Practice Set 1-solutions

1. The magnetic field at a distance R from a long straight wire carrying a steady current I is proportional to **[NET 2012]**

A. IR

B. I/R^2

C. I^2/R^2

D. I/R

Solution: The correct option is **(d)**

2. The vector potential $\vec{A}$ due to a magnetic moment $\vec{m}$ at a point $\vec{r}$ is given by $\vec{A} = \frac{\vec{m}\times\vec{r}}{r^3}$. If $\vec{m}$ is directed along the positive z-axis, the x - component of the magnetic field, at the point $\vec{r}$, is **[NET 2011]**

A. $\frac{3myz}{r^5}$

B. $-\frac{3mxy}{r^5}$

C. $\frac{3mxz}{r^5}$

D. $\frac{3m\left(z^2-xy\right)}{r^5}$

Solution:

$$\vec{m} = m\hat{z}$$

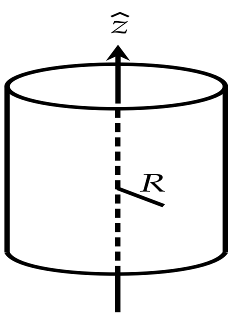
and

$$\vec{B} = \vec{\nabla}\times\vec{A} = \frac{m}{r^3}(2\cos\theta\hat{r} + \sin\theta\hat{\theta}) = \frac{1}{r^3}[3(\vec{m}\cdot\hat{r})\hat{r} - \vec{m}]$$

$$\vec{B} = \frac{1}{r^3}\left[3m\hat{z}\cdot\left(\frac{x\hat{x}+y\hat{y}+z\hat{z}}{r}\right)\frac{\vec{r}}{r} - m\hat{z}\right]$$

$$B_x = \frac{3mxz}{r^5}$$

3. An infinite solenoid with its axis of symmetry along the z-direction carries a steady current I. The vector potential $\vec{A}$ at a distance R from the axis **[NET 2012]**



A. is constant inside and varies as R outside the solenoid

B. varies as R inside and is constant outside the solenoid

C. varies as $\frac{1}{R}$ inside and as R outside the solenoid

D. varies as R inside and as $\frac{1}{R}$ outside the solenoid

Solution: The correct option is **(d)**

4. The force between two long and parallel wires carrying currents I_1 and I_2 and separated by a distance D is proportional to

[NET 2013]

A. $I_1 I_2 / D$

B. $(I_1 + I_2)/D$

C. $(I_1 I_2 / D)^2$

D. $I_1 I_2 / D^2$

Solution: The correct option is **(a)**

5. A time-dependent current $\vec{I}(t) = Kt\hat{z}$ (where K is a constant) is switched on at $t = 0$ in an infinite current-carrying wire. The magnetic vector potential at a perpendicular distance a from the wire is given (for time $t > a/c$) by

[NET 2014]

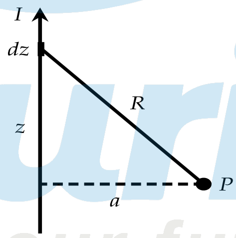
A. $\hat{z}\frac{\mu_0 K}{4\pi c}\int_{-\sqrt{c^2t^2-a^2}}^{\sqrt{c^2t^2-a^2}} dz \frac{ct-\sqrt{a^2+z^2}}{(a^2+z^2)^{1/2}}$

B. $\hat{z}\frac{\mu_0 K}{4\pi}\int_{-ct}^{ct} dz \frac{t}{(a^2+z^2)^{1/2}}$

C. $\hat{z}\frac{\mu_0 K}{4\pi c}\int_{-ct}^{ct} dz \frac{ct-\sqrt{a^2+z^2}}{(a^2+z^2)^{1/2}}$

D. $\hat{z}\frac{\mu_0 K}{4\pi}\int_{-\sqrt{c^2t^2-a^2}}^{\sqrt{c^2t^2-a^2}} dz \frac{t}{(a^2+z^2)^{1/2}}$

Solution:



$$\vec{A} = \hat{z}\frac{\mu_0}{4\pi}\int_{-\infty}^{\infty}\frac{I(t_r)}{R}dz = \hat{z}\frac{\mu_0}{4\pi}\int_{-\infty}^{\infty}\frac{K(t-R/c)}{R}dz$$

$$\Rightarrow \vec{A} = \hat{z}\frac{\mu_0 K}{4\pi c}\int_{-\sqrt{c^2t^2-a^2}}^{\sqrt{c^2t^2-a^2}} dz \frac{ct-\sqrt{a^2+z^2}}{(a^2+z^2)^{1/2}}$$

6. A charged particle moves in a helical path under the influence of a constant magnetic field. The initial velocity is such that the component along the magnetic field is twice the component in the plane normal to the magnetic field. The ratio ℓ/R of the pitch ℓ to the radius R of the helical path is

[NET 2014]

A. $\pi/2$

B. 4π

C. 2π

D. π

Solution:

$$v_{\parallel} = 2v_{\perp}$$

$$\text{Pitch of the helix } l = v_{\parallel}T = v_{\parallel}\frac{2\pi R}{v_{\perp}}$$

$$= 2v_\perp \frac{2\pi R}{v_\perp} = 4\pi R$$

$$\frac{l}{R} = 4\pi$$

7. A proton moves with a speed of 300 m/s in a circular orbit in the xy-plan in a magnetic field 1 tesla along the positive z-direction. When an electric field of 1 V/m is applied along the positive y-direction, the center of the circular orbit

[NET 2014]

