Practice Set-2
Electric Potential

1. An insulating sphere of radius a carries a charge density

$$\rho(\vec{r}) = \rho_0\left(a^2 - r^2\right)\cos\theta; r < a$$

The leading order term for the electric field at a distance d, far away from the charge distribution, is proportional to

[GATE 2010]

A. d^{-1} **B.** d^{-2} **C.** d^{-3} **D.** d^{-4}

Solution:

$$V(r) = \left[\frac{1}{r}\int_V \rho\, d\tau + \frac{1}{r^2}\int \rho\cos\theta\, d\tau + \cdots\right]$$

$$\text{I}^{\text{st}} \text{ term, } \int \rho\, d\tau = \int_0^a\int_0^\pi\int_0^{2\pi} \rho_0\left(a^2 - r^2\right)\cos\theta \times r^2\sin\theta\, dr\, d\theta\, d\phi = 0$$

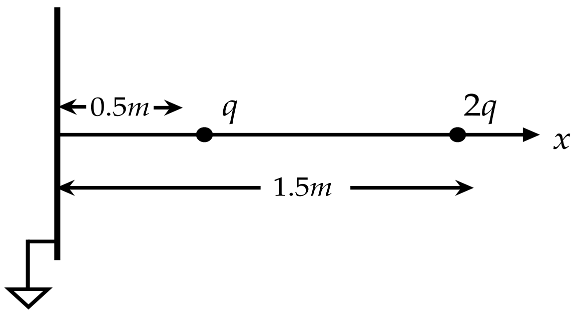
$$\text{II}^{\text{nd}} \text{ term, } \int \rho\cos\theta\, d\tau = \int_0^a\int_0^\pi\int_0^{2\pi} \rho_0\left(a^2 - r^2\right)\cos^2\theta \times r^2\sin\theta\, dr\, d\theta\, d\phi \neq 0$$

$$\Rightarrow \mathrm{V}\alpha\frac{1}{\mathrm{r}^2} \Rightarrow E\alpha\frac{1}{\mathrm{r}^3}$$

So the correct answer is **Option (C)**

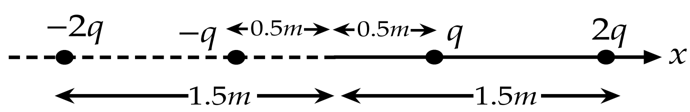
2. Two charges q and $2q$ are placed along the x -axis in front of a grounded, infinite conducting plane, as shown in the figure. They are located respectively at a distance of 0.5 m and 1.5 m from the plane. The force acting on the charge q is

[GATE 2011]



A. $\frac{1}{4\pi\varepsilon_0}\frac{7q^2}{2}$ **B.** $\frac{1}{4\pi\varepsilon_0}2q^2$ **C.** $\frac{1}{4\pi\varepsilon_0}q^2$ **D.** $\frac{1}{4\pi\varepsilon_0}\frac{q^2}{2}$

Solution: Using method of Images we can draw equivalent figure as shown below:



$$\begin{aligned} F &= \frac{q}{4\pi\varepsilon_0}\left[\frac{2q}{(1)^2}+\frac{q}{(1)^2}+\frac{2q}{(2)^2}\right] \\ &= \frac{q}{4\pi\varepsilon_0}\times\frac{7q}{2} \\ &= \frac{1}{4\pi\varepsilon_0}\frac{7q^2}{2} \end{aligned}$$

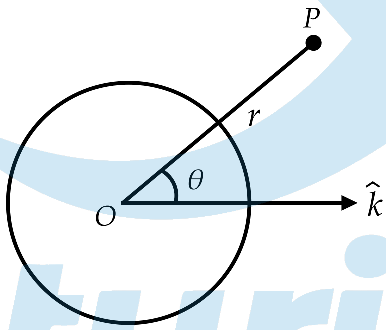
So the correct answer is **Option (A)**

3. A spherical conductor of radius a is placed in a uniform electric field $\vec{E}=E_0\hat{k}$. The potential at a point $P(r,\theta)$ for $r>a$, is given by

$$\Phi(r,\theta) = \text{ constant } - E_0 r\cos\theta + \frac{E_0 a^3}{r^2}\cos\theta$$

where r is the distance of P from the centre O of the sphere and θ is the angle OP makes with the z -axis The charge density on the sphere at $\theta=30^\circ$ is

