

# An Introduction to Multi-label Learning

(ML-KNN & BP-MLL)

Bobby Lumpkin



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## Why use novel approaches?

- Naive Approach: Train a sequence of independent binary classifiers (one per category)
- Doesn't capitalize on the information in the correlations between the different labels of each instance.

# Multi-label Paradigm: Definitions & Notation

- Let  $\mathcal{X}$  denote the domain of instances and  $\mathcal{Y} = \{1, \dots, Q\}$  be the finite set of labels.
- Given  $x \in \mathcal{X}$  and its associated  $Y \subseteq \mathcal{Y}$ , let  $\vec{y}_x$  be the category vector for  $x$  such that (for all  $\ell \in \mathcal{Y}$ )  $\vec{y}_x(\ell) = 1$  if  $\ell \in Y$ . Otherwise,  $\vec{y}_x(\ell) = 0$ .

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- Let  $H_0^\ell$  denote the event that test instance  $t$  does not have a label  $\ell$  and let  $H_1^\ell$  denote the event that it does have label  $\ell$ .



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- Let  $H_0^\ell$  denote the event that test instance  $t$  does not have a label  $\ell$  and let  $H_1^\ell$  denote the event that it does have label  $\ell$ .
- Let  $E_j^\ell$  ( $j \in \{1, \dots, K\}$ ) denote the event that, among the  $K$  nearest neighbors of  $t$ , there are exactly  $j$  instances which have label  $\ell$ .

# ML-KNN Algorithm: Overall Approach

**Overall Approach:** This ML-KNN algorithm takes a parametric, Bayesian approach towards estimating the Bayes Optimal Classifier. As with the single-label algorithm, it does this using the  $K$  nearest neighbors of an instance. Namely...

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$$\begin{aligned}\vec{y}_t(\ell) &= \operatorname{argmax}_{b \in \{0,1\}} \mathbb{P} \left( H_b^\ell | E_{\vec{C}_t(\ell)}^\ell \right), \quad \ell \in \mathcal{Y} \\ &= \operatorname{argmax}_{b \in \{0,1\}} \frac{\mathbb{P} \left( H_b^\ell \right) \cdot \mathbb{P} \left( E_{\vec{C}_t(\ell)}^\ell | H_b^\ell \right)}{\mathbb{P} \left( E_{\vec{C}_t(\ell)}^\ell \right)} \\ &= \operatorname{argmax}_{b \in \{0,1\}} \mathbb{P} \left( H_b^\ell \right) \cdot \mathbb{P} \left( E_{\vec{C}_t(\ell)}^\ell | H_b^\ell \right)\end{aligned}$$

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- Where we take a Bayesian approach towards estimating the prior probabilities,  $\mathbb{P} \left( H_b^\ell \right)$ , and conditional probabilities,  $\mathbb{P} \left( E_{\vec{C}_t(\ell)}^\ell | H_b^\ell \right)$ .

# ML-KNN Algorithm: Overall Approach continued...

**Definition:** Let  $\vec{r}_t$  denote the real-valued vector with  $\ell^{th}$  component:

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$\Rightarrow$  Thus, given training data,  $\mathcal{X}$ , and a test instance,  $t$ , we wish to compute  $[\vec{y}_t(\ell), \vec{r}_t(\ell)]$ .

# ML-KNN Algorithm: Computing the Prior Probabilities,

$$\widehat{\mathbb{P}(H_b^\ell)}$$

We model  $\mathbb{P}(H_1^\ell)$  with a  $\text{Beta}(s, s)$  prior and  $\text{Binomial}(m, \mathbb{P}(H_1^\ell))$  likelihood. (When  $s = 1$ ,  $\text{Beta}(s, s)$  reduces to the uniform distribution on  $[0, 1]$ .)



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$$\text{Beta} \left( s + \sum_{i=1}^m \vec{y}_{x_i}(\ell), s + m - \sum_{i=1}^m \vec{y}_{x_i}(\ell) \right).$$

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$\Rightarrow$  We will estimate  $\mathbb{P}(\mathbf{H}_1^\ell)$  with the expectation of it's posterior Beta distribution:

$$\widehat{\mathbb{P}(\mathbf{H}_1^\ell)} := \frac{s + \sum_{i=1}^m \vec{y}_{x_i}(\ell)}{2s + m}$$

where  $m$  is the number of training instances.

