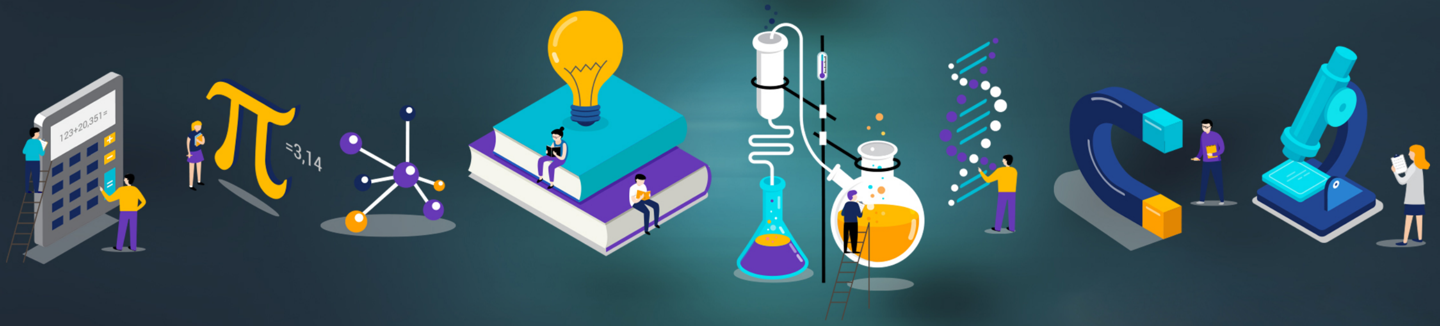

1. Mathematical Tools For QM - Solutions

Practice Set-1

1. Consider a particle in a one dimensional potential that satisfies $V(x) = V(-x)$. Let $|\psi_0\rangle$ and $|\psi_1\rangle$ denote the ground and the first excited states, respectively, and let $|\psi\rangle = \alpha_0|\psi_0\rangle + \alpha_1|\psi_1\rangle$ be a normalized state with α_0 and α_1 being real constants. The expectation value $\langle x\rangle$ of the position operator x in the state $|\psi\rangle$ is given by

[NET DEC 2011]

A. $\alpha_0^2\langle\psi_0|x|\psi_0\rangle + \alpha_1^2\langle\psi_1|x|\psi_1\rangle$ **B.** $\alpha_0\alpha_1[\langle\psi_0|x|\psi_1\rangle + \langle\psi_1|x|\psi_0\rangle]$

C. $\alpha_0^2 + \alpha_1^2$ **D.** $2\alpha_0\alpha_1$

Solution: Since $V(x) = V(-x)$ so potential is symmetric.

$$\langle\psi_0|x|\psi_0\rangle = 0, \langle\psi_1|x|\psi_1\rangle = 0$$
$$\langle\psi|x|\psi\rangle = (\alpha_0\langle\psi_0| + \alpha_1\langle\psi_1|) \times (\alpha_0|\psi_0\rangle + \alpha_1|\psi_1\rangle) = \alpha_0\alpha_1[\langle\psi_0|x|\psi_1\rangle + \langle\psi_1|x|\psi_0\rangle]$$

The correct option is **(b)**

2. The wave function of a particle at time $t = 0$ is given by $|\psi(0)\rangle = \frac{1}{\sqrt{2}}(|u_1\rangle + |u_2\rangle)$, where $|u_1\rangle$ and $|u_2\rangle$ are the normalized eigenstates with eigenvalues E_1 and E_2 respectively, $(E_2 > E_1)$. The shortest time after which $|\psi(t)\rangle$ will become orthogonal to $|\psi(0)\rangle$ is

[NET DEC 2011]

A. $\frac{-\hbar\pi}{2(E_2-E_1)}$ **B.** $\frac{\hbar\pi}{E_2-E_1}$

C. $\frac{\sqrt{2}\hbar\pi}{E_2-E_1}$ **D.** $\frac{2\hbar\pi}{E_2-E_1}$

Solution: $|\psi(0)\rangle = \frac{1}{\sqrt{2}}(|u_1\rangle + |u_2\rangle) \Rightarrow |\psi(t)\rangle = \frac{1}{\sqrt{2}}\left(|u_1\rangle e^{\frac{-iE_1t}{\hbar}} + |u_2\rangle e^{\frac{-iE_2t}{\hbar}}\right)$