A. remains stationary

B. moves at 1 m/s along the negative x-direction

C. moves at 1 m/s along the positive z - direction

D. moves at 1 m/s along the positive x - direction

Solution: Change particle will deflect in $+x$-direction with

$$v = \frac{E}{B} = \frac{1}{1} = 1 \text{ m/s}.$$

The correct option is **(d)**

8. Given a uniform magnetic field $B = B_0\hat{k}$ (where B_0 is a constant), a possible choice for the magnetic vector potential A is

[NET 2015]

A. $B_0 y\hat{i}$ **B.** $-B_0 y\hat{i}$

C. $B_0(x\hat{j} + y\hat{i})$ **D.** $B_0(x\hat{i} + y\hat{j})$

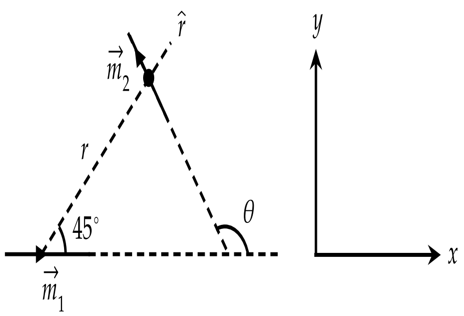
Solution: (a) $\vec{\nabla} \times \vec{A} = -B_0\hat{k}$
(b) $\vec{\nabla} \times \vec{A} = B_0\hat{k}$
(c) $\vec{\nabla} \times \vec{A} = 0$
(d) $\vec{\nabla} \times \vec{A} = 0$
The correct option is **(b)**

9. A small magnetic needle is kept at $(0,0)$ with its moment along the x-axis. Another small magnetic needle is at the point $(1,1)$ and is free to rotate in the xy - plane. In equilibrium the angle θ between their magnetic moments is such that

[NET 2015]

A. $\tan\theta = \frac{1}{3}$ **B.** $\tan\theta = 0$

C. $\tan\theta = 3$ **D.** $\tan\theta = 1$



Solution:

$$U=\frac{\mu_0}{4\pi r^3}\left[\vec{m}_1\cdot\vec{m}_2-3\left(\vec{m}_1\cdot\hat{r}\right)\left(\vec{m}_2\cdot\hat{r}\right)\right]$$
$$U=\frac{\mu_0 m_1 m_2}{4\pi r^3}\left[\cos\theta-3\cos 45^0\cos\left(\theta-45^0\right)\right]$$

For stable position energy is minimum i.e.

$$\frac{\partial U}{\partial\theta}=0\Rightarrow\frac{\mu_0 m_1 m_2}{4\pi r^3}\left[-\sin\theta+\frac{3}{\sqrt{2}}\sin\left(\theta-45^\circ\right)\right]=0$$
$$\Rightarrow\sin\theta=\frac{3}{\sqrt{2}}\left(\frac{\sin\theta}{\sqrt{2}}-\frac{\cos\theta}{\sqrt{2}}\right)\Rightarrow\tan\theta=3$$

The correct option **(c)**

10. A dipole of moment $\vec{p}$, oscillating at frequency ω, radiates spherical waves. The vector potential at large distance is

$$\vec{A}(\vec{r})=\frac{\mu_0}{4\pi}i\omega\frac{e^{ikr}}{r}\vec{p}$$

To order $\left(\frac{1}{r}\right)$ the magnetic field $\vec{B}$ at a point $\vec{r}=r\hat{n}$ is

[NET 2015]

A. $-\frac{\mu_0}{4\pi}\frac{\omega^2}{C}(\hat{n}\cdot\vec{p})\hat{n}\frac{e^{ikr}}{r}$

B. $-\frac{\mu_0}{4\pi}\frac{\omega^2}{C}(\hat{n}\times\vec{p})\frac{e^{ikr}}{r}$

C. $-\frac{\mu_0}{4\pi}\omega^2 k(\hat{n}\cdot\vec{p})\vec{p}\frac{e^{ikr}}{r}$

D. $-\frac{\pi_0}{4\pi}\frac{\omega^2}{C}\vec{p}\frac{e^{ikr}}{r}$

Solution:

$$\text{Let }\vec{p}=p\hat{z}$$

then,$\vec{B}$ must be in $\hat{\phi}$ direction.

$$\hat{n}\times\vec{p}=\hat{r}\times\hat{z}=\hat{\phi}$$

The correct option is (b).

11. A loop of radius a, carrying a current I, is placed in a uniform magnetic field B. If the normal to the loop is denoted by $\hat{n}$, the force $\vec{F}$ and the torque $\vec{T}$ on the loop are

[NET 2015]

A. $\vec{F}=0$ and $\vec{T}=\pi a^2 I\hat{n}\times B$

B. $\vec{F}=\frac{\mu_0}{4\pi}\vec{I}\times\vec{B}$

C. $\vec{F}=\frac{\mu_0}{4\pi}\vec{I}\times\vec{B}$ and $\vec{T}=I\hat{n}\times\vec{B}$

D. $\vec{F}=0$ and $\vec{T}=\frac{1}{\mu_0\varepsilon_0}I\vec{B}$

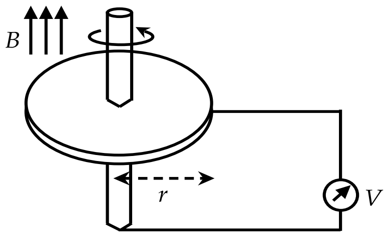
Solution:

$$\text{In uniform field }\vec{F}=0$$
$$\begin{aligned}\text{Torque }\vec{T}&=\vec{m}\times\vec{B}\\&=\pi a^2 I\hat{n}\times\vec{B}\end{aligned}$$

