

## Practise Set-1 Solutions

1. Consider two point charges  $q$  and  $\lambda q$  located at the points,  $x = a$  and  $x = \mu a$ , respectively. Assuming that the sum of the two charges is constant, what is the value of  $\lambda$  for which the magnitude of the electrostatic force is maximum?

[JEST 2015]

A.  $\mu$

B. 1

C.  $\frac{1}{\mu}$

D.  $1 + \mu$

**Solution:**

$$\begin{aligned}
 F &= \frac{1}{4\pi\epsilon_0} \frac{(\lambda q \times q)}{(\mu a - a)^2} \\
 &= \frac{1}{4\pi\epsilon_0} \frac{\lambda q^2}{a^2(\mu - 1)^2} \\
 &= \frac{1}{4\pi\epsilon_0 a^2(\mu - 1)^2} \frac{\lambda c^2}{(1 + \lambda)^2} \quad \because q + \lambda q = c \\
 &= c
 \end{aligned}$$

For maximum  $F$ ,  $\frac{dF}{d\lambda} = 0$

$$0 = \frac{1}{4\pi\epsilon_0 a^2(\mu - 1)^2} \left[ \frac{(1 + \lambda)^2 c^2 - \lambda c^2 \times 2(1 + \lambda)}{(1 + \lambda)^4} \right]$$

$$\Rightarrow (1 + \lambda)^2 c^2 = \lambda c^2 \times 2(1 + \lambda) \Rightarrow 1 + \lambda = 2\lambda$$

$$\Rightarrow \lambda = 1$$

Correct option is (B)

2. An electric field in a region is given by  $\vec{E}(x, y, z) = ax\hat{i} + cz\hat{j} + 6by\hat{k}$ . For which values of  $a, b, c$  does this represent an electrostatic field?

[JEST 2012]

A. 13, 1, 12

B. 17, 6, 1

C. 13, 1, 6

D. 45, 6, 1

**Solution:**

For electrostatic field,

$$\vec{\nabla} \times \vec{E} = 0$$

$$\vec{\nabla} \times \vec{E} = \begin{bmatrix} \hat{i} & \hat{j} & \hat{k} \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ ax & cz & 6by \end{bmatrix} = 0$$

$$\Rightarrow \vec{\nabla} \times \vec{E} = (6b - c)\hat{i} + \hat{j}[0 - 0] + \hat{k}[0] = 0$$

$$\Rightarrow (6b - c)\hat{i} = 0$$

$$\Rightarrow c = 6b$$

Correct option is (C)

3. The electric fields outside ( $r > R$ ) and inside ( $r < R$ ) a solid sphere with a uniform volume charge density are given by  $\vec{E}_{r>R} = \frac{1}{4\pi\epsilon_0} \frac{q}{r^2} \hat{r}$  and  $\vec{E}_{r<R} = \frac{1}{4\pi\epsilon_0} \frac{q}{R^3} r \hat{r}$  respectively, while the electric field outside a spherical shell with a uniform surface charge density is given by  $\vec{E}_{r>R} = \frac{1}{4\pi\epsilon_0} \frac{q}{r^2} \hat{r}$ ,  $q$  being the total charge. The correct ratio of the electrostatic energies for the second case to the first case is

[JEST 2013]

A. 1 : 3

B. 9 : 16

C. 3 : 8

D. 5 : 6

**Solution:**

Electrostatic energy in spherical shell ,

$$\begin{aligned}
 w_{sp} &= \frac{\epsilon_0}{2} \int_0^R |\vec{E}_1|^2 4\pi r^2 dr + \frac{\epsilon_0}{2} \int_R^\infty |\vec{E}_2|^2 4\pi r^2 dr \\
 &\Rightarrow \frac{\epsilon_0}{2} \int_R^\infty \frac{q^2}{(4\pi\epsilon_0)^2 r^4} 4\pi r^2 dr = \frac{q^2}{8\pi\epsilon_0} \left( -\frac{1}{r} \right)_R^\infty \\
 &= \frac{q^2}{8\pi\epsilon_0} \frac{1}{R}
 \end{aligned}$$