$|\psi(t)\rangle$ is orthogonal to $|\psi(0)\rangle \Rightarrow \langle\psi(0) \mid \psi(t)\rangle = 0 \Rightarrow \frac{1}{2}e^{\frac{-iE_1t}{\hbar}} + \frac{1}{2}e^{\frac{-iE_2t}{\hbar}} = 0$

$$\Rightarrow e^{\frac{-iE_1t}{\hbar}} + e^{\frac{-iE_2t}{\hbar}} = 0 \Rightarrow e^{\frac{-iE_1t}{\hbar}} = -e^{\frac{-iE_2t}{\hbar}} \Rightarrow e^{i\frac{(E_2-E_1)}{\hbar}} = -1$$

$$\Rightarrow \cos\frac{(E_2-E_1)t}{\hbar} = \cos\pi \Rightarrow t = \frac{\pi\hbar}{E_2-E_1}$$

The correct option is **(b)**

3. The commutator $\left[x^2, p^2\right]$ is

[NET JUNE 2012]

A. $2i\hbar xp$
B. $2i\hbar(xp+px)$
C. $2i\hbar px$
D. $2i\hbar(xp-px)$

Solution: $\left[x^2, p^2\right] = x\left[x, p^2\right] + \left[x, p^2\right]x = xp[x,p] + x[x,p]p + p[x,p]x + [x,p]px$

$$\left[x^2, p^2\right] = xp(i\hbar) + x(i\hbar)p + p(i\hbar)x + (i\hbar)px = 2i\hbar(xp+px)$$

The correct option is **(b)**

4. Which of the following is a self-adjoint operator in the spherical polar coordinate system (r, θ, ϕ) ?

[NET JUNE 2012]

A. $-\frac{i\hbar}{\sin^2\theta}\frac{\partial}{\partial\theta}$
B. $-i\hbar\frac{\partial}{\partial\theta}$
C. $-\frac{i\hbar}{\sin\theta}\frac{\partial}{\partial\theta}$
D. $-i\hbar\sin\theta\frac{\partial}{\partial\theta}$

Solution: $\frac{-i\hbar}{\sin\theta}\frac{\partial}{\partial\theta}$ is Hermitian.
The correct option is **c**

5. Given the usual canonical commutation relations, the commutator $[A, B]$ of $A = i\left(xp_y - yp_x\right)$ and $B = \left(yp_z + zp_y\right)$ is

[NET DEC 2012]

A. $\hbar\left(xp_z - p_xz\right)$
B. $-\hbar\left(xp_z - p_xz\right)$
C. $\hbar\left(xp_z + p_xz\right)$
D. $-\hbar\left(xp_z + p_xz\right)$

Solution:

$$[A,B] = \lfloor(ixp_y - iyp_x), (yp_z + zp_y)]$$
$$[A,B] = i\left[xp_y, yp_z\right] - i\left[yp_x, yp_z\right] + i\left\lfloor xp_y, zp_y\right\rfloor - i\left\lfloor yp_x, zp_y\right\rfloor$$
$$[A,B] = i\left[xp_y, yp_z\right] - 0 + 0 - i \mid yp_x, zp_y\rfloor = i\left[xp_y, yp_z\right] - i \mid yp_x, zp_y\rfloor$$
$$[A,B] = ix\left[p_y, yp_z\right] + i\left[x, yp_z\right]p_y - iy \mid p_x, zp_y\rfloor - i|y, zp_y|\,p_x$$
$$[A,B] = ix\left[p_y, yp_z\right] + 0 - 0 - i\lfloor y, zp_y\rfloor p_x = ix \mid p_y, yp_z\rfloor - i\lfloor y, zp_y\rfloor p_x$$
$$[A,B] = ix \times (-i\hbar)p_z - izi\hbar \times p_x = \hbar\left[xp_z + zp_x\right]$$
$$[A,B] = \hbar\left(xp_z + p_xz\right)$$

The correct option is **(c)**

6. If the operators A and B satisfy the commutation relation $[A, B] = I$, where I is the identity operator, then

[NET JUNE 2013]

A. $\left[e^A, B\right] = e^A$
B. $\left[e^A, B\right] = \left[e^B, A\right]$
C. $\left[e^A, B\right] = \left[e^{-B}, A\right]$
D. $\left[e^A, B\right] = I$

Solution:

$$[A,B]=I \text{ and } e^A=\left[1+\frac{A}{1}+\frac{A^2}{\lfloor 2}+\ldots\ldots.\right]$$

$$\left[e^A,B\right]=\left[1+\frac{A}{1}+\frac{A^2}{L2}+\ldots\ldots.B\right]=[1,B]+[A,B]+\frac{\left[A^2,B\right]}{\lfloor 2}+\frac{\left[A^3,B\right]}{\lfloor 3}\ldots.$$

$$\left[e^A,B\right]=0+I+\frac{A[A,B]+[A,B]A}{2!}+\frac{A\left[A^2,B\right]+\left[A^2,B\right]A}{3!}+\ldots.$$

$$\left[e^A,B\right]=1+A+\frac{A^2}{2!}+\ldots.=e^A \text{ where } [A,B]=I,\left[A^2,B\right]=2A \text{ and } \left[A^3,B\right]=3A^2.$$

