CHEM-630 Assignment 3
Matthew J. Urffer

murffer@utk.edu

January 22, 2013

1.11: Atoms

(a) Calculate the average atomic weight of the element when Ag has 51.35% ^{107}Ag and ^{109}Ag

$$< mass > = 0.5135 \times 106.90508\text{mu} + (1 - 0.5135) \times 108.90475\text{mu} \tag{1}$$
$$= 107.85592\text{mu} \tag{2}$$

(b) Identify an element which has 7 stable isotopes
Ruthenium, dysprosium, ytterbium, and mercury all have 7 stable isotopes.

(c) Identify a stable nuclide which shows wither a magic neutron number only or a magic proton number only. What sign of extra stability to you find in it?
The magic nucleon numbers are 2,8,20,28,50,82 and 126. A stable nuclide with a magic proton number of 8 is ^{18}O. In this isotope the 10 neutrons are paired up in even-even spin, and is therefore a more stable energy state than an even-odd paring. Even-odd pairings would occur with the magic nucleon numbers since the magic nucleon numbers are all even.

(d) Plat a graph of the average atomic weight of the elements up through Th minus the atomic number (avg atomic wt - Z) against the atomic number (Z). How is this line related to the stability band?
Data was parsed from the 2007 atomic weights of the elements (IUPAC Technical Report) for the average atomic weight. The Z of each element was then subtracted off the of the atomic weight, and plotted in Figure 1. The resulting line is essentially the center of the stability band.

Advanced Topic: Predicting Radiological Half-Lives

The prediction of the radiological half-life from first principles was investigated for alpha decay, following the presentation of Dommelen in Quantum Mechanics for Engineers. In general there are 10 orders of magnitudes for the half-lives of different nuclei, from nano seconds to quintillions of years (excluding beryllium, which is in the atto second range). A simple model for alpha decay involves treating the alpha particle as a wavelet packet inside the nucleus. This packet is trapped inside the nucleus because of the strong nuclear force, but there is always some probability that the particle can tunnel out, this probability is approximated by a transmission coefficient 3.

$$T \approx e^{-2\gamma_{12}} \tag{3}$$

where γ_{12} is shown for the case of a high and wide barrier (Coulomb repulsion outside nucleus for wide, strong nuclear force for high).

$$\gamma_{12} = \frac{1}{\hbar} \int_{x_1}^{x_2} |p_c|\, dx \tag{4}$$
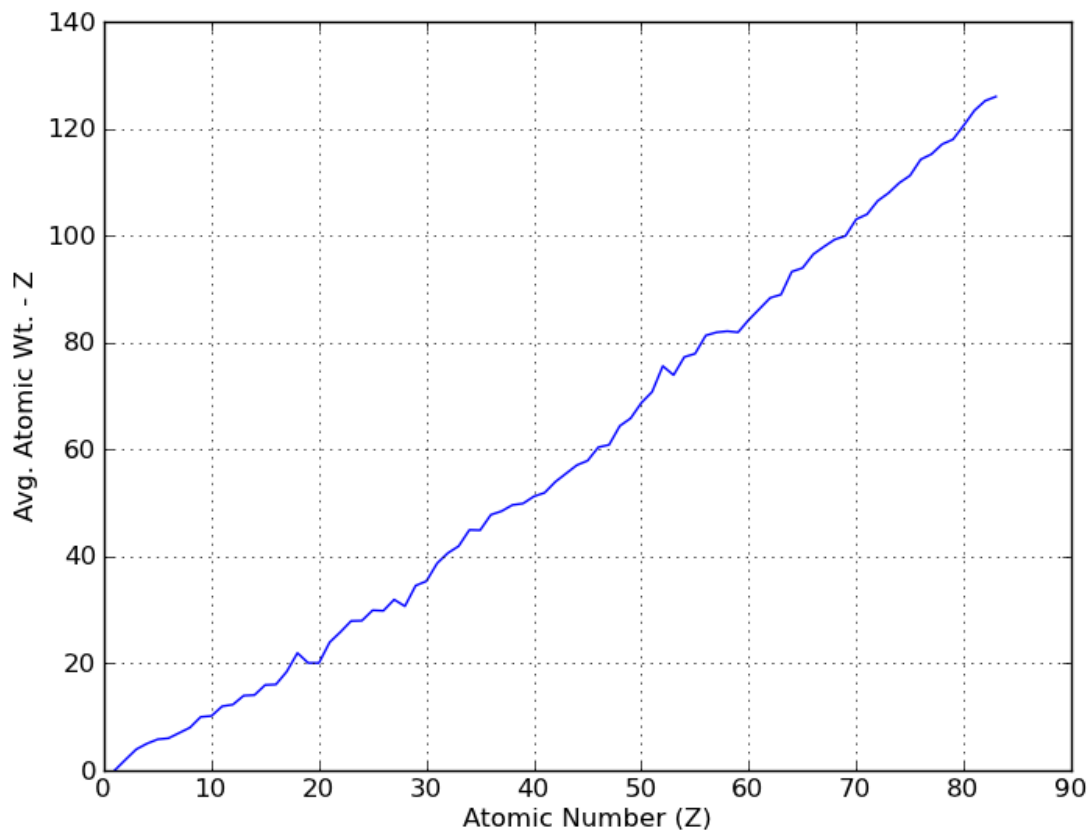$$|p_c| = \sqrt{2m(V - E)} \tag{5}$$