The correct option is **(a)**

12. A conducting circular disc of radius r and resistivity ρ rotates with an angular velocity ω in a magnetic field B perpendicular to it. A voltmeter is connected as shown in the figure below. Assuming its internal resistance to be infinite, the reading on the voltmeter

[NET 2016]



A. depends on ω, B, r and ρ

B. depends on ω, B and r but not on ρ

C. is zero because the flux through the loop is not changing

D. is zero because a current the flows in the direction of B

Solution: Force experienced by charge is

$$\vec{F} = q(\vec{v} \times \vec{B}) \text{ and } v = r\omega$$

13. A set of N concentric circular loops of wire, each carrying a steady current I in the same direction, is arranged in a plane. The radius of the first loop is $r_1 = a$ and the radius of the n^{th} loop is given by $r_n = nr_{n-1}$. The magnitude B of the magnetic field at the centre of the circles in the limit $N \to \infty$, is

[NET 2016]

A. $\mu_0 I\left(e^2-1\right)/4\pi a$ **B.** $\mu_0 I(e-1)/\pi a$

C. $\mu_0 I\left(e^2-1\right)/8a$ **D.** $\mu_0 I(e-1)/2a$

Solution:

$$B = \frac{\mu_0 I}{2}\left(\frac{1}{r_1} + \frac{1}{r_2} + \frac{1}{r_3} + \ldots\ldots.\frac{1}{r_n}\right)$$

$$r_1 = a$$

$$r_n = nr_{n-1}$$

$$r_1 = r_0 = a, r_2 = 2r_1 = 2a, r_3 = 3r_2 = 3.2a \text{ and } r_4 = 4r_3 = 4.3.2a$$

$$\Rightarrow B = \frac{\mu_0 I}{2a}\left(1 + \frac{1}{2} + \frac{1}{3.2} + \frac{1}{4.3.2} + \ldots\ldots\right)$$

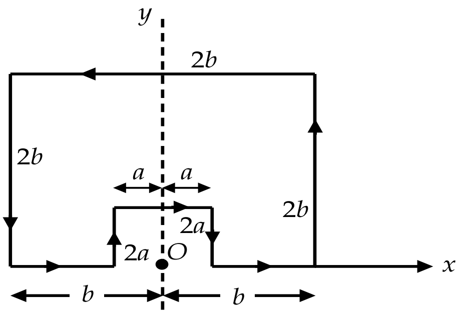
$$B = \frac{\mu_0 I}{2a}\left(\sum_{n=1}^{N} \frac{1}{\lfloor n}\right)$$

$$e^x = \sum_{n=0}^{\infty} \frac{x^n}{\lfloor n} \Rightarrow e = \sum_{n=0}^{\infty} \frac{1}{\lfloor n} = 1 + \sum_{n=1}^{\infty} \frac{1}{\lfloor n} \Rightarrow \sum_{n=1}^{\infty} \frac{1}{\lfloor n} = e - 1$$

$$\lim_{N\to\infty}\left(\sum_{n=l}^{N} \frac{1}{n}\right) = e - 1 \Rightarrow B = \frac{\mu_0 I}{2a}(e-1)$$

THe correct option is **(d)**

14. A constant current I is flowing in a piece of wire that is bent into a loop as shown in the figure.



The magnitude of the magnetic field at the point O is

[NET 2017]

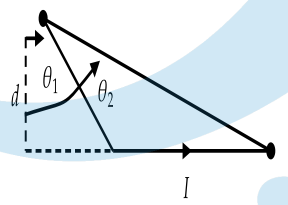
A. $\frac{\mu_0 I}{4\pi\sqrt{5}} \ln\left(\frac{a}{b}\right)$

B. $\frac{\mu_0 I}{4\pi\sqrt{5}} \left(\frac{1}{a}-\frac{1}{b}\right)$

C. $\frac{\mu_0 I}{4\pi\sqrt{5}} \left(\frac{1}{a}\right)$

D. $\frac{\mu_0 I}{4\pi\sqrt{5}} \left(\frac{1}{b}\right)$

Solution:



$$\vec{B} = \frac{\mu_0 I}{4\pi d}\left(\sin\theta_2 - \sin\theta_1\right)\hat{\phi}$$

Magnetic field due to left and right segment of 2a

$$B_{2a} = \frac{\mu_0 I}{4\pi a}\left(\frac{2a}{\sqrt{5}a}\right)\otimes$$

Field due to upper segment of $2a$

$$= \frac{\mu_0 I}{4\pi(2a)} \times \left(\frac{a}{\sqrt{5}a} + \frac{a}{\sqrt{5}a}\right)$$

Net field

$$B_{2a} = 2 \times \frac{\mu_0 I}{4\pi a} \times \frac{2}{\sqrt{5}} + \frac{\mu_0 I}{4\pi a} \times \frac{1}{\sqrt{5}}$$

$$B_{2a} = \frac{\mu_0 I}{4\pi a}\sqrt{5}\otimes(\text{ inward })$$

$$\text{similarly,} B_{2b} = \frac{\mu_0 I}{4\pi b}\sqrt{5}\odot(\text{outward})$$

Net field

$$B = B_{2a} - B_{2b} = \frac{\mu_0 I}{4\pi}\sqrt{5}\left(\frac{1}{a}-\frac{1}{b}\right)$$