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$\Rightarrow$  We estimate  $\widehat{\mathbb{P}}(H_0^\ell) := 1 - \widehat{\mathbb{P}}(H_1^\ell)$ .

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## Definition:

- (i) Let  $c$  be a vector of length  $K + 1$ , where  $c(j) =$  the number of training instances where  $\vec{C}_{x_i}(\ell) = j$  when  $\vec{y}_{x_i}(\ell) = 1$ .

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- (ii) Similarly, let  $c'$  be a vector of length  $K + 1$ , where  $c(j) =$  the number of training instances where  $\vec{C}_{x_i}(\ell) = j$  when  $\vec{y}_{x_i}(\ell) = 0$ .

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- \* We give  $\overrightarrow{\mathbb{P}(E_j^\ell | H_1^\ell)}$  (and  $\overrightarrow{\mathbb{P}(E_j^\ell | H_0^\ell)}$ ) a  $\text{Dirichlet}(K + 1, (s, \dots, s))$  prior distribution.
- \* We use a  $\text{Multinomial}(K + 1, (\frac{c(0)}{m_1}, \dots, \frac{c(K)}{m_1}))$  likelihood, where  $m_1 = \sum_{i=1}^m \vec{y}_{x_i}(\ell)$  (analogously for  $\overrightarrow{\mathbb{P}(E_j^\ell | H_0^\ell)}$ ).

# ML-KNN Algorithm: Computing the Conditional Probabilities, $\widehat{\mathbb{P}(E_j^\ell | H_b^\ell)}$ continued...

$\Rightarrow$  The posterior distribution for  $\overrightarrow{\mathbb{P}(E_j^\ell | H_1^\ell)}$  is

$$\text{Dirichlet}\left(K + 1, \left(s + c(0), \dots, s + c(K)\right)\right)$$

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$\Rightarrow$  Given  $j \in \{0, \dots, K\}$ , we estimate  $\mathbb{P}(E_j^\ell | H_1^\ell)$  and  $\mathbb{P}(E_j^\ell | H_0^\ell)$  with the expectations of their posterior distributions:

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$$\mathbb{P}(E_j^\ell | H_1^\ell) := \frac{(s + c(j))}{((K + 1)s + \sum_{n=0}^K c(n))}$$

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# Computing $\vec{y}_t$ and $\vec{r}_t$

Using our previous derivation:

$$\widehat{\vec{y}_t(\ell)} := \operatorname{argmax}_{b \in \{0,1\}} \left[ \widehat{\mathbb{P}(H_b^\ell)} \cdot \widehat{\mathbb{P}(E_{\vec{C}_T(\ell)}^\ell | H_b^\ell)} \right]$$

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**NOTE:** The larger the value for  $s$ , the less importance assigned to the training data: As  $s \rightarrow \infty$ ,  $\vec{r}_t(\ell) \rightarrow \frac{1}{2}$ .



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## ML-KNN Classification in scikit-multilearn:

→ MLkNN() from the scikit-multilearn module can be used to instantiate a ML-KNN object.

```
from skmultilearn.adapt import MLkNN  
  
classifier = MLkNN(k=3)  
  
# train  
classifier.fit(X_train, y_train)  
  
# predict  
predictions = classifier.predict(X_test)
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- “MLkNN” class methods like “fit()” and “predict()” mirror those for standard scikit-learn objects.

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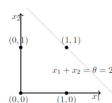
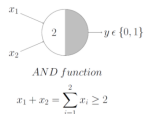
## BP-MLL Approach

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AND Function

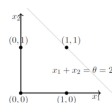
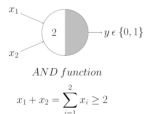




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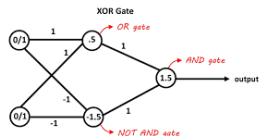
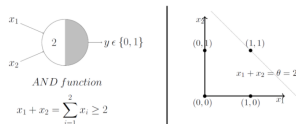
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- Since most subsequent work in the following two decades centered around single layer networks, the power of neural networks was restricted to linearly separable problems. This excluded the possibility of learning even simple functions like XOR, which required a second layer.

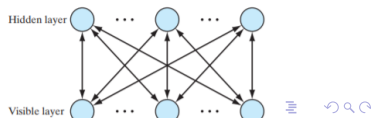
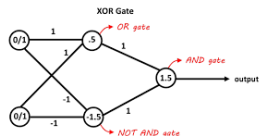
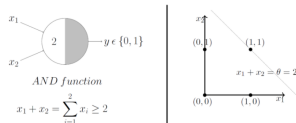
AND Function



# A Brief History

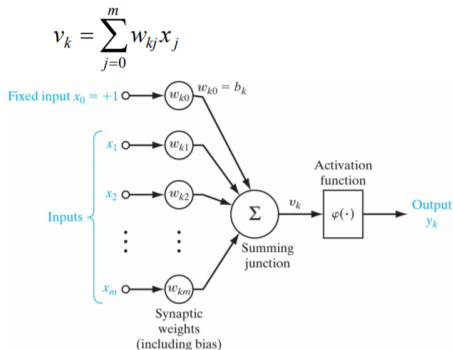
- Inspired by biological nervous systems, neural networks date back to the first half of the 20th century with works such as those by McCulloch and Pitts, which could model simple logical operations.
- Since most subsequent work in the following two decades centered around single layer networks, the power of neural networks was restricted to linearly separable problems. This excluded the possibility of learning even simple functions like XOR, which required a second layer.
- In the early 1980s, research on neural networks resurged largely due to successful learning algorithms for multi-layer neural networks and are used today for various tasks such as computer vision, associative memory, representation learning, NLP, etc..

AND Function



# Introduction to Feed-forward Networks: Perceptron Model

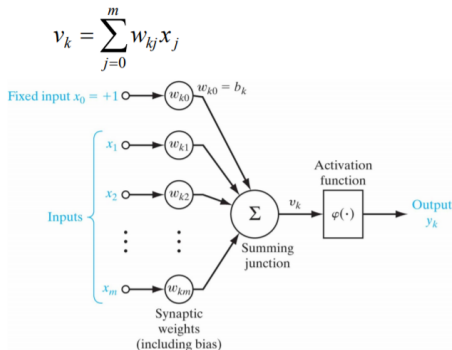
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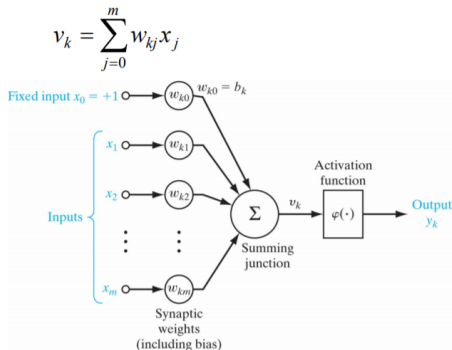
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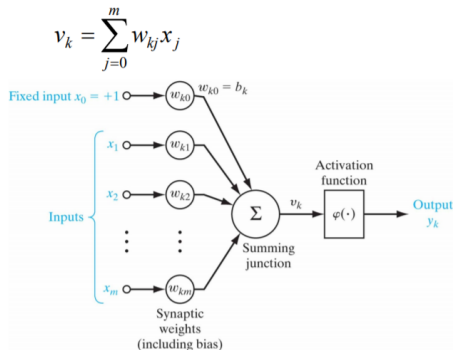
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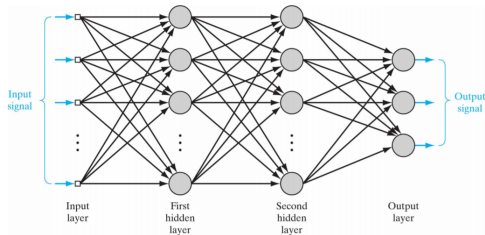
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- Neurons in the first layer represent components of the input vectors.
- The output of the neuron in the next layer is determined by applying a non-linear “activation function” to a linear combination of the input components, plus a bias.
- Rosenblatt’s Perceptron model uses a step function non-linearity, but other common activation functions include the sigmoid function ( $\sigma()$ ),  $\tanh()$ , ReLU, Leaky ReLU, softmax, etc..



# Multilayer Networks & Training

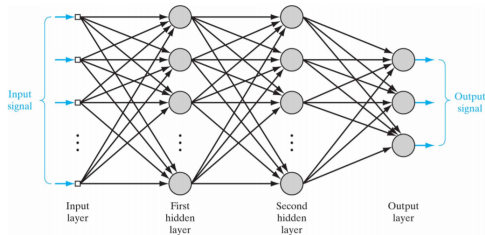
- Adding additional layers and units (like in the network pictured below) significantly expands the class of discrimination problems a network can learn.





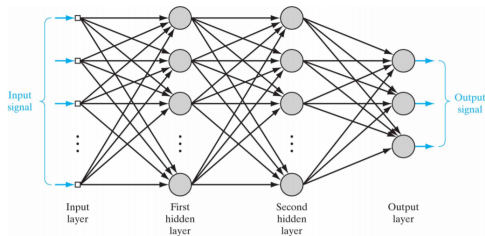
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- “Online” training involves evaluating an instance, and updating weights using gradient descent.
- For multi-label learning with  $Q$  instances, networks will have  $Q$  output layers, each with a  $\tanh()$  activation.



# Designing a Cost Function

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## Naive Approach (“BasicBP”):

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$$E = \sum_{i=1}^m E_i = \sum_{i=1}^m \frac{1}{|Y_i| |\bar{Y}_i|} \sum_{(k,l) \in Y_i \times \bar{Y}_i} \exp(-(c_k^i - c_l^i))$$

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so that the  $i^{th}$  error term is severely penalized if  $c_k^i$  is much smaller than  $c_l^i$ .

- Back-propagation for training is derived just as in the standard (MSE) case (details omitted here, but can be found in Zhang and Zhou's paper).

# Deep Learning APIs in Python: TensorFlow/Keras

**TensorFlow**: an open source python library for numerical computation and large-scale machine learning, created by the Google Brain team.

- One of the most widely used APIs for deep learning, along with PyTorch and Keras.
- Later versions of TensorFlow began incorporating the Keras API, since users found its high-level design to be simpler.

