Least greatest proofs

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For a set of numbers X , how do you formalize "there is a greatest X " or "there is a least X "?
Prove or disprove: There is a least prime number.
Prove or disprove: There is a greatest integer.
Approach 1, De Morgan's and universal generalization:
Approach 2, proof by contradiction:
Extra examples: Prove or disprove that \mathbb{N} , \mathbb{Q} each have a least and a greatest element.