1. Condensed Matter Physics

Practice set-1

1. flux quantum (fluxoid) is approximately equal to 2×10^{-7} gauss-cm². A type II superconductor is placed in a small magnetic field, which is then slowly increased till the field starts penetrating the superconductor. The strength of the field at this point is $\frac{2}{\pi} \times 10^5$ gauss.

A. The penetrating depth of this superconductor is

[NET/JRF(JUNE-2011)]

A. $100\ \text{A}^0$ **B.** $10\ \overset{0}{\text{A}}$ **C.** $1000\ \overset{0}{\text{A}}$ **D.** $314\ \overset{0}{\text{A}}$

Solution:

$$\text{Given Fluxoid } (\phi)_0 = 2 \times 10^{-7} \text{ gauss } - \text{cm}^2$$

$$\text{First Critical field } (H_{c1}) = \frac{2}{\pi} \times 10^5 \text{ gauss}$$

The relation between first critical field and penetration depth is

$$H_{c1} = \frac{\phi_0}{\pi\lambda^2} \therefore \lambda^2 = \frac{\phi_0}{\pi H_{c1}} = \frac{2 \times 10^{-7}}{\pi \times \frac{2}{\pi} \times 10^5}$$

$$= 10^{-12}\ \text{cm}^2 \Rightarrow \lambda = 10^{-6}\ \text{cm} = 100\ \overset{0}{\text{A}}$$

So the correct answer is **Option (A)**

B. The applied field is further increased till superconductivity is completely destroyed. The strength of the field is now $\frac{8}{\pi} \times 10^5$ gauss. The correlation length of the superconductor is

A. $20\ \overset{0}{\text{A}}$ **B.** $200\ \overset{0}{\text{A}}$ **C.** $628\ \overset{0}{\text{A}}$ **D.** $2000\ \overset{0}{\text{A}}$

Solution:

Given second critical field $(H_{c2}) = \frac{8}{\pi} \times 10^5$ gauss. The relation between second critical field and correlation length is $H_{c2} = \frac{\phi_0}{\pi\varepsilon^2}$.

$$\therefore \varepsilon^2 = \frac{\phi_0}{\pi H_{c2}} = \frac{2 \times 10^{-7}}{\pi \times \frac{8}{\pi} \times 10^5} = \frac{1}{4} \times 10^{-12} \text{ cm}^2 \Rightarrow \varepsilon$$
$$= \frac{1}{2} \times 10^{-6} \text{ cm} = \frac{100}{2} \times 10^{-10} \text{ m} = 50\text{Å}$$

None of the options is matched.

2. The potential of a diatomic molecule as a function of the distance r between the atoms is given by $V(r) = -\frac{a}{r^6} + \frac{b}{r^{12}}$. The value of the potential at equilibrium separation between the atoms is:

[NET/JRF(DEC-2011)]

A. $-4a^2/b$ **B.** $-2a^2/b$ **C.** $-a^2/2b$ **D.** $-a^2/4b$

Solution:

$$\text{Given } V(r) = -\frac{a}{r^6} + \frac{b}{r^{12}}.$$

$$\text{At equilibrium radius, } \left.\frac{dV(r)}{dr}\right|_{r=r_0} = 0$$

$$\therefore \frac{dV(r)}{dr} = +\frac{6a}{r_0^7} - \frac{12b}{r_0^{13}} = 0 \Rightarrow \frac{r_0^{13}}{r_0^7}$$
$$= \frac{12b}{6a} = \frac{2b}{a} \Rightarrow r_0^6 = \frac{2b}{a}$$

$$\therefore \text{ The value of potential at equilibrium is } V(r_0) = -\frac{a}{r_0^6} + \frac{b}{r_0^{12}} = -\frac{a^2}{2b} + \frac{a^2}{4b} = \frac{-a^2}{4b}$$

So the correct answer is **Option (D)**

3. If the number density of a free electron gas in three dimensions is increased eight times, its Fermi temperature will

[NET/JRF(DEC-2011)]

A. Increase by a factor of 4 **B.** Decrease by a factor of 4

C. Increase by a factor of 8 **D.** Decrease by a factor of 8

Solution:

The relation between Fermi energy and electron density is $E_F = \frac{\hbar^2}{2m}\left(3\pi^2 n\right)^{2/3}$.

$$\Rightarrow E_F' = \frac{\hbar^2}{2m}\left(3\pi^2 \times 8n\right)^{2/3} = 4E_F \Rightarrow T_F' = \frac{4E_F}{E_F} T_F = 4T_F$$

So the correct answer is **Option (A)**

4. The excitations of a three-dimensional solid are bosonic in nature with their frequency ω and wave-number k are related by $\omega \propto k^2$ in the large wavelength limit. If the chemical potential is zero, the behaviour of the specific heat of the system at low temperature is proportional to

[NET/JRF(DEC-2011)]

A. $T^{1/2}$ **B.** T **C.** $T^{3/2}$ **D.** T^3

Solution:
Solution: If the dispersion relation is $\omega \propto k^s$ in large wavelength. Then the specific heat is $C_v \propto T^{3/s}$.
Given $\omega \propto k^2 \therefore C_v \propto T^{3/2}$
So the correct answer is **Option (C)**

5. Consider a system of non-interacting particles in d dimensional obeying the dispersion relation $\varepsilon = Ak^s$, where ε is the energy, k is the wavevector, s is an integer and A is constant. The density of states, $N(\varepsilon)$, is proportional to

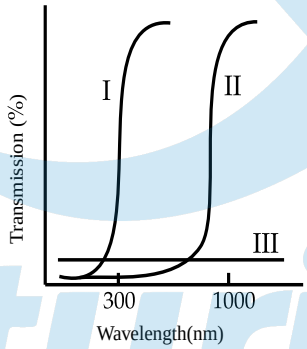
[NET/JRF(JUNE-2012)]

A. $\varepsilon^{\frac{s}{d}-1}$ **B.** $\varepsilon^{\frac{d}{s}-1}$ **C.** $\varepsilon^{\frac{d}{s}+1}$ **D.** $\varepsilon^{\frac{s}{d}+1}$

Solution: So the correct answer is **Option (B)**

6. The experimentally measured transmission spectra of metal, insulator and semiconductor thin films are shown in the figure. It can be inferred that I, II and III correspond, respectively, to respectively, to

[NET/JRF(JUNE-2012)]



A. Insulator, semiconductor and metal **B.** Semiconductor, metal and insulator
C. Metal, semiconductor and insulator **D.** Insulator, metal and semiconductor

Solution: So the correct answer is **Option (A)**

7. The dispersion relation of phonons in a solid is given by

$$\omega^2(k) = \omega_0^2 (3 - \cos k_x a - \cos k_y a - \cos k_z a)$$

The velocity of the phonons at large wavelength is

[NET/JRF(JUNE-2012)]

A. $\omega_0 a/\sqrt{3}$ **B.** $\omega_0 a$ **C.** $\sqrt{3}\omega_0 a$ **D.** $\omega_0 a/\sqrt{2}$

Solution:

For large $\lambda, (k_x a, k_y a, k_z a)$ are small.

$$\omega^2(k) = \omega_0^2 \left[3 - \left(1 - \frac{k_x^2 a^2}{2}\right) - \left(1 - \frac{k_y^2 a^2}{2}\right) - \left(1 - \frac{k_z^2 a^2}{2}\right)\right]$$
$$= \frac{\omega_0^2 a^2}{2} (k_x^2 + k_y^2 + k_z^2)$$

$$\omega^2(k) = \frac{\omega_0^2 a^2}{2} k^2 \Rightarrow \omega = \frac{\omega_0 a}{\sqrt{2}} k \Rightarrow v_g = \frac{d\omega}{dk} = \frac{\omega_0 a}{\sqrt{2}}$$

So the correct answer is **Option (D)**

8. A magnetic field sensor based on the Hall Effect is to be fabricated by implanting As into a Si film of thickness 1μm. The specifications require a magnetic field sensitivity of 500mV/ Tesla at an excitation current of 1 mA. The implantation dose is to be adjusted such that the average carrier density, after activation, is

[NET/JRF(DEC-2012)]

A. 1.25×10^{26} m^{-3} **B.** 1.25×10^{22} m^{-3}
C. 4.1×10^{21} m^{-3} **D.** 4.1×10^{20} m^{-3}

Solution:

$$n = \frac{IB}{teV_H} = \frac{10^{-3}}{10^{-6} \times 1.6 \times 10^{-19}} \times \frac{1}{500 \times 10^{-3}}$$
$$= 1.25 \times 10^{22} m^{-3} \text{ where } \frac{V_H}{B} = 500 \times 10^{-3} V/T$$

So the correct answer is **Option (B)**

9. In a band structure calculation, the dispersion relation for electrons is found to be

$$\varepsilon_k = \beta \left(\cos k_x a + \cos k_y a + \cos k_z a\right)$$

where β is a constant and a is the lattice constant. The effective mass at the boundary of the first Brillouin zone is

