

Digital Elevation Map Reconstruction from single-angle satellite images using photoclinometry

Abstract

This report presents a complete pipeline for reconstructing Digital Elevation Models (DEMs) of lunar surface regions from single grayscale images obtained from the Chandrayaan-2 TMC2 instrument. The approach is based on the classical Lambertian Shape-from-Shading (SfS) technique, but extended to handle non-frontal illumination using Sun azimuth and elevation angles derived from spacecraft metadata and allowing the use of tif files. The optimization is performed via the L-BFGS algorithm, enforcing integrability and smoothness constraints to ensure physical realism. The resulting DEMs accurately recover relative topography without requiring stereo imaging.

1 Introduction

Photoclinometry (or Shape-from-Shading, SfS) reconstructs a surface's shape from brightness variations caused by lighting direction. Unlike traditional stereo photogrammetry, SfS operates on a single image by leveraging the relationship between pixel intensity, surface normal, and illumination geometry.

Brightness \longrightarrow Surface Normal \longrightarrow Elevation (DEM).

This project applies SfS to Chandrayaan-2 TMC2 imagery, considering the Sun's true position in the lunar sky rather than assuming frontal lighting.



Figure 1: Pipeline overview for Lambertian SfS with non-frontal illumination.

2 Pipeline Overview

2.1 1. Data Preprocessing

The raw data consist of TMC2 grayscale images and corresponding metadata providing solar illumination parameters. The preprocessing workflow includes:

Preprocessing Steps

1. **Image Normalization:** Convert .tif images to floating-point grayscale intensity in [0,1].
2. **Patch Division:** Split large scenes into 512×512 tiles to enable parallel processing.
3. **Metadata Extraction:** Parse solar azimuth (ϕ_s) and elevation (θ_s) angles from the XML headers.
4. **Illumination Vector:** Compute the light direction as:

$$\mathbf{L} = (\sin \theta_s \cos \phi_s, \sin \theta_s \sin \phi_s, \cos \theta_s).$$

2.2 2. Lambertian Reflectance Model

The Lambertian model assumes that the observed brightness is proportional to the cosine of the angle between the surface normal and the light source:

$$I(x, y) = \rho (\mathbf{n}(x, y) \cdot \mathbf{L}),$$

where ρ is the albedo (assumed constant) and \mathbf{n} is the normalized surface normal:

$$\mathbf{n}(x, y) = \frac{(-p, -q, 1)}{\sqrt{1 + p^2 + q^2}},$$

with $p = \frac{\partial z}{\partial x}$ and $q = \frac{\partial z}{\partial y}$ representing the local surface gradients.

Thus, for any given \mathbf{L} derived from metadata, pixel intensity $I(x, y)$ provides a nonlinear constraint between brightness and local slope.

2.3 3. Energy Formulation and Constraints

To estimate (p, q) , an energy functional combining data fidelity, integrability, and smoothness terms is minimized:

$$E(p, q) = \iint \left(I(x, y) - \rho (\mathbf{n}(x, y) \cdot \mathbf{L}) \right)^2 + \lambda_{\text{int}}(p_y - q_x)^2 + \lambda_{\text{smo}}(\|\nabla p\|^2 + \|\nabla q\|^2) dxdy.$$

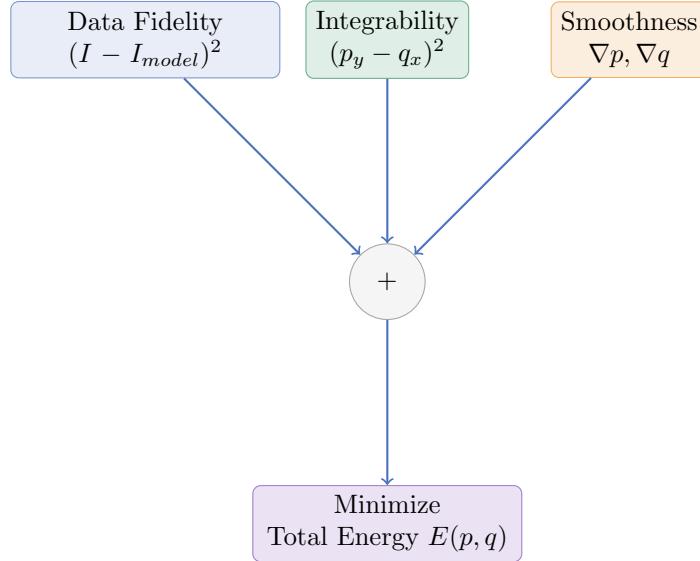


Figure 2: Structure of the energy minimized in the SfS optimization.

2.4 4. Optimization using L-BFGS

The minimization of $E(p, q)$ is performed using the L-BFGS algorithm, a quasi-Newton method that approximates curvature information efficiently:

$$x_{k+1} = x_k - \alpha_k H_k \nabla E(x_k),$$

where α_k is determined via line search satisfying Wolfe conditions. Convergence is achieved when $\|\nabla E\|$ becomes sufficiently small.

2.5 5. Integration to Height Field

Once the optimal slope fields (p^*, q^*) are obtained, the height map $z(x, y)$ is reconstructed by solving the Poisson equation:

$$\nabla^2 z = p_x + q_y.$$

This step integrates the gradient field into a continuous surface representing relative elevation. Boundary conditions are estimated from neighboring patches.

2.6 6. Postprocessing and Output

Refinement and Visualization

- **Median Filtering:** Removes small artifacts and noise.
- **DEM Normalization:** Scales to physically realistic elevation ranges.
- **Patch Stitching:** Combines multiple reconstructed tiles using bilinear interpolation at boundaries.

The final products include:

- Relative DEM (.tif / .npy)
- Slope and Aspect Visualizations

3 Conclusion

This report presents a practical implementation of Lambertian Shape-from-Shading with non-frontal illumination for lunar topography reconstruction. By integrating solar geometry directly from metadata and using robust optimization with integrability constraints, the pipeline generates high-quality relative DEMs from single images. The results demonstrate that realistic lighting models can be effectively combined with classical photoclinometry for planetary surface analysis.

References

- Brooks, M. J., & Horn, B. K. P. (1989). *Shape from Shading*. MIT Press.
- Rindfleisch, T. R. (1966). *Photometric Method for Lunar Topography*. Photometric Engineering, 32(2):262–277.
- Nocedal, J., & Wright, S. J. (1999). *Numerical Optimization*. Springer.