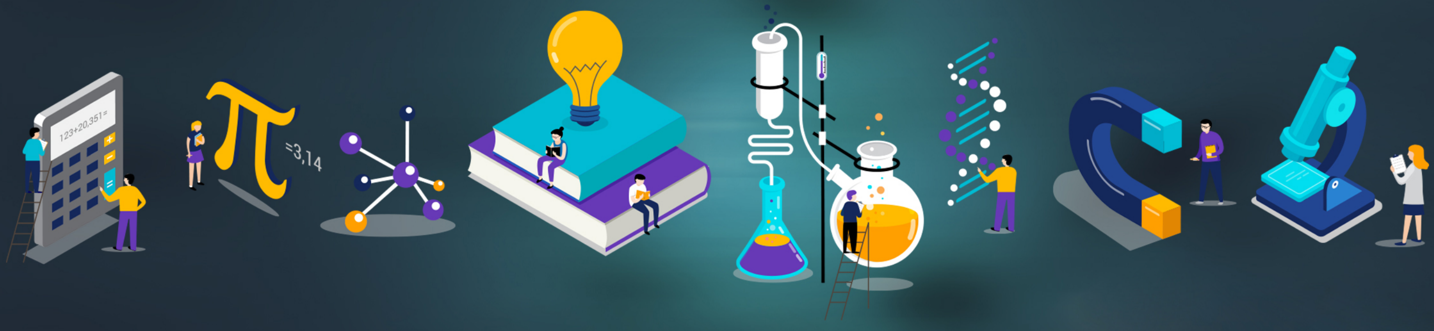


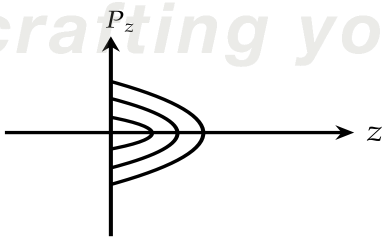
1. Dynamical System Solutions

Practice set- 1

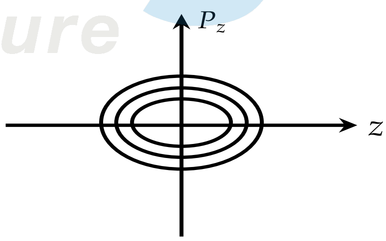
1. The trajectory on the zp_z - plane (phase-space trajectory) of a ball bouncing perfectly elastically off a hard surface at $z = 0$ is given by approximately by (neglect friction):

[NET JUNE 2011]

A.



B.



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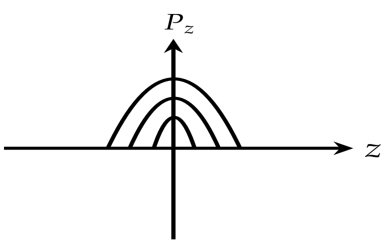
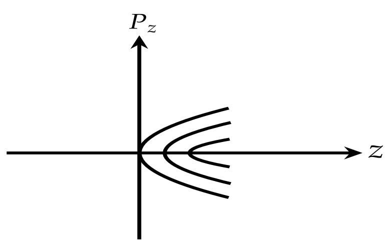


D.



Solution:

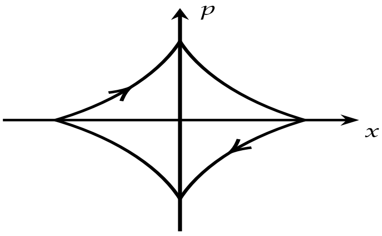
$$H = \frac{P_z^2}{2m} + mgz \text{ and } E = \frac{P_z^2}{2m} + mgz$$

The correct option is **(a)**

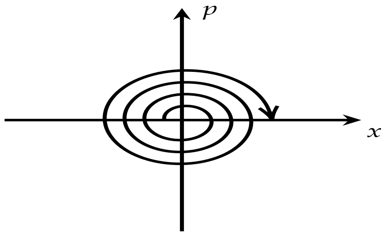
2. The bob of a simple pendulum, which undergoes small oscillations, is immersed in water. Which of the following figures best represents the phase space diagram for the pendulum?