[NET/JRF(DEC-2012)]

A. $\frac{2\hbar^2}{5\beta a^2}$ **B.** $\frac{4\hbar^2}{5\beta a^2}$ **C.** $\frac{\hbar^2}{2\beta a^2}$ **D.** $\frac{\hbar^2}{3\beta a^2}$

Solution:

$$\varepsilon_k = \beta \left(\cos k_x a + \cos k_y a + \cos k_z a\right),$$

$$\text{Effective mass } m^* = \frac{\hbar^2}{\left(\frac{d^2\varepsilon_k}{d^2k}\right)}$$

Brilliouin zone boundary is at $k_x = \pm\frac{\pi}{a}, k_y = \pm\frac{\pi}{a}, k_z = \pm\frac{\pi}{a}$

$$\text{Hence } \left(\frac{d^2\varepsilon_k}{d^2k}\right)\Big|_{\frac{\pi}{a},\frac{\pi}{a},\frac{\pi}{a}} = 3\beta a^2 \Rightarrow m^* = \frac{\hbar^2}{3\beta a^2}$$

So the correct answer is **Option (D)**

10. The radius of the Fermi sphere of free electrons in a monovalent metal with an fcc structure, in which the volume of the unit cell is a^3, is

[NET/JRF(DEC-2012)]

A. $\left(\frac{12\pi^2}{a^3}\right)^{1/3}$ **B.** $\left(\frac{3\pi^2}{a^3}\right)^{1/3}$ **C.** $\left(\frac{\pi^2}{a^3}\right)^{1/3}$ **D.** $\frac{1}{a}$

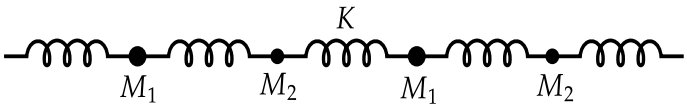
Solution:

$$\text{Radius of Fermi sphere is } k_F = \left(\frac{3\pi^2 N}{V}\right)^{1/3}, E_F = \left(\frac{\hbar^2}{2m}\right)\left(3\pi^2 n\right)^{2/3} = \left(\frac{\hbar^2 k_F^2}{2m}\right)$$

For fcc solid $\frac{N}{V}=\frac{4}{a^3}\Rightarrow k_F=\left(\frac{12\pi^2}{a^3}\right)^{1/3}$

So the correct answer is **Option (A)**

11. The phonon dispersion for the following one-dimensional diatomic lattice with masses M_1 and M_2 (as shown in the figure)



is given by

$$\omega^2(q)=K\left(\frac{1}{M_1}+\frac{1}{M_2}\right)\left[1\pm\sqrt{1-\frac{4M_1M_2}{(M_1+M_2)^2}\sin^2\left(\frac{qa}{2}\right)}\right]$$

where a is the lattice parameter and K is the spring constant. The velocity of sound is

[NET/JRF(JUNE-2013)]

A. $\sqrt{\frac{K(M_1+M_2)}{2M_1M_2}}a$ **B.** $\sqrt{\frac{K}{2(M_1+M_2)}}a$ **C.** $\sqrt{\frac{K(M_1+M_2)}{M_1M_2}}a$ **D.** $\sqrt{\frac{KM_1M_2}{2(M_1+M_2)^3}}a$

Solution:

For small value of q (i.e. long wavelength approximation limit).

We have $\sin\left(\frac{qa}{2}\right)\approx\frac{qa}{2}$

$$\therefore\omega^2(q)=\mathrm{K}\left(\frac{1}{M_1}+\frac{1}{M_2}\right)\left[1\pm\sqrt{1-\frac{4M_1M_2}{(M_1+M_2)^2}\sin^2\left(\frac{qa}{2}\right)}\right]$$

$$\Rightarrow\omega^2(q)=\mathrm{K}\left(\frac{1}{M_1}+\frac{1}{M_2}\right)\left[1\pm\sqrt{1-\frac{4M_1M_2}{(M_1+M_2)^2}\left(\frac{qa}{2}\right)^2}\right]$$

$$\Rightarrow\omega^2(q)=\mathrm{K}\left(\frac{1}{M_1}+\frac{1}{M_2}\right)\left[1\pm\left(1-\frac{1}{2}\times\frac{4M_1M_2}{(M_1+M_2)^2}\frac{q^2a^2}{4}\right)\right]$$

$$\Rightarrow\omega^2(q)=\mathrm{K}\left(\frac{1}{M_1}+\frac{1}{M_2}\right)\left[1\pm\left(1-\frac{M_1M_2}{(M_1+M_2)^2}\frac{q^2a^2}{2}\right)\right]$$

For Acoustical branch: $\omega^2(q)=\mathrm{K}\left(\frac{1}{M_1}+\frac{1}{M_2}\right)\left[1-\left(1-\frac{M_1M_2}{(M_1+M_2)^2}\frac{q^2a^2}{2}\right)\right]$

$$\Rightarrow\omega^2(q)=\mathrm{K}\left(\frac{M_1+M_2}{M_1M_2}\right)\left(\frac{M_1M_2}{(M_1+M_2)^2}\frac{q^2a^2}{2}\right)=\frac{\mathrm{K}a^2}{2(M_1+M_2)}q^2$$

$$\therefore\omega(q)=\sqrt{\frac{\mathrm{K}}{2(M_1+M_2)}}aq$$

Velocity of sound is $v_g=\frac{\omega}{q}=\sqrt{\frac{\mathrm{K}}{2(M_1+M_2)}}a$

So the correct answer is **Option (B)**

12. The electron dispersion relation for a one-dimensional metal is given by

$$\varepsilon_k = 2\varepsilon_0 \left[\sin^2 \frac{ka}{2} - \frac{1}{6}\sin^2 ka\right]$$

where k is the momentum, a is the lattice constant, ε_0 is a constant having dimensions of energy and $|ka| \leq \pi$. If the average number of electrons per atom in the conduction band is $1/3$, then the Fermi energy is

[NET/JRF(JUNE-2013)]

A. $\varepsilon_0/4$ **B.** ε_0 **C.** $2\varepsilon_0/3$ **D.** $5\varepsilon_0/3$

Solution: So the correct answer is **Option (A)**

13. If the energy dispersion of a two-dimensional electron system is $E = u\hbar k$ where u is the velocity and k is the momentum, then the density of states $D(E)$ depends on the energy as

[NET/JRF(JUNE-2013)]

A. $1/\sqrt{E}$ **B.** $\sqrt{E}$ **C.** E **D.** constant

Solution:

In two dimensional system, the number of allowed k-states in range k and $k+dk$ is

$$g(k)dk = \left(\frac{L}{2\pi}\right)^2 2\pi k dk$$

Given dispersion relation is $E = u\hbar k \therefore k = \dfrac{E}{u\hbar} \Rightarrow dk = \dfrac{dE}{u\hbar}$

$$\therefore g(E)dE = \left(\frac{L}{2\pi}\right)^2 2\pi \times \frac{E}{u\hbar} \times \frac{dE}{u\hbar} = \left(\frac{L}{2\pi}\right)^2 \frac{2\pi}{(u\hbar)^2} E dE$$

$$\Rightarrow \rho(E) = \frac{g(E)dE}{dE} = \frac{1}{(u\hbar)^2}\frac{L^2}{2\pi}E$$

So the correct answer is **Option (C)**

14. The energy of an electron in a band as a function of its wave vector k is given by $E(k) = E_0 - B(\cos k_x a + \cos k_y a + \cos k_z a)$, where E_0, B and a are constants. The effective mass of the electron near the bottom of the band is

[NET/JRF(DEC-2013)]

A. $\frac{2\hbar^2}{3Ba^2}$ **B.** $\frac{\hbar^2}{3Ba^2}$ **C.** $\frac{\hbar^2}{2Ba^2}$ **D.** $\frac{\hbar^2}{Ba^2}$

Solution:

Near the bottom of the band the $k \to 0$

$\cos k_x a \approx 1 - \frac{1}{2}(k_x a)^2, \cos k_y a \approx 1 - \frac{1}{2}(k_y a)^2, \cos k_z a \approx 1 - \frac{1}{2}(k_z a)^2$

$$E(k) = E_0 - B(\cos k_x a + \cos k_y a + \cos k_z a)$$
$$= E_0 - B\left(1 - \frac{1}{2}(k_x a)^2 + 1 - \frac{1}{2}(k_y a)^2 + 1 - \frac{1}{2}(k_z a)^2\right)$$

$$= E_0 - B\left(3 - \frac{1}{2}a^2\left(k_x + k_x + k_x\right)^2\right) = E_0 - 3B - \frac{1}{2}Ba^2k^2$$

Effective mass of the electron is $m^* = \dfrac{\hbar^2}{d^2E/dk^2} = \dfrac{\hbar^2}{Ba^2}$

