data from form.
var a1 = parseFloat(document.getElementById("a1").value);
var a2 = parseFloat(document.getElementById("a2").value);
var a3 = parseFloat(document.getElementById("a3").value);
var b1 = parseFloat(document.getElementById("b1").value);
var b2 = parseFloat(document.getElementById("b2").value);
var b3 = parseFloat(document.getElementById("b3").value);
var c1 = parseFloat(document.getElementById("c1").value);
var c2 = parseFloat(document.getElementById("c2").value);
var c3 = parseFloat(document.getElementById("c3").value);
var d1 = parseFloat(document.getElementById("d1").value);
var d2 = parseFloat(document.getElementById("d2").value);
var d3 = parseFloat(document.getElementById("d3").value);
var n = parseInt(document.getElementById("n").value);
//Program performs iteration
if(a1!=0 || b2!=0 || c3!=0)
{
x1=0; x2=0; x3=0;
for (i=1; i<=n; i++)
{
 x1=(d1-(b1*x2)-(c1*x3))/a1;
 x2=(d2-(a2*x1)-(c2*x3))/b2;
 x3=(d3-(a3*x1)-(b3*x2))/c3;
 var x1=x1.toFixed(4);
 var x2=x2.toFixed(4);
 var x3=x3.toFixed(4);
//Program prints out the result of the iteration
document.write("<tr><td>");
document.siedel.value= x1;;
document.write("*** ");
document.write("<td>");
document.siedel.value= x2;
document.write(" *** ");
document.write("<td>");
document.siedel.value= x3;
document.write("<br>");
 }
}
else
{
document.write("no satisfying solution. The tolerance condition failed. ajj
        must not be zero, rearrange your equations so that no main
        diagonal element is zero");
}
</script>
```

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