Practise Set-1 Solutions

1. Consider two point charges q and λq located at the points, $x = a$ and $x = \mu a$, respectively. Assuming that the sum of the two charges is constant, what is the value of λ for which the magnitude of the electrostatic force is maximum?

[JEST 2015]

A. μ **B.** 1 **C.** $\frac{1}{\mu}$ **D.** $1+\mu$

Solution:

$$F = \frac{1}{4\pi\varepsilon_0}\frac{(\lambda q \times q)}{(\mu a - a)^2}$$
$$= \frac{1}{4\pi\varepsilon_0}\frac{\lambda q^2}{a^2(\mu-1)^2}$$
$$= \frac{1}{4\pi\varepsilon_0 a^2(\mu-1)^2}\frac{\lambda c^2}{(1+\lambda)^2} \quad \because q+\lambda q$$
$$= c$$

$$\text{For maximum } F, \frac{dF}{d\lambda} = 0$$

$$0 = \frac{1}{4\pi\varepsilon_0 a^2(\mu-1)^2}\left[\frac{(1+\lambda)^2c^2 - \lambda c^2 \times 2(1+\lambda)}{(1+\lambda)^4}\right]$$

$$\Rightarrow (1+\lambda)^2 c^2 = \lambda c^2 \times 2(1+\lambda) \Rightarrow 1+\lambda$$
$$= 2\lambda$$
$$\Rightarrow \lambda = 1$$

Correct option is (B)

2. An electric field in a region is given by $\vec{E}(x,y,z) = ax\hat{i} + cz\hat{j} + 6by\hat{k}$. For which values of a,b,c does this represent an electrostatic field?

[JEST 2012]

A. $13,1,12$ **B.** $17,6,1$ **C.** $13,1,6$ **D.** $45,6,1$

Solution:

For electrostatic field,

$$\vec{\nabla}\times\vec{E} = 0$$

$$\vec{\nabla}\times\vec{E} = \begin{bmatrix} \hat{i} & \hat{j} & \hat{k} \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ ax & cz & 6by \end{bmatrix} = 0$$

$$\Rightarrow \vec{\nabla}\times\vec{E} = (6b-c)\hat{i} + \hat{j}[0-0] + \hat{k}[0] = 0$$
$$\Rightarrow (6b-c)\hat{i} = 0$$
$$\Rightarrow c = 6b$$

Correct option is (C)

3. The electric fields outside $(r > R)$ and inside $(r < R)$ a solid sphere with a uniform volume charge density are given by $\vec{E}_{r>R} = \frac{1}{4\pi\varepsilon_0}\frac{q}{r^2}\hat{r}$ and $\vec{E}_{r<R} = \frac{1}{4\pi\varepsilon_0}\frac{q}{R^3}r\hat{r}$ respectively, while the electric field outside a spherical shell with a uniform surface charge density is given by $\vec{E}_{r>R} = \frac{1}{4\pi\varepsilon_0}\frac{q}{r^2}\hat{r}, q$ being the total charge. The correct ratio of the electrostatic energies for the second case to the first case is

[JEST 2013]

A. $1:3$ **B.** $9:16$ **C.** $3:8$ **D.** $5:6$

Solution:

Electrostatic energy in spherical shell ,

$$w_{sp} = \frac{\varepsilon_0}{2}\int_0^R \left|\vec{E}_1\right|^2 4\pi r^2 dr + \frac{\varepsilon_0}{2}\int_R^\infty \left|\vec{E}_2\right|^2 4\pi r^2 dr$$

$$\Rightarrow \frac{\varepsilon_0}{2}\int_R^\infty \frac{q^2}{(4\pi\in_0)^2 r^4} 4\pi r^2 dr = \frac{q^2}{8\pi\in_0}\left(-\frac{1}{r}\right)_R^\infty$$

$$= \frac{q^2}{8\pi\in_0}\frac{1}{R}$$

Electrostatic energy in solid sphere,

$$w_s = \frac{\varepsilon_0}{2}\int_0^R |E_1|^2 4\pi r^2 dr + \frac{\varepsilon_0}{2}\int_R^\infty |E_2|^2 4\pi r^2 dr$$

$$\Rightarrow \frac{q^2}{8\pi\in_0} \times \frac{1}{R^6}\left[\frac{r^5}{5}\right]_0^R + \frac{q^2}{8\pi\in_0}\left[-\frac{1}{r}\right]_R^\infty$$

$$w_s = \frac{q^2}{5\times 8\pi\in_0}\cdot\frac{1}{R} + \frac{q^2}{8\pi\in_0 R} = \frac{6q^2}{40\pi\in_0 R}$$

$$\text{Now } \frac{W_{\text{spherical}}}{W_{\text{sphere}}} = \frac{\frac{q^2}{8\pi\in_0 R}}{\frac{6q^2}{40\pi\in_0 R}}$$

$$= \frac{5}{6}$$

Correct option is (D)

