College of Professional Studies
Northeastern University San Jose

MPS Analytics

Course: ALY 6050 - Enterprise Analytics

Assignment:

MODULE PROJECT - 6

Submitted on:

March 31, 2023

Submitted to:

Professor: Azadeh Mobasher

Submitted by:

ARCHIT BARUA

Part 1

Introduction

In this assignment, we are faced with a real-world logistics problem encountered by the Rockhill Shipping & Transport Company. Our task is to help the company's manager, Allen, negotiate a new shipping contract with Chimotoxic, a chemical manufacturer, to transport hazardous waste products from six different plants to three waste disposal sites. The main objective is to determine the most cost-effective shipping routes while ensuring that the hazardous waste is transported safely and in compliance with local regulations.

To solve this problem, we plan to use linear programming to model and find the optimal shipping routes and their corresponding costs. Additionally, we will explore the possibility of dropping and picking up loads at intermediate points and determine whether this approach offers any cost savings. Our analysis will require us to estimate the costs of shipping barrels of hazardous waste from each of the six plants to each of the three waste disposal sites.

Through this exercise, we will gain practical experience in applying linear programming techniques to solve complex logistics problems in a real-world setting. Our goal is to develop a contract proposal to submit to Chimotoxic for waste proposal. We will determine the optimal shipping routes and their respective optimal costs for both direct shipping and intermediate shipping points. By working together, we can help Allen make informed decisions when negotiating future shipping contracts with hazardous materials.

Analysis

Given

The three waste disposal sites at Orangeburg, Florence, and Macon can respectively accommodate a maximum of 65, 80, and 105 barrels per week.

	Waste Proposal Site		
Plant:	***Orangeburg***	***Florence***	***Macon***
Denver	$12	$15	$17
Morganton	14	9	10
Morrisville	13	20	11
Pineville	17	16	19
Rockhill	7	14	12
Statesville	22	16	18

Table 1: Shipping costs, per barrel of waste from six plants to three waste disposal sites

Plant:	***Waste per Week (bbl)***
Denver	45
Morganton	26
Morrisville	42
Pineville	53
Rockhill	29
Statesville	38

Constraints **(Direct Shipping)**

Constaints	LHS		RHS
Denver	45	=	45
Morganton	26	=	26
Morrisville	42	=	42
Pineville	53	=	53
Rockhill	29	=	29
Statesville	38	=	38
Orangeburg	65	<=	65
Florence	80	<=	80
Macon	88	<=	105

The objective is to minimize the total shipping cost.

DIRECT SHIPPING

	Waste Proposal Site			
Plant:	*Orangeburg*	*Florence*	*Macon*	*Waste per Week (bbl)*
Denver	$12	$15	$17	45
Morganton	$14	$9	$10	26
Morrisville	$13	$20	$11	42
Pineville	$17	$16	$19	53
Rockhill	$7	$14	$12	29
Statesville	$22	$16	$18	38
Maximum Cap	65	80	105	

Constaints	LHS		RHS
Denver	45	=	45
Morganton	26	=	26
Morrisville	42	=	42
Pineville	53	=	53
Rockhill	29	=	29
Statesville	38	=	38
Orangeburg	65	<=	65
Florence	80	<=	80
Macon	88	<=	105

	Waste Proposal Site		
Plant:	*Orangeburg*	*Florence*	*Macon*
Denver	36	9	0
Morganton	0	0	26
Morrisville	0	0	42
Pineville	0	53	0
Rockhill	29	0	0
Statesville	0	18	20

Total cost	$2,988
total waste shipped from	233
total waste shipped to	233

The LP method in Excel Solver was applied to solve the logistics problem. The solution obtained indicated that the optimal shipping routes for transporting hazardous waste material from six plants to three waste disposal sites were as follows:

Denver: 36 barrels to Orangeburg and 9 barrels to Florence

Morganton: 26 barrels to Macon

Morrisville: 42 barrels to Macon

Pineville: 53 barrels to Florence

Rockhill: 29 barrels to Orangeburg

Statesville: 18 barrels to Florence and 20 barrels to Macon

The solution met the production requirements for each plant and the waste disposal capacity of each site. The total waste material shipped was 233 barrels, which was equal to the total amount of waste generated. The total waste material shipped from and to the sites were also equal, with 233 barrels of waste material being shipped in total. Specifically, 65 barrels were shipped to Orangeburg, 80 barrels to Florence, and 88 barrels to Macon, which were within the waste disposal capacity of each site.