The correct option is **(b)**

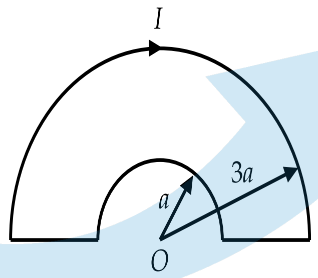
15. A circular current carrying loop of radius a carries a steady current. A constant electric charge is kept at the centre of the loop. The electric and magnetic fields, $\vec{E}$ and $\vec{B}$ respectively, at a distance d vertically above the centre of the loop satisfy

[NET 2017]

A. $\vec{E} \perp \vec{B}$
B. $\vec{E} = 0$
C. $\vec{\nabla}(\vec{E} \cdot \vec{B}) = 0$
D. $\vec{\nabla} \cdot (\vec{E} \times \vec{B}) = 0$

Solution: $\vec{E} \times \vec{B} = 0 \Rightarrow \vec{\nabla} \cdot (\vec{E} \times \vec{B}) = 0$
The correct option is **(c)**

16. The loop shown in the figure below carries a steady current I



The magnitude of the magnetic field at the point O is

[NET 2018]

A. $\frac{\mu_0 I}{2a}$
B. $\frac{\mu_0 I}{6a}$
C. $\frac{\mu_0 I}{4a}$
D. $\frac{\mu_0 I}{3a}$

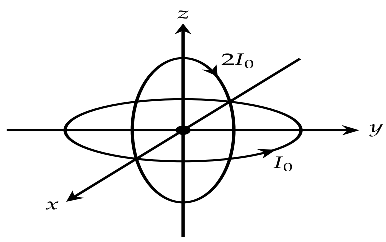
Solution:

$$B_a = \frac{1}{2}\frac{\mu_0 I}{2a} \odot,$$
$$B_{3a} = \frac{1}{2}\frac{\mu_0 I}{2(3a)} \otimes$$
$$B = B_a - B_{3a} = \frac{\mu_0 I}{4a}\left(1 - \frac{1}{3}\right) = \frac{\mu_0 I}{6a}$$

THe correct option is **(b)**

17. Two current-carrying circular loops, each of radius R, are placed perpendicular to each other, as shown in the figure.
The loop in the xy - plane carries a current I_0 while that in the xz-plane carries a current $2I_0$. The resulting magnetic field $\vec{B}$ at the origin is

[NET 2018 dec]



A. $\frac{\mu_0 I_0}{2R}[2\hat{j}+\hat{k}]$

B. $\frac{\mu_0 I_0}{2R}[2\hat{j}-\hat{k}]$

C. $\frac{\mu_0 I_0}{2R}[-2\hat{j}+\hat{k}]$

D. $\frac{\mu_0 I_0}{2R}[-2\hat{j}-\hat{k}]$

Solution:

Field due to loop in xy plane is

$$\vec{B}_1 = \frac{\mu_0 I_0}{2R}\hat{z}$$

Field due to loop in xz plane is

$$\vec{B}_2 = \frac{\mu_0 (2I_0)}{2R}(-\hat{y})$$

Resultant field

$$\vec{B} = \vec{B}_1 + \vec{B}_2 = \frac{\mu_0 I_0}{2R}(-2\hat{y}+\hat{z})$$

The correct option is **(c)**

Practice Set - 2 solutions

1. Two magnetic dipoles of magnitude m each are placed in a plane as shown in figure The energy of interaction is given by

[GATE 2010]

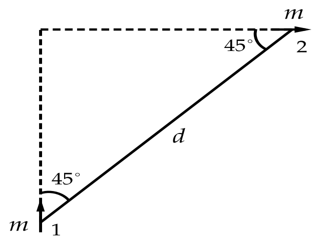


Figure 1

A. Zero

B. $\frac{\mu_0 m^2}{4\pi d^3}$

C. $\frac{3\mu_0 m^2}{2\pi d^3}$

D. $-\frac{3\mu_0 m^2}{8\pi d^3}$

Solution:

$$U = \frac{\mu_0}{4\pi r^3}\left[\vec{m}_1 \cdot \vec{m}_2 - 3\left(\vec{m}_1 \cdot \hat{r}\right)\left(\vec{m}_2 \cdot \hat{r}\right)\right]$$

Since $\vec{m}_1 \perp \vec{m}_2 \Rightarrow \vec{m}_1 \cdot \vec{m}_2 = 0$

$$U = \frac{\mu_0}{4\pi d^3}\left[-3 \times m\cos 45^0 \times m\cos 45^0\right]$$

$$\Rightarrow U = -\frac{3\mu_0 m^2}{8\pi d^3}$$

The correct option is **(d)**

2. If a force $\vec{F}$ is derivable from a potential function $V(r)$, where r is the distance from the origin of the coordinate system, it follows that

[GATE 2011]

A. $\vec{\nabla} \times \vec{F} = 0$

B. $\vec{\nabla} \cdot \vec{F} = 0$

C. $\vec{\nabla} V = 0$

D. $\nabla^2 V = 0$

Solution: The correct option is **(a)**

3. A uniform surface current is flowing in the positive y-direction over an infinite sheet lying in $x-y$ plane. The direction of the magnetic field is

[GATE 2011]