4. If $\vec{E}_1 = xy\hat{i} + 2yz\hat{j} + 3xz\hat{k}$ and $\vec{E}_2 = y^2\hat{i} + \left(2xy + z^2\right)\hat{j} + 2yz\hat{k}$ then

[JEST 2013]

A. Both are impossible electrostatic fields.

B. Both are possible electrostatic fields.

C. Only $\vec{E}_1$ is a possible electrostatic field.

D. Only $\vec{E}_2$ is a possible electrostatic field.

Solution:

For electrostatic field $\vec{\nabla} \times \vec{E} = 0$

$$\vec{\nabla} \times \vec{E}_2 = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ y^2 & 2xy + z^2 & 2yz \end{vmatrix} = (2z - 2z)\hat{i} + 0 + (2y - 2y)\hat{z} = 0$$

$$\vec{\nabla}\times\vec{E}_1=\begin{vmatrix}\hat{i} & \hat{j} & \hat{k}\\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z}\\ xy & 2yz & 3xz\end{vmatrix}=(0-2y)\hat{i}+0+x\hat{j}\neq 0$$

Correct option is (D)

5. A charge q is placed at the centre of an otherwise neutral dielectric sphere of radius a and relative permittivity ε_r. We denote the expression $q/4\pi\varepsilon_0 r^2$ by $E(r)$. Which of the following statements is false? **[JEST 2013]**

A. The electric field inside the sphere, $r<a$, is given by $E(r)/\varepsilon_r$

B. The field outside the sphere, $r>a$, is given by $E(r)$

C. The total charge inside a sphere of radius $r>a$ is given by q.

D. The total charge inside a sphere of radius $r<a$ is given by q.

Solution: Correct option is (D)

6. Two large nonconducting sheets one with a fixed uniform positive charge and another with a fixed uniform negative charge are placed at a distance of 1 meter from each other. The magnitude of the surface charge densities are $\sigma_+=6.8\mu\mathrm{C/m^2}$ for the positively charged sheet and $\sigma_-=4.3\mu\mathrm{C/m^2}$ for the negatively charged sheet. What is the electric field in the region between the sheets? **[JEST 2014]**

A. 6.30×10^5 N/C **B.** 3.84×10^5 N/C **C.** 1.40×10^5 N/C **D.** 1.16×10^5 N/C

Solution:

$$\text{Electric field between the sheet is }=\frac{\sigma_+}{2\varepsilon_0}+\frac{\sigma_-}{2\varepsilon_0}$$
$$=\frac{6.8\times10^{-6}}{2\varepsilon_0}+\frac{4.3\times10^{-6}}{2\varepsilon_0}$$
$$\frac{11.2\times10^{-6}}{2\times8.86\times10^{-12}}=0.626\times10^6\Rightarrow 6.3\times10^5\ \mathrm{N/C}$$

Correct option is (A)

7. A circular loop of radius R, carries a uniform line charge density λ. The electric field, calculated at a distance z directly above the center of the loop, is maximum if z is equal to, **[JEST 2015]**

A. $\frac{R}{\sqrt{3}}$ **B.** $\frac{R}{\sqrt{2}}$ **C.** $\frac{R}{2}$ **D.** $2R$

Solution:

$$E=\frac{1}{4\pi\varepsilon_0}\frac{(\lambda\times2\pi R)z}{(R^2+z^2)^{3/2}}$$

$$\text{For maximum }E,\ \frac{dE}{dz}=0$$

$$\Rightarrow\frac{\lambda\times2\pi R}{4\pi\varepsilon_0}\left[\frac{(R^2+z^2)^{3/2}-z\times3/2\sqrt{R^2+z^2}\times2z}{(R^2+z^2)^3}\right]=0$$
$$\Rightarrow(R^2+z^2)^{3/2}=3z^2\sqrt{R^2+z^2}$$
$$\Rightarrow R^2+z^2=3z^2$$

$$\Rightarrow R^2 = 2z^2$$
$$\Rightarrow z = \frac{R}{\sqrt{2}}$$

Correct option is (B)

Practise Set-2 Solutions

1. Four equal point charges are kept fixed at the four vertices of a square. How many neutral points (i.e. points where the electric field vanishes) will be found inside the square?