Electrostatic energy in solid sphere,

$$\begin{aligned}
 w_s &= \frac{\epsilon_0}{2} \int_0^R |E_1|^2 4\pi r^2 dr + \frac{\epsilon_0}{2} \int_R^\infty |E_2|^2 4\pi r^2 dr \\
 &\Rightarrow \frac{q^2}{8\pi\epsilon_0} \times \frac{1}{R^6} \left[ \frac{r^5}{5} \right]_0^R + \frac{q^2}{8\pi\epsilon_0} \left[ -\frac{1}{r} \right]_R^\infty \\
 w_s &= \frac{q^2}{5 \times 8\pi\epsilon_0} \cdot \frac{1}{R} + \frac{q^2}{8\pi\epsilon_0 R} = \frac{6q^2}{40\pi\epsilon_0 R} \\
 \text{Now } \frac{W_{\text{spherical}}}{W_{\text{sphere}}} &= \frac{\frac{q^2}{8\pi\epsilon_0 R}}{\frac{6q^2}{40\pi\epsilon_0 R}} = \frac{5}{6}
 \end{aligned}$$

Correct option is (D)

4. If  $\vec{E}_1 = xy\hat{i} + 2yz\hat{j} + 3xz\hat{k}$  and  $\vec{E}_2 = y^2\hat{i} + (2xy + z^2)\hat{j} + 2yz\hat{k}$  then

[JEST 2013]

- A. Both are impossible electrostatic fields.  
 B. Both are possible electrostatic fields.  
 C. Only  $\vec{E}_1$  is a possible electrostatic field.  
 D. Only  $\vec{E}_2$  is a possible electrostatic field.

**Solution:**For electrostatic field  $\vec{\nabla} \times \vec{E} = 0$ 

$$\vec{\nabla} \times \vec{E}_2 = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ y^2 & 2xy + z^2 & 2yz \end{vmatrix} = (2z - 2z)\hat{i} + 0 + (2y - 2y)\hat{z} = 0$$

$$\vec{\nabla} \times \vec{E}_1 = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ xy & 2yz & 3xz \end{vmatrix} = (0 - 2y)\hat{i} + 0 + x\hat{j} \neq 0$$

Correct option is (D)

5. A charge  $q$  is placed at the centre of an otherwise neutral dielectric sphere of radius  $a$  and relative permittivity  $\epsilon_r$ . We denote the expression  $q/4\pi\epsilon_0 r^2$  by  $E(r)$ . Which of the following statements is false? [JEST 2013]

- A. The electric field inside the sphere,  $r < a$ , is given by  $E(r)/\epsilon_r$
- B. The field outside the sphere,  $r > a$ , is given by  $E(r)$
- C. The total charge inside a sphere of radius  $r > a$  is given by  $q$ .
- D. The total charge inside a sphere of radius  $r < a$  is given by  $q$ .

**Solution:** Correct option is (D)

6. Two large nonconducting sheets one with a fixed uniform positive charge and another with a fixed uniform negative charge are placed at a distance of 1 meter from each other. The magnitude of the surface charge densities are  $\sigma_+ = 6.8\mu\text{C}/\text{m}^2$  for the positively charged sheet and  $\sigma_- = 4.3\mu\text{C}/\text{m}^2$  for the negatively charged sheet. What is the electric field in the region between the sheets? [JEST 2014]

- A.  $6.30 \times 10^5 \text{ N/C}$
- B.  $3.84 \times 10^5 \text{ N/C}$
- C.  $1.40 \times 10^5 \text{ N/C}$
- D.  $1.16 \times 10^5 \text{ N/C}$