The correct option is **(a)**

7. Suppose Hamiltonian of a conservative system in classical mechanics is $H=\omega xp$, where ω is a constant and x and p are the position and momentum respectively. The corresponding Hamiltonian in quantum mechanics, in the coordinate representation, is

[NET DEC 2014]

A. $-i\hbar\omega\left(x\frac{\partial}{\partial x}-\frac{1}{2}\right)$

B. $-i\hbar\omega\left(x\frac{\partial}{\partial x}+\frac{1}{2}\right)$

C. $-i\hbar\omega x\frac{\partial}{\partial x}$

D. $-\frac{i\hbar\omega}{2}\times\frac{\partial}{\partial x}$

Solution: Classically $H=\omega xp$, quantum mechanically H must be Hermitian,
So, $H=\frac{\omega}{2}(xp+px)$ and $H\psi=\frac{\omega}{2}(xp\psi+px\psi)$

$$\Rightarrow H\psi=\frac{\omega}{2}\left(x(-i\hbar)\frac{\partial\psi}{\partial x}+\frac{-i\hbar\partial(x\psi)}{\partial x}\right)=\frac{\omega}{2}(-i\hbar)\left(x\frac{\partial\psi}{\partial x}+x\frac{\partial\psi}{\partial x}+\psi\right)$$

$$\Rightarrow H\psi=\frac{-i\hbar\omega}{2}\left(2x\frac{\partial\psi}{\partial x}+\psi\right)=-i\hbar\omega\left(x\frac{\partial}{\partial x}+\frac{1}{2}\right)\psi$$

The correct option is **(b)**

8. Let x and p denote, respectively, the coordinate and momentum operators satisfying the canonical commutation relation $[x,p]=i$ in natural units $(\hbar=1)$. Then the commutator $[x,pe^{-p}]$ is

[NET DEC 2014]

A. $i(1-p)e^{-p}$

B. $i\left(1-p^2\right)e^{-p}$

C. $i\left(1-e^{-p}\right)$

D. ipe $^{-p}$

Solution: $\because [x,p]=i$

$$\left[x,pe^{-p}\right]=[x,p]e^{-p}+p\left[x,e^{-p}\right]=ie^{-p}+p\left[x,1-p+\frac{p^2}{\lfloor 2}-\frac{p^3}{\lfloor 3}\ldots..\right]$$

$$=ie^{-p}+p\left[[x,1]-[x,p]+\left[x,\frac{p^2}{L2}\right]\ldots\right]=ie^{-p}+p\left[0-i+\frac{2ip}{\lfloor 2}-\frac{3ip^2}{\lfloor 3}\ldots\ldots\right]$$

$$\Rightarrow\left[x,pe^{-p}\right]=ie^{-p}-i\left[p-p^2+\frac{p^3}{\lfloor 2}\ldots.\right]=ie^{-p}-ipe^{-p}=i(1-p)e^{-p}$$

The correct option is **(a)**

9. The wavefunction of a particle in one-dimension is denoted by $\psi(x)$ in the coordinate representation and by $\phi(p)=\int\psi(x)e^{\frac{-ipx}{\hbar}}dx$ in the momentum representation. If the action of an operator $\hat{T}$ on $\psi(x)$ is given by $\hat{T}\psi(x)=\psi(x+a)$, where a is a constant then $\hat{T}\phi(p)$ is given by

[NET JUNE 2015]

A. $-\frac{i}{\hbar}\operatorname{ap}\phi(p)$ **B.** $e^{\frac{-iap}{\hbar}}\phi(p)$

C. $e^{\frac{+iap}{\hbar}}\phi(p)$ **D.** $\left(1+\frac{i}{\hbar}ap\right)\phi(p)$

Solution: $\phi(p)=\int\psi(x)e^{\frac{-ipx}{\hbar}}dx$

$$T\psi(x)=\psi(x+a)$$
$$T\phi(p)=\int T\psi(x)e^{\frac{-ipx}{\hbar}}dx=\int\psi(x+a)e^{\frac{-ipx}{\hbar}}dx=e^{\frac{ipa}{\hbar}}\int\psi(x+a)e^{\frac{-ip(x+a)}{\hbar}}dx$$
$$\Rightarrow T\phi(p)=e^{\frac{ipa}{\hbar}}\phi(p)$$

The correct option is **(c)**

10. Two different sets of orthogonal basis vectors $\left\{\begin{pmatrix}1\\0\end{pmatrix},\begin{pmatrix}0\\1\end{pmatrix}\right\}$ and $\left\{\frac{1}{\sqrt{2}}\begin{pmatrix}1\\1\end{pmatrix},\frac{1}{\sqrt{2}}\begin{pmatrix}1\\-1\end{pmatrix}\right\}$ are given for a two dimensional real vector space. The matrix representation of a linear operator $\hat{A}$ in these basis are related by a unitary transformation. The unitary matrix may be chosen to be