So the correct answer is **Option (D)**

15. A uniform linear monoatomic chain is modeled by a spring-mass system of masses m separated by nearest neighbour distance a and spring constant $m\omega_0^2$. The dispersion relation for this system is

[NET/JRF(DEC-2013)]

A. $\omega(k) = 2\omega_0\left(1 - \cos\left(\frac{ka}{2}\right)\right)$ **B.** $\omega(k) = 2\omega_0 \sin^2\left(\frac{ka}{2}\right)$

C. $\omega(k) = 2\omega_0 \sin\left(\frac{ka}{2}\right)$ **D.** $\omega(k) = 2\omega_0 \tan\left(\frac{ka}{2}\right)$

Solution:

The dispersion relation for uniform linear mono-atomic chain of atoms is

$$\omega(k) = 2\omega_0 \sin\left(\frac{ka}{2}\right)$$

So the correct answer is **Option (C)**

16. The pressure of a nonrelativistic free Fermi gas in three-dimensions depends, at $T = 0$, on the density of fermions n as

[NET/JRF(JUNE-2014)]

A. $n^{5/3}$ **B.** $n^{1/3}$ **C.** $n^{2/3}$ **D.** $n^{4/3}$

Solution:

The Fermi energy in three dimension is defined as

$$E_F = \frac{\hbar^2}{2m}\left(\frac{3\pi^2 N}{V}\right)^{2/3} = \frac{\hbar^2}{2m}\left(3\pi^2 n\right)^{2/3}$$

Where, n is the electron concentration or density of free Fermi gas. The total energy of free Fermi gas in 3D is

$$E = \frac{3}{5}NE_F = \frac{3}{5}N \times \frac{\hbar^2}{2m}\left(\frac{3\pi^2 N}{V}\right)^{2/3}$$

The pressure of a nonrelativistic free Fermi gas is defined as

$$p_F = -\left(\frac{\partial E}{\partial V}\right)_N = -\frac{3}{5}N \times \frac{\hbar^2}{2m}\left(3\pi^2 N\right)^{2/3} \times \left(-\frac{2}{3}\right)V^{-5/3}$$
$$= \frac{2}{5}nE_F = \frac{2}{5}n \times \frac{\hbar^2}{2m}\left(3\pi^2 n\right)^{2/3} = \frac{2}{5}\frac{\hbar^2}{2m}\left(3\pi^2\right)^{2/3} n^{5/3}$$

So the correct answer is **Option (A)**

17. Consider an electron in bec lattice with lattice constant a. A single particle wavefunction that satisfies the Bloch theorem will have the form $f(\vec{r})\exp\left(\vec{ik}\cdot\vec{r}\right)$, with $f(\vec{r})$ being

[NET/JRF(JUNE-2014)]

A. $1+\cos\left[\frac{2\pi}{a}(x+y-z)\right]+\cos\left[\frac{2\pi}{a}(-x+y+z)\right]+\cos\left[\frac{2\pi}{a}(x-y+z)\right]$

B. $1+\cos\left[\frac{2\pi}{a}(x+y)\right]+\cos\left[\frac{2\pi}{a}(y+z)\right]+\cos\left[\frac{2\pi}{a}(z+x)\right]$

C. $1+\cos\left[\frac{\pi}{a}(x+y)\right]+\cos\left[\frac{\pi}{a}(y+z)\right]+\cos\left[\frac{\pi}{a}(z+x)\right]$

D. $1+\cos\left[\frac{\pi}{a}(x+y-z)\right]+\cos\left[\frac{\pi}{a}(-x+y+z)\right]+\cos\left[\frac{\pi}{a}(x-y+z)\right]$

Solution:

The primitive translational vector for BCC is

$$\vec{a}'=\frac{a}{2}(-\hat{i}+\hat{j}+\hat{k}),\vec{b}'=\frac{a}{2}(\hat{i}-\hat{j}+\hat{k}),\vec{c}'=\frac{a}{2}(\hat{i}+\hat{j}-\hat{k})$$

Bloch function defined as

$$\psi_k(\vec{r})=u_k(\vec{r})e^{i\vec{k}\cdot\vec{r}}=f(\vec{r})e^{i\vec{k}\vec{r}}$$

Here $f(\vec{r})$ is atomic wavefunction, which has the periodicity of the lattice i.e.

$$u_k(\vec{r}+a)=u_k(\vec{r})$$

Given Bloch function

$$f(\vec{r})=1+\cos\left[\frac{2\pi}{a}(x+y)\right]+\cos\left[\frac{2\pi}{a}(y+z)\right]+\cos\left[\frac{2\pi}{a}(z+x)\right]$$

$$f\left(\vec{r}+\vec{a}'\right)=1+\cos\left[\frac{2\pi}{a}\left(x+y-\frac{a}{2}+\frac{a}{2}\right)\right]+\cos\left[\frac{2\pi}{a}\left(y+z+\frac{a}{2}+\frac{a}{2}\right)\right]+\cos\left[\frac{2\pi}{a}\left(z+x+\frac{a}{2}-\frac{a}{2}\right)\right]$$

$$f\left(\vec{r}+\vec{a}'\right)=1+\cos\left[\frac{2\pi}{a}(x+y)\right]+\cos\left[\frac{2\pi}{a}(y+z)+2\pi\right]+\cos\left[\frac{2\pi}{a}(z+x)\right]$$

$$f\left(\vec{r}+\vec{a}'\right)=1+\cos\left[\frac{2\pi}{a}(x+y)\right]+\cos\left[\frac{2\pi}{a}(y+z)\right]+\cos\left[\frac{2\pi}{a}(z+x)\right]=f(\vec{r})$$

$$f\left(\vec{r}+\vec{a}'\right)=f(\vec{r})$$

Similarly,

$$f\left(\vec{r}+\vec{b}'\right)=f(\vec{r})\quad\text{and}\quad f\left(\vec{r}+\vec{c}'\right)=f(\vec{r})$$

Other functions do not satisfy the periodicity

So the correct answer is **Option (B)**

18. The dispersion relation for electrons in an f.c.c. crystal is given, in the tight binding approximation, by

$$\varepsilon(k)=-4\varepsilon_0\left[\cos\frac{k_xa}{2}\cos\frac{k_ya}{2}+\cos\frac{k_ya}{2}\cos\frac{k_za}{2}+\cos\frac{k_za}{2}\cos\frac{k_xa}{2}\right]$$

where a is the lattice constant and ε_0 is a constant with the dimension of energy. The x component of the velocity of the electron at $\left(\frac{\pi}{a},0,0\right)$ is

[NET/JRF(JUNE-2014)]

A. $-2\varepsilon_0 a/\hbar$ **B.** $2\varepsilon_0 a/\hbar$ **C.** $-4\varepsilon_0 a/\hbar$ **D.** $4\varepsilon_0 a/\hbar$

Solution:

Group velocity of electron in dispersive medium is expressed as

$$\vec{v} = \frac{1}{\hbar}\frac{d\varepsilon}{dk} = \frac{1}{\hbar}\left[\frac{d\varepsilon}{dk_x}\hat{i} + \frac{d\varepsilon}{dk_y}\hat{j} + \frac{d\varepsilon}{dk_z}\hat{k}\right] = \vec{v}_x\hat{i} + \vec{v}_y\hat{j} + \vec{v}_z\hat{k}$$

$$\vec{v} = \frac{2\varepsilon_0 a}{\hbar}\left[\begin{array}{l}\left(\sin\frac{k_x a}{2}\cos\frac{k_y a}{2} + \cos\frac{k_z a}{2}\sin\frac{k_x a}{2}\right)\hat{i} + \left(\cos\frac{k_x a}{2}\sin\frac{k_y a}{2} + \sin\frac{k_y a}{2}\cos\frac{k_z a}{2}\right)\hat{j} + \\ \left(\sin\frac{k_z a}{2}\cos\frac{k_y a}{2} + \cos\frac{k_x a}{2}\sin\frac{k_z a}{2}\right)\hat{k}\end{array}\right]$$

At $\left(\frac{\pi}{a}, 0, 0\right)$

$$\vec{v} = \frac{2\varepsilon_0 a}{\hbar}\left[\left(\sin\frac{\pi}{2}\cos 0 + \cos 0\sin\frac{\pi}{2}\right)\hat{i} + \left(\cos\frac{\pi}{2}\sin 0 + \sin 0\cos 0\right)\hat{j} + \left(\cos 0\sin 0 + \sin 0\cos\frac{\pi}{2}\right)\hat{k}\right]$$

$$\vec{v} = \frac{4\varepsilon_0 a}{\hbar}[\hat{i} + 0\hat{j} + 0\hat{k}] = [0\hat{i} + 0\hat{j} + 0\hat{k}] = \vec{v}_x\hat{i} + \vec{v}_y\hat{j} + \vec{v}_z\hat{k}$$

$$\vec{v}_x = \frac{4\varepsilon_0 a}{\hbar}, \vec{v}_y = 0, \vec{v}_z = 0$$

The x - component of velocity is $v_x = \frac{4\varepsilon_0 a}{\hbar}$

So the correct answer is **Option (D)**

19. Consider two crystalline solids, one of which has a simple cubic structure, and the other has a tetragonal structure. The effective spring constant between atoms in the c-direction is half the effective spring constant between atoms in the a and b directions. At low temperatures, the behaviour of the lattice contribution to the specific heat will depend as a function of temperature T as

[NET/JRF(DEC-2014)]

A. T^2 for the tetragonal solid, but as T^3 for the simple cubic solid

B. T for the tetragonal solid, and as T^3 for the simple cubic solid

C. T for both solids

D. T^3 for both solids

Solution:

Solution: The specific heat of solid in three dimensions is proportional to T^3 and it is independent of crystal structure.

$$\begin{array}{llll} \text{In} & 3D & : & C_V \propto T^3 \\ \text{In} & 2D & : & C_V \propto T^2 \\ \text{In} & 1D & : & C_V \propto T \end{array}$$

So the correct answer is **Option (D)**

20. A superconducting ring carries a steady current in the presence of a magnetic field $\vec{B}$ normal to the plane of the ring. Identify the INCORRECT statement.