A. along $\hat{i}$ for $z > 0$ and along $-\hat{i}$ for $z < 0$

B. along $\hat{k}$ for $z > 0$ and along $-\hat{k}$ for $z < 0$

C. along $-\hat{i}$ for $z > 0$ and along $\hat{i}$ for $z < 0$

D. along $-\hat{k}$ for $z > 0$ and along $\hat{k}$ for $z < 0$

Solution: The correct option is **(a)**

4. A magnetic dipole of dipole moment $\vec{m}$ is placed in a non-uniform magnetic field $\vec{B}$. If the position vector of the dipole is $\vec{r}$, the torque acting on the dipole about the origin is

[GATE 2011]

A. $\vec{r} \times (\vec{m} \times \vec{B})$

B. $\vec{r} \times \vec{\nabla}(\vec{m} \cdot \vec{B})$

C. $\vec{m} \times \vec{B}$

D. $\vec{m} \times \vec{B} + \vec{r} \times \nabla(\vec{m} \cdot \vec{B})$

Solution: The correct option is **(c)**

5. Which of the following expressions for a vector potential $\vec{A}$ DOES NOT represent a uniform magnetic field of magnitude B_0 along the z-direction?

[GATE 2011]

A. $\vec{A} = (0, B_0 x, 0)$

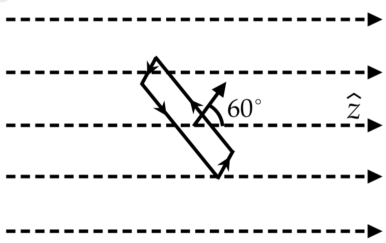
B. $\vec{A} = (-B_0 y, 0, 0)$

C. $\vec{A} = \left(\frac{B_0 x}{2}, \frac{B_0 y}{2}, 0\right)$

D. $\vec{A} = \left(-\frac{B_0 y}{2}, \frac{B_0 x}{2}, 0\right)$

Solution: $\vec{B} \neq \vec{\nabla} \times \vec{A}$
The correct option is **(c)**

6. In a constant magnetic field of 0.6 Tesla along the z direction, find the value of the path integral $\oint \vec{A} \cdot \vec{dl}$ in the units of (Tesla m^2) on a square loop of side length $(1/\sqrt{2})$ meters. The normal to the loop makes an angle of 60° to the z-axis, as shown in the figure.



The answer should be up to two decimal places.

[GATE]

Solution:

$$\begin{aligned}
\oint \vec{A} \cdot \vec{dl} &= \int_S (\vec{\nabla} \times \vec{A}) d\vec{a} \\
&= \int_S \vec{B} \cdot d\vec{a} \\
&= BA\cos 60^0 \\
&= 0.6 \times \left(\frac{1}{\sqrt{2}}\right)^2 \times \frac{1}{2} \\
&= 0.15 T.m^2
\end{aligned}$$

7. The value of the magnetic field required to maintain non-relativistic protons of energy 1MeV in a circular orbit of radius 100 mm is Tesla

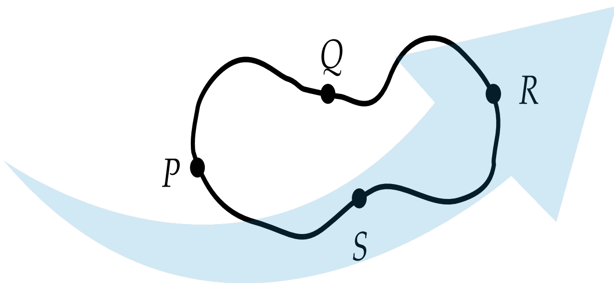
[GATE 2014]

Solution:

$$
\begin{aligned}
E &= \frac{q^2B^2R^2}{2m_p} \Rightarrow 1.6\times10^{-13} = \frac{\left(1.6\times10^{-19}\right)^2 B^2(0.1)^2}{2\left(1.67\times10^{-27}\right)} \\
\Rightarrow B^2 &= \frac{1.6\times10^{-13}\times 2\left(1.67\times10^{-27}\right)}{\left(1.6\times10^{-19}\right)^2(0.1)^2} \\
\Rightarrow B^2 &= \frac{10^{-13}\times2\left(1.67\times10^{-27}\right)}{\left(1.6\times10^{-38}\right)(0.01)} = \frac{3.34\times10^{-40}}{1.6\times10^{-40}} = 2.08 \\
\Rightarrow B &= \sqrt{2.08}\ \text{Tesla} \ = 1.44\ \text{Tesla}
\end{aligned}
$$

8. Given that the magnetic flux through the closed loop $PQRSP$ is ϕ. If $\int_P^R \vec{A}\cdot d\vec{l} = \phi_1$ along PQR, the value of $\int^R \vec{A}\cdot d\vec{l}$ along PSR is

[GATE 2015]



A. (a) $\phi-\phi_1$

B. $\phi_1-\phi$

C. $-\phi_1$

D. ϕ_1

Solution:

$$
\begin{aligned}
\phi &= \int_s \vec{B}\cdot d\vec{a} \\
&= \oint \vec{A}\cdot d\vec{l} \\
&= \int_P^R \vec{A}\cdot d\vec{l} + \int_R^P \vec{A}\cdot d\vec{l} \\
\phi &= \phi_1 - \int_P^R \vec{A}\cdot d\vec{l} \\
\int_P^R \vec{A}\cdot d\vec{l} &= \phi_1 - \phi
\end{aligned}
$$

The correct option is **(b)**

9. Which of the following magnetic vector potentials gives rise to a uniform magnetic field $B_0\hat{k}$?

[GATE 2016]

