



Context-Free Grammars

$$G_1 = (\underbrace{\{X, Y, Z\}}_{\text{variables}}, \underbrace{\{0, 1, 2\}}_{\text{terminals}}, \underbrace{R}_{\text{rules}}, \underbrace{X}_{\text{start variable}})$$

R = X can be replaced with either 0 or 1 or 2

$$X \rightarrow 2X2 \mid Y$$

$$X \Rightarrow 2X2 \Rightarrow 2Y2 \Rightarrow 200Y112$$

$$\Rightarrow \underline{200112} \leftarrow \text{in } L(G_1)$$

$$Y \rightarrow 00Y11 \mid \epsilon$$

$$22001122$$

$$X \Rightarrow 2X2 \Rightarrow 22X22 \Rightarrow 22Y22$$

$$\Rightarrow 2200Y1122 \Rightarrow 22001122 \checkmark$$

in $L(G_1)$

$$Z \rightarrow 1X$$

unreachable

When X is the start variable

$$L(G_1) = \{2^n(00)^k(11)^k2^n \mid n \geq 0, k \geq 0\} - \{2^n0^m1^m2^n \mid n \geq 0, m \geq 0, \\ m \text{ is even}\}$$

If we change the start variable to Z : $L(G_1') = \{X, Y, Z, \{0, 1, 2\}, R, Z\}$

$$L(G_1') = \{12^n(00)^k(11)^k2^n \mid n \geq 0, k \geq 0\}$$

$G_2 = (\{S, T\}, \{0, 1, 2\}, R, S)$ that is what we want
desire
covet

$R = \text{BAD}$ WRONG

$S \rightarrow 0S1 \mid 1T \mid T0$ ↗ do not use

$T \rightarrow 1T \mid OT$

← Generates the language with all strings over $\{0, 1\}^*$ except for $\{0^n 1^n \mid n \geq 0\}$???

Any string that contains the symbol 2 is not in the language.

Doesn't allow us to terminate the derivation!
Generates \emptyset in this case.

In the language: Not in the language:
 $10 \quad 000$ $01 \quad 000111 \quad \epsilon$

$R =$ CORRECT

$S \rightarrow 0S1 \mid 1T \mid T0$

$T \rightarrow 1T \mid OT \mid \epsilon$ YEE

GOOD YAY HAWW

01 000111
 $S \rightarrow 0S1 \times$ $S \rightarrow 0S1 \Rightarrow 00S11 \Rightarrow 000S11 \times$

1000
 $S \rightarrow 1T \Rightarrow 1OT \Rightarrow 1OOT \Rightarrow 100OT$

We need to stop ~~100~~, but the rules don't allow us to!

Design a CFG to generate the set of strings over $\{0, 1, 2\}$ where
(the number of 0's + the number of 1's) \geq (the number of 2's)

$$G_3 = (\{J, K, L\}, \{0, 1, 2\}, R, J)$$

$R =$

$$J \rightarrow K \mid KJ \mid L$$

$$K \rightarrow \epsilon \mid 2KL \mid LK2$$

$$L \rightarrow 0 \mid 1 \mid LL$$

22110

$$\begin{aligned} J \Rightarrow K &\Rightarrow 2KL \Rightarrow 22KL \Rightarrow 22LL \\ &\Rightarrow 221L \Rightarrow 2211 \end{aligned}$$

20|02

$$\begin{aligned} K &\Rightarrow 2KL \Rightarrow 2LKJ \Rightarrow \\ &\cancel{2L2L \Rightarrow 202L \Rightarrow \cancel{\text{X}}} \end{aligned}$$

$$\begin{aligned} J \Rightarrow KJ &\Rightarrow 2KLJ \Rightarrow 2LJ \Rightarrow 20J \\ &\Rightarrow 20K \Rightarrow 20LK2 \Rightarrow 20L2 \\ &\Rightarrow 2002 \quad \checkmark \end{aligned}$$



G_{123}

Design a CFG to generate the language $L(G_1) \cup L(G_2) \cup L(G_3)$

$$G_1 = (\{X, Y, Z\}, \{0, 1, 2\}, R_1, X)$$

$$\begin{aligned} R_1 = \\ X &\rightarrow 2X2 \mid Y \\ Y &\rightarrow 00Y11 \mid \epsilon \\ Z &\rightarrow 1X \end{aligned}$$

$$G_2 = (\{S, T\}, \{0, 1, 2\}, R_2, S)$$

$$\begin{aligned} R_2 = \\ S &\rightarrow OS1 \mid IT \mid TO \\ T &\rightarrow IT \mid OT \mid \epsilon \end{aligned}$$

$$G_3 = (\{J, K, L\}, \{0, 1, 2\}, R_3, J)$$

$$\begin{aligned} R_3 = \\ J &\rightarrow K \mid KJ \mid L \\ K &\rightarrow 2KL \mid LK2 \mid \epsilon \\ L &\rightarrow 0 \mid 1 \mid LL \end{aligned}$$

$$G_{123} = \left(\{X, Y, Z, S, T, J, K, L\}, \{0, 1, 2\}, R_{123}, A \right)$$

$$R_{123} =$$

$$\begin{array}{|l|} \hline A \rightarrow X \mid S \mid J \\ \hline \begin{array}{l} X \rightarrow 2X2 \mid Y \\ Y \rightarrow 00Y11 \mid \epsilon \\ Z \rightarrow 1X \end{array} \\ \hline \end{array}$$

$$\begin{array}{|l|} \hline S \rightarrow OS1 \mid IT \mid TO \\ T \rightarrow IT \mid OT \mid \epsilon \\ \hline \end{array}$$

$$\begin{array}{|l|} \hline J \rightarrow K \mid KJ \mid L \\ K \rightarrow 2KL \mid LK2 \mid \epsilon \\ L \rightarrow 0 \mid 1 \mid LL \\ \hline \end{array}$$