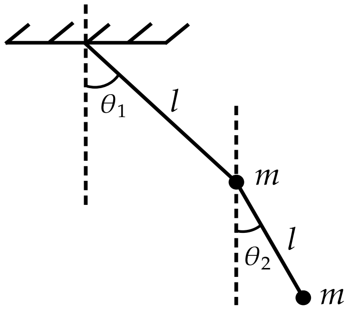
Practice set-1

1. A double pendulum consists of two point masses m attached by strings of length l as shown in the figure: The kinetic energy of the pendulum is

[NET/JRF(DEC-2011)]



A. $\frac{1}{2}ml^2\left[\dot{\theta}_1^2+\dot{\theta}_2^2\right]$

B. $\frac{1}{2}ml^2\left[2\dot{\theta}_1^2+\dot{\theta}_2^2+2\dot{\theta}_1\dot{\theta}_2\cos\left(\theta_1-\theta_2\right)\right]$

C. $\frac{1}{2}ml^2\left[\dot{\theta}_1^2+2\dot{\theta}_2^2+2\dot{\theta}_1\dot{\theta}_2\cos\left(\theta_1-\theta_2\right)\right]$

D. $\frac{1}{2}ml^2\left[2\dot{\theta}_1^2+\dot{\theta}_2^2+2\dot{\theta}_1\dot{\theta}_2\cos\left(\theta_1+\theta_2\right)\right]$

Solution:

Let co-ordinate (x_1, y_1) and (x_2, y_2).

$$K.E. = \frac{1}{2}m\left(\dot{x}_1^2+\dot{y}_1^2\right)+\frac{1}{2}m\left(\dot{x}_2^2+\dot{y}_2^2\right)$$

$$x_1 = l\sin\theta_1, y_1 = l\cos\theta_1 \Rightarrow \dot{x}_1$$

$$= l\cos\theta_1\dot{\theta}_1, \dot{y}_1 = -l\sin\theta_1\dot{\theta}_1$$

$$x_2 = l\sin\theta_1+l\sin\theta_2, y_2 = l\cos\theta_1+l\cos\theta_2$$

$$\Rightarrow \dot{x}_2 = l\cos\theta_1\dot{\theta}_1+l\cos\theta_2\dot{\theta}_2, \dot{y}_2$$

$$= l\left(-\sin\theta_1\dot{\theta}_1\right)+l\left(-\sin\theta_2\right)\dot{\theta}_2$$

Put the value of $\dot{x}_1, \dot{y}_1, \dot{x}_2, \dot{y}_2$ in K.E equation, one will get

$$T = \frac{1}{2}ml^2\left[2\dot{\theta}_1^2+\dot{\theta}_2^2+2\dot{\theta}_1\dot{\theta}_2\cos\left(\theta_1-\theta_2\right)\right]$$

So the correct answer is **Option (B)**

2. A particle of mass m moves inside a bowl. If the surface of the bowl is given by the equation $z = \frac{1}{2}a\left(x^2+y^2\right)$, where a is a constant, the Lagrangian of the particle is

[NET/JRF(DEC-2011)]

A. $\frac{1}{2}m\left(\dot{r}^2+r^2\dot{\phi}^2-gar^2\right)$

B. $\frac{1}{2}m\left[\left(1+a^2r^2\right)\dot{r}^2+r^2\dot{\phi}^2\right]$

C. $\frac{1}{2}m\left(\dot{r}^2+r^2\dot{\theta}^2+r^2\sin^2\theta\dot{\phi}^2-gar^2\right)$

D. $\frac{1}{2}m\left[\left(1+a^2r^2\right)\dot{r}^2+r^2\dot{\phi}^2-gar^2\right]$

Solution:

$$L = \frac{1}{2}m\left(\dot{x}^2+\dot{y}^2+\dot{z}^2\right)-mgz,$$

$$\text{where } z=\frac{1}{2}a\left(x^2+y^2\right)$$

$$\text{It has cylindrical symmetry. Thus } x=r\cos\phi, y=r\sin\phi, z=\frac{1}{2}a\left(r^2\right)$$

$$\dot{x}=\dot{r}\cos\phi-r\sin\phi\dot{\phi}, \dot{y}$$

$$=\dot{r}\sin\phi+r\cos\phi\dot{\phi} \text{ and } \dot{z}=a(r\dot{r})$$

$$\text{So, } L=\frac{1}{2}m\left[\left(1+a^2r^2\right)\dot{r}^2+r^2\dot{\phi}^2-gar^2\right]$$

So the correct answer is **Option (D)**

3. The Lagrangian of a particle of mass m moving in one dimension is given by

$$L=\frac{1}{2}m\dot{x}^2-bx$$

where b is a positive constant. The coordinate of the particle $x(t)$ at time t is given by: (in following c_1 and c_2 are constants)

[NET/JRF(JUNE-2013)]

A. $-\frac{b}{2m}t^2+c_1t+c_2$

B. c_1t+c_2

C. $c_1\cos\left(\frac{bt}{m}\right)+c_2\sin\left(\frac{bt}{m}\right)$

D. $c_1\cosh\left(\frac{bt}{m}\right)+c_2\sinh\left(\frac{bt}{m}\right)$

Solution:

$$\text{Equation of motion } \frac{d}{dt}\left(\frac{\partial L}{\partial \dot{x}}\right)-\frac{\partial L}{\partial x}=0 \Rightarrow \frac{d}{dt}(m\dot{x})+b$$

$$=0 \Rightarrow m\ddot{x}+b=0 \Rightarrow m\ddot{x}=-b$$

$$\frac{d^2x}{dt^2}=-\frac{b}{m} \Rightarrow \frac{dx}{dt}=-\frac{b}{m}t+c_1 \Rightarrow x$$

$$=-\frac{b}{m}\frac{t^2}{2}+c_1t+c_2$$

So the correct answer is **Option (A)**

4. A particle moves in a potential $V=x^2+y^2+\frac{z^2}{2}$. Which component(s) of the angular momentum is/are constant(s) of motion?

[NET/JRF(DEC-2013)]

A. None

B. L_x, L_y and L_z

C. only L_x and L_y

D. only L_z

Solution:

$$\text{A particle moves in a potential } V=x^2+y^2+\frac{z^2}{2}$$

$$V(r,\theta,\phi)=r^2\sin^2\theta\cos^2\phi+r^2\sin^2\theta\sin^2\phi+\frac{r^2}{2}\cos^2\theta$$

$$V(r,\theta,\phi)=r^2\sin^2\theta+\frac{r^2}{2}\cos^2\theta$$

Now ϕ is cyclic-co-ordinate $\left(p_\phi\right)$ i.e L_z is constant of motion.