**Solution:**

$$\begin{aligned} \text{Electric field between the sheet is} &= \frac{\sigma_+}{2\epsilon_0} + \frac{\sigma_-}{2\epsilon_0} \\ &= \frac{6.8 \times 10^{-6}}{2\epsilon_0} + \frac{4.3 \times 10^{-6}}{2\epsilon_0} \\ &= \frac{11.2 \times 10^{-6}}{2 \times 8.86 \times 10^{-12}} = 0.626 \times 10^6 \Rightarrow 6.3 \times 10^5 \text{ N/C} \end{aligned}$$

Correct option is (A)

7. A circular loop of radius  $R$ , carries a uniform line charge density  $\lambda$ . The electric field, calculated at a distance  $z$  directly above the center of the loop, is maximum if  $z$  is equal to, [JEST 2015]

- A.  $\frac{R}{\sqrt{3}}$
- B.  $\frac{R}{\sqrt{2}}$
- C.  $\frac{R}{2}$
- D.  $2R$

**Solution:**

$$\begin{aligned} E &= \frac{1}{4\pi\epsilon_0} \frac{(\lambda \times 2\pi R)z}{(R^2 + z^2)^{3/2}} \\ \text{For maximum } E, \frac{dE}{dz} &= 0 \\ \Rightarrow \frac{\lambda \times 2\pi R}{4\pi\epsilon_0} \left[ \frac{(R^2 + z^2)^{3/2} - z \times 3/2 \sqrt{R^2 + z^2} \times 2z}{(R^2 + z^2)^3} \right] &= 0 \\ \Rightarrow (R^2 + z^2)^{3/2} &= 3z^2 \sqrt{R^2 + z^2} \\ \Rightarrow R^2 + z^2 &= 3z^2 \end{aligned}$$

$$\Rightarrow R^2 = 2z^2$$

$$\Rightarrow z = \frac{R}{\sqrt{2}}$$

Correct option is (B)



## Practise Set-2 Solutions

1. Four equal point charges are kept fixed at the four vertices of a square. How many neutral points (i.e. points where the electric field vanishes) will be found inside the square?

[NET/JRF(DEC-2011)]

- A. 1                      B. 4                      C. 5                      D. 7

**Solution:**

Inside the square, there is only one point where field vanishes It is at the center of the square

So the correct answer is **Option (A)**

2. A static charge distribution gives rise to an electric field of the form  $\vec{E} = \alpha \left(1 - e^{-r/R}\right) \frac{\hat{r}}{r^2}$ , where  $\alpha$  and  $R$  are positive constants. The charge contained within a sphere of radius  $R$ , centred at the origin is

[NET/JRF(DEC-2011)]

- A.  $\pi\alpha\epsilon_0 \frac{e}{R^2}$                       B.  $\pi\alpha\epsilon_0 \frac{e^2}{R^2}$                       C.  $4\pi\alpha\epsilon_0 \frac{R}{e}$                       D.  $\pi\alpha\epsilon_0 \frac{R^2}{e}$

**Solution:**

$$\begin{aligned}
 Q_{enc} &= \epsilon_0 \oint \vec{E} \cdot d\vec{a} \\
 &= \alpha\epsilon_0 \int \left(1 - e^{-r/R}\right) \frac{\hat{r}}{r^2} \cdot (r^2 \sin\theta d\theta d\phi \hat{r}) \\
 &= \alpha\epsilon_0 \times \int_0^\pi \int_0^{2\pi} \left(1 - e^{-r/R}\right) \sin\theta d\theta d\phi \\
 \text{at } r = R, \quad Q_{enc} &= 4\pi\alpha\epsilon_0 \left(1 - \frac{1}{e}\right). \text{ So none of the options given are correct.}
 \end{aligned}$$

None of the options given are correct

3. Charges  $Q, Q$  and  $-2Q$  are placed on the vertices of an equilateral triangle  $ABC$  of sides of length  $a$ , as shown in the figure. The dipole moment of this configuration of charges, irrespective of the choice of origin, is

[NET/JRF(JUNE-2012)]