A. $B_0 z\hat{k}$

B. $-B_0 x\hat{j}$

C. $\frac{B_0}{2}(-y\hat{i}+x\hat{j})$

D. $\frac{B_0}{2}(y\hat{i}+x\hat{j})$

Solution: (a) $\vec{\nabla}\times\vec{A}=0$
(b) $\vec{\nabla}\times\vec{A}=-B_0\hat{k}$
(c) $\vec{\nabla}\times\vec{A}=B_0\hat{k}$

(d) $\vec{\nabla} \times \vec{A} = 0$
The correct option is **(c)**

10. The magnitude of the magnetic dipole moment associated with a square shaped loop carrying a steady current I is m. If this loop is changed to a circular shape with the same current I passing through it, the magnetic dipole moment becomes $\frac{pm}{\pi}$. The value of p is

[GATE 2016]

Solution: Magnetic dipole moment associated with a square shaped loop (let side is a) carrying a steady current I is $m = Ia^2$.
Magnetic dipole moment associated with a circular shaped loop (let radius is r) carrying a steady current I is $m' = I\pi r^2$. Here,

$$\begin{aligned} 4a &= 2\pi r \\ r &= \frac{2a}{\pi} \\ m' &= I\pi r^2 = I\pi\left(\frac{2a}{\pi}\right)^2 \\ &= \frac{4Ia^2}{\pi} = \frac{4m}{\pi} \end{aligned}$$

11. An infinite solenoid carries a time varying current $I(t) = At^2$, with $A \neq 0$. The axis of the solenoid is along the $\hat{z}$ direction. $\hat{r}$ and $\hat{\theta}$ are the usual radial and polar directions in cylindrical polar coordinates. $\vec{B} = B_r\hat{r} + B_\theta\hat{\theta} + B_z\hat{z}$ is the magnetic field at a point outside the solenoid. Which one of the following statements is true?

[GATE 2017]

A. $B_r = 0, B_\theta = 0, B_z = 0$

B. $B_r \neq 0, B_\theta \neq 0, B_z = 0$

C. $B_r \neq 0, B_\theta \neq 0, B_z \neq 0$

D. $B_r = 0, B_\theta = 0, B_z \neq 0$

Solution: The correct option is **(d)**

12. An infinitely long straight wire is carrying a steady current I. The ratio of magnetic energy density at distance r_1 to that at $r_2 (= 2r_1)$ from the wire is

[GATE 2018]

Solution:

$$u_B = \frac{B^2}{2\mu_0} \propto \frac{1}{r^2} \Rightarrow \frac{u_{B1}}{u_{B2}} = \frac{r_2^2}{r_1^2} = \frac{(2r_1)}{r_1^2} = 4$$

13. A constant and uniform magnetic field $\vec{B} = B_0\hat{k}$ pervades all space. Which one of the following is the correct choice for the vector potential in Coulomb gauge?

[GATE 2018]

A. $-B_0(x+y)\hat{i}$

B. $B_0(x+y)\hat{j}$

C. $B_0 x\hat{j}$

D. $-\frac{1}{2}B_0(x\hat{i} - y\hat{j})$

Solution: Check option (c),

$$\vec{\nabla} \cdot \vec{A} = 0, \vec{B} = \vec{\nabla} \times \vec{A} = B_0\hat{k}$$

The correct option is **(c)**

14. A solid cylinder of radius R has total charge Q distributed uniformly over its volume. It is rotating about its axis with angular speed ω. The magnitude of the total magnetic moment of the cylinder is

[GATE 2019]

A. (a) $QR^2\omega$

B. $\frac{1}{2}QR^2\omega$

C. $\frac{1}{4}QR^2\omega$

D. $\frac{1}{8}QR^2\omega$

Solution:

Magnetic moment due to disc

$$\mu = \frac{\pi\sigma\omega R^4}{4}$$

Due to cylinder

$$d\mu = \frac{\pi\omega R^4}{4}(\rho dz) \quad (\sigma \to \rho dz)$$

$$\mu = \frac{\pi\omega R^4}{4}\int_0^L \frac{Q}{\pi R^2 L}dz = \frac{Q\omega R^4}{4}$$

15. An infinitely long wire parallel to the x-axis is kept at $z = d$ and carries a current I in the positive x direction above a superconductor filling the region $z \leq 0$ (see figure). The magnetic field $\vec{B}$ inside the superconductor is zero so that the field just outside the superconductor is parallel to its surface. The magnetic field due to this configuration at a point $(x, y, z > 0)$ is

[GATE 2019]

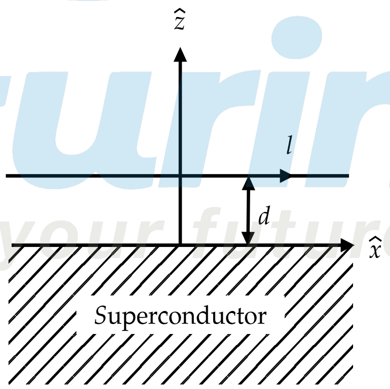


Figure 2

A. $\left(\frac{\mu_0 I}{2\pi}\right)\frac{-(z-d)\hat{j}+y\hat{k}}{[y^2+(z-d)^2]}$

B. $\left(\frac{\mu_0 I}{2\pi}\right)\left[\frac{-(z-d)\hat{j}+y\hat{k}}{y^2+(z-d)^2}+\frac{(z+d)\hat{j}-y\hat{k}}{y^2+(z+d)^2}\right]$

C. $\left(\frac{\mu_0 I}{2\pi}\right)\left[\frac{-(z-d)\hat{j}+y\hat{k}}{y^2+(z-d)^2}-\frac{(z+d)\hat{j}-y\hat{k}}{y^2+(z+d)^2}\right]$