So the correct answer is **Option (D)**

5. A pendulum consists of a ring of mass M and radius R suspended by a massless rigid rod of length l attached to its rim. When the pendulum oscillates in the plane of the ring, the time period of oscillation is

[NET/JRF(DEC-2013)]

A. $2\pi\sqrt{\frac{l+R}{g}}$

B. $\frac{2\pi}{\sqrt{g}}\left(l^2+R^2\right)^{1/4}$

C. $2\pi\sqrt{\frac{2R^2+2Rl+l^2}{g(R+l)}}$

D. $\frac{2\pi}{\sqrt{g}}\left(2R^2+2Rl+l^2\right)^{1/4}$

Solution:

The moment of inertia about pivotal point is given by

$$I = I_{c,m} + Md^2 = MR^2 + M(l+R)^2$$

If ring is displaced by angle θ then potential energy is $-Mg(l+R)\cos\theta$.

The Lagrangian is given by

$$\begin{aligned} L &= \frac{1}{2}I\dot{\theta}^2 - V(\theta) \\ &= \frac{1}{2}\left(MR^2 + M(l+R)^2\right)\dot{\theta}^2 + Mg(l+R)\cos\theta \end{aligned}$$

$$\frac{d}{dt}\left(\frac{\partial L}{\partial \dot{\theta}}\right) - \left(\frac{\partial L}{\partial \theta}\right) = 0 \Rightarrow \left(MR^2 + M(l+R)^2\right)\ddot{\theta} + Mg(l+R)\sin\theta = 0$$

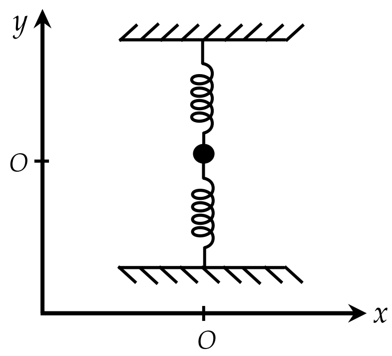
For small oscillation $\sin\theta = \theta \Rightarrow \left(MR^2 + M(l+R)^2\right)\ddot{\theta} + Mg(l+R)\theta = 0$

Time period is given by $2\pi\sqrt{\dfrac{2R^2+2Rl+l^2}{g(R+l)}}$.

So the correct answer is **Option (C)**

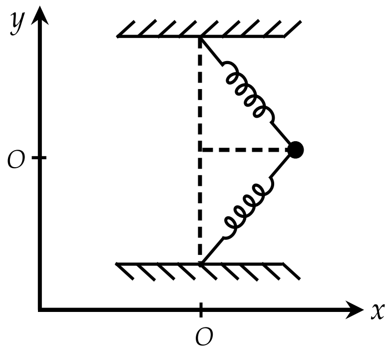
6. Consider a particle of mass m attached to two identical springs each of length l and spring constant k (see the figure). The equilibrium configuration is the one where the springs are unstretched. There are no other external forces on the system. If the particle is given a small displacement along the x-axis, which of the following describes the equation of motion for small oscillations?

[NET/JRF(DEC-2013)]



A. $m\ddot{x} + \frac{kx^3}{l^2} = 0$ **B.** $m\ddot{x} + kx = 0$ **C.** $m\ddot{x} + 2kx = 0$ **D.** $m\ddot{x} + \frac{kx^2}{l} = 0$

Solution:



The lagrangian of system is given by

$$L = \frac{1}{2}m\dot{x}^2 - V(x)$$

The potential energy is given by

$$V(x) = \frac{k}{2}\left[\left(x^2+l^2\right)^{\frac{1}{2}} - l\right]^2 + \frac{k}{2}\left[\left(x^2+l^2\right)^{\frac{1}{2}} - l\right]^2$$

$$V(x) = k\left[\left(x^2+l^2\right)^{\frac{1}{2}} - l\right]^2$$

For small oscillation one can approximate potential by Taylor expansion

$$V(x) = kl^2\left[\left(1+\frac{x^2}{l^2}\right)^{\frac{1}{2}} - 1\right]^2 \Rightarrow V(x)$$

$$= kl^2\left[\left(1+\frac{1}{2}\frac{x^2}{l^2} - \frac{1}{8}\frac{x^4}{l^4}\right) - 1\right]^2$$

$$V(x) = \frac{kl^2}{4}\left(\frac{x^2}{l^2}\right)^2 \Rightarrow V(x) = k\left(\frac{x^4}{4l^2}\right)$$

So Lagrangian of system is given by $L = \frac{1}{2}m\dot{x}^2 - k\left(\frac{x^4}{4l^2}\right)$

The Lagranges equation of motion $\frac{d}{dt}\left(\frac{\partial L}{\partial \dot{x}}\right) - \left(\frac{\partial L}{\partial x}\right) = 0 \Rightarrow m\ddot{x} + \frac{kx^3}{l^2} = 0$

So the correct answer is **Option (A)**

7. The equation of motion of a system described by the time-dependent Lagrangian

$$L = e^{\gamma t}\left[\frac{1}{2}m\dot{x}^2 - V(x)\right] \text{ is}$$

[NET/JRF(DEC-2014)]

A. $m\ddot{x} + \gamma m\dot{x} + \frac{dV}{dx} = 0$

B. $m\ddot{x} + \gamma m\dot{x} - \frac{dV}{dx} = 0$

C. $m\ddot{x} - \gamma m\dot{x} + \frac{dV}{dx} = 0$

D. $m\ddot{x} + \frac{dV}{dx} = 0$

Solution:

$$\because L=e^{\gamma t}\left[\frac{1}{2}m\dot{x}^2-V(x)\right]\Rightarrow\frac{\partial L}{\partial\dot{x}}$$
$$=e^{\gamma t}m\dot{x}\text{ and }\frac{\partial L}{\partial x}=-\frac{\partial V}{\partial x}e^{\gamma t}$$
$$\because\frac{d}{dt}\left(\frac{\partial L}{\partial\dot{x}}\right)-\frac{\partial L}{\partial x}=0\Rightarrow\frac{d}{dt}\left(e^{\gamma t}m\dot{x}\right)+\frac{\partial V}{\partial x}e^{\gamma t}$$
$$=m\ddot{x}e^{rt}+m\dot{x}\gamma e^{\gamma t}+\frac{\partial V}{\partial x}e^{rt}=0$$
$$\left(m\ddot{x}+m\gamma\dot{x}+\frac{\partial V}{\partial x}\right)e^{\gamma t}=0\Rightarrow m\ddot{x}+\gamma m\dot{x}+\frac{\partial V}{\partial x}=0$$

So the correct answer is **Option (A)**

8. A particle of unit mass moves in the xy-plane in such a way that $\dot{x}(t)=y(t)$ and $\dot{y}(t)=-x(t)$. We can conclude that it is in a conservative force-field which can be derived from the potential

[NET/JRF(JUNE-2015)]