[NET JUNE 2012]

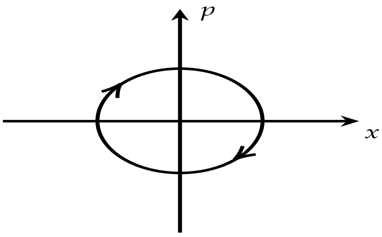
A.



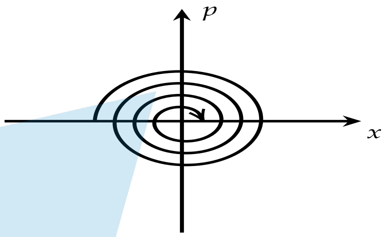
B.



C.



D.

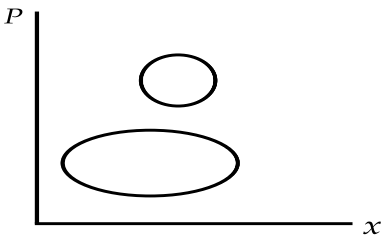


Solution: When simple pendulum oscillates in water it is damped oscillation so amplitude continuously decrease and finally it stops.
The correct option **(d)**

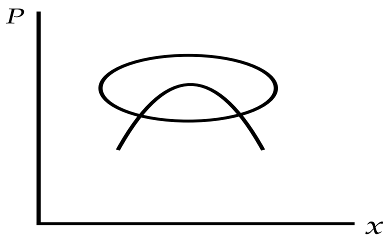
3. Which of the following set of phase-space trajectories is not possible for a particle obeying Hamilton's equations of motion?

[NET DEC 2012]

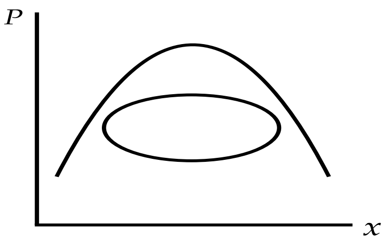
A.



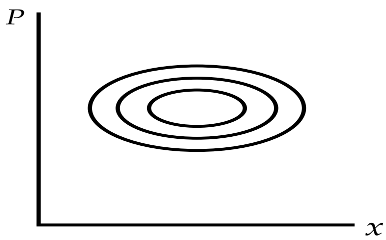
B.



C.



D.



Solution: Phase curve does not cut each other
The correct option is **(b)**

4. The Hamiltonian of a classical particle moving in one dimension is $H = \frac{p^2}{2m} + \alpha q^4$ where α is a positive constant and p and q are its momentum and position respectively. Given that its total energy $E \le E_0$ the available volume of phase space depends on E_0 as

[NET DEC 2014]

A. $E_0^{3/4}$

B. E_0

C. $\sqrt{E_0}$

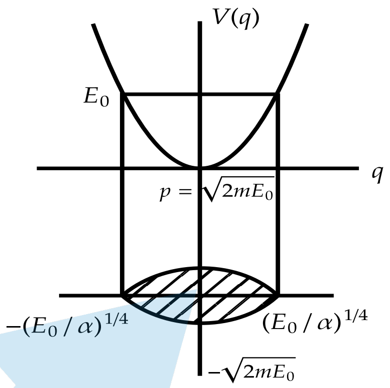
D. is independent of E_0

Solution:

$H = \frac{p^2}{2m} + \alpha q^4$ Phase area $= \oint p \cdot dq$

$$A = \oint p \cdot dq = \pi\sqrt{2mE} \times \left(\frac{E}{\alpha}\right)^{\frac{1}{4}}$$

$$A \propto E_0^{1/2} \cdot E_0^{1/4} \Rightarrow A \propto E_0^{3/4}$$

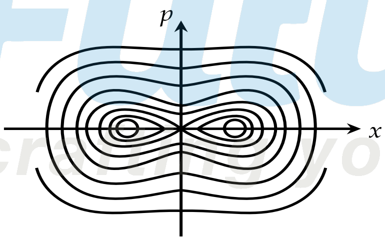


The correct option is **(a)**

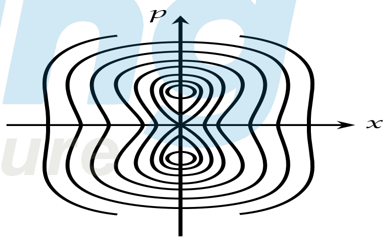
5. Which of the following figures is a schematic representation of the phase space trajectories (i.e., contours of constant energy) of a particle moving in a one-dimensional potential $V(x) = \frac{-1}{2}x^2 + \frac{1}{4}x^4$

[NET JUNE 2015]

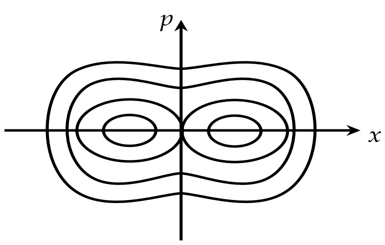
A.



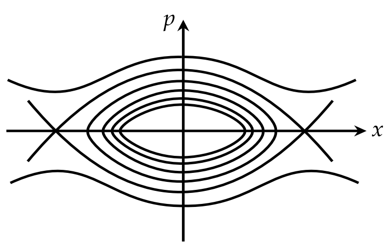
B.



C.



D.



Solution:

$V(x) = \frac{-x^2}{2} + \frac{x^4}{4}$

$$\frac{\partial V}{\partial x} = 0 \Rightarrow x = 0, x = \pm 1$$

$$\frac{\partial^2 V}{\partial x^2} = -ve \text{ for } x = 0 \text{ (unstable point)}$$

$$= + \text{ ve for } x = \pm 1 \text{ (stable point)}$$

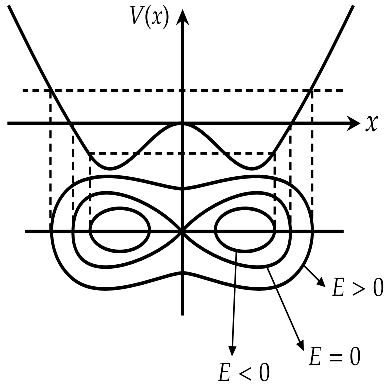


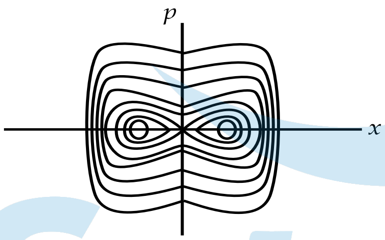
Figure 1.1

The correct option is **(a)**

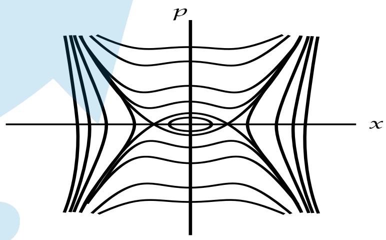
6. A particle moves in one dimension in a potential $V(x) = -k^2x^4 + \omega^2x^2$ where k and ω are constants. Which of the following curves best describes the trajectories of this system in phase space?

[NET DEC 2017]

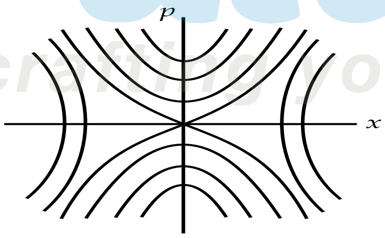
A.