Figure 1: Average Atomic Weight - Z

. Under this approximation the probability for passing through the barrier is the integration from inside the nucleus to outside the nucleus. While the probability of a successful tunnel per attempt is quite low the wavelet bounces around the potential barrier inside the nucleus on the order of $2r_0/v_\alpha$ times per second. When the integration is preformed there is very good agreement for the even-even nuclei, with slightly less for all nuclei, as shown in Figure 2.
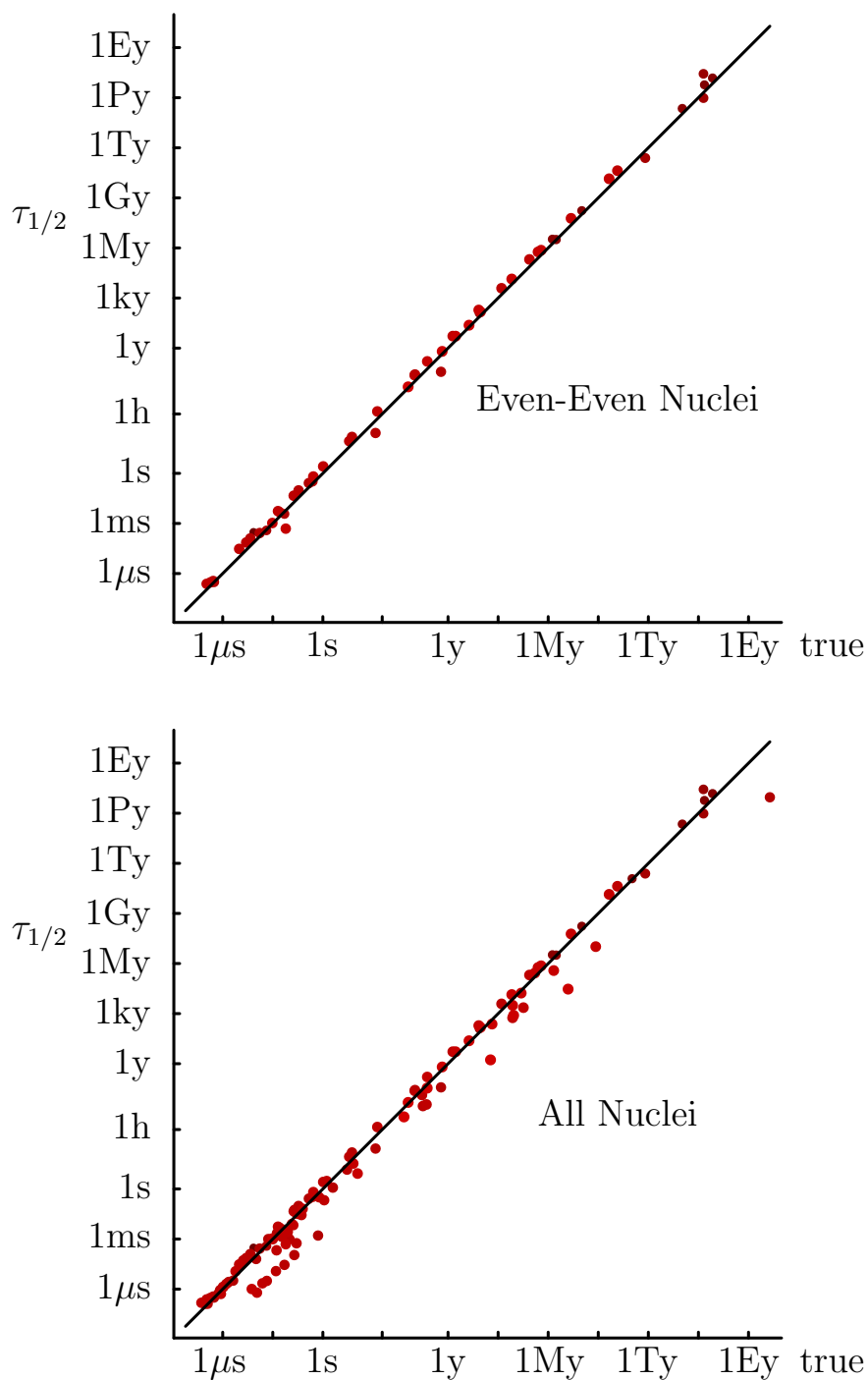
Gamow / Gurney & Condon Theory

Figure 2: Theory predict half-life agreement. Reproduced from Dommelen, Quantum Mechanics for Engineers