The minimum shipping cost was $2,988. Linear programming proved useful in determining the optimal shipping routes that minimized the total shipping cost of hazardous waste while ensuring safety and compliance with local regulations. The obtained solution will help Rockhill Shipping & Transport Company develop a contract proposal to submit to Chimotoxic for waste proposal.

Transshipment

Aside from shipping directly from the plants to the waste disposal sites, Allen, the manager of Rockhill Shipping & Transport Company, is also considering using the plants and waste disposal sites as intermediate shipping points. In this scenario, a truck would drop off the load at a plant or a disposal site, where another truck would pick it up and transport it to the final destination. Chimotoxic, the chemical manufacturer, has agreed to handle all the handling costs for the plants and disposal sites, meaning that Rockhill will only incur transportation costs.

Given the possibility of intermediate shipping points, Allen wants to make an informed decision regarding the most cost-effective shipping method. He wants to determine if dropping and picking up loads at intermediate points would offer cost savings compared to shipping directly to the waste disposal sites. Linear programming techniques can be used to model and solve this problem, and the obtained solution will help Allen make an informed decision regarding the optimal shipping method to use.

	Plant					
Plant:	**Denver**	**Morganton**	**Morrisville**	**Pineville**	**Rockhill**	**Statesville**
Denver	$---	$3	$4	$9	$5	$4
Morganton	6	---	7	6	9	4
Morrisville	5	7	---	3	4	9
Pineville	5	4	3	---	3	11
Rockhill	5	9	5	3	---	14
Statesville	4	7	11	12	8	---

Table 2: Shipping costs, per barrel of waste from each plant to another plant

	Waste Proposal Site		
Waste Disposal Site:	**Orangeburg**	**Florence**	**Macon**
Orangeburg	$---	$12	$10
Florence	12	---	15
Macon	10	15	---

Table 3: Shipping costs, per barrel of waste between the three waste disposal sites

Transshipment

Cost	Plants & Waste Proposal Sites								
	Denver (D)	Morganton (M)	Morrisville (MV)	Pineville (P)	Rockhill (R)	Statesville (S)	Orangeburg (O)	Florence (F)	Macon (M)
Denver (D)	3000	3	4	9	5	4	12	15	17
Morganton (M)	6	3000	7	6	9	4	14	9	10
Morrisville (MV)	5	7	3000	3	4	9	13	20	11
Pineville (P)	5	4	3	3000	3	11	17	16	19
Rockhill (R)	5	9	5	3	3000	14	7	14	12
Statesville (S)	4	7	11	12	8	3000	22	16	18
Orangeburg (O)	3000	3000	3000	3000	3000	3000	3000	12	10
Florence (F)	3000	3000	3000	3000	3000	3000	12	3000	15
Macon (M)	3000	3000	3000	3000	3000	3000	10	15	3000

To ensure that Excel's Solver function can operate effectively without interfering with the results, a value of $3000 was assigned to cells where no transport cost value was provided. This value was chosen to satisfy the requirement of having a value in those cells while being high enough not to influence the optimization process. By assigning a uniform value, the Solver function could focus on finding the optimal shipping routes while still considering the possibility of dropping and picking up loads at intermediate points. This technique helped to ensure the accuracy and effectiveness of the optimization process in finding the most cost-effective shipping routes for Rockhill Shipping & Transport Company.