A. $\frac{1}{2}\left(x^2+y^2\right)$ **B.** $\frac{1}{2}\left(x^2-y^2\right)$ **C.** $x+y$ **D.** $x-y$

Solution:

$$\because\dot{x}=y\text{ and }\dot{y}=-x$$
$$\Rightarrow\quad\ddot{x}=\dot{y}=-x\quad\text{ and }\ddot{y}=-\dot{x}=-y$$
$$\Rightarrow\quad\ddot{x}+x=0\quad\text{ and }\ddot{y}+y=0$$
$$\text{that is possible for }L=\frac{1}{2}m\dot{x}^2+\frac{1}{2}m\dot{y}^2-\frac{1}{2}\left(x^2+y^2\right)$$
$$\Rightarrow V=\frac{1}{2}\left(x^2+y^2\right)$$

So the correct answer is **Option (A)**

9. The Lagrangian of a particle moving in a plane s given in Cartesian coordinates as

$$L=\dot{x}\dot{y}-x^2-y^2$$

In polar coordinates the expression for the canonical momentum p_r (conjugate to the radial coordinate r) is

[NET/JRF(DEC-2015)]

A. $\dot{r}\sin\theta+r\dot{\theta}\cos\theta$ **B.** $\dot{r}\cos\theta+r\dot{\theta}\sin\theta$

C. $2\dot{r}\cos\theta-r\dot{\theta}\sin2\theta$ **D.** $\dot{r}\sin2\theta+r\dot{\theta}\cos2\theta$

Solution:

$$L=\dot{x}\dot{y}-x^2-y^2=\dot{x}\dot{y}-\left(x^2+y^2\right)$$
$$x=r\cos\theta,y=r\sin\theta\Rightarrow\dot{x}=\dot{r}\cos\theta-r\sin\theta\dot{\theta},$$
$$\dot{y}=\dot{r}\sin\theta+r\cos\theta\dot{\theta}$$
$$L=\dot{r}^2\sin\theta\cos\theta-r^2\sin\theta\cos\theta\dot{\theta}^2+\dot{r}r\cos^2\theta\dot{\theta}-\dot{r}r\sin^2\theta\dot{\theta}$$
$$P_r=\frac{\partial L}{\partial\dot{r}}\Rightarrow2\dot{r}\sin\theta\cos\theta+r\dot{\theta}\left(\cos^2\theta-\sin^2\theta\right)$$
$$\Rightarrow P_r=\dot{r}\sin2\theta+r\dot{\theta}\cos2\theta$$

So the correct answer is **Option (D)**

10. The dynamics of a particle governed by the Lagrangian

$$L = \frac{1}{2}m\dot{x}^2 - \frac{1}{2}kx^2 - kx\dot{x}t \text{ describes}$$

[NET/JRF(DEC-2016)]

A. An undamped simple harmonic oscillator
B. A damped harmonic oscillator with a time varying damping factor
C. An undamped harmonic oscillator with a time dependent frequency
D. A free particle

Solution:

$$L = \frac{1}{2}m\dot{x}^2 - \frac{1}{2}kx^2 - kx\dot{x}t$$
$$\frac{\partial L}{\partial \dot{x}} = m\dot{x} - kxt, \frac{\partial L}{\partial x} = -kx - k\dot{x}t$$
$$\frac{d}{dt}\left(\frac{\partial L}{\partial \dot{x}}\right) - \frac{\partial L}{\partial x} = 0 \Rightarrow m\ddot{x} - k\dot{x}t - kx + kx + k\dot{x}t$$
$$= 0 \Rightarrow m\ddot{x} = 0$$

So motion is equivalent to free particle

So the correct answer is **Option (D)**

11. The parabolic coordinates (ξ, η) are related to the Cartesian coordinates (x, y) by $x = \xi\eta$ and $y = \frac{1}{2}(\xi^2 - \eta^2)$. The Lagrangian of a two-dimensional simple harmonic oscillator of mass m and angular frequency ω is

[NET/JRF(DEC-2016)]

A. $\frac{1}{2}m\left[\dot{\xi}^2 + \dot{\eta}^2 - \omega^2(\xi^2 + \eta^2)\right]$
B. $\frac{1}{2}m(\xi^2 + \eta^2)\left[\left(\dot{\xi}^2 + \dot{\eta}^2\right) - \frac{1}{4}\omega^2(\xi^2 + \eta^2)\right]$
C. $\frac{1}{2}m(\xi^2 + \eta^2)\left[\dot{\xi}^2 + \dot{\eta}^2 - \frac{1}{2}\omega^2\xi\eta\right]$
D. $\frac{1}{2}m(\xi^2 + \eta^2)\left[\dot{\xi}^2 + \dot{\eta}^2 - \frac{1}{4}\omega^2\right]$

Solution:

For two dimensional Harmonic oscillation

$$L = \frac{1}{2}m(\dot{x}^2 + \dot{y}^2) - \frac{1}{2}m\omega^2(x^2 + y^2)$$
$$x = \xi\eta, \quad y = \frac{1}{2}(\xi^2 - \eta^2)$$
$$\dot{x} = \dot{\xi}\eta + \xi\dot{\eta}, \quad \dot{y} = \xi\dot{\xi} - \eta\dot{\eta}$$
$$L = \frac{1}{2}m\left[(\dot{\xi}\eta + \xi\dot{\eta})^2 + (\xi\dot{\xi} - \eta\dot{\eta})^2\right] - \frac{1}{2}m\omega^2\left[\xi^2\eta^2 + \frac{1}{4}(\xi^2 - \eta^2)^2\right]$$

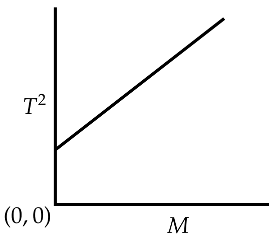
$$
\begin{aligned}
L &= \frac{1}{2}m\left(\dot{\xi}^2\eta^2+\xi^2\dot{\eta}^2+\xi^2\dot{\xi}^2+\eta^2\dot{\eta}^2\right)-\frac{1}{8}m\omega^2\left(\xi^4+\eta^4+2\xi^2\eta^2\right)\\
&= \frac{1}{2}m\left(\xi^2+\eta^2\right)\left(\dot{\eta}^2+\dot{\xi}^2\right)-\frac{1}{8}m\omega^2\left(\xi^2+\eta^2\right)^2\\
&= \frac{1}{2}m\left(\xi^2+\eta^2\right)\left[\dot{\eta}^2+\dot{\xi}^2-\frac{1}{4}\omega^2\left(\xi^2+\eta^2\right)\right]
\end{aligned}
$$

So the correct answer is **Option (B)**

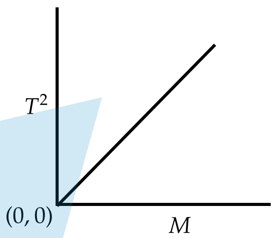
12. The spring constant k of a spring of mass m_s is determined experimentally by loading the spring with mass M and recording the time period T, for a single oscillation. If the experiment is carried out for different masses, then the graph that correctly represents the result is

[NET/JRF(DEC-2017)]

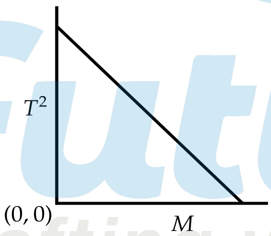
A.