[NET/JRF(DEC-2014)]

A. The flux passing through the superconductor is quantized in units of hc/e,

B. The current and the magnetic field in the superconductor are time independent.

C. The current density $\vec{j}$ and $\vec{B}$ are related by the equation $\vec{\nabla}\times\vec{j}+\Lambda^2\vec{B}=0$, where Λ is a constant

D. The superconductor shows an energy gap which is proportional to the transition temperature of the superconductor

Solution:

The flux quantization in superconducting ring is $\phi = n\phi_o$

where $\phi_o = \dfrac{hc}{2e}$ in CGS units and $\phi_o = \dfrac{h}{2e}$ in MKS units.

So the correct answer is **Option (A)**

21. The critical magnetic fields of a superconductor at temperatures $4K$ and $8K$ are 11 mA/m and 5.5 mA/m respectively. The transition temperature is approximately

[NET/JRF(JUNE-2015)]

A. $8.4K$ **B.** 10.6 K **C.** 12.9 K **D.** 15.0 K

Solution:

The relation between critical field and critical temperature is

$$H_C(T) = H_0\left[1-\left(\frac{T}{T_C}\right)^2\right]$$

Let at $T = T_1, H_C(T_1), T = T_2, H_C(T) = H_C(T_2)$

Thus we get $H_C(T_1) = H_0\left[1-\left(\frac{T_1}{T_C}\right)^2\right], H_C(T_2) = H_0\left[1-\left(\frac{T_2}{T_C}\right)^2\right]$

$$\frac{H_C(T_1)}{H_C(T_2)} = \frac{1-\left(\frac{T_1}{T_C}\right)^2}{1-\left(\frac{T_2}{T_C}\right)^2} \Rightarrow T_C = \sqrt{\frac{\frac{H_C(T_1)}{H_C(T_2)}T_2^2 - T_1^2}{\frac{H_C(T_1)}{H_C(T_2)}-1}} \Rightarrow T_C$$

$$= \sqrt{\frac{2(8)^2-(4)^2}{2-1}} \approx 10.6$$

where $T_1 = 4k, T_2 = 8k, H_C(T_1) = 11$ mA/m and $H_C(T_2) = 5.5$ mA/m

So the correct answer is **Option (B)**

22. The low-energy electronic excitations in a two-dimensional sheet of grapheme is given by $E(\vec{k}) = \hbar vk$, where v is the velocity of the excitations. The density of states is proportional to

[NET/JRF(JUNE-2015)]

A. E **B.** $E^{\frac{3}{2}}$ **C.** $E^{\frac{1}{2}}$ **D.** E^2

Solution:

The number of k - states in range k and $k+dk$ in two dimension is

$$g(k)dk = \left(\frac{L}{2\pi}\right)^2 2\pi k dk$$

$$\therefore E = \hbar v k \Rightarrow dE = \hbar v dk \Rightarrow g(E)dE$$

$$= \left(\frac{L}{2\pi}\right)^2 2\pi \times \frac{E}{\hbar v} \times \frac{dE}{\hbar v} = \left(\frac{L}{2\pi}\right)^2 \frac{2\pi}{(\hbar v)^2} E dE$$

The density of state is

$$\rho(E) = \frac{g(E)dE}{dE} = \left(\frac{L}{2\pi}\right)^2 \frac{2\pi}{(\hbar v)^2} E \Rightarrow \rho(E) \propto E$$

So the correct answer is **Option (A)**

23. The dispersion relation of electrons in a 3 -dimensional lattice in the tight binding approximation is given by,

$$\varepsilon_k = \alpha \cos k_x a + \beta \cos k_y a + \gamma \cos k_z a$$

where a is the lattice constant and α, β, γ are constants with dimension of energy. The effective mass tensor at the corner of the first Brillouin zone $\left(\frac{\pi}{a}, \frac{\pi}{a}, \frac{\pi}{a}\right)$ is

[NET/JRF(DEC-2015)]

A. $\frac{\hbar^2}{a^2}\begin{pmatrix} -1/\alpha & 0 & 0 \\ 0 & -1/\beta & 0 \\ 0 & 0 & 1/\gamma \end{pmatrix}$

B. $\frac{\hbar^2}{a^2}\begin{pmatrix} -1/\alpha & 0 & 0 \\ 0 & -1/\beta & 0 \\ 0 & 0 & -1/\gamma \end{pmatrix}$

C. $\frac{\hbar^2}{a^2}\begin{pmatrix} 1/\alpha & 0 & 0 \\ 0 & 1/\beta & 0 \\ 0 & 0 & 1/\gamma \end{pmatrix}$

D. $\frac{\hbar^2}{a^2}\begin{pmatrix} 1/\alpha & 0 & 0 \\ 0 & 1/\beta & 0 \\ 0 & 0 & -1/\gamma \end{pmatrix}$

Solution:

The effective mass as a tensor quantity can be written as

$$m''_{ij} = \begin{bmatrix} m^*_{xx} & m^*_{xy} & m^*_{xz} \\ m^*_{yx} & m^*_{yy} & m^*_{yz} \\ m^*_{zx} & m^*_{zy} & m^*_{zz} \end{bmatrix} \text{ where } m^*_{ij} = \frac{\hbar^2}{\left(\frac{\partial^2 E}{\partial k_i \partial k_j}\right)}$$

$$\text{since } \varepsilon_k = \alpha \cos k_x a + \beta \cos k_y a + \gamma \cos k_z a$$

$$\therefore m^*_{xx} = \frac{\hbar^2}{\left(\frac{\partial^2 \varepsilon}{\partial k_x \partial k_x}\right)} = \frac{-\hbar^2}{\alpha a^2 \cos k_x a}, \quad m^*_{yy} = \frac{\hbar^2}{\left(\frac{\partial^2 \varepsilon}{\partial k_y^2}\right)} = \frac{-\hbar^2}{\beta a^2 \cos k_y a}$$

$$m^*_{zz} = \frac{\hbar^2}{\left(\frac{\partial^2 \varepsilon}{\partial k_z^2}\right)} = \frac{-\hbar^2}{\gamma a^2 \cos k_z a}, \text{ other terms are zero}$$

$$\text{Now, at } \left(\frac{\pi}{a}, \frac{\pi}{a}, \frac{\pi}{a}\right); m_{xx} = \frac{\hbar^2}{\alpha a^2}, m^*_{yy} = \frac{\hbar^2}{\beta a^2}, m^*_{zz}$$

$$= \frac{\hbar^2}{\gamma a^2} \Rightarrow m^*_{ij} = \frac{\hbar^2}{a^2}\begin{bmatrix} 1/\alpha & 0 & 0 \\ 0 & 1/\beta & 0 \\ 0 & 0 & 1/\gamma \end{bmatrix}$$

So the correct answer is **Option (C)**

24. A thin metal film of dimension 2 mm $\times$ 2 mm contains 4×10^{12} electrons. The magnitude of the Fermi wavevector of the system, in the free electron approximation, is

[NET/JRF(DEC-2015)]

A. $2\sqrt{\pi} \times 10^7$ cm^{-1} **B.** $\sqrt{2\pi} \times 10^7$ cm^{-1}

C. $\sqrt{\pi} \times 10^7$ cm^{-1} **D.** $2\pi \times 10^7$ cm^{-1}

Solution:

This is the case of two dimensional metal box. The Fermi wave vector of electron in $2-D$ is

$$\begin{aligned} k_F = (2\pi n)^{\frac{1}{2}} &= \left(2\pi \frac{N}{L^2}\right)^{\frac{1}{2}}; L^2 = 2mm \times 2mm \\ &= 4 \times 10^{-2} \text{ cm}^2 \\ \Rightarrow k_F &= \sqrt{2\pi}\left(\frac{4 \times 10^{12}}{4 \times 10^{-2} \text{ cm}^2}\right)^{\frac{1}{2}} = \sqrt{2\pi}\left(10^{14} \text{ cm}^{-2}\right)^{\frac{1}{2}} \\ &= \sqrt{2\pi} \times 10^7 \text{ cm}^{-1} \end{aligned}$$

So the correct answer is **Option (B)**

25. For an electron moving through a one-dimensional periodic lattice of periodicity a, which of the following corresponds to an energy eigenfunction consistent with Bloch's theorem?