[NET/JRF(DEC-2011)]

A. 1 **B.** 4 **C.** 5 **D.** 7

Solution:

Inside the square, there is only one point where field vanishes It is at the center of the square

So the correct answer is **Option (A)**

2. A static charge distribution gives rise to an electric field of the form $\vec{E} = \alpha\left(1 - e^{-r/R}\right)\frac{\hat{r}}{r^2}$, where α and R are positive constants. The charge contained within a sphere of radius R, centred at the origin is

[NET/JRF(DEC-2011)]

A. $\pi\alpha\varepsilon_0\frac{e}{R^2}$ **B.** $\pi\alpha\varepsilon_0\frac{e^2}{R^2}$ **C.** $4\pi\alpha\varepsilon_0\frac{R}{e}$ **D.** $\pi\alpha\varepsilon_0\frac{R^2}{e}$

Solution:

$$\begin{aligned} Q_{enc} &= \varepsilon_0 \oint \vec{E}\cdot d\vec{a} \\ &= \alpha\varepsilon_0 \int \left(1 - e^{-r/R}\right)\frac{\hat{r}}{r^2}\cdot\left(r^2\sin\theta\, d\theta\, d\phi\, \hat{r}\right) \\ &= \alpha\varepsilon_0 \times \int_0^{\pi}\int_0^{2\pi}\left(1 - e^{-r/R}\right)\sin\theta\, d\theta\, d\phi \end{aligned}$$

at $r = R$, $\quad Q_{enc} = 4\pi\alpha\varepsilon_0\left(1 - \frac{1}{e}\right)$. So none of the options given are correct.

None of the options given are correct

3. Charges Q, Q and $-2Q$ are placed on the vertices of an equilateral triangle ABC of sides of length a, as shown in the figure. The dipole moment of this configuration of charges, irrespective of the choice of origin, is

[NET/JRF(JUNE-2012)]

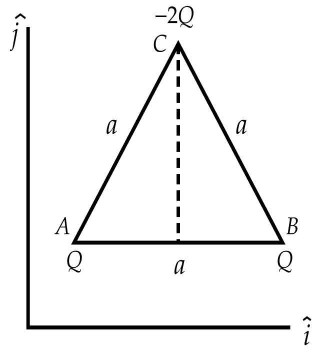


Figure 1

A. $+2aQ\hat{i}$ **B.** $+\sqrt{3}aQ\hat{j}$ **C.** $-\sqrt{3}aQ\hat{j}$ **D.** 0

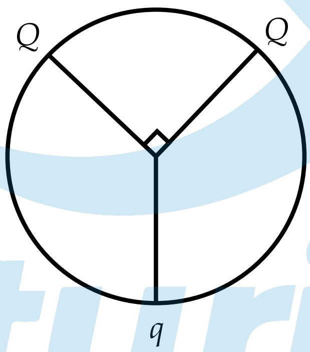
Solution:

Let coordinates of A is (l,m), then

$$\vec{p} = q_i\vec{r}_i = Q[l\hat{i}+m\hat{j}] + Q[(l+a)\hat{\imath}+m\hat{j}] - 2Q\left[\left(l+\frac{a}{2}\right)\hat{\imath}+\left(m+\frac{\sqrt{3}a}{2}\right)\hat{j}\right]$$
$$\vec{p} = Q[l\hat{\imath}+m\hat{j}] + Q[(l+a)\hat{\imath}+m\hat{j}] - Q[(2l+a)\hat{\imath}+(2m+\sqrt{3}a)\hat{j}]$$
$$\vec{p} = -\sqrt{3}aQ\hat{j}$$

So the correct answer is **Option (C)**

4. Three charges are located on the circumference of a circle of radius R as shown in the figure below. The two charges Q subtend an angle 90° at the centre of the circle. The charge q is symmetrically placed with respect to the charges Q. If the electric field at the centre of the circle is zero, what is the magnitude of Q ?
[NET/JRF(DEC-2012)]



A. $q/\sqrt{2}$ **B.** $\sqrt{2}q$ **C.** $2q$ **D.** $4q$

Solution:

$$E_1 = E_2 = \frac{1}{4\pi\varepsilon_0}\frac{Q}{R^2} \text{ and } E_3 = \frac{1}{4\pi\varepsilon_0}\frac{q}{R^2}$$

