



## 1. Rigid body dynamics solutions

### Practice set 1 solutions

1. An annulus of mass  $M$  made of a material of uniform density has inner and outer radii  $a$  and  $b$  respectively. Its principle moment of inertia along the axis of symmetry perpendicular to the plane of the annulus is:

[NET DEC 2011]

- A.  $\frac{1}{2}M \frac{(b^4 + a^4)}{(b^2 - a^2)}$  B.  $\frac{1}{2}M\pi (b^2 - a^2)$   
C.  $\frac{1}{2}M (b^2 - a^2)$  D.  $\frac{1}{2}M (b^2 + a^2)$

**Solution:** The correct option is (d)

2. Two bodies of equal mass  $m$  are connected by a massless rigid rod of length  $l$  lying in the  $xy$ -plane with the centre of the rod at the origin. If this system is rotating about the  $z$ -axis with a frequency  $\omega$ , its angular momentum is

[NET DEC 2012]

- A.  $ml^2\omega/4$  B.  $ml^2\omega/2$   
C.  $ml^2\omega$  D.  $2ml^2\omega$

**Solution:**

Since rod is massless i.e.  $M = 0$ .

Moment of inertia of the system  $I = m_1 r_1^2 + m_2 r_2^2, m_1 = m_2 = m$  and  $r_1 = r_2 = \frac{l}{2}$

$$I = \frac{ml^2}{4} + \frac{ml^2}{4} \Rightarrow I = \frac{ml^2}{2}$$

$$\text{Angular momentum, } J = I\omega \text{ and } J = \frac{ml^2\omega}{2}$$

The correct option is (b)

3. Two masses  $m$  each, are placed at the points  $(x, y) = (a, a)$  and  $(-a, -a)$  and two masses,  $2m$  each, are placed at the points  $(a, -a)$  and  $(-a, a)$ . The principal moments of inertia of the system are

[NET DEC 2015]

- A.  $2m^2, 4ma^2$                       B.  $4ma^2, 8ma^2$   
 C.  $4ma^2, 4ma^2$                       D.  $8ma^2, 8ma^2$

**Solution:**

$$I_{xx} = \sum_i m_i (y_i^2 + z_i^2) = \sum_i m_i y_i^2 \quad \because z_i = 0$$

$$\Rightarrow I_{xx} = ma^2 + ma^2 + 2ma^2 + 2ma^2 \Rightarrow I_{xx} = 6ma^2$$

$$\text{Similarly, } I_{yy} = 6ma^2 \quad \text{and} \quad I_{zz} = 12ma^2$$

$$I_{xz} = I_{zx} = 0, I_{yz} = I_{zy} = 0$$

$$I_{xy} = I_{yx} = -m_i \sum_z x_i y_i = -m$$

$$I_{xy} = I_{yx} = -m_i \sum_i x_i y_i = -ma^2 - ma^2 + 2ma^2 + 2ma^2 \Rightarrow I_{xy} = I_{yx} = 2ma^2$$

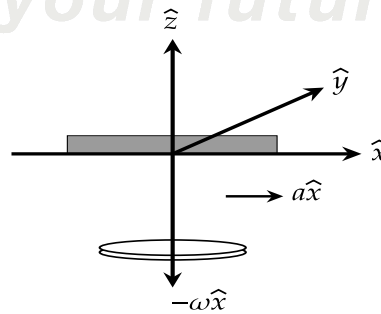
Moment of inertia tensor  $I = \begin{pmatrix} 6ma^2 & 2ma^2 & 0 \\ 2ma^2 & 6ma^2 & 0 \\ 0 & 0 & 12ma^2 \end{pmatrix}$  Eigen value of matrices is principal moment of inertia, which is given by

$$\lambda_1 = 4ma^2 = I_x, \lambda_2 = 8ma^2 = I_y, \lambda_3 = 12ma^2 = I_z$$

So,  $I_x = 4ma^2$  and  $I_y = 8ma^2$

The correct option is (b)

4. A disc of mass  $m$  is free to rotate in a plane parallel to the  $xy$  plane with an angular velocity  $-\omega \hat{z}$  about a massless rigid rod suspended from the roof of a stationary car (as shown in the figure below). The rod is free to orient itself along any direction.



The car accelerates in the positive  $x$ -direction with an acceleration  $a > 0$ . Which of the following statements is true for the coordinates of the centre of mass of the disc in the reference frame of the car?

[NET DEC 2017]

- A. only the  $x$  and the  $z$  coordinates change                      B. only the  $y$  and the  $z$  coordinates change  
 C. only the  $x$  and the  $y$  coordinates change                      D. all the three coordinates change

**Solution:** The correct option is (d)

## Practice set 2

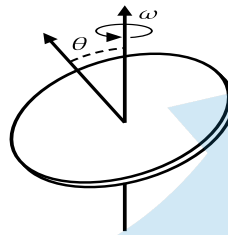
1. A uniform solid cylinder is released on a horizontal surface with speed 5 m/s without any rotation (slipping without rolling). The cylinder eventually starts rolling without slipping. If the mass and radius of the cylinder are 10gm and 1 cm respectively, the final linear velocity of the cylinder is..... m/s. (up to two decimal places).

[GATE 2017]

**Solution:**  $mvr = mv_{cm}r + I_{cm}\omega = mv_{cm}r + \frac{1}{2}mr^2 \frac{v_{cm}}{r} \Rightarrow v = \frac{3}{2}v_{cm} \Rightarrow v_{cm} = \frac{2}{3}v = \frac{10}{3} = 3.33\text{m/sec}$

2. A uniform circular disc of mass  $m$  and radius  $R$  is rotating with angular speed  $\omega$  about an axis passing through its centre and making an angle  $\theta = 30^\circ$  with the axis of the disc. If the kinetic energy of the disc is  $\alpha m\omega^2 R^2$ , the value of  $\alpha$  is (up to two decimal places).

[GATE 2018]



**Solution:**

The kinetic energy of the disc is,

$$T = \frac{1}{2} \vec{L} \cdot \vec{\omega}$$

Where  $\vec{L}$  is angular momentum and  $\vec{\omega}$  is angular velocity

$$T = \frac{1}{2} |\vec{L}| |\vec{\omega}| \cos 30^\circ = \frac{1}{2} I \omega \cdot \omega \frac{\sqrt{3}}{2} = \frac{1}{2} \left( \frac{mR^2}{2} \right) \omega^2 \times \frac{\sqrt{3}}{2}$$

$$T = \frac{\sqrt{3}}{8} m \omega^2 R^2 = 0.21 m \omega^2 R^2 \Rightarrow \alpha m \omega^2 R^2 = 0.21 m \omega^2 R^2$$

Hence,  $\alpha = 0.21$



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