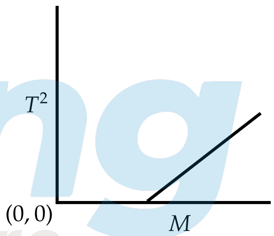
B.



C.



D.



Solution:

The Langrangian of system.

$$
\begin{aligned}
L &= \frac{1}{2}\cdot\frac{m_s}{3}\dot{x}^2+\frac{1}{2}M\dot{x}^2-\frac{1}{2}kx^2,\ \frac{d}{dt}\left(\frac{\partial L}{\partial \dot{x}}\right)-\frac{\partial L}{\partial x}=0\\
\frac{d}{dt}\frac{\partial L}{\partial x} &= 0 \Rightarrow \left(\frac{m_s}{3}+M\right)\ddot{x}=-kx\\
T &= 2\pi\sqrt{\frac{M+\frac{m_s}{3}}{k}} \Rightarrow T^2=4\pi\frac{\left(M+\frac{m_s}{3}\right)}{k}
\end{aligned}
$$

So the correct answer is **Option (A)**

13. The motion of a particle in one dimension is described by the Langrangian $L=\frac{1}{2}\left(\left(\frac{dx}{dt}\right)^2-x^2\right)$ in suitable units. The value of the action along the classical path from $x=0$ at $t=0$ to $x=x_0$ at $t=t_0$, is

[NET/JRF(DEC-2018)]

A. $\frac{x_0^2}{2\sin^2 t_0}$ **B.** $\frac{1}{2}x_0^2\tan t_0$ **C.** $\frac{1}{2}x_0^2\cot t_0$ **D.** $\frac{x_0^2}{2\cos^2 t_0}$

Solution:

$$L=\frac{1}{2}\dot{x}^2-\frac{1}{2}x^2$$

From Lagrangian equation of motion, $\frac{d}{dt}\left(\frac{\partial L}{\partial \dot{x}}\right)-\frac{\partial L}{\partial x}=0$

$$\ddot{x}+x=0$$
$$\text{The solution is} = A\sin t+B\cos t$$
$$t=0,\quad x=0,\quad B=0$$
$$x=A\sin t$$
$$t=t_0,\quad x=x_0, A=\frac{x_0}{\sin t_0}$$
$$x=\frac{x_0}{\sin t_0}\sin t,\quad \dot{x}=\frac{x_0}{\sin t_0}\cos t$$
$$A=\int_0^{t_0}L dt=\int_0^{t_0}\frac{1}{2}\dot{x}^2 dt-\int_0^{t_0}\frac{1}{2}x^2 dt$$
$$=\frac{1}{2}\frac{x_0^2}{\sin^2 t_0}\int_0^{t_0}\cos^2 t dt-\frac{1}{2}\frac{x_0^2}{\sin^2 t_0}\int_0^{t_0}\sin^2 t dt$$
$$=\frac{1}{2}\frac{x_0^2}{\sin^2 t_0}\left[\int_0^{t_0}\cos^2 t dt-\int_0^{t}\sin^2 t dt\right]$$
$$=\frac{1}{2}\frac{x_0^2}{\sin^2 t_0}\int_t^{t_0}\cos 2t dt=\frac{1}{2}\frac{x_0^2}{\sin^2 t_0}\frac{\sin 2t_0}{2}\Bigg|_0^{t_0}=\frac{x_0^2}{2}\cot t_0$$

So the correct answer is **Option (C)**

14. Two particles of masses m_1 and m_2 are connected by a massless thread of length l as shown in figure below.
The particle of mass in on the plane undergoes a circular motion with radius r_0 and angular momentum L. When a small radial displacement $\in$ (whew $\in \ll < r_0$) is applied, its radial coordinate is found to oscillate about r_0. The frequency of the oscillations is

[NET/JRF(JUNE-2019)]

A. $\sqrt{\frac{7m_2g}{(m_1+\frac{m_2}{2})r_0}}$ **B.** $\sqrt{\frac{7m_2g}{(m_1+m_2)r_0}}$ **C.** $\sqrt{\frac{3m_2g}{(m_1+\frac{m_2}{2})r_0}}$ **D.** $\sqrt{\frac{3m_2g}{(m_1+m_2)r_0}}$

Solution:

$$L=\frac{1}{2}(m_1+m_2)\dot{r}+\frac{1}{2}m_1r^2\dot{\theta}^2-m_2g(l-r)$$
$$\text{Lagrangian equation of motion; }\frac{d}{dt}\left(\frac{\partial L}{\partial \dot{r}}\right)-\frac{\partial L}{\partial r}=0$$
$$(m_1+m_2)\ddot{r}-m_1r\dot{\theta}^2+m_2g=0$$

Hence angular momentum is conserved

$$m_1r^2\dot{\theta}=m_1r_0^2\dot{\theta}_0\Rightarrow\dot{\theta}=\frac{r_0^2\dot{\theta}_0}{r^2}$$
$$\text{For circular motion } mr_0\dot{\theta}_0^2=m_2g$$
$$\text{so } r\dot{\theta}^2=\frac{m_2}{m_1}\left(\frac{r_0}{r}\right)^3 g$$

$$(m_1+m_2)\ddot{r}-m_2\left(\frac{r_0}{r}\right)^3 g+m_2g=0$$

Put $r=r_0+\in \ddot{r}=\ddot{\varepsilon}$

$$(m_1+m_2)\ddot{\in}-m_2\left(\frac{r_0}{r_0+\varepsilon}\right)^3 g+m_2g \Rightarrow (m_1+m_2)\ddot{\in}-m_2r_0^3(r_0+\varepsilon)^{-3}g+m_2g$$

$$(m_1+m_2)\ddot{\star}-m_2r_0^3gr_0^{-3}\left(1+\frac{\varepsilon}{r_0}\right)^{-3}+m_2g=0$$

$$(m_1+m_2)\ddot{+}\frac{m_2 3\varepsilon}{r_0}=0 \Rightarrow \omega=\sqrt{\frac{3m_2g}{(m_1+m_2)r_0}}$$

So the correct answer is **Option (D)**

15. Which of the following terms, when added to the Lagrangian $L(x,y,\dot{x},\dot{y})$ of a system with two degrees of freedom will not change the equations of motion?
(check question)

[NET/JRF(DEC-2019)]

