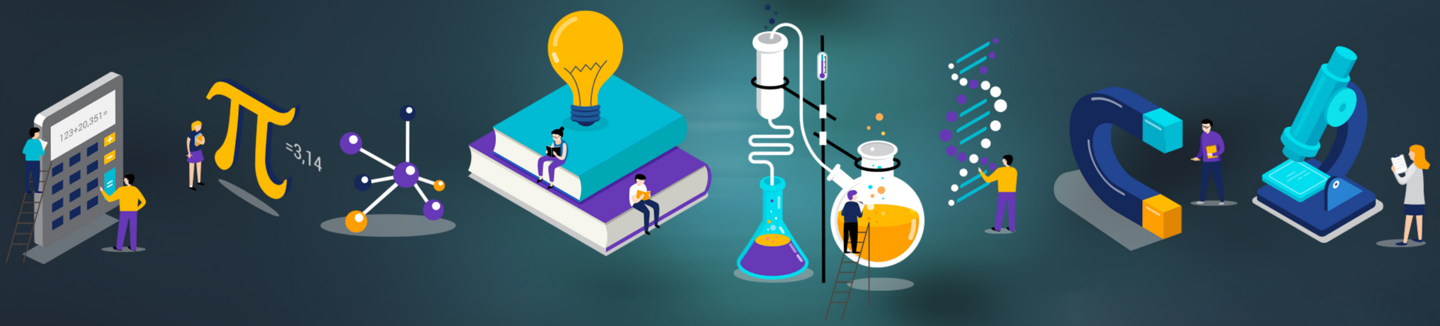


1. Rigid Body Dynamics -Solutions

Practice Set-1

1. An annulus of mass M made of a material of uniform density has inner and outer radii a and b respectively. Its principle moment of inertia along the axis of symmetry perpendicular to the plane of the annulus is: **[NET DEC 2011]**

A. $\frac{1}{2}M\frac{(b^4+a^4)}{(b^2-a^2)}$ **B.** $\frac{1}{2}M\pi\left(b^2-a^2\right)$

C. $\frac{1}{2}M\left(b^2-a^2\right)$ **D.** $\frac{1}{2}M\left(b^2+a^2\right)$

Solution: The correct option is **(d)**

2. Two bodies of equal mass m are connected by a massless rigid rod of length l lying in the xy-plane with the centre of the rod at the origin. If this system is rotating about the z-axis with a frequency ω, its angular momentum is **[NET DEC 2012]**

A. $ml^2\omega/4$ **B.** $ml^2\omega/2$

C. $ml^2\omega$ **D.** $2ml^2\omega$

Solution:

Since rod is massless i.e. $M=0$.

Moment of inertia of the system $I=m_1r_1^2+m_2r_2^2, m_1=m_2=m$ and $r_1=r_2=\frac{l}{2}$

$$I=\frac{ml^2}{4}+\frac{ml^2}{4}\Rightarrow I=\frac{ml^2}{2}$$

Angular momentum, $J=I\omega$ and $J=\frac{ml^2\omega}{2}$

The correct option is **(b)**

3. Two masses m each, are placed at the points $(x,y)=(a,a)$ and $(-a,-a)$ and two masses, $2m$ each, are placed at the points $(a,-a)$ and $(-a,a)$. The principal moments of inertia of the system are

[NET DEC 2015]

A. $2m^2, 4ma^2$ **B.** $4ma^2, 8ma^2$

C. $4ma^2, 4ma^2$ **D.** $8ma^2, 8ma^2$

Solution:

$$I_{xx}=\sum_i m_i\left(y_i^2+z_i^2\right)=\sum m_i y_i^2 \quad \because z_i=0$$