Resultant of E_1 and E_2 is $E = \sqrt{E_1^2+E_2^2} = \sqrt{2}E_1,$

Thus $E_3 = E$

$$Q = \frac{q}{\sqrt{2}}$$

So the correct answer is **Option (A)**

5. A point charges q of mass m is kept at a distance d below a grounded infinite conducting sheet which lies in the xy - plane. For what value of d will the charge remains stationary?
[NET/JRF(DEC-2012)]

A. $q/4\sqrt{mg\pi\varepsilon_0}$ **B.** $q/\sqrt{mg\pi\varepsilon_0}$

C. There is no finite value of d **D.** $\sqrt{mg\pi\varepsilon_0}/q$

Solution: There is attractive force between point charge q and grounded conducting sheet that can be calculate from method of images i.e.

$$\frac{1}{4\pi\varepsilon_0}\frac{q^2}{(2d)^2} = mg$$
$$d = \frac{q}{4\sqrt{mg\pi\varepsilon_0}}$$

So the correct answer is **Option (A)**

6. A solid sphere of radius R has a charge density, given by

$$\rho(r) = \rho_0\left(1-\frac{ar}{R}\right)$$

where r is the radial coordinate and ρ_0, a and R are positive constants. If the magnitude of the electric field at $r = R/2$ is 1.25 times that at $r = R$, then the value of a is

[NET/JRF(DEC-2014)]

A. 2 **B.** 1 **C.** $1/2$ **D.** $1/4$

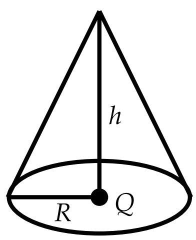
Solution:

$$\oint_S \vec{E}\cdot d\vec{a} = \frac{1}{\varepsilon_0}Q_{enc} \Rightarrow |\vec{E}|\times 4\pi r^2 = \frac{1}{\varepsilon_0}\int_0^r \rho_0\left(1-\frac{ar}{R}\right)4\pi r^2 dr$$
$$\Rightarrow |\vec{E}|\times 4\pi r^2 = \frac{4\pi\rho_0}{\varepsilon_0}\int_0^r\left(r^2-\frac{ar^3}{R}\right)dr$$
$$= \frac{4\pi\rho_0}{\varepsilon_0}\left(\frac{r^3}{3}-\frac{ar^4}{4R}\right) \Rightarrow |\vec{E}| = \frac{\rho_0}{\varepsilon_0}\left(\frac{r}{3}-\frac{ar^2}{4R}\right)$$
$$\because E_{r=R/2} = 1.25E_{r=R} \Rightarrow \frac{\rho_0}{\varepsilon_0}\left(\frac{R/2}{3}-\frac{aR^2/4}{4R}\right) = 1.25\frac{\rho_0}{\varepsilon_0}\left(\frac{R}{3}-\frac{aR^2}{4R}\right)$$
$$\Rightarrow \left(\frac{1}{6}-\frac{a}{16}\right) = \frac{5}{4}\left(\frac{1}{3}-\frac{a}{4}\right) \Rightarrow \left(\frac{1}{6}-\frac{a}{16}\right)$$
$$= \left(\frac{5}{12}-\frac{5a}{16}\right) \Rightarrow \frac{5a}{16}-\frac{a}{16} = \frac{5}{12}-\frac{1}{6}$$
$$\Rightarrow \frac{4a}{16} = \frac{5-2}{12} \Rightarrow \frac{a}{4} = \frac{3}{12} \Rightarrow a = 1$$

So the correct answer is **Option (B)**

7. Consider a charge Q at the origin of 3 - dimensional coordinate system. The flux of the electric field through the curved surface of a cone that has a height h and a circular base of radius R (as shown in the figure) is

[NET/JRF(DEC-2015)]



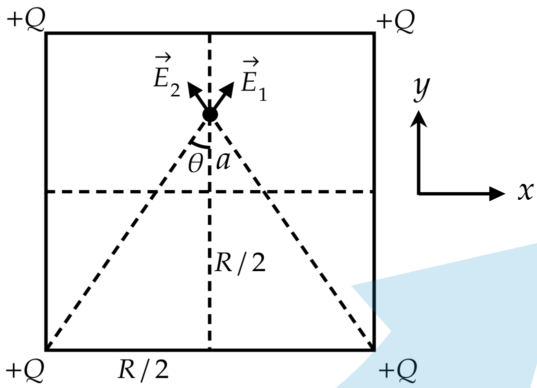
A. $\frac{Q}{\varepsilon_0}$ **B.** $\frac{Q}{2\varepsilon_0}$ **C.** $\frac{hQ}{R\in_0}$ **D.** $\frac{QR}{2h\in_0}$