D. $\left(\frac{\mu_0 I}{2\pi}\right)\left[\frac{y\hat{j}+(z-d)\hat{k}}{y^2+(z-d)^2}+\frac{y\hat{j}-(z+d)\hat{k}}{y^2+(z+d)^2}\right]$

Solution: Verify that $\vec{B} = 0$, when $d = 0$
The correct option is **(b)**

16. The vector potential inside a long solenoid with n turns per unit length and carrying current I, written in cylindrical coordinates is $\vec{A}(s,\phi,z) = \frac{\mu_0 nI}{2}s\hat{\phi}$. If the term $\frac{\mu_0 nI}{2}s(\alpha\cos\phi\hat{\phi}+\beta\sin\phi\hat{s})$, where $\alpha \neq 0, \beta \neq 0$ is added to $\vec{A}(S,\phi,z)$, the magnetic field remains the same if

[GATE 2019]

A. $\alpha=\beta$ **B.** $\alpha=-\beta$

C. $\alpha=2\beta$ **D.** $\alpha=\frac{\beta}{2}$

Solution:

$$\text{Solution: } \vec{B}=\vec{\nabla}\times\vec{A}=\frac{1}{r}\begin{vmatrix}\hat{r} & r\hat{\phi} & \hat{z}\\ \frac{\partial}{\partial r} & \frac{\partial}{\partial \phi} & \frac{\partial}{\partial z}\\ A_r & rA_\phi & 0\end{vmatrix}=\mu_0 nI\hat{z}$$

$$\vec{B}'=\vec{\nabla}\times\vec{A}'=\frac{1}{r}\begin{vmatrix}\hat{r} & r\hat{\phi} & \hat{z}\\ \frac{\partial}{\partial r} & \frac{\partial}{\partial \phi} & \frac{\partial}{\partial z}\\ A_r & rA_\phi & 0\end{vmatrix}=\mu_0 nI\left[(\alpha\cos\phi+1)-\frac{\beta\cos\phi}{2}\right]\hat{z}$$

$$\text{Equate } \vec{B}'=\vec{B}\Rightarrow\left[(\alpha\cos\phi+1)-\frac{\beta\cos\phi}{2}\right]=\mu_0 nI$$

$$\Rightarrow\alpha\cos\phi=\frac{\beta}{2}\cos\phi\Rightarrow\alpha=\frac{\beta}{2}$$

The correct option is **(d)**

17. A magnetic field $\vec{B}=B_0(\hat{i}+2\hat{j}-4\hat{k})$ exists at point. If a test charge moving with a velocity, $\vec{v}=v_0(3\hat{i}-\hat{j}+2\hat{k})$ experiences no force at a certain point, the electric field at that point in SI units is

[JEST 2012]

A. $\vec{E}=-v_0B_0(3\hat{i}-2\hat{j}-4\hat{k})$ **B.** $\vec{E}=-v_0B_0(\hat{i}+\hat{j}+7\hat{k})$

C. $\vec{E}=v_0B_0(14\hat{j}+7\hat{k})$ **D.** $\vec{E}=-v_0B_0(14\hat{j}+7\hat{k})$

Solution:

$$\vec{F}=q[\vec{E}+\vec{v}\times\vec{B}]=0\Rightarrow\vec{E}=-(\vec{v}\times\vec{B})$$
$$\Rightarrow\vec{E}=-v_0B_0\{(4-4)\hat{i}+(2+12)\hat{j}+(6+1)\hat{k}\}$$
$$=-v_0B_0(14\hat{j}+7\hat{k})$$

18. A small magnet is dropped down a long vertical copper tube in a uniform gravitational field. After a long time, the magnet

[JEST 2012]

A. attains a constant velocity **B.** moves with a constant acceleration

C. moves with a constant deceleration **D.** executes simple harmonic motion

Solution: The correct option is **(a)**

19. A thin uniform ring carrying charge Q and mass M rotates about its axis. What is the gyromagnetic ratio (defined as ratio of magnetic dipole moment to the angular momentum) of this ring?

[JEST 2013]

A. $\frac{Q}{2\pi M}$ **B.** $\frac{Q}{M}$

C. $\frac{Q}{2M}$ **D.** $\frac{Q}{\pi M}$

Solution: Magnetic dipole moment $M'=IA=\frac{Q}{T}\pi r^2\Rightarrow\frac{Q}{2\pi T}\times 2\pi\times\pi r^2=\frac{Q\omega r^2}{2}$
Angular momentum $J=Mr^2\omega\Rightarrow\frac{M'}{J}=\frac{Q}{2M}$

20. The electric and magnetic field caused by an accelerated charged particle are found to scale as $E \propto r^{-n}$ and $B \propto r^{-m}$ at large distances. What are the value of n and m ?