$$\Rightarrow I_{xx}=ma^2+ma^2+2ma^2+2ma^2 \Rightarrow I_{xx}=6ma^2$$

Similarly, $I_{yy}=6ma^2$ and $I_{zz}=12ma^2$

$$I_{xz}=I_{zx}=0, I_{yz}=I_{zy}=0$$

$$I_{xy}=I_{yx}=-m_i\sum_z x_i y_i=-m$$

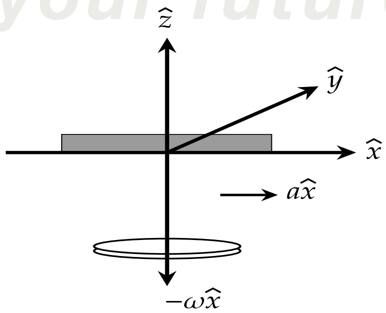
$$I_{xy}=I_{yx}=-m_i\sum_i x_i y_i=-ma^2-ma^2+2ma^2+2ma^2 \Rightarrow I_{xy}=I_{yx}=2ma^2$$

Moment of inertia tensor $I=\begin{pmatrix} 6ma^2 & 2ma^2 & 0 \\ 2ma^2 & 6ma^2 & 0 \\ 0 & 0 & 12ma^2 \end{pmatrix}$ Eigen value of matrices is principal moment of inertia, which is given by

$$\lambda_1=4ma^2=I_x, \lambda_2=8ma^2=I_y, \lambda_3=12ma^2=I_z$$

So, $I_x=4ma^2$ and $I_y=8ma^2$
The correct option is **(b)**

4. A disc of mass m is free to rotate in a plane parallel to the xy plane with an angular velocity $-\omega\hat{z}$ about a massless rigid rod suspended from the roof of a stationary car (as shown in the figure below). The rod is free to orient itself along any direction.



The car accelerates in the positive x-direction with an acceleration $a>0$. Which of the following statements is true for the coordinates of the centre of mass of the disc in the reference frame of the car?

[NET DEC 2017]

A. only the x and the z coordinates change **B.** only the y and the z coordinates change

C. only the x and the y coordinates change **D.** all the three coordinates change

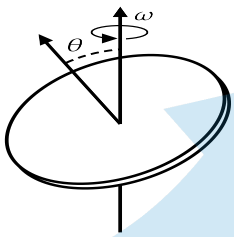
Solution: The correct option is **(d)**

Practice set-2

1. A uniform solid cylinder is released on a horizontal surface with speed 5 m/s without any rotation (slipping without rolling). The cylinder eventually starts rolling without slipping. If the mass and radius of the cylinder are 10gm and 1 cm respectively, the final linear velocity of the cylinder is............. m/s. (up to two decimal places). **[GATE 2017]**

Solution: $mvr = mv_{cm}r + I_{cm}\omega = mv_{cm}r + \frac{1}{2}mr^2\frac{v_{cm}}{r} \Rightarrow v = \frac{3}{2}v_{cm} \Rightarrow v_{cm} = \frac{2}{3}v = \frac{10}{3} = 3.33m/\text{sec}$

2. A uniform circular disc of mass m and radius R is rotating with angular speed ω about an axis passing through its centre and making an angle $\theta = 30^\circ$ with the axis of the disc. If the kinetic energy of the disc is $\alpha m\omega^2 R^2$, the value of α is (up to two decimal places). **[GATE 2018]**



Solution:

The kinetic energy of the disc is,

$$T = \frac{1}{2}\vec{L}\cdot\vec{\omega}$$

Where $\vec{L}$ is angular momentum and ω is angular velocity

$$T = \frac{1}{2}|\vec{L}||\vec{\omega}|\cos 30^\circ = \frac{1}{2}I\omega\cdot\omega\frac{\sqrt{3}}{2} = \frac{1}{2}\left(\frac{mR^2}{2}\right)\omega^2 \times \frac{\sqrt{3}}{2}$$

$$T = \frac{\sqrt{3}}{8}m\omega^2R^2 = 0.21m\omega^2R^2 \Rightarrow \alpha m\omega^2R^2 = 0.21m\omega^2R^2$$

Hence, $\alpha = 0.21$

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