	Plants & Waste Proposal Sites												
Decision	**Denver (D)**	**Morganton (M)**	**Morrisville (MV)**	**Pineville (P)**	**Rockhill (R)**	**Statesville (S)**	**Orangeburg (O)**	**Florence (F)**	**Macon (M)**	**Transient:**		**Waste Generation:**	**Outward - Inward (Retaining Supply):**
Denver (D)	0	45	0	0	0	0	0	0	0	45	=	45	45
Morganton (M)	0	0	0	0	0	0	0	42	46	88	=	26	26
Morrisville (MV)	0	0	0	0	0	0	0	0	42	42	=	42	42
Pineville (P)	0	17	0	0	36	0	0	0	0	53	=	53	53
Rockhill (R)	0	0	0	0	0	0	65	0	0	65	=	29	29
Statesville (S)	0	0	0	0	0	0	0	38	0	38	=	38	38
Orangeburg (O)	0	0	0	0	0	0	0	0	0	0	≤	65	
Florence (F)	0	0	0	0	0	0	0	0	0	0	≤	80	
Macon (M)	0	0	0	0	0	0	0	0	0	0	≤	105	
Shipped To:	0	62	0	0	36	0	65	80	88				
	=	=	=	=	=	=	≤	≤	≤				
Disposal:	45	26	42	53	29	38	65	80	105				
Outward - Inward (Retaining Supply):							65	80	88				

Total Cost	$2,674
total waste generation	233
total waste disposal	250

After utilizing Excel's Solver function, we have obtained the optimal solution for minimizing the trans-shipment cost associated with shipping hazardous waste for Rockhill Shipping & Transport Company. This solution proposes that all of the company's plants and waste disposal sites be used as intermediary nodes. By implementing the proposed solution, hazardous waste can be shipped in compliance with local regulations while also minimizing transportation costs. The cost associated with the minimized trans-shipment of hazardous waste is $2,674.00 per week.

Furthermore, the optimal solution specifies the number of units of waste barrels that should be shipped from each plant to the waste disposal sites. Specifically, the optimal number of units of waste barrels to be shipped are:

Denver: 45 to Morganton

Morganton: 42 to Florence, 46 to Macon

Morrisville: 42 to Macon

Pineville: 17 to Morganton, 36 to Rockhill

Rockhill: 65 to Orangeburg

Statesville: 38 to Florence

By following the optimal shipping routes and the proposed number of units of waste barrels to be shipped, Rockhill Shipping & Transport Company can ensure the safe and compliant transportation of hazardous waste while minimizing transportation costs. This optimal solution will assist Rockhill Shipping & Transport Company in developing a contract proposal to submit to Chimotoxic for waste disposal.

Conclusion

Based on the analysis, it is recommended that the manager choose the transshipment plan for shipping hazardous waste. This is because it is more cost-effective compared to directly shipping from the plants to the waste sites. The cost for direct shipping is $2,988, while the cost for transshipment is only $2,674, which is a significant difference of $314 per week. Therefore, the transshipment plan offers a substantial cost-saving opportunity for the company.

In conclusion, it is advisable for the manager to opt for the transshipment plan for the transportation of hazardous waste. This plan involves dropping and picking up loads at various plants and waste sites, which would cost the company $314 less per week compared to direct shipping from plants to waste sites. By implementing this plan, the company can reduce costs and improve efficiency in the transportation of hazardous waste.

Part 2

Introduction

As an investor, it is important to carefully allocate funds in a portfolio to achieve desirable returns while minimizing risk. In this scenario, the investor has selected six asset types with their corresponding expected returns and covariance matrix. The goal is to determine the optimal allocation of $10,000 among these assets to achieve a minimum baseline expected return of 11% while minimizing risk. This problem can be solved using a quadratic model, which involves equations with at least one squared variable. By varying the baseline expected return and obtaining corresponding solutions for risk and portfolio return, we can observe any patterns in the relationship between risk and portfolio return. Therefore, this project aims to use mathematical models to assist the investor in making informed decisions for their portfolio.