[NET/JRF(DEC-2015)]

A. $\psi(x) = A\exp\left(i\left[\frac{\pi x}{a} + \cos\left(\frac{\pi x}{2a}\right)\right]\right)$ **B.** $\psi(x) = A\exp\left(i\left[\frac{\pi x}{a} + \cos\left(\frac{2\pi x}{a}\right)\right]\right)$

C. $\psi(x) = A\exp\left(i\left[\frac{2\pi x}{a} + i\cosh\left(\frac{2\pi x}{a}\right)\right]\right)$ **D.** $\psi(x) = A\exp\left(i\left[\frac{\pi x}{a} + i\left|\frac{\pi x}{2a}\right|\right]\right)$

Solution:

According to block theorem, $\psi(x+a) = \psi(x)$

$$\begin{aligned} \psi(x+a) &= A\exp\left\{i\left[\frac{\pi}{a}(x+a) + \cos\left(\frac{2\pi}{a}(x+a)\right)\right]\right\} \\ &= A\exp\left\{i\left[\left(\frac{\pi x}{a} + \pi\right) + \cos\left(\frac{2\pi x}{a} + 2\pi\right)\right]\right\} \\ &= A\exp\left[i\left\{\frac{\pi}{a}(x+a) + \cos\frac{2\pi x}{a}\right\}\right] \\ &= A\exp\left[i\left(\frac{\pi x}{a} + \cos\frac{2\pi x}{a}\right)\right] \end{aligned}$$

So the correct answer is **Option (B)**

26. The band energy of an electron in a crystal for a particular k-direction has the form $\varepsilon(k) = A - B\cos 2ka$, where A and B are positive constants and $0 < ka < \pi$. The electron has a hole-like behaviour over the following range of k :

[NET/JRF(JUNE-2016)]

A. $\frac{\pi}{4} < ka < \frac{3\pi}{4}$ **B.** $\frac{\pi}{2} < ka < \pi$

C. $0 < ka < \frac{\pi}{4}$ **D.** $\frac{\pi}{2} < ka < \frac{3\pi}{4}$

Solution:

$$\varepsilon(k) = A - B\cos 2ka, \frac{d\varepsilon}{dk} = 2Ba\sin 2ka, \frac{d^2\varepsilon}{dk^2} = 4Ba^2\cos 2ka$$

$$\text{Effective mass } m^* = \frac{\hbar^2}{d^2\varepsilon/dk^2} = \frac{\hbar^2}{4Ba^2\cos 2ka}$$

Effective mass of electron (m_e^*) and effective mass of holes (m_h^*) are opposite in sign i.e.,

$$(m_h' = -m_e^*)$$

Now, in the range $0 < ka < \frac{\pi}{4}$, m^* is positive

While in the range $\frac{\pi}{4} < ka < \frac{3\pi}{4}$, m^* is negative

Thus, electron has hole like behaviour in the region $\frac{\pi}{4} < ka < \frac{3\pi}{4}$

So the correct answer is **Option (A)**

27. Consider a one-dimensional chain of atoms with lattice constant a. The energy of an electron with wave-vector k is $\varepsilon(k) = \mu - \gamma\cos(ka)$, where μ and γ are constants. If an electric field E is applied in the positive x-direction, the time dependent velocity of an electron is
(In the following B is the constant)

[NET/JRF(DEC-2016)]

A. Proportional to $\cos\left(B - \frac{eE}{\hbar}at\right)$

B. Proportional to E

C. Independent of E

D. Proportional to $\sin\left(B - \frac{eE}{\hbar}\text{ at }\right)$

Solution:

In the presence of electric field E , we can write

$$\vec{F} = -e\vec{E} \Rightarrow \frac{d\vec{p}}{dt} = -e\vec{E} \Rightarrow \hbar\frac{dk}{dt} = -eE$$

$$\text{Integration gives, } k(t) = k(0) - \frac{eE}{\hbar}t$$

$$\text{The group velocity } v = \frac{d\omega}{dk} = \frac{1}{\hbar}\frac{\partial\varepsilon(k)}{dk}$$

$$\text{Since, } \varepsilon(k) = \mu - \gamma\cos(ka), \qquad \therefore \quad \frac{\partial\varepsilon(k)}{\partial k} = \gamma a\sin ka$$

$$\text{Thus, } v = \frac{\gamma a}{\hbar}\sin(ka)$$

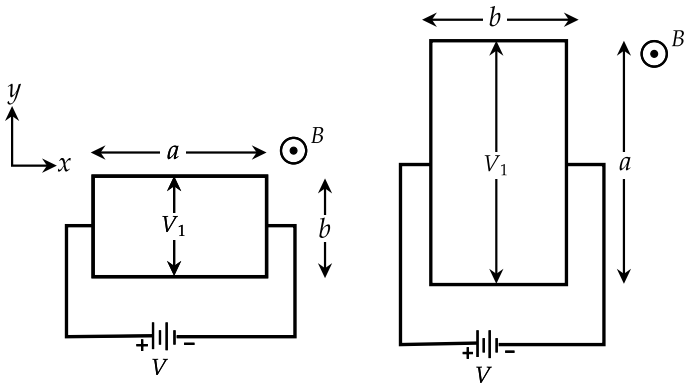
Time dependent velocity of electron is

$$v(t) = \frac{\gamma a}{\hbar}\sin[k(t)a] = \frac{\gamma a}{\hbar}\sin\left[\left(k(0) - \frac{eE}{\hbar}t\right)a\right]$$

$$= \frac{\gamma a}{\hbar}\sin\left[k(0)a - \frac{eE}{\hbar}at\right] \Rightarrow v(t) = \frac{\gamma a}{\hbar}\sin\left[B - \frac{eE}{\hbar}at\right]$$

So the correct answer is **Option (D)**

28. A thin rectangular conducting plate of length a and width b is placed in the xy-plane in two different orientations as shown in the figures below. In both cases a magnetic field B is applied in the z-direction and a current flows in the x direction due to the applied voltage V.



If the Hall voltage across the y-direction in the two cases satisfy $V_2 = 2V_1$ the ratio $a : b$ must be

[NET/JRF(DEC-2016)]

A. $1 : 2$ **B.** $1 : \sqrt{2}$ **C.** $2 : 1$ **D.** $\sqrt{2} : 1$

Solution:

Since, Hall voltage is given by $V_H = \frac{IB}{\rho w}$, where w is width of conducting plate.

$$\text{Since, in case (I), } V = I_1 R_1 \text{ and } R_1 = \frac{\rho l_1}{A_1} = \frac{\rho a}{a \times b} = \frac{\rho}{b}$$

$$V = \frac{I_1 \rho}{b} \Rightarrow I_1 = \frac{bV}{\rho}$$

$$\text{Then, } V_H = V_1 = \frac{I_1 B}{\rho w} = \frac{bVB}{\rho^2 w} = \frac{bVB}{\rho^2 a} \qquad (\because w = a)$$

$$\text{And also in case (II), } R_2 = \frac{\rho l_2}{A_2} = \frac{\rho b}{a \times b} = \frac{\rho}{a}$$

$$V = I_2 R_2 \Rightarrow I_2 = \frac{V}{R_2} = \frac{Va}{\rho}$$

$$\text{Then, } V_H = V_2 = \frac{I_2 B}{\rho w} = \frac{VaB}{\rho^2 b}$$

$$\text{Since, } V_2 = 2V_1 \Rightarrow \frac{a^2}{b^2} = \frac{2}{1} \Rightarrow a : b = \sqrt{2} : 1$$

So the correct answer is **Option (D)**

29. The electrical conductivity of copper is approximately 95% of the electrical conductivity of silver, while the electron density in silver is approximately 70% of the electron density in copper. In Drude's model, the approximate ratio τ_{Cu}/τ_{Ag} of the mean collision time in copper (τ_{Cu}) to the mean collision time in silver (τ_{Ag}) is

[NET/JRF(JUNE-2017)]