A. $x\ddot{x}-y\ddot{y}$ **B.** $x\ddot{y}-y\ddot{x}$ **C.** $x\dot{y}-y\dot{x}$ **D.** $y\dot{x}^2+x\dot{y}^2$

Solution:

$$L(x,y,\dot{x},\dot{y})$$
$$\frac{d}{dt}\left(\frac{\partial L}{\partial \dot{x}}\right)-\frac{\partial L}{\partial x}=0, \frac{d}{dt}\left(\frac{\partial L}{\partial \dot{y}}\right)-\frac{\partial L}{\partial y}=0$$
$$L'=L(x,y,\dot{x},\dot{y})+x\ddot{y}-y\ddot{x}$$
$$\frac{d'}{dt'}\left(\frac{\partial L'}{\partial \dot{x}}\right)-\frac{\partial L'}{\partial x}=\frac{d}{dt}\left(\frac{\partial L}{\partial \dot{x}}\right)-\frac{\partial L}{\partial x}+\ddot{y}$$
$$=0=0+\ddot{y}=0 \Rightarrow \dot{y}=c_1$$
$$\frac{d}{dt}\left(\frac{\partial L}{\partial y}\right)-\frac{\partial L'}{\partial y}=\frac{d}{dt}\left(\frac{\partial L}{\partial \dot{y}}\right)-\frac{\partial L}{\partial y}+\ddot{x}$$
$$=0=0-\ddot{x}=0 \Rightarrow \dot{x}=c_2$$

So the correct answer is **Option (B)**

16. A point mass m, is constrained to move on the inner surface of a paraboloid of revolution $x^2+y^2=az$ (where $a>0$ is a constant). When it spirals down the surface, under the influence of gravity (along $-z$ direction), the angular speed about the z-axis is proportional to

[NET/JRF(JUNE-2020)]

A. 1 (independent of z) **B.** z
C. z^{-1} **D.** z^{-2}

Solution:

Using Lagrangian in cylindrical coordinate

$$L=\frac{1}{2}m\left(\dot{r}^2+r^2\dot{\theta}^2+\dot{z}^2\right)-mgz$$

$$\text{with constraint } x^2+y^2=az \Rightarrow r^2=az \Rightarrow \dot{z}=\frac{2r\dot{r}}{a}$$

$$L=\frac{1}{2}m\left(\dot{r}^2+r^2\dot{\theta}^2+\left(\frac{2r\dot{r}}{a}\right)^2\right)-\frac{mgr^2}{a}$$

$$\theta \text{is cyclic coordinate so } \frac{\partial L}{\partial \theta}=0 \Rightarrow \frac{\partial L}{\partial \dot{\theta}}=J \Rightarrow mr^2\dot{\theta}=J \Rightarrow \dot{\theta} \propto \frac{1}{r^2} \propto \frac{1}{z}$$

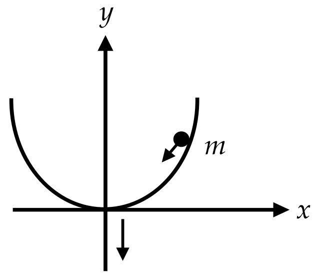
So the correct answer is **Option (C)**

Answer key

Q.No.	Answer	Q.No.	Answer
1	**B**	2	**D**
3	**A**	4	**D**
5	**C**	6	**A**
7	**A**	8	**A**
9	**D**	10	**D**
11	**B**	12	**A**
13	**C**	14	**D**
15	**B**	16	**C**

Practice set-2

1. A particle of mass m slides under the gravity without friction along the parabolic path $y = ax^2$, as shown in the figure. Here a is a constant.



The Lagrangian for this particle is given by

[GATE 2012]

A. $L = \frac{1}{2}m\dot{x}^2 - mgax^2$

B. $L = \frac{1}{2}m\left(1+4a^2x^2\right)\dot{x}^2 - mgax^2$

C. $L = \frac{1}{2}m\dot{x}^2 + mgax^2$

D. $L = \frac{1}{2}m\left(1+4a^2x^2\right)\dot{x}^2 + mgax^2$

Solution:

$$\text{Equation of constrain is given by } y = ax^2, K.E., T = \frac{1}{2}m\left(\dot{x}^2+\dot{y}^2\right)$$
$$\dot{y} = 2ax\dot{x} \Rightarrow T = \frac{1}{2}m\left(\dot{x}^2+4a^2x^2\dot{x}^2\right)$$
$$= \frac{1}{2}m\dot{x}^2\left(1+4ax^2\right)$$
$$V = mgy = mgax^2$$
$$\because L = T - V \Rightarrow L$$
$$= \frac{1}{2}m\left(1+4a^2x^2\right)\dot{x}^2 - mgax^2$$

So the correct answer is **Option (B)**

2. The Lagrange's equation of motion of the particle for above question is given by

[GATE 2012]

A. $\ddot{x} = 2gax$

B. $m\left(1+4a^2x^2\right)\ddot{x} = -2mgax - 4ma^2x\dot{x}^2$

C. $m\left(1+4a^2x^2\right)\ddot{x} = 2mgax + 4ma^2x\dot{x}^2$

D. $\ddot{x} = -2gax$

Solution:

$$\frac{d}{dt}\left(\frac{dL}{d\dot{x}}\right) = \frac{dL}{dx} \Rightarrow m\ddot{x}\left(1+4a^2x^2\right)$$
$$= -4ma^2x\dot{x}^2 - 2mgax$$

So the correct answer is **Option (B)**

3. The Lagrangian of a system with one degree of freedom q is given by $L = \alpha\dot{q}^2 + \beta q^2$, where α and β are non-zero constants. If p_q denotes the canonical momentum conjugate to q then which one of the following statements is CORRECT?

[GATE 2013]

A. $p_q = 2\beta q$ and it is a conserved quantity.

B. $p_q = 2\beta q$ and it is not a conserved quantity.

C. $p_q = 2\alpha\dot{q}$ and it is a conserved quantity.

D. $p_q = 2\alpha\dot{q}$ and it is not a conserved quantity.

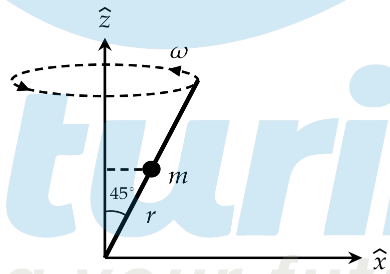
Solution:

$$\text{As, } \frac{\partial L}{\partial \dot{q}} = p_q \text{ but } \quad \frac{\partial L}{\partial q} \neq 0. \text{ Thus, it is not a conserved quantity.}$$

So the correct answer is **Option (D)**

4. A bead of mass m can slide without friction along a massless rod kept at 45° with the vertical as shown in the figure. The rod is rotating about the vertical axis with a constant angular speed ω. At any instant r is the distance of the bead from the origin. The momentum conjugate to r is

[GATE 2014]



A. $m\dot{r}$ **B.** $\frac{1}{\sqrt{2}}m\dot{r}$ **C.** $\frac{1}{2}m\dot{r}$ **D.** $\sqrt{2}m\dot{r}$

Solution:

$$L = \frac{1}{2}m\left(\dot{r}^2 + r^2\dot{\theta}^2 + r^2\sin^2\theta\dot{\phi}^2\right) - mgr\cos\theta$$

$$\text{Equation of constrain is } \theta = \frac{\pi}{4} \text{ and it is given } \dot{\phi} = \omega$$

$$L = \frac{1}{2}m\left(\dot{r}^2 + \frac{1}{2}r^2\omega^2\right) - \frac{1}{\sqrt{2}}mgr$$

$$\text{Thus the momentum conjugate to } r \text{ is } p_r = \frac{\partial L}{\partial \dot{r}} \Rightarrow p_r = m\dot{r}$$

So the correct answer is **Option (A)**

5. The Lagrangian of a system is given by $L = \frac{1}{2}ml^2\left[\dot{\theta}^2 + \sin^2\theta\dot{\varphi}^2\right] - mgl\cos\theta$, where m, l and g are constants. Which of the following is conserved?