Given

	Expected Returns
Bonds	0.07
High tech stocks	0.12
Foreign stocks	0.11
Call options	0.14
Put options	0.14
Gold	0.09

	Bonds	High tech stocks	Foreign stocks	Call options	Put options	Gold
Bonds	0.001	0.0003	-0.0003	0.00035	-0.00035	0.0004
High tech stocks		0.009	0.0004	0.0016	-0.0016	0.0006
Foreign stocks			0.008	0.0015	-0.0055	-0.0007
Call options				0.012	-0.0005	0.0008
Put options					0.012	-0.0008
Gold						0.005

Table 1: The Covariance matrix of assets' returns

Analysis

	Bonds	High tech stocks	Foreign stocks	Call options	Put options	Gold
Expected Returns	0.07	0.12	0.11	0.14	0.14	0.09

	Bonds	High tech stocks	Foreign stocks	Call options	Put options	Gold
Bonds	0.001	0.0003	-0.0003	0.00035	-0.00035	0.0004
High tech stocks	0.0003	0.009	0.0004	0.0016	-0.0016	0.0006
Foreign stocks	-0.0003	0.0004	0.008	0.0015	-0.0055	-0.0007
Call options	0.00035	0.0016	0.0015	0.012	-0.0005	0.0008
Put options	-0.00035	-0.0016	-0.0055	-0.0005	0.012	-0.0008
Gold	0.0004	0.0006	-0.0007	0.0008	-0.0008	0.005

The covariance matrix presented in the table above follows the principle that the diagonal elements correspond to the variance of each asset while the off-diagonal entries represent the covariance between pairs of assets. The matrix is symmetric, and only the upper triangle is shown in the figure, implying that Mij is equal to Mji. Therefore, we have filled in the blank covariance values based on this concept to complete the matrix.

1. Suppose that our investor wishes to invest $10,000 in this portfolio. Determine how he should allocate this investment to the individual assets in his portfolio in order to have a minimum baseline expected return of 11%, and at the same time, at a minimum risk.

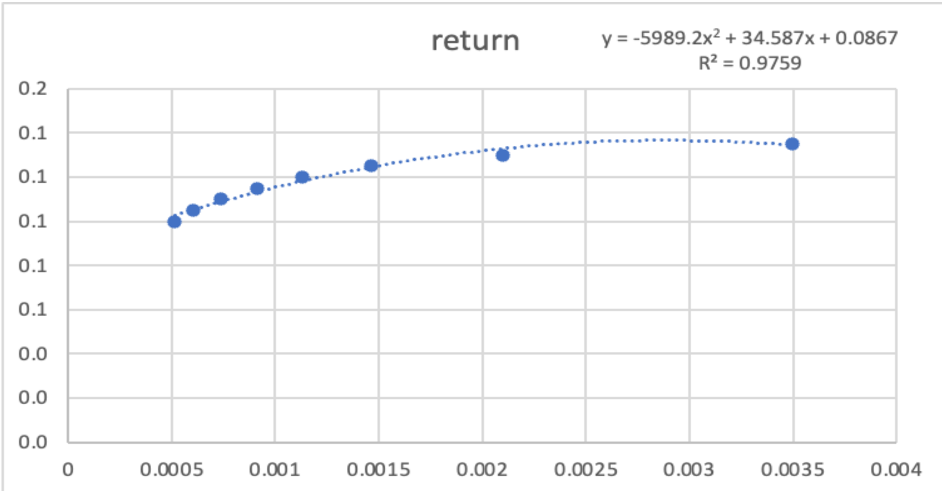
	Weights	Allocation	Return	0.11
Bonds	0.1898059	**$1,898.06**	Variance	0.000736
High tech stocks	0.108631	**$1,086.31**	SD	0.027123
Foreign stocks	0.270828	**$2,708.28**		
Call options	0.047943	**$479.43**		
Put options	0.25447	**$2,544.70**		
Gold	0.128323	**$1,283.23**		
Total	1	**$10,000**		