A. 0.44 **B.** 1.50 **C.** 0.33 **D.** 0.66

Solution:

$$\sigma = \frac{ne^2\tau}{m} \Rightarrow \frac{\sigma_{cu}}{\sigma_{Ag}} = \frac{n_{cu}}{n_{Ag}}\frac{\tau_{cu}}{\tau_{Ag}} \Rightarrow \frac{\tau_{cu}}{\tau_{Ag}} = \frac{\sigma_{cu}}{\sigma_{Ag}} \times \frac{n_{Ag}}{n_{cu}}$$

$$\Rightarrow \frac{\tau_{cu}}{\tau_{Ag}} = \frac{0.95\sigma_{Ag}}{\sigma_{Ag}} \times \frac{0.7n_{cu}}{n_{cu}} \approx 0.66$$

So the correct answer is **Option (D)**

30. The dispersion relation of a gas of spin $\frac{1}{2}$ fermions in two dimensions is $E = \hbar v|\vec{k}|$, where E is the energy, $\vec{k}$ is the wave vector and v is a constant with the dimension of velocity. If the Fermi energy at zero temperature is $\in_F$, the number of particles per unit area is

[NET/JRF(DEC-2017)]

A. $\frac{\varepsilon_F}{(4\pi v\hbar)}$ **B.** $\frac{\varepsilon_F^3}{(6\pi^2 v^3\hbar^3)}$ **C.** $\frac{\pi\in_F^{3/2}}{(3v^3\hbar^3)}$ **D.** $\frac{\varepsilon_F^2}{(2\pi v^2\hbar^2)}$

Solution:

$$E = \hbar v|\vec{k}| \Rightarrow k = \frac{E}{\hbar v} \Rightarrow dk = \frac{dE}{\hbar v}$$

$$g(E)dE = 2\left(\frac{L}{2\pi}\right)^2 .2\pi \cdot \frac{E}{\hbar v} \cdot \frac{dE}{\hbar v} = \left(\frac{L}{2\pi}\right)^2 \cdot \frac{4\pi}{(\hbar v)^2} E \cdot dE$$

at $T = 0K$

$$N = \int_0^{E_F} g(E)dE = \frac{L^2}{4\pi^2}\frac{4\pi}{(\hbar v)^2} \cdot \frac{\varepsilon_F^2}{2}$$

$$N = \frac{L^2}{2\pi\hbar^2 v^2}\varepsilon_F^2$$

$$n = \frac{N}{L^2} = \frac{\varepsilon_F^2}{2\pi v^2\hbar^2}$$

So the correct answer is **Option (D)**

31. The dispersion relation for the electrons in the conduction band of a semiconductor is given by $E = E_0 + \alpha k^2$ where α and E_0 are constants. If ω_c is the cyclotron resonance frequency of the conduction band electrons in a magnetic field B, the value of α is

[NET/JRF(JUNE-2018)]

A. $\frac{\hbar\omega_c}{4eB}$ **B.** $\frac{2\hbar^2\omega_c}{eB}$ **C.** $\frac{\hbar^2\omega_c}{eB}$ **D.** $\frac{\hbar^2\omega_c}{2eB}$

Solution:

$$\omega_c = \frac{eB}{m^*} \text{ where } m^* = \frac{\hbar^2}{d^2E/dk^2}$$

$$\text{Since } E = E_0 + \alpha k^2 \Rightarrow \frac{d^2E}{dk^2} = 2\alpha$$

$$\Rightarrow \omega_c = \frac{eB}{\hbar^2/2\alpha} = \frac{2\alpha}{\hbar^2}eB \Rightarrow \alpha = \frac{\hbar^2\omega_c}{2eB}$$

So the correct answer is **Option (D)**

Answer key			
Q.No.	Answer	Q.No.	Answer
1	**A**	2	**D**
3	**A**	4	**C**
5	**B**	6	**A**
7	**D**	8	**B**
9	**D**	10	**A**
11	**B**	12	**A**
13	**C**	14	**D**
15	**C**	16	**A**
17	**B**	18	**D**
19	**D**	20	**A**
21	**B**	22	**A**
23	**C**	24	**B**
25	**B**	26	**A**
27	**D**	28	**D**
29	**D**	30	**D**
31	**D**		

Practice set-2

1. The valence electrons do not directly determine the following property of a metal

[GATE 2010]

A. Electrical conductivity

B. Thermal conductivity

C. Shear modulus

D. Metallic luster

Solution: So the correct answer is **Option (C)**

2. The Hall coefficient, R_H, of sodium depends on

[GATE 2010]

A. The effective charge carrier mass and carrier density

B. The charge carrier density and relaxation time

C. The charge carrier density only

D. The effective charge carrier mass

Solution: So the correct answer is **Option (C)**

3. The Bloch theorem states that within a crystal, the wavefunction, $\psi(\vec{r})$, of an electron has the form

[GATE 2010]

A. $\psi(\vec{r}) = u(\vec{r})e^{i\vec{k}\cdot\vec{r}}$ where u$(\vec{r})$ is an arbitrary function and $\vec{k}$ is an arbitrary vector

B. $\psi(\vec{r}) = u(\vec{r})e^{i\vec{G}\cdot\vec{r}}$ where u$(\vec{r})$ is an arbitrary function and $\vec{G}$ is a reciprocal lattice vector

C. $\psi(\vec{r}) = u(\vec{r})e^{i\vec{G}\cdot\vec{r}}$ where $u(\vec{r}) = u(\vec{r}+\vec{\Lambda}), \vec{\Lambda}$ is a lattice vector and $\vec{G}$ is a reciprocal lattice vector

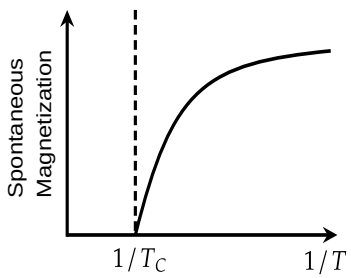
D. $\psi(\vec{r}) = u(\vec{r})e^{ik,r}$ where $u(\vec{r}) = u(\vec{r}+\vec{\Lambda}), \vec{\Lambda}$ is a lattice vector and $\vec{k}$ is an arbitrary vector

Solution: So the correct answer is **Option (D)**

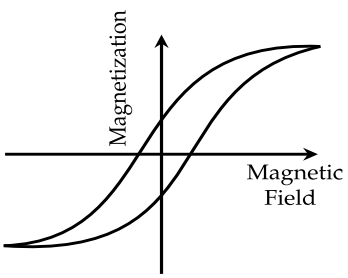
4. In an experiment involving a ferromagnetic medium, the following observations were made. Which one of the plots does NOT correctly represent the property of the medium? (T_C is the Curie temperature)

[GATE 2010]

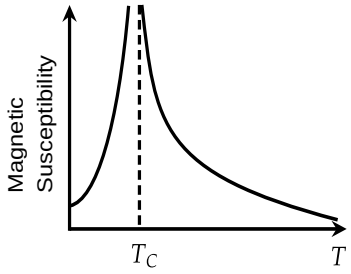
A.



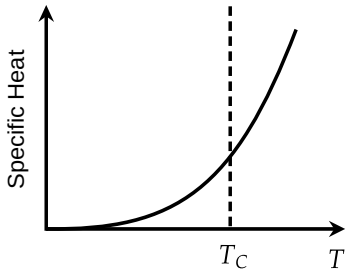
B.



C.



D.



Solution: So the correct answer is **Option (C)**

5. For a two-dimensional free electron gas, the electronic density n, and the Fermi energy E_F, are related by **[GATE 2010]**

A. $n = \frac{(2mE_F)^{3/2}}{3\pi^2\hbar^3}$ **B.** $n = \frac{mE_F}{\pi\hbar^2}$ **C.** $n = \frac{mE_F}{2\pi\hbar^2}$ **D.** $n = \frac{2^{1/3}(mE_F)^{1/3}}{\pi\hbar}$

Solution:

For two dimensional gas, the number of possible k-states between k and $k+dk$ is

$$g(k)dk = \left(\frac{L}{2\pi}\right)^2 2\pi k dk = 2\left(\frac{L}{2\pi}\right)^2 2\pi k dk \text{ it is multiplied by 2 for electron gas}$$

$$\text{Since } k^2 = \frac{2mE}{\hbar^2} \because 2kdk$$

$$= \frac{2m}{\hbar^2}dE \Rightarrow 2\pi k dk = \frac{2\pi m}{\hbar^2}dE$$

$$\therefore g(E)dE = 2\left(\frac{L}{2\pi}\right)^2 \cdot \frac{2\pi m}{\hbar^2}dE$$

The total number of electrons at $T = 0°\text{K}$ is

$$N = \int_0^{E_F} g(E)dE \times F(E) = \int_0^{E_F} g(E)dE$$

$$= 2\pi \cdot \frac{2m}{\hbar^2}\left(\frac{L}{2\pi}\right)^{2E_F} dE = 2\pi \cdot \frac{2m}{\hbar^2} \cdot \frac{L^2}{4\pi^2} \cdot E_F$$

$$\Rightarrow N = \frac{m}{\pi\hbar^2} \cdot L^2 E_F \Rightarrow E_F = \frac{\pi\hbar^2}{m}\left(\frac{N}{L^2}\right)$$

$$= \frac{\pi\hbar^2}{m} \cdot n \Rightarrow n = \frac{mE_F}{\pi\hbar^2}$$

So the correct answer is **Option (B)**

6. The temperature (T) dependence of magnetic susceptibility (χ) of a ferromagnetic substance with a Curie temperature (T_c) is given by **[GATE 2011]**

A. $\frac{C}{T-T_c}$, for $T < T_c$

B. $\frac{C}{T-T_c}$, for $T > T_c$

C. $\frac{C}{T+T_c}$, for $T > T_c$

D. $\frac{C}{T+T_c}$, for all temperatures where C is constant.

Solution: So the correct answer is **Option (B)**

Common Data for Questions 14 and 15:
The tight binding energy dispersion $(E-k)$ relation for electrons in a one-dimensional array of atoms having lattice constant a and total length L is

$$E = E_0 - \beta - 2\gamma\cos(ka)$$

where E_0, β and γ are constants and k is the wave vector.

