1. Nuclear Models Solutions

Practice Set-1

1. The root-mean-square (r.m.s) energy of a nucleon in a nucleus of atomic number A in its ground state varies as:

   a. $A^{4/3}$ **b.** $A^{1/3}$ **c.** $A^{-1/3}$ **d.** $A^{-2/3}$

   Solution: So the correct answer is **Option(c)**

2. Let E_S denotes the contribution of the surface energy per nucleon in the liquid drop model. The ratio $E_S\left({}^{27}_{13}\mathrm{Al}\right) : E_S\left({}^{64}_{30}\mathrm{Zn}\right)$ is

   [NET/JRF (JUNE-2016)]

   a. $2:3$ **b.** $4:3$ **c.** $5:3$ **d.** $3:2$

   Solution:

   $$E_S = \frac{B}{A} = \frac{A^{\frac{2}{3}}}{A} \propto A^{-\frac{1}{3}} \Rightarrow \frac{E_S(Al)}{E_S(Z_n)} = \frac{(27)^{-\frac{1}{3}}}{(64)^{-\frac{1}{3}}} = \frac{(64)^{\frac{1}{3}}}{(27)^{\frac{1}{3}}} = \frac{4}{3}$$

   So the correct answer is **Option(b)**

3. The binding energy of a light nucleus (Z,A) in MeV is given by the approximate formula

   $$B(A,Z) \approx 16A - 20A^{2/3} - \frac{3}{4}Z^2A^{-1/3} + 30\frac{(N-Z)^2}{A}$$

   where $N = A - Z$ is the neutron number. The value of Z of the most stable isobar for a given A is

   [NET/JRF (JUNE-2013)]

   a. $\frac{A}{2}\left(1-\frac{A^{2/3}}{160}\right)^{-1}$ **b.** $\frac{A}{2}$

   c. $\frac{A}{2}\left(1-\frac{A^{2/3}}{120}\right)^{-1}$ **d.** $\frac{A}{2}\left(1+\frac{A^{4/3}}{64}\right)^{-1}$

Solution:

$$\left.\frac{\partial B}{\partial Z}\right|_{Z=Z'}=0 \Rightarrow Z'=\frac{A}{2}\left(1-\frac{A^{2/3}}{160}\right)^{-1}$$

So the correct answer is **Option(a)**

4. If the binding energy B of a nucleus (mass number A and charge Z) is given by

$$B=a_V A-a_S A^{2/3}-a_{sym}\frac{(2Z-A)^2}{A}+\frac{a_c Z^2}{A^{1/3}}$$

where $a_V=16\mathrm{MeV}, a_S=16\mathrm{MeV}, a_{sym}=24\mathrm{MeV}$ and $a_C=0.75\mathrm{MeV}$, then for the most stable isobar for a nucleus with $A=216$ is

[NET/JRF (DEC-2014)]

a. 68 **b.** 72 **c.** 84 **d.** 92

Solution:

For the most stable isobar for a nucleus $\frac{dB}{dZ}=0 \Rightarrow -a_{sym}\frac{2(2Z-A)\times 2}{A}+\frac{2a_C Z}{A^{1/3}}=0$

$$\Rightarrow 24\frac{2(2Z-216)\times 2}{216}+0.75\frac{2Z}{(216)^{1/3}}=0 \Rightarrow \frac{4(2Z-216)}{9}+\frac{3}{4}\frac{2Z}{6}=0$$

$$\Rightarrow \frac{4(2Z-216)}{9}+\frac{Z}{4}=0 \Rightarrow 16(2Z-216)+9Z=0 \Rightarrow 41Z=216\times 16 \Rightarrow Z=82.3$$

So the correct answer is **Option(c)**

5. Of the nuclei of mass number $A=125$, the binding energy calculated from the liquid drop model (given that the coefficients for the Coulomb and the asymmetry energy are $a_c=0.7\mathrm{MeV}$ and $a_{sym}=22.5\mathrm{MeV}$ respectively) is a maximum for