The table presented illustrates the expected returns of the six assets in the portfolio, namely Bonds, High tech stocks, Foreign stocks, Call options, Put options, and Gold. The light yellow-colored decision variables in the table denote the investment weight of each asset, while the blue constraint represents the minimum portfolio return of 11%. The goal is to minimize portfolio variance, which can be computed using the formula = MMULT(TRANSPOSE(weights cells), MMULT(covariance matrix, weights cells)). Meanwhile, the portfolio return is calculated using = MMULT(returns cells, weight cells). We applied the GRG nonlinear method using Excel Solver and obtained the optimal investment weights for each asset, with Bonds, High tech stocks, Foreign stocks, Call options, Put options, and Gold having investment weight percentages of 18.98%, 10.86%, 27.08%, 4.79%, 25.45%, and 12.83%, respectively. Using these weights, we can allocate the investment of $10,000 across the six assets. The resulting portfolio return is 11%, while the minimum portfolio variance is 0.000736, and the minimum standard deviation is 0.0271.

2. Let the solution pair be denoted by (r, e), where “r” denotes the minimized risk and “e” denotes the expected portfolio return after the problem is solved. Use successive values of 10%, 10.5%, 11%, 11.5%, 12%, 12.5%, 13% and 13.5% as the baseline return values to obtain eight pairs of solutions (r, e). Plot “e” versus “r”. Explain whether there exists a pattern in this plot. In other words, explain, in your opinion, the type of mathematical relationship that “r” and “e” may have.

minimized risk	return
0.000513907	0.1
0.000603273	0.105
0.000735635	0.11
0.000910995	0.115
0.001129351	0.12
0.00146350	0.125
0.00209792	0.13
0.003496248	0.135

In order to obtain the minimized risk values for different baseline returns, we repeated the first part of the question multiple times. By varying the baseline returns, we computed the corresponding solutions for risk and portfolio return until we were able to derive all the minimized risk values against the baseline returns.

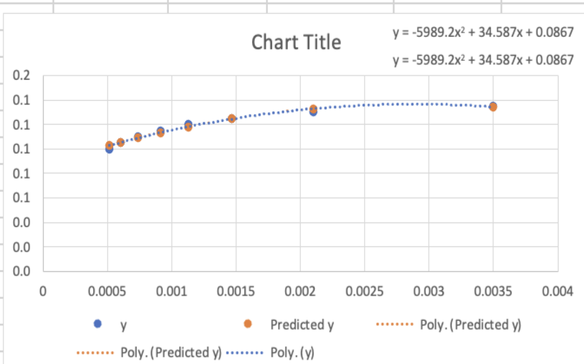


Using the 8 pairs of solutions for minimized risk and portfolio return, we generated a plot of 'return' versus 'risk' in Excel. The polynomial equation was chosen for the plot, and we obtained the equation y = -5989.2x2 + 34.587x +0.0867 to represent the relationship between the two variables. The R2 value of 0.9759 indicates that the regression model is a good fit for the data.

Quardratic Regression Fit

A	-5989.2
B	34.587
C	0.087
Objective	2.537E-05
y=ax^2+bx+c	

x	y	Predicted y	(y-Py)	(y-Py)^2
0.000513907	0.1	0.102893	-0.0028927	0.000008368
0.000603273	0.105	0.105386	-0.0003857	0.000000149
0.000735635	0.11	0.108902	0.0010977	0.000001205
0.000910995	0.115	0.113238	0.0017619	0.000003104
0.001129351	0.12	0.118122	0.0018780	0.000003527
0.00146350	0.125	0.12449	0.0005098	0.000000260
0.00209792	0.13	0.132901	-0.0029007	0.000008414
0.003496248	0.135	0.134414	0.0005858	0.000000343
				0.000025370



To confirm the mathematical relationship between the variables, we calculated the value of (y - ax2 + bx + c)T (y - ax2 + bx + c). The resulting objective value of 2.54e-05, as indicated in the table, is significantly small. Hence, we can infer that our quadratic regression model is an excellent fit for the data.