[GATE 2016]

A. $\dot{\varphi}\sin^2\theta$ **B.** $\dot{\varphi}\sin\theta$ **C.** $\frac{\dot{\varphi}}{\sin\theta}$ **D.** $\frac{\dot{\varphi}}{\sin^2\theta}$

Solution:

Solution: As φ is cyclic coordinate, so $\frac{\partial L}{\partial \dot{\varphi}} = p_\varphi = ml^2 \sin^2 \dot{\varphi}$, is a constant since m, l and g are constants. Thus $\dot{\varphi} \sin^2 \theta$ is conserved.

So the correct answer is **Option (A)**

6. If the Lagrangian $L_0 = \frac{1}{2}m\left(\frac{dq}{dt}\right)^2 - \frac{1}{2}m\omega^2 q^2$ is modified to $L = L_0 + \alpha q\left(\frac{dq}{dt}\right)$, which one of the following is TRUE?

[GATE 2017]

A. Both the canonical momentum and equation of motion do not change

B. Canonical momentum changes, equation of motion does not change

C. Canonical momentum does not change, equation of motion changes

D. Both the canonical momentum and equation of motion change

Solution:

$$\text{For Lagrangian } L_0 = \frac{1}{2}m\left(\frac{dq}{dt}\right)^2 - \frac{1}{2}m\omega^2 q^2$$

$$\text{canonical momentum is } p = m\dot{q}$$

and equation of motion is given by

$$\frac{d}{dt}\left(\frac{\partial L}{\partial \dot{q}}\right) - \left(\frac{\partial L}{\partial q}\right) = 0 \Rightarrow m\ddot{q} + m\omega^2 q = 0$$

$$\text{For Lagrangian } L = L_0 + \alpha q\left(\frac{dq}{dt}\right) \Rightarrow L$$

$$= \frac{1}{2}m\left(\frac{dq}{dt}\right)^2 - \frac{1}{2}m\omega^2 q^2 + \alpha q\dot{q}$$

$$\text{Canonical momentum is } p = m\dot{q} + \alpha q$$

Equation of motion is,

$$\frac{d}{dt}\left(\frac{\partial L}{\partial \dot{q}}\right) - \left(\frac{\partial L}{\partial q}\right) = 0 \Rightarrow m\ddot{q} + m\omega^2 q = 0$$

So the correct answer is **Option (B)**

7. A double pendulum consists of two equal masses m suspended by two strings of length l. What is the Lagrangian of this system for oscillations in a plane? Assume the angles θ_1, θ_2 made by the two strings are small (you can use $\cos\theta = 1 - \theta^2/2$).
Note: $\omega_0 = \sqrt{g/l}$.

[JEST 2014]

A. $L \approx ml^2\left(\dot{\theta}_1^2 + \frac{1}{2}\dot{\theta}_2^2 - \omega_0^2\theta_1^2 - \frac{1}{2}\omega_0^2\theta_2^2\right)$

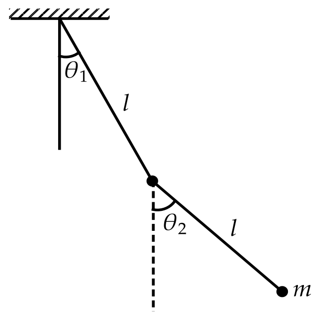
B. $L \approx ml^2\left(\dot{\theta}_1^2 + \frac{1}{2}\dot{\theta}_2^2 + \dot{\theta}_1\dot{\theta}_2 - \omega_0^2\theta_1^2 - \frac{1}{2}\omega_0^2\theta_2^2\right)$

C. $L \approx ml^2\left(\dot{\theta}_1^2 + \frac{1}{2}\dot{\theta}_2^2 - \dot{\theta}_1\dot{\theta}_2 - \omega_0^2\theta_1^2 - \frac{1}{2}\omega_0^2\theta_2^2\right)$

D. $L \approx ml^2\left(\frac{1}{2}\dot{\theta}_1^2 + \frac{1}{2}\dot{\theta}_2^2 + \dot{\theta}_1\dot{\theta}_2 - \omega_0^2\theta_1^2 - \omega_0^2\theta_2^2\right)$

Solution:

$$
\begin{aligned}
x_1 &= l\sin\theta_1, \\
y_1 &= l\cos\theta_1 \\
x_2 &= x_1 + l\sin\theta_2 \\
y_2 &= y_1 + l\cos\theta_2 \\
\dot{x}_2 &= l\cos\theta_1\dot{\theta}_1 + l\cos\theta_2\dot{\theta}_2, \\
\dot{y}_2 &= -l\sin\theta_1\dot{\theta}_1 - l\sin\theta_2\dot{\theta}_2 \\
\dot{x}_2^2 + \dot{y}_2^2 &= l^2\cos^2\theta_1\dot{\theta}_1^2 + l^2\cos^2\theta_2\dot{\theta}_2^2 \\
&\quad + 2l^2\cos\theta_1\dot{\theta}_1\cos\theta_2\dot{\theta}_2 + l^2\sin\theta_1^2\dot{\theta}_1^2 \\
&\quad + l^2\sin\theta_2^2\dot{\theta}_2^2 + 2l^2\sin\theta_1\sin\theta_2\dot{\theta}_1\dot{\theta}_2 \\
\Rightarrow \dot{x}_2^2 + \dot{y}_2^2 &= l^2\dot{\theta}_1^2 + l^2\dot{\theta}_2^2 + 2l^2\cos(\theta_1-\theta_2)\dot{\theta}_1\dot{\theta}_2
\end{aligned}
$$



also $\dot{x}_1^2 + \dot{y}_1^2 = l^2\dot{\theta}_1^2$

$$
\begin{aligned}
L = T - V &= \frac{1}{2}m\left(\dot{x}_1^2 + \dot{y}_1^2 + \dot{x}_2^2 + \dot{y}_2^2\right) - mgy_1 - mgy_2 \\
\Rightarrow L &= \frac{1}{2}m\left(l^2\dot{\theta}_1^2 + l^2\dot{\theta}_1^2 + l^2\dot{\theta}_2^2 + 2l^2\cos(\theta_1-\theta_2)\dot{\theta}_1\dot{\theta}_2\right) \\
&\quad + 2mgl\cos\theta_1 + mgl\cos\theta_2 \\
\Rightarrow L &= ml^2\left[\dot{\theta}_1^2 + \frac{1}{2}\dot{\theta}_2^2 + \dot{\theta}_1\dot{\theta}_2 + \frac{2g}{2l}\left(1 - \frac{\theta_1^2}{2}\right) + \frac{1}{2}\frac{g}{l}\left(1 - \frac{\theta_2^2}{2}\right)\right] \\
&\quad [\because \cos(\theta_1-\theta_2) \approx 1] \\
\Rightarrow L &= ml^2\left[\dot{\theta}_1^2 + \frac{1}{2}\dot{\theta}_2^2 + \dot{\theta}_1\dot{\theta}_2 + \frac{g}{l} - \frac{g}{l}\frac{\theta_1^2}{2} + \frac{g}{2l} - \frac{g}{2l}\frac{\theta_2^2}{2}\right]
\end{aligned}
$$

comparing given options, option (B) is correct i.e.