[NET/JRF (DEC-2015)]

a. ${}^{125}_{54}\mathrm{Xe}$ **b.** ${}^{124}_{53}I$ **c.** ${}^{125}_{52}\mathrm{Te}$ **d.** ${}^{125}_{51}\mathrm{Sb}$

Solution:

$$Z_0=\frac{4a_a+a_cA^{-1/3}}{2a_cA^{-1/3}+8a_aA^{-1}}=\frac{4a_aA+a_cA^{2/3}}{8a_a+2a_cA^{2/3}} \Rightarrow Z_0=\frac{4\times 22.5\times 125+0.7\left(5^3\right)^{2/3}}{8\times 22.5+2\times 0.7\left(5^3\right)^{2/3}}$$

$$\Rightarrow Z_0=\frac{11250+17.5}{180+35}=\frac{11267.5}{215}=52.4 \Rightarrow Z_0\approx 52$$

So the correct answer is **Option(c)**

6. The Bethe-Weizsacker formula for the binding energy (in MeV) of a nucleus of atomic number Z and mass number A is

$$15.8A-18.3A^{2/3}-0.714\frac{Z(Z-1)}{A^{1/3}}-23.2\frac{(A-2Z)^2}{A}$$

The ratio Z/A for the most stable isobar of a $A=64$ nucleus, is nearest to

a. 0.30 **b.** 0.35 **c.** 0.45 **d.** 0.50

Solution:

$$Z_0 = \frac{A}{2+\frac{a_c}{2a_a}A^{2/3}} \Rightarrow \frac{Z_0}{A} = \frac{1}{2+\frac{a_c}{2a_a}A^{2/3}}$$

given $a_c = 0.714$ and $a_a = 23.2$

$$\therefore \quad \frac{Z_0}{A} = \frac{1}{2+\frac{0.714}{2\times 23.2}A^{2/3}} = \frac{1}{2+0.015A^{2/3}} = \frac{1}{2+0.015(64)^{2/3}} = 0.45$$

So the correct answer is **Option (c)**

7. Let us approximate the nuclear potential in the shell model by a three dimensional isotropic harmonic oscillator. Since the lowest two energy levels have angular momenta $l=0$ and $l=1$ respectively, which of the following two nuclei have magic numbers of protons and neutrons?

[NET/JRF (JUNE-2015)]

a. ${}_{2}^{4}\mathrm{He}$ and ${}_{8}^{16}\mathrm{O}$

b. ${}_{1}^{2}D$ and ${}_{4}^{8}Be$

c. ${}_{2}^{4}\mathrm{He}$ and ${}_{4}^{8}\mathrm{Be}$

d. ${}_{2}^{4}\mathrm{He}$ and ${}_{6}^{12}\mathrm{C}$

Solution:

$${}_2He^4 \text{ has } Z=2, N=2$$

$$\text{and } {}_8O^{16} \text{ has } Z=8, N=8 \text{ magic numbers } (2,8,20,28,50,82,126)$$

So the correct answer is **Option (a)**

8. According to the shell model the spin and parity of the two nuclei ${}_{51}^{125}\mathrm{Sb}$ and ${}_{38}^{89}\mathrm{Sr}$ are, respectively,

[NET/JRF (DEC-2011)]

a. $\left(\frac{5}{2}\right)^+$ and $\left(\frac{5}{2}\right)^+$

b. $\left(\frac{5}{2}\right)^+$ and $\left(\frac{7}{2}\right)^+$

c. $\left(\frac{7}{2}\right)^+$ and $\left(\frac{5}{2}\right)^+$

d. $\left(\frac{7}{2}\right)^+$ and $\left(\frac{7}{2}\right)^+$

Solution:

${}_{51}^{125}\mathrm{Sb}; Z=51$ and $N=74$

$$Z=51$$

$$\left(s_{1/2}\right)^2\left(p_{3/2}\right)^4\left(p_{1/2}\right)^2\left(d_{5/2}\right)^6\left(s_{1/2}\right)^2\left(d_{3/2}\right)^4\left(f_{7/2}\right)^8\left(p_{3/2}\right)^4\left(f_{5/2}\right)^6\left(p_{1/2}\right)^2\left(g_{9/2}\right)^{10}\left(g_{7/2}\right)^1$$

$$\Rightarrow j=\frac{7}{2} \text{ and } l=4. \text{ Thus spin and parity } = \left(\frac{7}{2}\right)^+$$

${}_{38}^{89}Sr; Z=38$ and $N=51$

$$N=51:$$

$$\left(s_{1/2}\right)^2\left(p_{3/2}\right)^4\left(p_{1/2}\right)^2\left(d_{5/2}\right)^6\left(s_{1/2}\right)^2\left(d_{3/2}\right)^4\left(f_{7/2}\right)^8\left(p_{3/2}\right)^4\left(f_{5/2}\right)^6\left(p_{1/2}\right)^2\left(g_{9/2}\right)^{10}\left(g_{7/2}\right)^1$$

$$\Rightarrow j=\frac{7}{2} \text{ and } l=4. \text{ Thus spin and parity } = \left(\frac{7}{2}\right)^+$$

So the correct answer is **Option (d)**

9. According to the shell model, the total angular momentum (in units of $\hbar$) and the parity of the ground state of the ${}_{3}^{7}Li$ nucleus is

[NET/JRF (DEC-2013)]

a. $\frac{3}{2}$ with negative parity

b. $\frac{3}{2}$ with positive parity

c. $\frac{1}{2}$ with positive parity

d. $\frac{7}{2}$ with negative parity

Solution:

$$Z = 3, N = 4$$
$$\text{For odd } Z = 3; \left(s_{1/2}^2\right)\left(p_{3/2}^1\right) \Rightarrow j = 3/2, l = 1 \text{ and parity} = (-1)^1 = -1.$$

So the correct answer is **Option (a)**

10. According to the shell model, the nuclear magnetic moment of the $^{27}_{13}$Al nucleus is (Given that for a proton $g_l = 1, g_s = 5.586$, and for a neutron $g_l = 0, g_s = -3.826$)

[NET/JRF (JUNE-2016)]

a. $-1.913\mu_N$ **b.** $14.414\mu_N$ **c.** $4.793\mu_N$ **d.** 0

Solution:

$$_{13}Al^{27} : Z = 13, N = 14 \text{ for } Z = 13, S_{1/2}^2, P_{3/2}^4, P_{1/2}^2, d_{5/2}^5 \Rightarrow j = \frac{5}{2}, l = 2$$
$$\text{Magnetic moment, } \mu = \frac{1}{2}[2j - 1 + g_S]\mu_N = \frac{1}{2}\left[2 \times \frac{5}{2} - 1 + 5.586\right]\mu_N \Rightarrow \mu = 4.793\mu_N$$

So the correct answer is **Option (c)**

11. The spin-parity assignments for the ground and first excited states of the isotope $^{57}_{28}$Ni, in the single particle shell model, are

[NET/JRF (DEC-2017)]

a. $\left(\frac{1}{2}\right)^-$ and $\left(\frac{3}{2}\right)^-$ **b.** $\left(\frac{5}{2}\right)^+$ and $\left(\frac{7}{2}\right)^+$

c. $\left(\frac{3}{2}\right)^+$ and $\left(\frac{5}{2}\right)^+$ **d.** $\left(\frac{3}{2}\right)^-$ and $\left(\frac{5}{2}\right)^-$

Solution:

Spin parity for $_{28}$Ni^{57} for ground state and first excited state

For $_{28}Ni^{57}$: $P = 28, N = 29 \to$ will decide the j^P

So, for $N = 29$, ground state configuration,

$$1s_{1/2}^2 1p_{3/2}^4 1p_{1/2}^2 1d_{5/2}^6 2s_{1/2}^2 1d_{3/2}^4 1f_{7/2}^8 2p_{3/2}^1$$

$$\text{So, } j = \frac{3}{2}, l = 1$$

Spin parity for ground state of $_{28}$Ni^{57} $\to \left(\frac{3}{2}\right)^-$

For first excited state,

$$1s_{1/2}^2 1p_{3/2}^4 1p_{1/2}^2 1d_{5/2}^6 2s_{1/2}^2 1d_{3/2}^4 1f_{7/2}^8 2p_{3/2}^1 \to 1f_{5/2}$$

$$P = \frac{5}{2}, l = 3 \Rightarrow \text{spin parity} \to \left(\frac{5}{2}\right)^-$$

So the correct answer is **Option (d)**

12. The first excited state of the rotational spectrum of the nucleus $^{238}_{92}U$ has an energy 45keV above the ground state. The energy of the second excited state (in keV) is

[NET/JRF (DEC-2017)]

a. 150 **b.** 120 **c.** 90 **d.** 60

Solution:

As per the shell model (Collective Model)

Rotational Energies,

$$E_r = \frac{\hbar^2}{2I}J(J+1), I \to \text{ is moment of inertia where only even value of } J \text{ are allowed}$$

$$\text{i.e., } J = 0^+, 2^+, 4^+, 6^+, \ldots\ldots.$$

Now, for ground state $J = 0^+, E = 0\text{keV}$

For first excited state, $J = 2^+, E = 45\text{keV}$ (given)

$$\text{So, } 45\text{keV} = \frac{\hbar^2}{2I} \cdot 2 \cdot 3 \text{ or,} \frac{\hbar^2}{2I} = \frac{45}{6}\text{keV}..(i)$$

Now, for second excited state, $J = 4^+$

$$E_2 = \frac{\hbar^2}{2I} \cdot 4 \cdot 5 \text{ (put value of } \frac{\hbar^2}{2I} \text{ from (i))}$$

$$\text{or, } E_2 = \frac{45}{6} \times 20 = \frac{900}{6} = 150\text{keV}.$$

So the correct answer is **150**

13. The low lying energy levels due to the vibrational excitations of an even-even nucleus are shown in the figure below,
The spin-parity j^p of the level E_1 is

[NET/JRF (DEC-2018)]

a. 1^+ **b.** 1^- **c.** 2^- **d.** 2^+

Solution: Quadrupole oscillations are the lowest order nuclear vibrational mode. The quanta of vibrational energy are called phonons. A quadrupole phonon carries 2 units of angular momentum. Therefore, the parity is $P = (-1)^2 = +ve$
Also, the even-even ground state is O^+. The 1 phonon excited state is 2^+. The 2 phonons excited states are $0^+, 2^+, 4^+$. Thus correct option is (a)

$$\left.\begin{array}{l} 1.35 \text{ ——- } 0^+ \\ 1.25 \text{ ——- } 2^+ \\ 1.17 \text{ ——- } 4^+ \end{array}\right\} \text{2-phonons}$$

0.56 ——-2^+ : 1-phonon
0 ——-0^+:Ground state
meV
So the correct answer is **Option (d)**

14. An excited state of a ^8_4Be nucleus decays into two α-particles which are in a spinparity 0^+state. If the mean life-time of this decay is 10^{-22} s, the spin-parity of the excited state of the nucleus is

a. 2^+ **b.** 3^+ **c.** 0^- **d.** 4^-

Solution: The parity angular momentum selection rule in α-decay says that, if the initial and final particles are same, the I_α must be even; if the parties are different, then I_α must be odd.
The ground state is 0^+thus spin-parity of excited state must be 2^+. Thus correct option is (a)