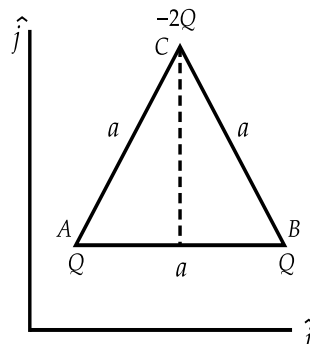


Figure 1

A.  $+2aQ\hat{i}$

B.  $+\sqrt{3}aQ\hat{j}$

C.  $-\sqrt{3}aQ\hat{j}$

D. 0

**Solution:**

Let coordinates of A is  $(l, m)$ , then

$$\vec{p} = q_i \vec{r}_i = Q[l\hat{i} + m\hat{j}] + Q[(l+a)\hat{i} + m\hat{j}] - 2Q \left[ \left(l + \frac{a}{2}\right)\hat{i} + \left(m + \frac{\sqrt{3}a}{2}\right)\hat{j} \right]$$

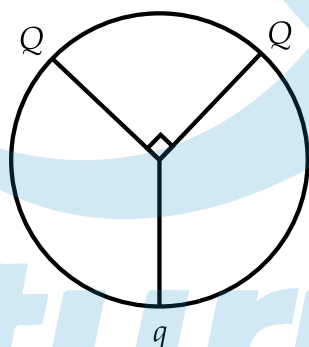
$$\vec{p} = Q[l\hat{i} + m\hat{j}] + Q[(l+a)\hat{i} + m\hat{j}] - Q[(2l+a)\hat{i} + (2m + \sqrt{3}a)\hat{j}]$$

$$\vec{p} = -\sqrt{3}aQ\hat{j}$$

So the correct answer is **Option (C)**

4. Three charges are located on the circumference of a circle of radius  $R$  as shown in the figure below. The two charges  $Q$  subtend an angle  $90^\circ$  at the centre of the circle. The charge  $q$  is symmetrically placed with respect to the charges  $Q$ . If the electric field at the centre of the circle is zero, what is the magnitude of  $Q$  ?

[NET/JRF(DEC-2012)]



A.  $q/\sqrt{2}$

B.  $\sqrt{2}q$

C.  $2q$

D.  $4q$

**Solution:**

$$E_1 = E_2 = \frac{1}{4\pi\epsilon_0} \frac{Q}{R^2} \text{ and } E_3 = \frac{1}{4\pi\epsilon_0} \frac{q}{R^2}$$

$$\text{Resultant of } E_1 \text{ and } E_2 \text{ is } E = \sqrt{E_1^2 + E_2^2} = \sqrt{2}E_1,$$

$$\text{Thus } E_3 = E$$

$$Q = \frac{q}{\sqrt{2}}$$

So the correct answer is **Option (A)**

5. A point charges  $q$  of mass  $m$  is kept at a distance  $d$  below a grounded infinite conducting sheet which lies in the  $xy$  - plane. For what value of  $d$  will the charge remains stationary?

[NET/JRF(DEC-2012)]

A.  $q/4\sqrt{mg\pi\epsilon_0}$

B.  $q/\sqrt{mg\pi\epsilon_0}$

C. There is no finite value of  $d$

D.  $\sqrt{mg\pi\epsilon_0}/q$

**Solution:** There is attractive force between point charge  $q$  and grounded conducting sheet that can be calculate from method of images i.e.

$$\frac{1}{4\pi\epsilon_0} \frac{q^2}{(2d)^2} = mg$$

$$d = \frac{q}{4\sqrt{mg\pi\epsilon_0}}$$

So the correct answer is **Option (A)**

6. A solid sphere of radius  $R$  has a charge density, given by

$$\rho(r) = \rho_0 \left(1 - \frac{ar}{R}\right)$$

where  $r$  is the radial coordinate and  $\rho_0, a$  and  $R$  are positive constants. If the magnitude of the electric field at  $r = R/2$  is 1.25 times that at  $r = R$ , then the value of  $a$  is

[NET/JRF(DEC-2014)]