```
import tensorflow as tf
mnist = tf.keras.datasets.mnist

(x_train, y_train), (x_test, y_test) = mnist.load_data()
x_train, x_test = x_train / 255.0, x_test / 255.0

model = tf.keras.models.Sequential([
    tf.keras.layers.Flatten(),
    tf.keras.layers.Dense(512, activation=tf.nn.relu),
    tf.keras.layers.Dropout(0.2),
    tf.keras.layers.Dense(10, activation=tf.nn.softmax)
])
model.compile(optimizer='adam',
              loss='sparse_categorical_crossentropy',
              metrics=['accuracy'])

model.fit(x_train, y_train, epochs=5)
model.evaluate(x_test, y_test)
```

# TensorFlow Implementation of BP-MLL

## BP-MLL in TensorFlow:

- Data Scientist, Lukas Huwald, published an implementation of the bp-mlm cost function for the TensorFlow API as part of the module “bpmll”.
- After installation, the bp-mlm loss function can be utilized just as any other TensorFlow loss function.

```
# create simple mlp
model = Sequential()
model.add(Dense(128, input_dim=dim_no, activation='relu', kernel_initializer='glorot_uniform'))
model.add(Dense(64, activation='relu', kernel_initializer='glorot_uniform'))
model.add(Dense(class_no, activation='sigmoid', kernel_initializer='glorot_uniform'))
model.compile(loss=bp_mll_loss, optimizer='adagrad', metrics=[])

# train a few epochs
model.fit(X_train, Y_train, epochs=100)
```



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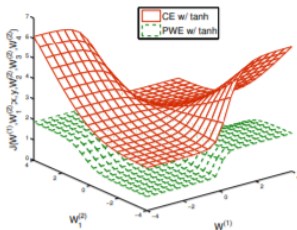
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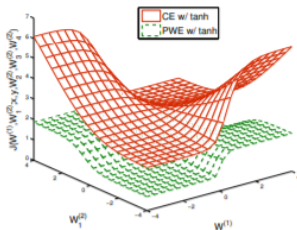
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- **Generalization:** When compared against a “standard” feed forward network with dropout regularization, adaptive learning rates and ReLU hidden layer activations, the test-set performance of BP-MLL is inferior on benchmark datasets for large scale text categorization (?).

# References (Original Method Papers)