[JEST 2013]

A. $n = 1, m = 2$ **B.** $n = 2, m = 1$

C. $n = 1, m = 1$ **D.** $n = 2, m = 2$

Solution:

For large distance

$$F = \frac{qa\sin\theta}{r}$$
$$, B = \frac{qa\sin\theta}{r}$$
$$\Rightarrow E \propto \frac{1}{r},$$
$$B \propto \frac{1}{r}$$
$$\text{So} \quad m = n = 1$$

The correct option is **(c)**

21. A system of two circular co-axial coils carrying equal currents I along same direction having equal radius R and separated by a distance R (as shown in the figure below). The magnitude of magnetic field at the midpoint P is given by

[JEST 2014]

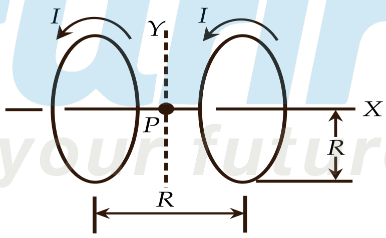


Figure 3

A. (a) $\frac{\mu_0 I}{2\sqrt{2}R}$ **B.** $\frac{4\mu_0 I}{5\sqrt{5}R}$

C. $\frac{8\mu_0 I}{5\sqrt{5}R}$ **D.** 0

Solution:

$$\because B = \frac{\mu_0 I R^2}{2\left(R^2 + d^2\right)^{\frac{3}{2}}}$$
$$B_1 = \frac{\mu_0 I R^2}{2\left(R^2 + \frac{R^2}{4}\right)^{\frac{3}{2}}}$$
$$B_2 = \frac{\mu_0 I R^2}{2\left(R^2 + \frac{R^2}{4}\right)^{\frac{3}{2}}} \because d = \frac{R}{2}$$
$$B = B_1 + B_2$$

$$= \frac{\mu_0 I \times 2}{2R\left(\frac{5}{4}\right)^{\frac{3}{2}}}$$

$$B = \frac{\mu_0 I 4^{\frac{3}{2}}}{R \quad 5^{\frac{3}{2}}} = \frac{8\mu_0 I}{5\sqrt{5}R}$$

The correct option is **(c)**

22. A charged particle is released at time $t = 0$, from the origin in the presence of uniform static electric and magnetic fields given by $E = E_0\hat{y}$ and $B = B_0\hat{z}$ respectively. Which of the following statements is true for $t > 0$?

[JEST 2015]

A. The particle moves along the x-axis.

B. The particle moves in a circular orbit.

C. The particle moves in the (x, y) plane.

D. Particle moves in the (y, z) plane

Solution: In a cycloid charged particle will be always confined in a plane perpendicular to B.
The correct option is **(c)**

23. The strength of magnetic field at the center of a regular hexagon with sides of length a carrying a steady current I is:

[JEST 2016]

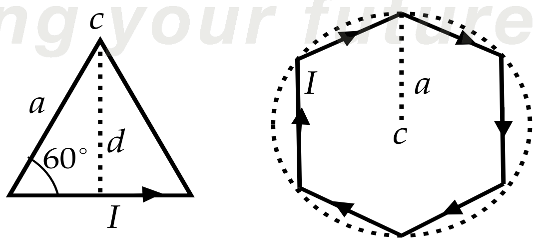
A. $\frac{\mu_0 I}{\sqrt{3}\pi a}$

B. $\frac{\sqrt{6}\mu_0 I}{\pi a}$

C. $\frac{3\mu_0 I}{\pi a}$

D. $\frac{\sqrt{3}\mu_0 I}{\pi a}$

Solution:



$$d = a\cos 30^\circ = \frac{\sqrt{3}}{2}a$$

$$\because B = \frac{\mu_0 I}{4\pi d}(\sin\theta_2 - \sin\theta_1)$$

$$\Rightarrow B_1 = \frac{\mu_0 I}{4\pi d} 2\sin 30^\circ = \frac{\mu_0 I}{4\pi\frac{\sqrt{3}}{2}a} 2\sin 30^\circ = \frac{\mu_0 I}{2\sqrt{3}\pi a}$$

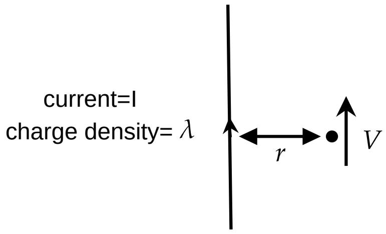
$$\Rightarrow B = 6B_1 = 6 \times \frac{\mu_0 I}{2\sqrt{3}\pi a}$$

$$= \frac{3\mu_0 I}{\sqrt{3}\pi a}$$

$$= \frac{\sqrt{3}\mu_0 I}{\pi a}$$

24. A wire with uniform line charge density λ per unit length carries a current I as shown in the figure. Take the permittivity and permeability of the medium to be $\varepsilon_0 = \mu_0 = 1$.A particle of charge q is at a distance r and is travelling along a trajectory parallel to the wire. What is the speed of the charge?

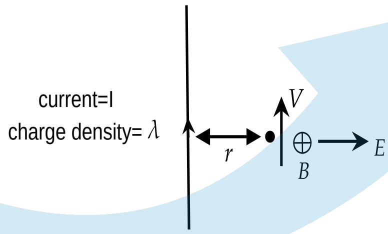
[JEST 2019]



A. $\frac{\lambda}{I}$ **B.** $\frac{\lambda}{2I}$

C. $\frac{\lambda}{3I}$ **D.** $\frac{4\lambda}{I}$

Solution:



$$E = \frac{\lambda}{2\pi\varepsilon_0 r} \text{ and } B = \frac{\mu_0 I}{2\pi r}$$

Net force on q is zero i.e.

$$\vec{F} = 0$$
$$\Rightarrow q[\vec{E} + (\vec{v} \times \vec{B})] = 0$$
$$E = vB \Rightarrow \frac{\lambda}{2\pi\varepsilon_0 r} = v\frac{\mu_0 I}{2\pi r}$$
$$v = \frac{\lambda}{I} \quad \because \varepsilon_0 = \mu_0 = 1$$

The correct option is **(a)**

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