A. 2

B. 1

C.  $1/2$

D.  $1/4$

**Solution:**

$$\oint_S \vec{E} \cdot d\vec{a} = \frac{1}{\epsilon_0} Q_{enc} \Rightarrow |\vec{E}| \times 4\pi r^2 = \frac{1}{\epsilon_0} \int_0^r \rho_0 \left(1 - \frac{ar}{R}\right) 4\pi r^2 dr$$

$$\Rightarrow |\vec{E}| \times 4\pi r^2 = \frac{4\pi\rho_0}{\epsilon_0} \int_0^r \left(r^2 - \frac{ar^3}{R}\right) dr$$

$$= \frac{4\pi\rho_0}{\epsilon_0} \left(\frac{r^3}{3} - \frac{ar^4}{4R}\right) \Rightarrow |\vec{E}| = \frac{\rho_0}{\epsilon_0} \left(\frac{r}{3} - \frac{ar^2}{4R}\right)$$

$$\therefore E_{r=R/2} = 1.25 E_{r=R} \Rightarrow \frac{\rho_0}{\epsilon_0} \left(\frac{R/2}{3} - \frac{aR^2/4}{4R}\right) = 1.25 \frac{\rho_0}{\epsilon_0} \left(\frac{R}{3} - \frac{aR^2}{4R}\right)$$

$$\Rightarrow \left(\frac{1}{6} - \frac{a}{16}\right) = \frac{5}{4} \left(\frac{1}{3} - \frac{a}{4}\right) \Rightarrow \left(\frac{1}{6} - \frac{a}{16}\right) = \left(\frac{5}{12} - \frac{5a}{16}\right)$$

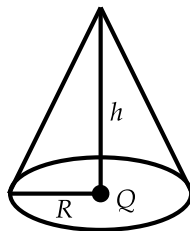
$$= \left(\frac{5}{12} - \frac{5a}{16}\right) \Rightarrow \frac{5a}{16} - \frac{a}{16} = \frac{5}{12} - \frac{1}{6}$$

$$\Rightarrow \frac{4a}{16} = \frac{5-2}{12} \Rightarrow \frac{a}{4} = \frac{3}{12} \Rightarrow a = 1$$

So the correct answer is **Option (B)**

7. Consider a charge  $Q$  at the origin of 3 - dimensional coordinate system. The flux of the electric field through the curved surface of a cone that has a height  $h$  and a circular base of radius  $R$  (as shown in the figure) is

[NET/JRF(DEC-2015)]



A.  $\frac{Q}{\epsilon_0}$

B.  $\frac{Q}{2\epsilon_0}$

C.  $\frac{hQ}{R\epsilon_0}$

D.  $\frac{QR}{2h\epsilon_0}$

**Solution:** So the correct answer is **Option (B)**

8. Four equal charges of  $+Q$ , each are kept at the vertices of a square of side  $R$ . A particle of mass  $m$  and charge  $+Q$  is placed in the plane of the square at a short distance  $a (\ll R)$  from the centre. If the motion of the particle is confined to the plane, it will undergo small oscillations with an angular frequency

[NET/JRF(JUNE-2016)]

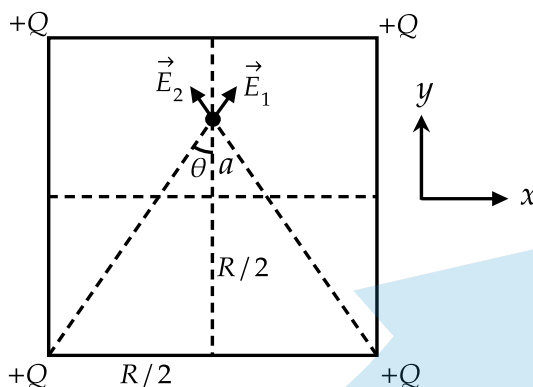
A.  $\sqrt{\frac{Q^2}{2\pi\epsilon_0 R^3 m}}$