[GATE 2011]



A. $3\sqrt{3}\varepsilon_0 E_0/2$ **B.** $3\varepsilon_0 E_0/2$ **C.** $\sqrt{3}\varepsilon_0 E_0/2$ **D.** $\varepsilon_0 E_0/2$

Solution:

$$\begin{aligned} \sigma &= -\ \varepsilon_0 \frac{\partial V}{\partial r}\Big|_{r=a} \\ &= -\varepsilon_0\left[-E_0\cos\theta - \frac{2E_0a^3}{r^3}\cos\theta\right]_{r=a} \\ \sigma &= -\varepsilon_0\left[-E_0\cos\theta - 2E_0\cos\theta\right] \Rightarrow \sigma \\ &= +3E_0\varepsilon_0\cos\theta = +3E_0\varepsilon_0\cos 30^\circ \\ &= \frac{3\sqrt{3}}{2}\varepsilon_0 E_0 \end{aligned}$$

So the correct answer is **Option (A)**

4. For a scalar function φ satisfying the Laplace equation, $\vec{\nabla}\varphi$ has

[GATE 2013]

A. Zero curl and non-zero divergence

B. Non-zero curl and zero divergence

C. Zero curl and zero divergence

D. Non-zero curl and non-zero divergence

Solution:

$$\nabla^2 \varphi = 0 \Rightarrow \vec{\nabla} \cdot (\vec{\nabla} \varphi)$$
$$= 0 \text{ and } \Rightarrow \vec{\nabla} \times (\vec{\nabla} \varphi) = 0$$

So the correct answer is **Option (C)**

5. A charge distribution has the charge density given by $\rho = Q\{\delta(x - x_0) - \delta(x + x_0)\}$. For this charge distribution the electric field at $(2x_0, 0, 0)$

[GATE 2013]

A. $\frac{2Q\hat{x}}{9\pi\varepsilon_0 x_0^2}$ **B.** $\frac{Q\hat{x}}{4\pi\varepsilon_0 x_0^3}$ **C.** $\frac{Q\hat{x}}{4\pi\varepsilon_0 x_0^2}$ **D.** $\frac{Q\hat{x}}{16\pi\varepsilon_0 x_0^2}$

Solution:

$$\text{Potential } V(r) = \frac{1}{4\pi\varepsilon_0}\left[\int_{-a}^{a} \frac{\rho(x')}{x} dx' + \int_{-a}^{a} \frac{\rho(x')}{x^2} x' dx' + \int_{-a}^{a} \frac{\rho(x')}{x^3} x'^2 dx' + \ldots\right]$$

First term, total charge

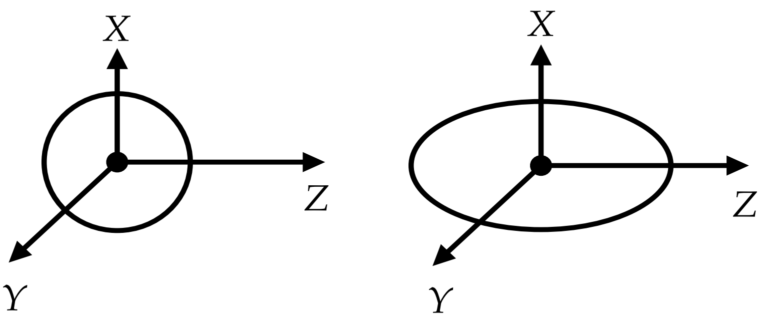
$$Q_T = \int \rho(x')\, dx' = Q\int_{-x_0}^{x_0} \delta(x' - x_0)\, dx' - Q\int_{-x_0}^{x_0} \delta(x' + x_0)\, dx'$$
$$= Q - Q = 0$$