$$
L = ml^2\left(\dot{\theta}_1^2 + \frac{1}{2}\dot{\theta}_2^2 + \dot{\theta}_1\dot{\theta}_2 - \frac{\omega_0^2\dot{\theta}_1^2}{2} - \frac{1}{4}\omega_0\dot{\theta}_2^2\right)
$$

So the correct answer is **Option (B)**

8. A bike stuntman rides inside a well of frictionless surface given by $z = a\left(x^2+y^2\right)$, under the action of gravity acting in the negative z direction. $\vec{g} = -g\hat{z}$. What speed should be maintain to be able to ride at a constant height z_0 without falling down?

[JEST 2015]

A. $\sqrt{gz_0}$

B. $\sqrt{3gz_0}$

C. $\sqrt{2gz_0}$

D. The biker will not be able to maintain a constant height, irrespective of speed.

Solution:

$$
z = a\left(x^2+y^2\right)
$$

Using equation of constrain, we must solve the given system in cylindrical co-ordinate.

$$
\begin{aligned}
z = ar^2, \dot{z} = 2ar\dot{r} \Rightarrow L \\
&= \frac{1}{2}m\left(\dot{r}^2 + r^2\dot{\theta} + \dot{z}^2\right) - mgz \\
\Rightarrow L &= \frac{1}{2}m\left(\dot{r}^2 + r^2\dot{\theta} + 4a^2r^2\dot{r}^2\right) - mgar^2 \\
&= \frac{1}{2}m\left[\dot{r}^2\left(1 + 4a^2r^2\right) + r^2\dot{\theta}^2\right] - mgar^2
\end{aligned}
$$

Equation of motion

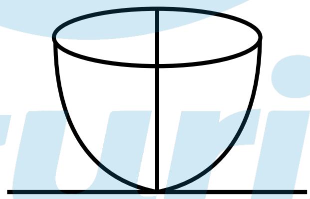
$$\frac{d}{dt}\left(\frac{\partial L}{\partial \dot{r}}\right) - \frac{\partial L}{\partial r} = 0$$

$$\Rightarrow m\ddot{r}\left(1 + 4a^2r^2\right) + m\dot{r}^2 4a^2 r - mr\dot{\theta}^2 + 2mgar = 0$$

At $z = z_0, \dot{r} = 0, \quad r = r_0,$ so,$mr_0\dot{\theta}^2 = 2mgar_0$

$$
\begin{aligned}
\dot{\theta}^2 = 2ga \Rightarrow \dot{\theta} = \sqrt{2ga}, \frac{v}{r_0} \\
&= \sqrt{2ga}, v = \sqrt{2ga} \cdot r_0 \\
v &= \sqrt{2ga} \cdot \left(\frac{z_0}{a}\right)^{1/2} \\
&= \sqrt{2gz_0}
\end{aligned}
$$

$(\because z_0 = ar_0^2)$



So the correct answer is **Option** (C)

9. The Lagrangian of a particle is given by $L = \dot{q}^2 - q\dot{q}$. Which of the following statements is true? **[JEST 2015]**

A. This is a free particle
B. The particle is experiencing velocity dependent damping
C. The particle is executing simple harmonic motion
D. The particle is under constant acceleration.

Solution:

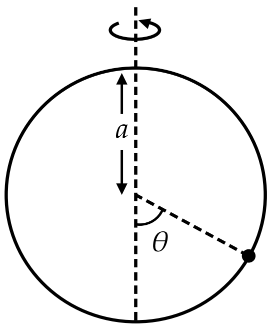
$$
\begin{aligned}
\because L = \dot{q}^2 - q\dot{q} \Rightarrow \frac{\partial L}{\partial \dot{q}} \\
&= 2\dot{q} - q \Rightarrow \frac{d}{dt}\left(\frac{\partial L}{\partial \dot{q}}\right) \\
&= 2\ddot{q} - \dot{q} \\
\because \frac{d}{dt}\left(\frac{\partial L}{\partial \dot{q}}\right) - \frac{\partial L}{\partial q} &= 0 \Rightarrow 2\ddot{q} - \dot{q} + \dot{q} \\
&= 0 \Rightarrow 2\ddot{q} = 0 \Rightarrow \frac{d^2q}{dt^2}
\end{aligned}
$$

$$= 0 \Rightarrow \frac{dq}{dt} = C \Rightarrow q = Ct + \alpha$$

So the correct answer is **Option (A)**

10. A hoop of radius a rotates with constant angular velocity ω about the vertical axis as shown in the figure. A bead of mass m can slide on the hoop without friction. If $g < \omega^2 a$ at what angle θ apart from 0 and π is the bead stationary (i.e., $\frac{d\theta}{dt} = \frac{d^2\theta}{dt^2} = 0$)?

[JEST 2016]



A. $\tan\theta = \frac{\pi g}{\omega^2 a}$ **B.** $\sin\theta = \frac{g}{\omega^2 a}$ **C.** $\cos\theta = \frac{g}{\omega^2 a}$ **D.** $\tan\theta = \frac{g}{\pi\omega^2 a}$

Solution:

The Lagrangian of the system is

$$L = \frac{1}{2}ma^2\left(\dot\theta^2 + \sin^2\theta\dot\phi^2\right) + mga\cos\theta$$

The equation of motion is,

$$\frac{d}{dt}\left(\frac{\partial L}{\partial\dot\theta}\right) - \left(\frac{\partial L}{\partial\theta}\right)$$
$$= 0 \Rightarrow ma^2\ddot\theta - ma^2\left(\sin\theta\cos\theta\dot\phi^2\right) + mga\sin\theta = 0$$

When bead is stationary, then

$$\frac{d\theta}{dt} = \frac{d^2\theta}{dt^2}$$
$$= 0 \Rightarrow -ma^2\left(\sin\theta\cos\theta\dot\phi^2\right) + mga\sin\theta = 0$$
$$\Rightarrow \dot\phi = \omega \text{ and } g < \omega^2 a, \text{ then } \cos\theta = \frac{g}{\omega^2 a}$$

So the correct answer is **Option (C)**

11. A bead of mass M slides along a parabolic wire described by $z = 2\left(x^2 + y^2\right)$. The wire rotates with angular velocity Ω about the z - axis. At what value of Ω does the bead maintain a constant nonzero height under the action of gravity along $-\hat{z}$?