B.  $\sqrt{\frac{Q^2}{\pi\epsilon_0 R^3 m}}$

C.  $\sqrt{\frac{\sqrt{2}Q^2}{\pi\epsilon_0 R^3 m}}$

D.  $\sqrt{\frac{Q^2}{4\pi\epsilon_0 R^3 m}}$

**Solution:**

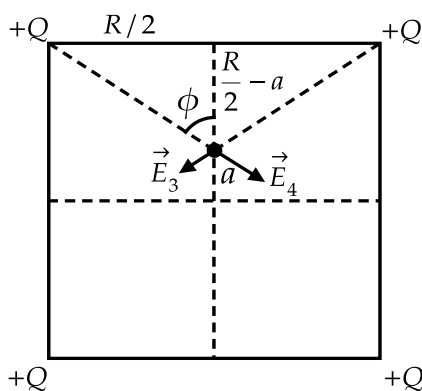


$$E_1 = E_2 = \frac{kQ}{\left[\left(a + \frac{R}{2}\right)^2 + \frac{R^2}{4}\right]}$$

Resultant field  $E_{12,y} = 2E_1 \cos \theta$

$$E_{12,y} = \frac{2kQ}{\left[\left(a + \frac{R}{2}\right)^2 + \frac{R^2}{4}\right]^{\frac{3}{2}}} \left(a + \frac{R}{2}\right) \approx \frac{2kQ}{\left[\frac{R^2}{4}\right]^{\frac{3}{2}}} \left(a + \frac{R}{2}\right)$$

$$E_{12,y} = \frac{4\sqrt{2}kQ}{R^3} \left(a + \frac{R}{2}\right)$$





$$\text{Similarly; } E_3 = E_4 = \frac{kQ}{\left[\left(\frac{R}{2} - a\right)^2 + \frac{R^2}{4}\right]}$$

$$\text{Resultant } E_{34,y} = 2E_3 \cos \phi = \frac{2kQ}{\left[\left(\frac{R}{2} - a\right)^2 + \frac{R^2}{4}\right]^{\frac{3}{2}}} \left(\frac{R}{2} - a\right)$$

$$\Rightarrow E_{34,y} = \frac{4\sqrt{2}kQ}{R^3} \left(\frac{R}{2} - a\right)$$

$$\text{Resultant } E = \frac{4\sqrt{2}kQ}{R^3} \left[ \left(\frac{R}{2} - a\right) - \left(\frac{R}{2} + a\right) \right] = -\frac{8\sqrt{2}kQ}{R^3} a$$

$$E = \frac{-8\sqrt{2}}{R^3} \times \frac{1}{4\pi\epsilon_0} Qa$$

$$E = -\frac{2\sqrt{2}Q}{\pi\epsilon_0 R^3} a$$

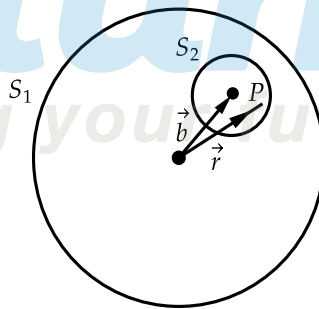
$$\Rightarrow F = QE = -\frac{2\sqrt{2}Q^2}{\pi\epsilon_0 R^3} a$$

$$\omega = \sqrt{\frac{2\sqrt{2}Q^2}{\pi\epsilon_0 m R^3}}$$

So the correct answer is **Option (C)**

9. Consider a sphere  $S_1$  of radius  $R$  which carries a uniform charge of density  $\rho$ . A smaller sphere  $S_2$  of radius  $a < \frac{R}{2}$  is cut out and removed from it. The centres of the two spheres are separated by the vector  $\vec{b} = \frac{\hat{n}R}{2}$ , as shown in the figure. The electric field at a point  $P$  inside  $S_2$  is

[NET/JRF(JUNE-2016)]



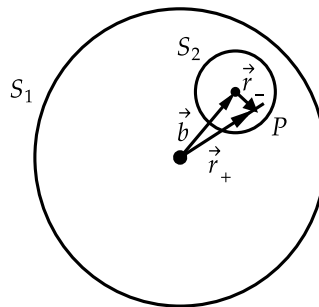
A.  $\frac{\rho R}{3\epsilon_0} \hat{n}$