Second term, dipole moment

$$p = \int x' \rho(x')\, dx' = Q\int_{-x_0}^{x_0} x' \delta(x' - x_0)\, dx' - Q\int_{-x_0}^{x_0} x' \delta(x' + x_0)\, dx'$$
$$= Qx_0 - Q \times -x_0 = 2Qx_0$$
$$V = \frac{2Qx_0}{4\pi\varepsilon_0 x^2} \Rightarrow \vec{E} = -\frac{\partial V}{\partial x}\hat{x}$$
$$= \frac{4Qx_0}{4\pi\varepsilon_0 x^3}\hat{x} = \frac{4Qx_0}{4\pi\varepsilon_0 (2x_0)^3}\hat{x}$$
$$= \frac{Q}{8\pi\varepsilon_0 x_0^2}\hat{x}$$

6. A charge $-q$ is distributed uniformly over a sphere, with a positive charge q at its center in (i). Also in (ii), a charge $-q$ is distributed uniformly over an ellipsoid with a positive charge q at its center. With respect to the origin of the coordinate system, which one of the following statements is correct?

[GATE 2015]



(i) (ii)

A. The dipole moment is zero in both (i) and (ii)

B. The dipole moment is non-zero in (i) but zero in (ii)

C. The dipole moment is zero in (i) but non-zero in (ii)

D. The dipole moment is non-zero in both (i) and (ii)

Solution:

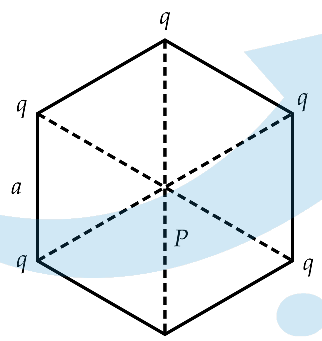
$$\vec{p} = \sum q_i \vec{r}_i = 0 \text{ in both cases.}$$

So the correct answer is **Option (A)**

7. Identical charges q are placed at five vertices of a regular hexagon of side a. The magnitude of the electric field and the electrostatic potential at the centre of the hexagon are respectively

[GATE 2017]

A. 0,0 **B.** $\frac{q}{4\pi\varepsilon_0 a^2}, \frac{q}{4\pi\varepsilon_0 a}$ **C.** $\frac{q}{4\pi\varepsilon_0 a^2}, \frac{5q}{4\pi\varepsilon_0 a}$ **D.** $\frac{\sqrt{5}q}{4\pi\varepsilon_0 a^2}, \frac{\sqrt{5}q}{4\pi\varepsilon_0 a}$



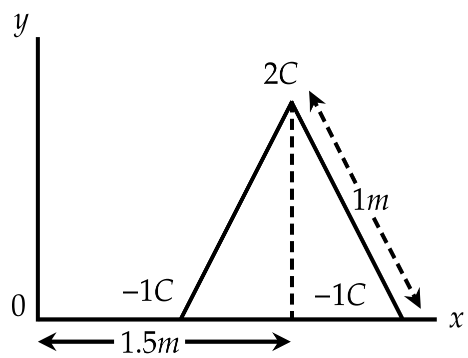
Solution:

$$\text{The resultant field at P is } E = \frac{q}{4\pi\varepsilon_0 a^2}$$
$$\text{The electrostatic potential at P is } V = \frac{5q}{4\pi\varepsilon_0 a}$$

So the correct answer is **Option (C)**

8. Three charges $(2C, -1C, -1C)$ are placed at the vertices of an equilateral triangle of side $1m$ as shown in the figure. The component of the electric dipole moment about the marked origin along the $\hat{y}$ direction is———Cm.

[GATE 2017]



Solution:

$$\vec{p} = -1(1\hat{x}) - 1(2\hat{x}) + 2(1.5\hat{x} + \sqrt{1-0.25}\hat{y})$$

$$\text{Along the } \hat{y} \text{ direction } = 2 \times \sqrt{1-0.25} = 1.73$$

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