Solution: So the correct answer is **Option (B)**

8. Four equal charges of $+Q$, each are kept at the vertices of a square of side R. A particle of mass m and charge $+Q$ is placed in the plane of the square at a short distance $a(\ll R)$ from the centre. If the motion of the particle is confined to the plane, it will undergo small oscillations with an angular frequency
[NET/JRF(JUNE-2016)]

A. $\sqrt{\frac{Q^2}{2\pi\varepsilon_0 R^3 m}}$ **B.** $\sqrt{\frac{Q^2}{\pi\varepsilon_0 R^3 m}}$ **C.** $\sqrt{\frac{\sqrt{2}Q^2}{\pi\varepsilon_0 R^3 m}}$ **D.** $\sqrt{\frac{Q^2}{4\pi\varepsilon_0 R^3 m}}$

Solution:

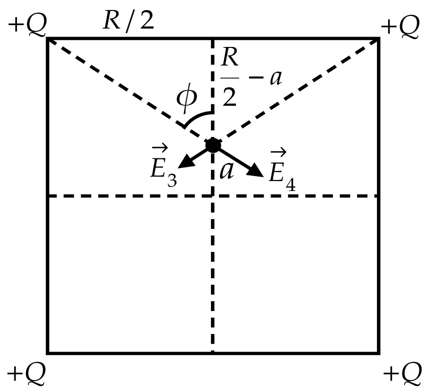


$$E_1 = E_2 = \frac{kQ}{\left[\left(a+\frac{R}{2}\right)^2 + \frac{R^2}{4}\right]}$$

Resultant field $E_{12,y} = 2E_1 \cos\theta$

$$E_{12,y} = \frac{2kQ}{\left[\left(a+\frac{R}{2}\right)^2 + \frac{R^2}{4}\right]^{\frac{3}{2}}}\left(a+\frac{R}{2}\right) \approx \frac{2kQ}{\left[\frac{R^2}{2}\right]^{\frac{3}{2}}}\left(a+\frac{R}{2}\right)$$

$$E_{12,y} = \frac{4\sqrt{2}kQ}{R^3}\left(a+\frac{R}{2}\right)$$



$$\text{Similarly; } E_3 = E_4 = \frac{kQ}{\left[\left(\frac{R}{2}-a\right)^2 + \frac{R^2}{4}\right]}$$

$$\text{Resultant } E_{34,y} = 2E_3 \cos\phi = \frac{2kQ}{\left[\left(\frac{R}{2}-a\right)^2 + \frac{R^2}{4}\right]^{\frac{3}{2}}}\left(\frac{R}{2}-a\right)$$

$$\Rightarrow \quad E_{34,y} = \frac{4\sqrt{2}kQ}{R^3}\left(\frac{R}{2}-a\right)$$

$$\text{Resultant } E = \frac{4\sqrt{2}kQ}{R^3}\left[\left(\frac{R}{2}-a\right) - \left(\frac{R}{2}+a\right)\right] = -\frac{8\sqrt{2}kQ}{R^3}a$$

$$E = \frac{-8\sqrt{2}}{R^3} \times \frac{1}{4\pi\varepsilon_0}Qa$$

$$E = -\frac{2\sqrt{2}Q}{\pi\varepsilon_0 R^3}a$$

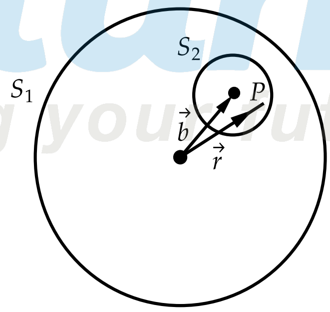
$$\Rightarrow \quad F = QE = -\frac{2\sqrt{2}Q^2}{\pi\varepsilon_0 R^3}a$$