B.  $\frac{\rho R}{3\epsilon_0 a} (\vec{r} - \hat{n}a)$

C.  $\frac{\rho R}{6\epsilon_0} \hat{n}$

D.  $\frac{\rho a}{3\epsilon_0 R} \vec{r}$

**Solution:**



Electric field at  $P$  due to  $S_1$  is  $\vec{E}_1 = \frac{\rho}{3\epsilon_0} \vec{r}_+$

Electric field at  $P$  due to  $S_2$  (assume  $-\rho$ ) is  $\vec{E}_2 = \frac{-\rho}{3\epsilon_0} \vec{r}_-$

$$\text{Thus } \vec{E} = \vec{E}_1 + \vec{E}_2 = \frac{\rho}{3\epsilon_0} (\vec{r}_+ - \vec{r}_-);$$

$$\because \vec{b} + \vec{r}_- = \vec{r}_+ \Rightarrow \vec{r}_+ - \vec{r}_- = \vec{b}$$

$$\vec{E} = \frac{\rho}{3\epsilon_0} \vec{b} = \frac{\rho R}{6\epsilon_0} \hat{n} \left( \because \vec{b} = \frac{R}{2} \hat{n} \right)$$

So the correct answer is **Option (C)**

10. The charge per unit length of a circular wire of radius  $a$  in the  $xy$ -plane, with its centre at the origin, is  $\lambda = \lambda_0 \cos \theta$ , where  $\lambda_0$  is a constant and the angle  $\theta$  is measured from the positive  $x$ -axis. The electric field at the centre of the circle is

[NET/JRF(DEC-2016)]

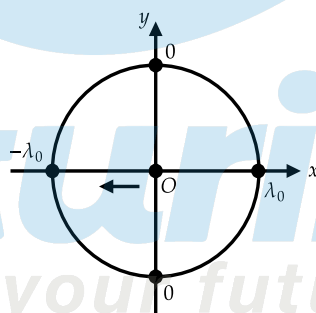
A.  $\vec{E} = -\frac{\lambda_0}{4\epsilon_0\alpha} \hat{i}$

B.  $\vec{E} = \frac{\lambda_0}{4\epsilon_0\alpha} \hat{i}$

C.  $\vec{E} = -\frac{\lambda_0}{4\epsilon_0\alpha} \hat{j}$

D.  $\vec{E} = \frac{\lambda_0}{4\pi\epsilon_0\alpha} \hat{k}$

**Solution:**



Electric field due to a charged element at  $P$  is

$$E = -(dE \cos \theta) \hat{i} - (dE \sin \theta) \hat{j}$$

So, the total electric field at the centre is

$$\begin{aligned} \vec{E} &= -\hat{i} \int dE \cos \theta - \hat{j} \int dE \sin \theta \\ \vec{E} &= -\hat{i} \int \frac{\lambda d\ell}{4\pi\epsilon_0 a^2} \cos \theta - \hat{j} \int \frac{\lambda d\ell}{4\pi\epsilon_0 a^2} \sin \theta \\ &= -\frac{\hat{i}\lambda_0}{4\pi\epsilon_0 a^2} a \int_0^{2\pi} \cos^2 \theta d\theta - \frac{\hat{j}\lambda_0}{4\pi\epsilon_0 a^2} a \int_0^{2\pi} \cos \theta \sin \theta d\theta \\ &= -\frac{\hat{i}\lambda_0}{4\pi\epsilon_0 a} \int_0^{2\pi} \cos^2 \theta d\theta = -\hat{i} \frac{\lambda_0}{4\pi\epsilon_0 a} \pi = -\frac{\lambda_0}{4\epsilon_0 a} \hat{i} \end{aligned}$$

At centre  $O$ , direction of field is  $-\hat{x}$ .

So the correct answer is **Option (A)**