1. Properties of Nuclei Solutions

Practice Set-1

1. The radius of a $^{64}_{29}$Cu nucleus is measured to be 4.8×10^{-13} cm. The radius of a $^{27}_{12}$Mg nucleus can be estimated to be

   a. 2.86×10^{-13} cm
   b. 5.2×10^{-13} cm
   c. 3.6×10^{-13} cm
   d. 8.6×10^{-13} cm

Solution:

$$\text{Since } R = R_0(A)^{1/3} \Rightarrow \frac{R_{Mg}}{R_{Cu}} = \left(\frac{A_{Mg}}{A_{Cu}}\right)^{1/3} = \left(\frac{27}{64}\right)^{1/3}$$

$$\Rightarrow \frac{R_{Mg}}{R_{Cu}} = \frac{3}{4} \Rightarrow R_{Mg} = \frac{3}{4} \times 4.8 \times 10^{-13} = 3.6 \times 10^{-13} \text{ cm.}$$

So the correct answer is **Option (c)**

2. The intrinsic electric dipole moment of a nucleus $^{A}_{Z}X$

   a. Increases with Z, but independent of A
   b. Decreases with Z, but independent of A
   c. Is always zero
   d. Increases with Z and A

Solution: So the correct answer is **Option (c)**

3. In deep inelastic scattering electrons are scattered off protons to determine if a proton has any internal structure. The energy of the electron for this must be at least

   a. 1.25×10^{9}eV
   b. 1.25×10^{12}eV
   c. 1.25×10^{6}eV
   d. 1.25×10^{8}eV

Solution:

The internal structure of proton can only be determined if the wavelength of the incoming electron is nearly equal to the size of the proton

$$\text{i.e. } \lambda = R = 1.2A^{1/3}(\text{fm}) = 1.2\text{fm} = 1.2\times10^{-15}\text{ m}$$
$$\text{According to de-Broglie relation, } \lambda = \frac{h}{p} = \frac{h}{\sqrt{2mE}}$$
$$\text{This can be also written as } E^2 = h^2\lambda^2/c^2 + m_0^2c^4$$

So the correct answer is **Option (b)**

4. The difference in the Coulomb energy between the mirror nuclei $^{49}_{24}$Cr and $^{49}_{25}$Mn is 6.0MeV. Assuming that the nuclei have a spherically symmetric charge distribution and that $\frac{e^2}{4\pi\varepsilon_0}$ is approximately $1.0\text{MeV} - fm$, the radius of the $^{49}_{25}$Mn nucleus is (a) 4.9×10^{-13} m (b) 4.9×10^{-15} m (c) 5.1×10^{-13} m (d) 5.1×10^{-15} m

Solution:

$$R = \frac{3e^2}{5\cdot\Delta W}\left(Z_1^2 - Z_2^2\right) = \frac{3\times1\times10^{-15}}{5\times6}\left(25^2 - 24^2\right)$$
$$= 4.9\times10^{-15}\text{ m}$$

So the correct answer is **Option (b)**

Practice Set-2

1. Inside a large nucleus, a nucleon with mass $939 \mathrm{MeVc}^{-2}$ has Fermi momentum $1.40 \mathrm{fm}^{-1}$ at absolute zero temperature. Its velocity is Xc, where the value of X is ———(up to two decimal places). ($\hbar c = 197 \mathrm{MeV}-\mathrm{fm}$)

Solution:

Here, Fermi-momentum or fermi radius, $k_F = 1.40 \mathrm{fm}^{-1}$ and $\hbar c = 197 \mathrm{Mev}-\mathrm{fm}$

Now, Fermi velocity -

$$V_F = \frac{P}{m} = \frac{\hbar k_F}{m} = \frac{(\hbar c) k_F \cdot c}{mc^2} = \frac{(197) \times 1 \cdot 40 \times c}{939} = \frac{275 \cdot 8c}{939} = 0.29c$$

So the correct answer is **0.29**

2. The mean kinetic energy of a nucleon in a nucleus of atomic weight A varies as A^n, where n is ———(upto two decimal places)

Solution:

$$\langle T \rangle = \frac{\int_0^R -\frac{\hbar^2}{2m}\left(\frac{d^2}{dr^2} + \frac{1}{r}\frac{d}{dr}\right) 4\pi r^2 dr}{\int_0^R 4\pi r^2 dr} = \frac{-\frac{\hbar^2}{2m} 4\pi \int_0^R (2+2) dr}{\int_0^R 4\pi r^2 dr} = \frac{-\frac{\hbar^2}{2m} 4\pi \times 4R}{4\pi R^3/3}$$

$$\Rightarrow \langle T \rangle \propto \frac{1}{R^2} = \frac{1}{\left(R_0 A^{\frac{1}{3}}\right)^2} = \frac{1}{A^{\frac{2}{3}}} = A^{-\frac{2}{3}} \Rightarrow n = -\frac{2}{3} = -0.667 = -0.67$$

So the correct answer is **-0.67**

3. According to the Fermi gas model of nucleus, the nucleons move in a spherical volume of radius $R\left(= R_0 A^{\frac{1}{3}}\right.$, where A is the mass number and R_0 is an empirical constant with the dimensions of length). The Fermi energy of the nucleus E_F is proportional to

a. R_0^2 **b.** $\frac{1}{R_0}$ **c.** $\frac{1}{R_0^2}$ **d.** $\frac{1}{R_0^3}$

Solution:

$$\text{Fermi energy } E_F = \frac{\hbar^2}{2m}\left(3\pi^2 \frac{N}{V}\right)^{2/3}$$

$$V = \frac{4\pi}{3} R^3 = \frac{4\pi}{3}\left(R_0 A^{1/3}\right)^3 = \frac{4\pi}{3} R_0^3 A$$

$$\therefore \quad E_F = \frac{\hbar^2}{2m}\left(\frac{3\pi^2 N}{\frac{4\pi}{3} R_0^3 A}\right)^{2/3} = \frac{\hbar^2}{2m}\left(\frac{9\pi N}{4A} \cdot \frac{1}{R_0^3}\right)^{2/3} \quad \Rightarrow E_F \propto \frac{1}{R_0^2}$$

So the correct answer is **Option (c)**

4. The stable nucleus that has $\frac{1}{3}$ the radius of ^{189}Os nucleus is,

a. $^{7}\mathrm{Li}$ **b.** $^{16}\mathrm{O}$ **c.** $^{4}\mathrm{He}$ **d.** ^{14}N

Solution:

$$R = \frac{1}{3}R_{Os} \Rightarrow R_0(A)^{1/3} = \frac{1}{3}R_0(189)^{1/3} \Rightarrow A = 7$$

So the correct answer is **Option (a)**

5. The binding energy of the k-shell electron in a Uranium atom $(Z = 92, A = 238)$ will be modified due to (i) screening caused by other electrons and (ii) the finite extent of the nucleus as follows:

   a. Increases due to (i), remains unchanged due to (ii)

   b. Decreases due to (i), decreases due to (ii)

   c. Increases due to (i), increases due to (ii)

   d. Decreases due to (i), remains unchanged due to (ii)

Solution: So the correct answer is **Option (b)**