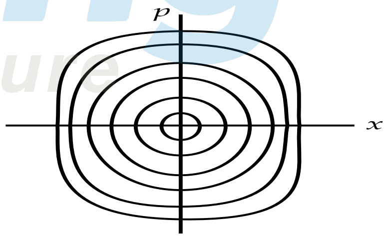
B.



C.



D.



Solution:

$V(x) = -k^2x^4 + \omega^2x^2$

For equation point

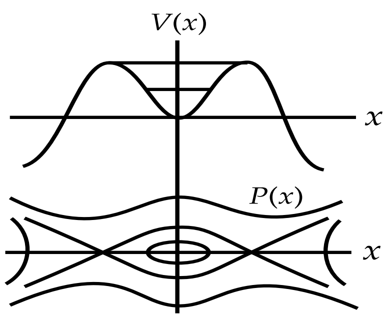
$\frac{\partial V}{\partial x} = 0 \Rightarrow -4k^2x^3 + 2\omega^2x = 0, x = 0$ or $x^2 = \frac{\omega^2}{2k^2}$

Now, $\frac{d^2V}{dx^2} = -12k^2x^2 + 2\omega^2$ At, $x = 0$

$\frac{d^2V}{dx^2} = 2\omega^2, x = 0$ is minimum.

And, $\frac{d^2V}{dx^2} = -12k^2\frac{\omega^2}{2k^2} + 2\omega^2 = -4\omega^2$, at $x^2 = \frac{\omega^2}{2k^2}$

Hence, $x = \pm\sqrt{\frac{\omega^2}{2k^2}}$ is maxima.



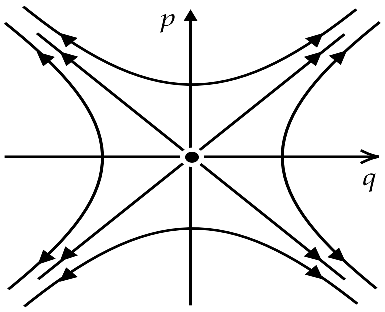
The correct option is **(c)**

Practice set -2

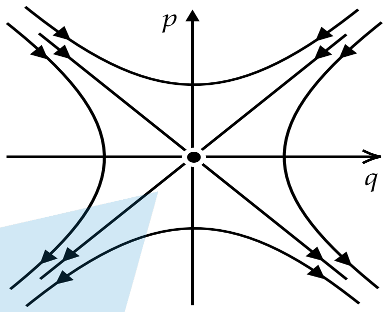
1. The Hamiltonian of particle of mass m is given by $H = \frac{p^2}{2m} - \frac{\alpha q^2}{2}$. Which one of the following figure describes the motion of the particle in phase space?

[GATE 2014]

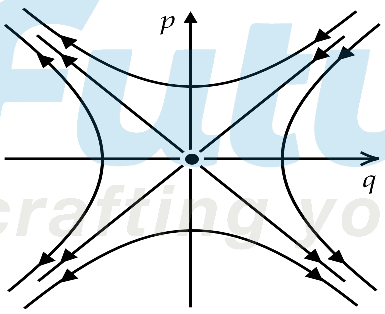
A.



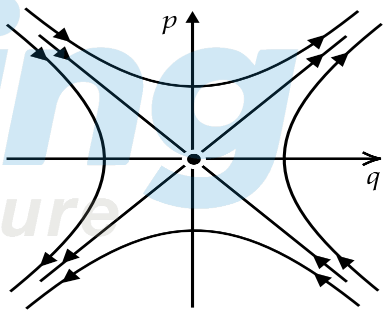
B.



C.



D.