7. The density of states of electrons (including spin degeneracy) in the band is given by

[GATE 2011]

A. $\frac{L}{\pi\gamma a\sin(ka)}$ **B.** $\frac{L}{2\pi\gamma a\sin(ka)}$ **C.** $\frac{L}{2\pi\gamma a\cos(ka)}$ **D.** $\frac{L}{\pi\gamma a\cos(ka)}$

Solution:

$$\begin{aligned} D(E) &= 2\times 2\left(\frac{L}{2\pi}\right)\cdot\frac{1}{dE/dk} \\ &= 2\left(\frac{L}{2\pi}\right)\cdot\frac{2\times 1}{2a\gamma\sin(ka)} \\ &= \frac{2\times L}{2\pi\gamma a\sin(ka)} \end{aligned}$$

So the correct answer is **Option (A)**

8. The effective mass of electrons in the band is given by

[GATE 2011]

A. (a) $\frac{\hbar^2}{\gamma a^2\cos(ka)}$ **B.** $\frac{\hbar^2}{2\gamma a^2\cos(ka)}$ **C.** $\frac{\hbar^2}{\gamma a^2\sin(ka)}$ **D.** $\frac{\hbar^2}{2\gamma a^2\sin(ka)}$

Solution:

$$\begin{aligned} \text{Effective mass } m^\circ &= \frac{\hbar^2}{\left(\frac{d^2E}{dk^2}\right)} = \frac{\hbar^2}{2a^2\gamma\cos(ka)} \\ &= \frac{\hbar^2}{2\gamma a^2\cos(ka)} \end{aligned}$$

So the correct answer is **Option (B)**

9. Which one of the following CANNOT be explained by considering a harmonic approximation for the lattice vibrations in solids?

[GATE 2012]

A. Deby's T^3 law **B.** Dulong Petit's law

C. Optical branches in lattices **D.** Thermal expansion

Solution: Solution: Thermal expansion in solid can only be explained if solid behave as a anharmonic oscillator.

So the correct answer is **Option (D)**

10. The group velocity at the boundary of the first Brillouin zone is

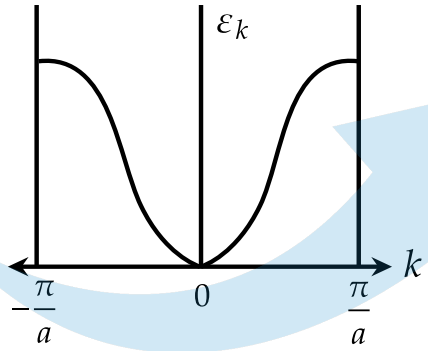
[GATE 2012]

A. 0 **B.** 1 **C.** $\sqrt{\frac{Aa^2}{2}}$ **D.** $\frac{1}{2}\sqrt{\frac{Aa^2}{2}}$

Solution: Solution: At the first Brillouin zone the frequency is maximum and the group velocity which is the derivative of the angular frequency is zero.

So the correct answer is **Option (A)**

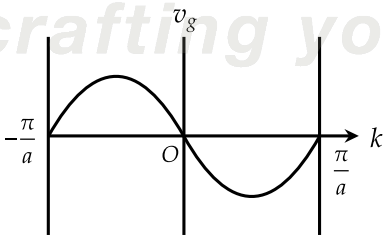
11. The energy ε_k for band electrons as a function of the wave vector k in the first Brillouin zone $\left(-\frac{\pi}{a} \le k \le \frac{\pi}{a}\right)$ of a one dimensional monoatomic lattice is shown as (a is lattice constant)



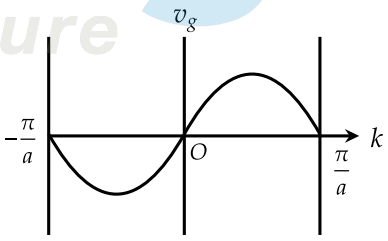
The variation of the group velocity v_g is most appropriately represented by

[GATE 2014]

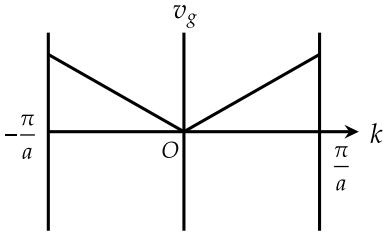
A.



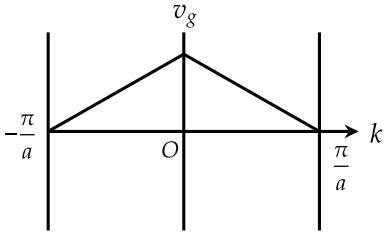
B.



C.



D.



Solution:

$$E = (E_0 - \gamma\beta(\cos ka))$$

$$V_g = \frac{1}{\hbar}\frac{dE}{dk} = \frac{a\gamma\beta}{\hbar}\sin ka$$

So the correct answer is **Option (B)**

12. The energy dependence of the density of states for a two dimensional non-relativistic electron gas is given by, $g(E) = CE^n$, where C is constant. The value of n is————————

[GATE 2015]

Solution:

We know that

$g(E) \propto E^{1/2}$ for $3-D$, $\quad g(E) \propto E^0$ for $2-D, g(E) \propto E^{-1/2}$ for $1-D$

$\Rightarrow n = 0$ for $2-D$

So the correct answer is 0

13. The dispersion relation for phonons in a one dimensional monoatomic Bravais lattice with lattice spacing a and consisting of ions of masses M is given by $\omega(k) = \sqrt{\frac{2c}{M}[1-\cos(ka)]}$, where ω is the frequency of oscillation, k is the wavevector and C is the spring constant. For the long wavelength modes ($\lambda >> a$), the ratio of the phase velocity to the group velocity is————————

[GATE 2015]

Solution:

$$\omega(k) = \sqrt{\frac{2C}{M}[1-\cos(ka)]}$$

For long wavelength modes ($\lambda >> a$)

$$\because \cos(ka) \cong 1 - \frac{(ka)^2}{2} \Rightarrow \omega(k) = \sqrt{\frac{2C}{M}\left[1-1+\frac{(ka)^2}{2}\right]} = a\sqrt{\frac{C}{M}}k$$

$$\text{Phase velocity } v_P = \frac{\omega}{k} = a\sqrt{\frac{C}{M}} \text{ and Group velocity}$$

$$v_g = \frac{d\omega}{dk} = a\sqrt{\frac{C}{M}} \Rightarrow \frac{v_P}{v_g} = 1$$

14. In a Hall effect experiment, the hall voltage for an intrinsic semiconductor is negative. This is because (symbols carry usual meaning)

[GATE 2015]

A. $n \approx p$ **B.** $n > p$ **C.** $\mu_n > \mu_p$ **D.** $m_p^* > m_n^*$

Solution:

The Hall voltage is $\quad V_H = R_H JB$

where J : current density, B : magnetic field and R_H : Hall constant

$$R_H = \frac{1}{e}\frac{p\mu_p^2 - n\mu_n^2 + (p-n)\mu_n^2\mu_p^2B^2}{(n\mu_n + p\mu_p)^2 + (p-n)^2\mu_n^2\mu_p^2B^2}$$

For intrinsic semiconductor $(n = p = n_i)$ $\quad R_H = \dfrac{1}{en_i}\dfrac{\mu_p - \mu_n}{\mu_p + \mu_n}$

In Intrinsic semiconductor $\mu_n > \mu_p$, therefore Hall voltage is negative.

So the correct answer is **Option (C)**

15. Consider a metal which obeys the Sommerfield model exactly. If E_F is the Fermi energy of the metal at $T = 0K$ and R_H is its Hall coefficient, which of the following statements is correct?