So the correct answer is **Option (a)**

Answer key

Q.No.	Answer	Q.No.	Answer
1	**c**	2	**b**
3	**a**	4	**c**
5	**c**	6	**c**
7	**a**	8	**d**
9	**a**	10	**c**
11	**d**	12	
13	**d**	14	**a**
15			

Practice Set-2

1. The semi-empirical mass formula for the binding energy of nucleus contains a surface correction term. This term depends on the mass number A of the nucleus as

[GATE-2011]

a. $A^{-1/3}$ **b.** $A^{1/3}$ **c.** $A^{2/3}$ **d.** A

Solution: So the correct answer is **Option (c)**

2. The nuclear spin and parity of $^{40}_{20}$Ca in its ground state is

[GATE-2019]

a. 0^{+} **b.** 0^{-} **c.** 1^{+} **d.** 1^{-}

Solution:

$$^{40}_{20}\text{Ca is an even-even nuclei, therefore } I=0, P=+ve$$
$$\therefore \quad \text{Spin-parity } =0^{+}$$

So the correct answer is **Option (a)**

3. In the nuclear shell model, the spin parity of $^{15}_{7}N$ is given by

[GATE-2010]

a. $\frac{1^-}{2}$ **b.** $\frac{1^+}{2}$ **c.** $\frac{3^-}{2}$ **d.** $\frac{3^+}{2}$

Solution:

$$Z=7; (s_{1/2})^2 (p_{3/2})^4 (p_{1/2})^1 \text{ and } N=8$$
$$l=1, J=\frac{1}{2} \Rightarrow \text{ parity } =(-1)^1=-1, \quad \text{spin} - \text{parity } = \left(\frac{1}{2}\right)^{-}$$

So the correct answer is **Option (a)**

4. According to the nuclear shell model, the respective ground state spin-parity values of $^{15}_{8}$O and $^{17}_{8}$O nuclei are

[GATE-2016]

a. $\frac{1^+}{2}, \frac{1^-}{2}$ **b.** $\frac{1}{2}^{-}, \frac{5^+}{2}$ **c.** $\frac{3^-}{2}, \frac{5^+}{2}$ **d.** $\frac{3^-}{2}, \frac{1^-}{2}$

Solution:

$$^{15}_{8}O; Z=8 \text{ and } N=7; \quad N=7: (s_{1/2})^2 (p_{3/2})^4 (p_{1/2})^1$$
$$\Rightarrow j=\frac{1}{2} \text{ and } l=1. \text{ Thus spin and parity } = \left(\frac{1}{2}\right)^{-}$$
$$^{17}_{8}O; Z=8 \text{ and } N=9; \quad N=9: (s_{1/2})^2 (p_{3/2})^4 (p_{1/2})^2 (d_{5/2})^1$$
$$\Rightarrow j=\frac{5}{2} \text{ and } l=2. \text{ Thus spin and parity } = \left(\frac{5}{2}\right)^{+}$$

So the correct answer is **Option (b)**

5. J^P for the ground state of the $^{13}C_6$ nucleus is

[GATE-2017]

a. 1^+ **b.** $\frac{3^-}{2}$ **c.** $\frac{3^+}{2}$ **d.** $\frac{1^-}{2}$

Solution:

$$^{13}C_6 : Z = 6, N = 7, N = 7 : \left(s_{1/2}\right)^2 \left(p_{3/2}\right)^4 \left(p_{1/2}\right)^1$$

$$\Rightarrow j = \frac{1}{2} \text{ and } l = 1.$$

$$\text{Thus, spin and parity} = \left(\frac{1}{2}\right)^-$$

So the correct answer is **Option (d)**

6. According to the single particles nuclear shell model, the spin-parity of the ground state of $^{17}_{8}$O is

[GATE-2011]

a. $\frac{1}{2}$ **b.** $\frac{3}{2}$ **c.** $\frac{3}{2}$ **d.** $\frac{5^+}{2}$