[JEST 2017]

A. $\sqrt{3g}$ **B.** $\sqrt{g}$ **C.** $\sqrt{2g}$ **D.** $\sqrt{4g}$

Solution:

$$L = \frac{1}{2}m\left(\dot r^2 + r^2\dot\theta^2 + 16r^2\dot r^2\right) - 2mgr^2 \Rightarrow L$$
$$= \frac{1}{2}m\left(\dot r^2\left(1 + 16r^2\right) + r^2\dot\theta^2\right) - 2mgr^2$$

The equation of motion is given by

$$\frac{d}{dt}\left(\frac{\partial L}{\partial \dot r}\right)-\frac{\partial L}{\partial r}$$
$$=0 \Rightarrow m\ddot r\left(1+16r^2\right)+16m\dot r^2 r-mr\dot\theta^2+4mgr=0$$

At equilibrium, $r=r_0, \dot r=0, \ddot r=0$

So, $-mr_0\dot\theta^2+4mgr_0=0 \Rightarrow \dot\theta=\Omega=\sqrt{4g}$

So the correct answer is **Option (D)**

12. A possible Lagrangian for a free particle is

[JEST 2017]

A. $L=\dot q^2-q^2$ **B.** $L=\dot q^2-q\dot q$ **C.** $L=\dot q^2-q$ **D.** $L=\dot q^2-\frac{1}{q}$

Solution:

$$\frac{d}{dt}\left(\frac{\partial L}{\partial \dot q}\right)-\left(\frac{\partial L}{\partial q}\right)=0 \Rightarrow 2\ddot q-\dot q+\dot q$$
$$=0 \Rightarrow \ddot q=0$$

So the correct answer is **Option (B)**

13. A rod of mass m and length l is suspended from two massless vertical springs with a spring constants k_1 and k_2. What is the Lagrangian for the system, if x_1 and x_2 be the displacements from equilibrium position of the two ends of the rod?

[JEST 2017]

A. $\frac{m}{8}\left(\dot x_1^2+2\dot x_1\dot x_2+\dot x_2^2\right)-\frac{1}{2}k_1x_1^2-\frac{1}{2}k_2x_2^2$

B. $\frac{m}{2}\left(\dot x_1^2+\dot x_1\dot x_2+\dot x_2^2\right)-\frac{1}{4}\left(k_1+k_2\right)\left(x_1^2+x_2^2\right)$

C. $\frac{m}{6}\left(\dot x_1^2+x_1\dot x_2+\dot x_2^2\right)-\frac{1}{2}k_1x_1^2-\frac{1}{2}k_2x_2^2$

D. $\frac{m}{2}\left(\dot x_1^2-2\dot x_1\dot x_2+\dot x_2^2\right)-\frac{1}{4}\left(k_1-k_2\right)\left(x_1^2+x_2^2\right)$

Solution:

$$T=\frac{1}{2}MV_{c,m}^2+\frac{1}{2}I_{c.m}\omega^2$$
$$=\frac{1}{2}m\left(\frac{\dot x_1+\dot x_2}{2}\right)^2+\frac{1}{2}\frac{ml^2}{12}\dot\theta^2$$

Potential energy is, $V=\frac{1}{2}kx_1^2+\frac{1}{2}kx_2^2$

$$\sin\theta=\frac{x_2-x_1}{l} \text{ for small oscillation} \theta$$
$$=\frac{x_2-x_1}{l}=\dot\theta$$
$$=\frac{\dot x_2-\dot x_1}{l}$$
$$L=\frac{1}{2}m\left(\frac{\dot x_1+\dot x_2}{2}\right)^2+\frac{1}{2}\frac{ml^2}{12}\left(\frac{\dot x_1-\dot x_2}{l}\right)^2-\frac{1}{2}kx_1^2-\frac{1}{2}kx_2^2$$
$$=\frac{m}{6}\left(\dot x_1^2+x_1\dot x_2+\dot x_2^2\right)-\frac{1}{2}k_1x_1^2-\frac{1}{2}k_2x_2^2$$

So the correct answer is **Option (C)**

14. Consider the Lagrangian

$$L = 1 - \sqrt{1-\dot{q}^2} - \frac{q^2}{2}$$

of a particle executing oscillations whose amplitude is A. If p denotes the momentum of the particle, then $4p^2$ is

[JEST 2015]

A. (a) $\left(A^2-q^2\right)\left(4+A^2-q^2\right)$ **B.** $\left(A^2+q^2\right)\left(4+A^2-q^2\right)$

C. $\left(A^2-q^2\right)\left(4+A^2+q^2\right)$ **D.** $\left(A^2+q^2\right)\left(4+A^2+q^2\right)$

Solution: So the correct answer is **Option (A)**

15. Consider the motion of a particle in two dimensions given by the Lagrangian

$$L = \frac{m}{2}\left(\dot{x}^2+\dot{y}^2\right) - \frac{\lambda}{4}(x+y)^2$$

where $\lambda > 0$. The initial conditions are given as $y(0) = 0, x(0) = 42$ meters, $\dot{x}(0) = \dot{y}(0) = 0$. What is the value of $x(t) - y(t)$ at $t = 25$ seconds in meters?

[JEST 2019]

Solution:

$$L = \frac{m}{2}\left(\dot{x}^2+\dot{y}^2\right) - \frac{\lambda}{4}(x+y)^2$$

The equation of motion is

$$\frac{d}{dt}\left(\frac{\partial L}{\partial \dot{x}}\right) - \left(\frac{\partial L}{\partial x}\right) = 0 \Rightarrow m\ddot{x} + \frac{\lambda}{2}x + \frac{\lambda}{2}y = 0 \quad (1)$$

$$\frac{d}{dt}\left(\frac{\partial L}{\partial \dot{y}}\right) - \left(\frac{\partial L}{\partial y}\right) = 0 \Rightarrow m\ddot{y} + \frac{\lambda}{2}y + \frac{\lambda}{2}x = 0 \quad (2)$$

Subtracting equation (1) from (2) gives $m(\ddot{x}-\ddot{y}) = 0 \Rightarrow \ddot{x}-\ddot{y} = 0$

Integrating both sides with t gives

$$\dot{x} - \dot{y} = c_1$$

From the equation $\dot{x}(0) = \dot{y}(0) = 0$, there $c_1 = 0$

$$\text{Hence, } \dot{x} - \dot{y} = 0 \quad (3)$$

Integrating both sides of this equation with t gives

$$x - y = c_2$$

Putting $x(0) = 42, y(0) = 0$ gives

$$42 - 0 = c_2 \Rightarrow 42$$

Therefore, $x - y = 42$

The value of $x - y$ is independent of t.

Therefore, at $t = 25s$

$$x(t) - y(t) = 42$$

Answer key			
Q.No.	Answer	Q.No.	Answer
1	**B**	2	**B**
3	**D**	4	**A**
5	**A**	6	**B**
7	**B**	8	**C**
9	**A**	10	**C**
11	**D**	12	**B**
13	**C**	14	**A**
15	**42**		

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