[GATE 2016]

A. $R_H \propto E_F^{\frac{3}{2}}$

B. $R_H \propto E_F^{\frac{2}{3}}$

C. $R_H \propto E_F^{\frac{-3}{2}}$

D. R_H is independent of E_F.

Solution:

$$R_H = \frac{1}{ne}, \quad \text{where } E_F = \frac{\hbar^2}{2m}\left(3\pi^2 n\right)^{2/3} \Rightarrow n$$

$$= \left(\frac{2m}{\hbar^2}\right)^{3/2} \cdot \frac{(E_F)^{3/2}}{3\pi^2} \Rightarrow R_H \propto E_F^{-3/2}$$

So the correct answer is **Option (C)**

16. A one-dimensional linear chain of atoms contains two types of atoms of masses m_1 and m_2 (where $m_2 > m_1$), arranged alternately. The distance between successive atoms is the same. Assume that the harmonic approximation is valid. At the first Brillouin zone boundary, which of the following statements is correct?

[GATE 2016]

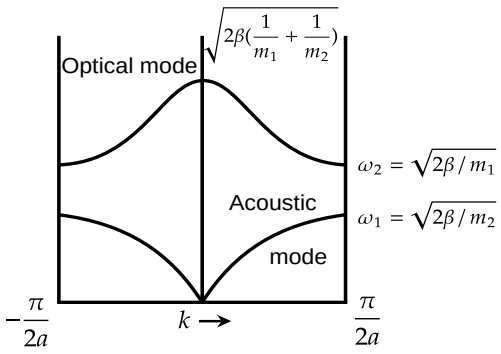
A. The atoms of mass m_2 are at rest in the optical mode, while they vibrate in the acoustical mode.

B. The atoms of mass m_1 are at rest in the optical mode, while they vibrate in the acoustical mode.

C. Both types of atoms vibrate with equal amplitudes in the optical as well as in the acoustical modes.

D. Both types of atoms vibrate, but with unequal, non-zero amplitudes in the optical as well as in the acoustical modes.

Solution: In optical mode, at Brillouin zone boundary atom of heavier mass (m_2) is at rest, whereas in Acoustic mode, atoms of lighter mass (m_1) is at rest.



So the correct answer is **Option (A)**

17. Consider a 2 - dimensional electron gas with a density of 10^{19} m^{-2}. The Fermi energy of the system is..... eV (up to two decimal places).
$\left(m_e = 9.31 \times 10^{-31}\text{ kg}, h = 6.626 \times 10^{-34}\text{Js}, \quad e = 1.602 \times 10^{-19}\text{C}\right)$

[GATE 2017]

Solution:

$$E_F = \left(\frac{\hbar^2}{2m}\right)(2\pi n) = \frac{\left(1.055 \times 10^{-34} J \cdot s\right)^2}{2 \times 9.31 \times 10^{-31}} \times 2 \times 3.142 \times 10^{19}$$
$$= 0.3756 \times 10^{-18}\text{ J} = 0.2345 \times 10\text{eV} = 2.34\text{eV}$$

18. At low temperatures (T), the specific heat of common metals is described by (with α and β as constants)

[GATE 2018]

A. $\alpha T + \beta T^3$ **B.** βT^3 **C.** $\exp(-\alpha/T)$ **D.** $\alpha T + \beta T^5$

Solution:

$$C = C_e + C_{pn} = \alpha T + \beta T^3$$

So the correct answer is **Option (A)**

19. The energy dispersion for electrons in one dimensional lattice with lattice parameter a is given by $E(k) = E_0 - \frac{1}{2}W\cos ka$, where W and E_0 are constants. The effective mass of the electron near the bottom of the band is

[GATE 2018]

A. $\frac{2\hbar^2}{Wa^2}$ **B.** $\frac{\hbar^2}{Wa^2}$ **C.** $\frac{\hbar^2}{2Wa^2}$ **D.** $\frac{\hbar^2}{4Wa^2}$

Solution:

$$E(k) = E_0 - \frac{1}{2}W\cos(ka)$$
$$\frac{dE}{dk} = \frac{aW}{2}\sin(ka) \Rightarrow \frac{d^2E}{dk^2} = \frac{a^2W}{2}\cos(ka)$$
$$\therefore m^* = \frac{\hbar^2}{\frac{d^2E}{dk^2}} = \frac{\hbar^2}{\frac{a^2W}{2}\cos(ka)} = \frac{2\hbar^2}{Wa^2}$$

[At bottom of the band, $k = 0$]

So the correct answer is **Option (A)**

20. In a certain two-dimensional lattice, the energy dispersion of the electrons is

$$\varepsilon(\vec{k}) = -2t\left[\cos k_x a + 2\cos\frac{1}{2}k_x a\cos\frac{\sqrt{3}}{2}k_y a\right]$$

where $\vec{k} = (k_x, k_y)$ denotes the wave vector, a is the lattice constant and t is a constant in units of eV. In this lattice the effective mass tensor m_{ij} of electrons calculated at the center of the Brillouin zone has the form $m_{ij} = \frac{\hbar^2}{ta^2}\begin{pmatrix}\alpha & 0\\ 0 & \alpha\end{pmatrix}$. The value of α (rounded off to two decimal places) is————————

[GATE 2019]

Solution:

Effective mass tensor matrix 4

$$m_{ij} = \begin{bmatrix}\frac{1}{m_{xx}} & \frac{1}{m_{xy}}\\ \frac{1}{m_{yx}} & \frac{1}{m_{yy}}\end{bmatrix} = \begin{bmatrix}\frac{1}{m_{xx}} & 0\\ 0 & \frac{1}{m_{yy}}\end{bmatrix}$$

$$\text{When } m_{xx} = \frac{\hbar^2}{\partial^2 E/\partial k_x^2} \text{ and } m_{yy} = \frac{\hbar^2}{\partial^2 E/\partial k_y^2}$$

$$\text{Now } \frac{\partial E}{\partial k_x} = 2t\left[a\sin k_x a + a\sin\left(\frac{1}{2}k_x a\right)\cos\left(\frac{\sqrt{3}}{2}k_y a\right)\right]$$

$$\frac{\partial^2 E}{dk_x^2} = 2t\left[a^2\cos(k_x a) + \frac{a^2}{2}\cos\left(\frac{1}{2}k_x a\right)\cos\left(\frac{\sqrt{3}}{2}k_y a\right)\right]$$

At the Brillouin zone centre i.e. at $k_x = k_y = 0$

$$\therefore \frac{\partial^2 E}{\partial k_x^2} = 2ta^2\left(1+\frac{1}{2}\right) = 3ta^2$$

$$\text{Similarly, } \frac{\partial E}{\partial k_y} = 2t\left[\sqrt{3}a\cos\left(\frac{1}{2}k_x a\right)\sin\left(\frac{\sqrt{3}}{2}k_y a\right)\right]$$

$$\frac{\partial^2 E}{\partial k_y^2} = 2t\left[\frac{3a^2}{2}\cos\left(\frac{1}{2}k_x a\right)\cos\left(\frac{\sqrt{3}}{2}k_y a\right)\right]$$

At the Brillouin zone centre i.e. at $k_x = k_y = 0$

$$\frac{\partial^2 E}{\partial_y^2} = 3ta^2$$

$$\text{Thus } m_{xx} = \frac{\hbar^2}{\partial^2 E/\partial k_x^2} = \frac{\hbar^2}{3ta^2} \text{ and } m_{yy}$$

$$= \frac{\hbar^2}{\partial^2 E/\partial k_y^2} = \frac{\hbar^2}{3ta^2}$$

$$m_{ij} = \begin{bmatrix}\frac{\hbar^2}{3ta^2} & 0\\ 0 & \frac{\hbar^2}{3ta^2}\end{bmatrix} = \frac{\hbar^2}{ta^2}\begin{bmatrix}\frac{1}{3} & 0\\ 0 & \frac{1}{3}\end{bmatrix}$$

$$\text{Thus } \alpha = \frac{1}{3} = 0.333$$

21. An ideal gas of non-relativistic fermions in 3-dimensions is at 0 K. When both the number density and mass of the particles are doubled, then the energy per particle is multiplied by a factor

[JEST 2014]

A. $2^{1/2}$ **B.** 1 **C.** $2^{1/3}$ **D.** $2^{-1/3}$

Solution:

$$E_F = \frac{\hbar^2}{2m}\left(3\pi^2 n\right)^{\frac{2}{3}} \quad \text{at } T = 0K$$

$$\because n' = 2n \text{ and } m' = 2m \Rightarrow E_F' = \frac{\hbar^2}{4m}\left(3\pi^2 2n\right)^{2/3} = \frac{\hbar^2}{2m}\left(3\pi^2 n\right)^{2/3} \times 2^{-1/3}$$

So the correct answer is **Option (D)**

Answer key			
Q.No.	Answer	Q.No.	Answer
1	**C**	2	**C**
3	**D**	4	**C**
5	**B**	6	**B**
7	**A**	8	**B**
9	**D**	10	**A**
11	**B**	12	**0**
13	**1**	14	**C**
15	**C**	16	**A**
17	**2.34eV**	18	**A**
19	**A**	20	**0.333**
21	**D**		

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