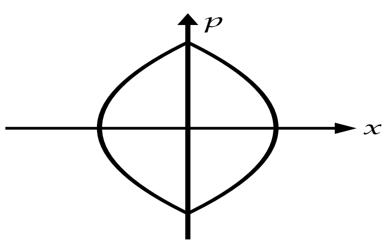


Solution: The corrrect option is **(d)**

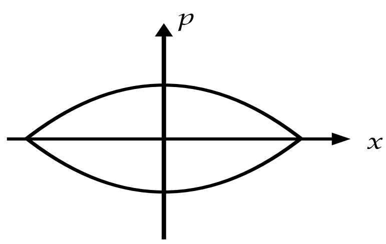
2. A particle moves in one dimension under a potential $V(x) = \alpha|x|$ with some non-zero total energy. Which one of the following best describes the particle trajectory in the phase space?

[GATE 2018]

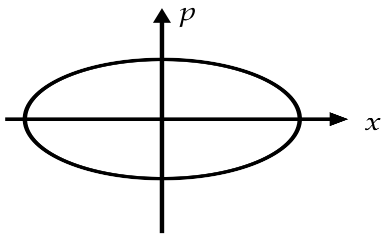
A.



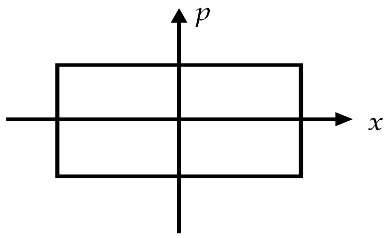
B.



C.

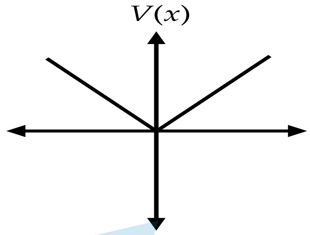


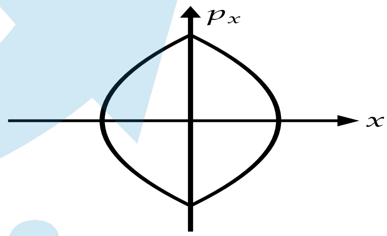
D.



Solution:

$E=\frac{p^2}{2m}+\alpha|x|$ For $x>0, E=\frac{p^2}{2m}+\alpha x \Rightarrow p^2=2m(E-\alpha x)$ For $x<0, E=\frac{p^2}{2m}-\alpha x \Rightarrow p^2=2m(E+\alpha x)$





The correct option is **(a)**

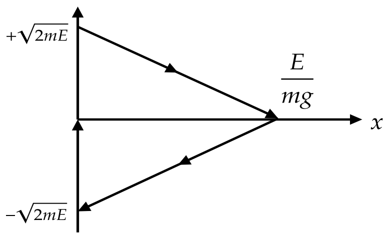
3. A ball bouncing of a rigid floor is described by the potential energy function

$$V(x)=\begin{cases} mgx & \text{for} \quad x>0 \\ \infty & \text{for} \quad x\leq 0 \end{cases}$$

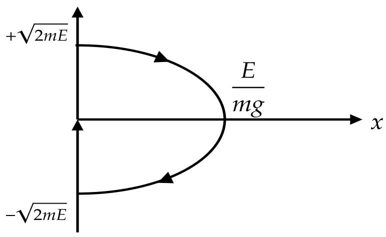
Which of the following schematic diagrams best represents the phase space plot of the ball?

[GATE 2019]

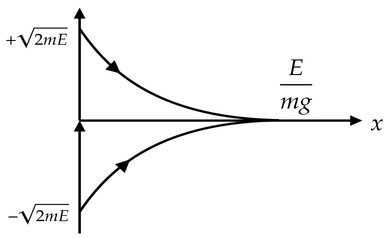
A.



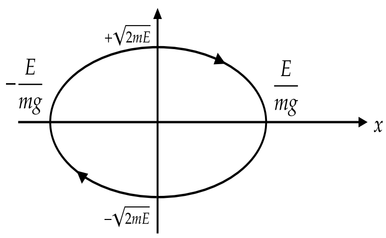
B.



C.



D.



Solution: $E = \frac{p^2}{2m} + mgx \Rightarrow p^2 = 2m(E - mgx)$ which is equation of parabola
The correct option is **(b)**

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