

# Digital Elevation Map Reconstruction from single-angle satellite images using photoclinometry

## Abstract

This report presents a complete pipeline for reconstructing Digital Elevation Models (DEMs) of lunar surface regions from single grayscale images obtained from the Chandrayaan-2 TMC2 instrument. The approach is based on the classical Lambertian Shape-from-Shading (SfS) technique, but extended to handle non-frontal illumination using Sun azimuth and elevation angles derived from spacecraft metadata and allowing the use of tif files. The optimization is performed via the L-BFGS algorithm, enforcing integrability and smoothness constraints to ensure physical realism. The resulting DEMs accurately recover relative topography without requiring stereo imaging.

## 1 Introduction

**Photoclinometry** (or **Shape-from-Shading, SfS**) reconstructs a surface's shape from brightness variations caused by lighting direction. Unlike traditional stereo photogrammetry, SfS operates on a single image by leveraging the relationship between pixel intensity, surface normal, and illumination geometry.

Brightness  $\longrightarrow$  Surface Normal  $\longrightarrow$  Elevation (DEM).

This project applies SfS to Chandrayaan-2 TMC2 imagery, considering the Sun's true position in the lunar sky rather than assuming frontal lighting.



Figure 1: Pipeline overview for Lambertian SfS with non-frontal illumination.

## 2 Pipeline Overview

### 2.1 1. Data Preprocessing

The raw data consist of TMC2 grayscale images and corresponding metadata providing solar illumination parameters. The preprocessing workflow includes:

#### Preprocessing Steps

1. **Image Normalization:** Convert .tif images to floating-point grayscale intensity in  $[0,1]$ .
2. **Patch Division:** Split large scenes into  $512 \times 512$  tiles to enable parallel processing.
3. **Metadata Extraction:** Parse solar azimuth ( $\phi_s$ ) and elevation ( $\theta_s$ ) angles from the XML headers.
4. **Illumination Vector:** Compute the light direction as:

$$\mathbf{L} = (\sin \theta_s \cos \phi_s, \sin \theta_s \sin \phi_s, \cos \theta_s).$$

## 2.2 2. Lambertian Reflectance Model

The Lambertian model assumes that the observed brightness is proportional to the cosine of the angle between the surface normal and the light source:

$$I(x, y) = \rho(\mathbf{n}(x, y) \cdot \mathbf{L}),$$

where  $\rho$  is the albedo (assumed constant) and  $\mathbf{n}$  is the normalized surface normal:

$$\mathbf{n}(x, y) = \frac{(-p, -q, 1)}{\sqrt{1 + p^2 + q^2}},$$

with  $p = \frac{\partial z}{\partial x}$  and  $q = \frac{\partial z}{\partial y}$  representing the local surface gradients.

Thus, for any given  $\mathbf{L}$  derived from metadata, pixel intensity  $I(x, y)$  provides a nonlinear constraint between brightness and local slope.

## 2.3 3. Energy Formulation and Constraints

To estimate  $(p, q)$ , an energy functional combining data fidelity, integrability, and smoothness terms is minimized:

$$E(p, q) = \iint \left( I(x, y) - \rho(\mathbf{n}(x, y) \cdot \mathbf{L}) \right)^2 + \lambda_{\text{int}}(p_y - q_x)^2 + \lambda_{\text{smo}}(\|\nabla p\|^2 + \|\nabla q\|^2) dx dy.$$

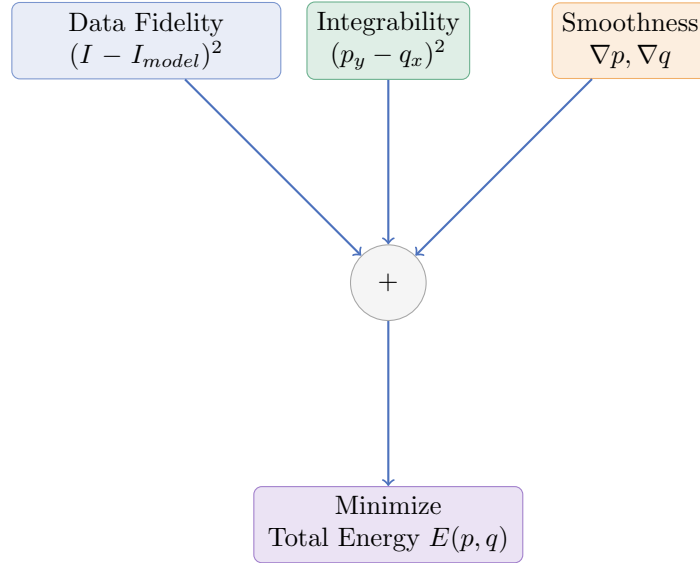


Figure 2: Structure of the energy minimized in the SfS optimization.

## 2.4 4. Optimization using L-BFGS

The minimization of  $E(p, q)$  is performed using the L-BFGS algorithm, a quasi-Newton method that approximates curvature information efficiently:

$$x_{k+1} = x_k - \alpha_k H_k \nabla E(x_k),$$

where  $\alpha_k$  is determined via line search satisfying Wolfe conditions. Convergence is achieved when  $\|\nabla E\|$  becomes sufficiently small.

## 2.5 5. Integration to Height Field

Once the optimal slope fields  $(p^*, q^*)$  are obtained, the height map  $z(x, y)$  is reconstructed by solving the Poisson equation:

$$\nabla^2 z = p_x + q_y.$$

This step integrates the gradient field into a continuous surface representing relative elevation. Boundary conditions are estimated from neighboring patches.

## 2.6 6. Postprocessing and Output

### Refinement and Visualization

- **Median Filtering:** Removes small artifacts and noise.
- **DEM Normalization:** Scales to physically realistic elevation ranges.
- **Patch Stitching:** Combines multiple reconstructed tiles using bilinear interpolation at boundaries.

The final products include:

- Relative DEM (.tif / .npz)
- Slope and Aspect Visualizations

## 3 Conclusion

This report presents a practical implementation of Lambertian Shape-from-Shading with non-frontal illumination for lunar topography reconstruction. By integrating solar geometry directly from metadata and using robust optimization with integrability constraints, the pipeline generates high-quality relative DEMs from single images. The results demonstrate that realistic lighting models can be effectively combined with classical photogrammetry for planetary surface analysis.

## References

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