$$\omega = \sqrt{\frac{2\sqrt{2}Q^2}{\pi\varepsilon_0 m R^3}}$$

So the correct answer is **Option (C)**

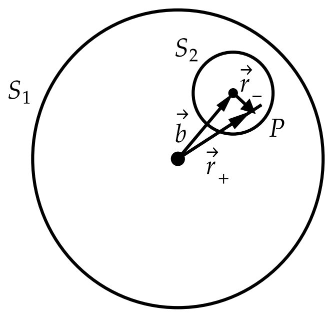
9. Consider a sphere S_1 of radius R which carries a uniform charge of density ρ. A smaller sphere S_2 of radius $a < \frac{R}{2}$ is cut out and removed from it. The centres of the two spheres are separated by the vector $\vec{b} = \frac{\hat{n}R}{2}$, as shown in the figure. The electric field at a point P inside S_2 is

[NET/JRF(JUNE-2016)]



A. $\frac{\rho R}{3\varepsilon_0}\hat{n}$ **B.** $\frac{\rho R}{3\varepsilon_0 a}(\vec{r} - \hat{n}a)$ **C.** $\frac{\rho R}{6\varepsilon_0}\hat{n}$ **D.** $\frac{\rho a}{3\varepsilon_0 R}\vec{r}$

Solution:



Electric field at P due to S_1 is $\vec{E}_1 = \frac{\rho}{3\varepsilon_0}\vec{r}_+$

Electric field at P due to S_2 (assume $-\rho$) is $\vec{E}_2 = \frac{-\rho}{3\varepsilon_0}\vec{r}_-$

$$\text{Thus } \vec{E} = \vec{E}_1 + \vec{E}_2 = \frac{\rho}{3\varepsilon_0}\left(\vec{r}_+ - \vec{r}_-\right);$$
$$\because \vec{b} + \vec{r}_- = \vec{r}_+ \Rightarrow \vec{r}_+ - \vec{r} = \vec{b}$$
$$\vec{E} = \frac{\rho}{3\varepsilon_0}\vec{b} = \frac{\rho R}{6\varepsilon_0}\hat{n}\left(\because \vec{b} = \frac{R}{2}\hat{n}\right)$$

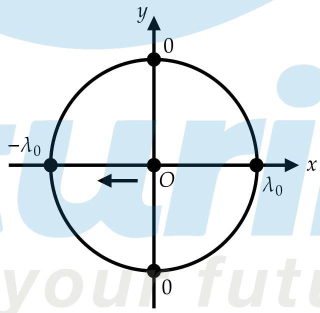
So the correct answer is **Option (C)**

10. The charge per unit length of a circular wire of radius a in the xy-plane, with its centre at the origin, is $\lambda = \lambda_0 \cos\theta$, where λ_0 is a constant and the angle θ is measured from the positive x-axis. The electric field at the centre of the circle is

[NET/JRF(DEC-2016)]

A. $\vec{E} = -\frac{\lambda_0}{4\epsilon_0\alpha}\hat{i}$ **B.** $\vec{E} = \frac{\lambda_0}{4\epsilon_0\alpha}\hat{i}$ **C.** $\vec{E} = -\frac{\lambda_0}{4\epsilon_0\alpha}\hat{j}$ **D.** $\vec{E} = \frac{\lambda_0}{4\pi\epsilon_0\alpha}\hat{k}$

Solution:



Electric field due to a charged element at P is

$$E = -(dE\cos\theta)\hat{i} - (dE\sin\theta)\hat{j}$$

So, the total electric field at the centre is

$$\vec{E} = -\hat{i}\int dE\cos\theta - \hat{j}\int dE\sin\theta$$
$$\vec{E} = -i\int\frac{\lambda d\ell}{4\pi\varepsilon_0 a^2}\cos\theta - j\int\frac{\lambda d\ell}{4\pi\varepsilon_0 a^2}\sin\theta$$
$$= -\frac{\hat{i}\lambda_0}{4\pi\varepsilon_0 a^2}a\int_0^{2\pi}\cos^2\theta d\theta - \frac{\hat{j}\lambda_0}{4\pi\varepsilon_0 a^2}a\int_0^{2\pi}\cos\theta\sin\theta d\theta$$
$$= -\frac{\hat{i}\lambda_0}{4\pi\varepsilon_0 a}\int_0^{2\pi}\cos^2\theta d\theta = -\hat{i}\frac{\lambda_0}{4\pi\varepsilon_0 a}\pi = -\frac{\lambda_0}{4\varepsilon_0 a}i$$

At centre O, direction of field is $-\hat{x}$.
So the correct answer is **Option (A)**