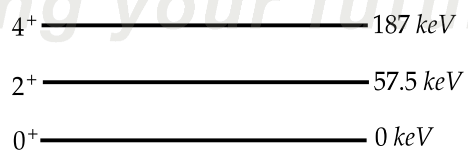
Solution:

$$Z = 8 \text{ and } N = 9; \left(s_{1/2}\right)^2 \left(p_{3/2}\right)^4 \left(p_{1/2}\right)^2 \left(d_{5/2}\right)^1$$

$$l = 2, J = \frac{5}{2} \Rightarrow \text{ parity } = (-1)^2 = +1, \text{ spin } - \text{ parity } = \left(\frac{5}{2}\right)^+$$

So the correct answer is **Option (d)**

7. The first three energy levels of $^{228}Th_{90}$ are shown below



The expected spin-parity and energy of the next level are given by

[GATE-2010]

a. $(6^+; 400\text{keV})$ **b.** $(6^+; 300\text{keV})$

c. $(2^+; 400\text{keV})$ **d.** $(4^+; 300\text{keV})$

Solution:

$$\frac{E_2}{E_1} = \frac{J_2(J_2+1)}{J_1(J_1+1)} \Rightarrow \frac{E_6}{E_4} = \frac{6(6+1)}{4(4+1)} \Rightarrow E_6 = 393\text{keV}$$

So the correct answer is **Option (a)**

8. In the nuclear shell model, the potential is modeled as $V(r) = \frac{1}{2}m\omega^2 r^2 - \lambda \vec{L} \cdot \vec{S}, \lambda > 0$. The correct spin-parity and isospin assignments for the ground state of $^{13}_{6}$C is

[GATE-2015]

a. $\frac{1^-}{2};\frac{-1}{2}$ **b.** $\frac{1^+}{2};\frac{-1}{2}$

c. $\frac{3^+}{2};\frac{1}{2}$ **d.** $\frac{3^-}{2};\frac{-1}{2}$

Solution:

$$^{13}C_6,\quad N=7, Z=6,\ \text{for}\ N=7;\quad \left(1S_{\frac{1}{2}}\right)^2\left(1P_{\frac{3}{2}}\right)^4\left(P_{\frac{1}{2}}\right)^1 \Rightarrow j=\frac{1}{2}\ \text{and}\ l=1$$

$$\text{Thus spin- parity is}\ \left(\frac{1}{2}\right)^-.$$

So the correct answer is **Option (a)**

9. The total angular momentum j of the ground state of the $^{17}_{8}O$ nucleus is

[GATE- 2020]

a. $\frac{1}{2}$ **b.** 1 **c.** $\frac{3}{2}$ **d.** $\frac{5}{2}$

Solution:

$$\text{For}\ ^{17}_{8}O : z=8\ \text{and}\ N=9$$
$$\text{For}\ N=9 : \left(1s_{1/2}\right)^2\left(1p_{3/2}\right)^4\left(1p_{1/2}\right)^2\left(1d_{5/2}\right)^1$$
$$\text{The angular momentum is}\ I=\frac{5}{2}$$

So the correct answer is **Option (d)**

10. For nucleus ^{164}Er, a $J^\pi=2^+$state is at 90keV. Assuming ^{164}Er to be a rigid rotor, the energy of its 4^+state is ————keV (up to one decimal place)

[GATE-2018]

Solution:

$$E_J = hcBJ(J+1)$$
$$E_{2^+} = hcB2(2+1)\ \text{and}\ E_{4^+} = hcB4(4+1)$$
$$\text{Then,}\ \frac{E_{4^+}}{E_{2^+}} = \frac{20}{6} \Rightarrow E_{4^+} = \frac{20}{6}\times 90\text{keV} = 300\text{keV}$$

Answer key			
Q.No.	Answer	Q.No.	Answer
1	**c**	2	**a**
3	**a**	4	**b**
5	**d**	6	**d**
7	**a**	8	**a**
9	**d**	10	**300**
11		12	
13		14	
15			

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