	Quardratic Regression Optimization					
A	-5989.243121		x	y	Predicted y	(y-Py)
B	34.58666782		0.000513907	0.1	0.1028499	0.0
C	0.087		0.000603273	0.105	0.1053428	0.000
			0.000735635	0.11	0.1088594	0.00
Objective	2.53548E-05		0.000910995	0.115	0.1131951	0.002
			0.001129351	0.12	0.1180789	0.00
			0.00146350	0.125	0.1244469	0.001
			0.00209792	0.13	0.1328571	0.00
			0.003496248	0.135	0.1343699	0.001

We further confirmed this by performing the quadratic regression optimization.

Excel Solver can also be utilized to obtain the optimized values of a, b, and c for the equation. The decision variables, in this case, are a, b, and c, with the objective being the minimization of the value of (y - ax2 + bx + c)T (y - ax2 + bx + c). After applying the GRG nonlinear method in Excel Solver, we obtained the optimized values for a, b, c, and objective value, as shown in the table above. The obtained values are highly similar to what we derived through the plot equation, indicating a good fit for our model.

Conclusion

In conclusion, we have analyzed the portfolio of six assets, namely Bonds, High tech stocks, Foreign stocks, Call options, Put options, and Gold. By using a quadratic model and applying Excel Solver, we have determined the optimal allocation of a $10,000 investment to achieve a minimum baseline expected return of 11% while minimizing risk. Our recommended distribution suggests that the investor should allocate 18.98% of the investment to Bonds, 10.86% to High tech stocks, 27.08% to Foreign stocks, 4.79% to Call options, 25.45% to Put options, and 12.83% to Gold. Under this allocation, the portfolio's variance is 0.000736. According to our analysis, if an individual has $10,000 to invest, our recommended allocation would be as follows: $1,898.06 on Bonds, $1,086.31 on High tech stocks, $2,708.28 on Foreign stocks, $479.43 on Call options, $2,544.70 on Put options, and $1,283.23 on Gold. These amounts are based on the optimal investment weights computed through our quadratic model using Excel Solver.

We have plotted 'return' versus 'risk' using the 8 pairs of solutions obtained for different baseline expected returns. The resulting polynomial equation, y = -5989.2x2 + 34.587x +0.0867, has an R2 value of 0.9759, indicating that the regression is a good fit for the data. By calculating the value of (y - ax2 + bx + c)T (y - ax2 + bx + c), we obtained an objective value of 2.54e-05, which further confirms that our quadratic regression model fits the data well. Overall, our analysis provides valuable insights for the investor to make informed decisions in managing their portfolio, achieving the desired expected return, and minimizing risk.

References

https://northeastern.instructure.com/courses/131426/pages/lesson-6-1-integer-binary-and-mixed-programming?module_item_id=8233888

https://northeastern.instructure.com/courses/131426/pages/lesson-6-2-nonlinear-programming?module_item_id=8233891

Rajan, S. (2020, July 24). *Cost functions for regression and its optimization techniques in machine learning*. Medium. Retrieved April 1, 2023, from https://towardsdatascience.com/cost-functions-of-regression-and-its-optimization-techniques-in-machine-learning-2f5931cd33f1

Waingankar, R. (2022, June 23). *MMULT in Excel*. WallStreetMojo. Retrieved April 1, 2023, from https://www.wallstreetmojo.com/mmult-in-excel/

YouTube. (2020, June 15). *Portfolio optimization seven security example with Excel solver*. YouTube. Retrieved April 1, 2023, from https://www.youtube.com/watch?v=sI5X2kJA_4k&ab_channel=MattMacarty

Zach. (2021, June 3). *How to perform quadratic regression in Excel*. Statology. Retrieved April 1, 2023, from https://www.statology.org/quadratic-regression-excel/