[NET JUNE 2015]

A. $\begin{pmatrix}0&-1\\1&0\end{pmatrix}$ **B.** $\begin{pmatrix}0&1\\1&0\end{pmatrix}$

C. $\frac{1}{\sqrt{2}}\begin{pmatrix}1&1\\1&-1\end{pmatrix}$ **D.** $\frac{1}{\sqrt{2}}\begin{pmatrix}1&0\\1&1\end{pmatrix}$

Solution: $u_1=\begin{pmatrix}1&0\\0&1\end{pmatrix},u_2=\frac{1}{\sqrt{2}}\begin{pmatrix}1&1\\1&-1\end{pmatrix}\Rightarrow u=u_1\otimes u_2=\frac{1}{\sqrt{2}}\begin{pmatrix}1&1\\1&-1\end{pmatrix}$

The correct option is **(c)**

11. A Hermitian operator $\hat{O}$ has two normalized eigenstates $|1\rangle$ and $|2\rangle$ with eigenvalues 1 and 2 , respectively. The two states $|u\rangle=\cos\theta|1\rangle+\sin\theta|2\rangle$ and $|v\rangle=\cos\phi|1\rangle+\sin\phi|2\rangle$ are such that $\langle v|\hat{O}|v\rangle=7/4$ and $\langle u\mid v\rangle=0$. Which of the following are possible values of θ and ϕ ?

[NET DEC 2015]

A. $\theta=-\frac{\pi}{6}$ and $\phi=\frac{\pi}{3}$ **B.** $\theta=\frac{\pi}{6}$ and $\phi=\frac{\pi}{3}$

C. $\theta=-\frac{\pi}{4}$ and $\phi=\frac{\pi}{4}$ **D.** $\theta=\frac{\pi}{3}$ and $\phi=-\frac{\pi}{6}$

Solution: $|u\rangle=\cos\theta|1\rangle+\sin\theta|2\rangle,\quad|v\rangle=\cos\phi|1\rangle+\sin\phi|2\rangle$

it is given $\hat{O}|1\rangle=|1\rangle,\quad\hat{O}|2\rangle=2|2\rangle\Rightarrow\langle v|\hat{O}|v\rangle=\frac{7}{4}$

$$\cos^2\phi+2\sin^2\phi=\frac{7}{4}\Rightarrow\cos^2\phi+\sin^2\phi=1\Rightarrow\sin^2\phi=\frac{7}{4}-1$$
$$\sin\phi=\frac{\sqrt{3}}{2}\Rightarrow\phi=\frac{\pi}{3}$$
$$\langle u\mid v\rangle=0\Rightarrow\cos\theta\cos\phi+\sin\theta\sin\phi=0\Rightarrow\cos(\theta-\phi)=0$$
$$\Rightarrow\theta-\phi=\frac{\pi}{2}\text{ or }\phi-\theta=\frac{\pi}{2}\Rightarrow\theta=\frac{\pi}{2}+\frac{\pi}{3}\text{ or }\theta=\frac{\pi}{3}-\frac{\pi}{2}\Rightarrow\theta=\frac{5\pi}{6}\text{ or }\theta=-\frac{\pi}{6}$$

12. If $\hat{L}_x,\hat{L}_y,\hat{L}_z$ are the components of the angular momentum operator in three dimensions the commutator $\left[\hat{L}_x,\hat{L}_x\hat{L}_y\hat{L}_z\right]$ may be simplified to

[NET JUNE 2016]

A. $i\hbar L_x\left(\hat{L}_z^2-\hat{L}_y^2\right)$
B. $i\hbar\hat{L}_z\hat{L}_y\hat{L}_x$
C. $i\hbar L_x\left(2\hat{L}_z^2-\hat{L}_y^2\right)$
D. 0

Solution:

$$\begin{aligned}
&:[L_x,L_xL_yL_z]=L_x[L_x,L_yL_z]+[L_x,L_x]L_yL_z\\
&=L_x[L_x,L_y]L_z+L_xL_y[L_x,L_z]+0=L_x[i\hbar L_z]L_z+L_xL_y(-i\hbar L_y)\\
&=i\hbar L_xL_z^2-i\hbar L_xL_y^2=i\hbar L_x\left(L_z^2-L_y^2\right)
\end{aligned}$$

The correct option is **(a)**

13. Consider the operator, $a=x+\frac{d}{dx}$ acting on smooth function of x. Then commutator $[\alpha,\cos x]$ is
[NET DEC 2016]

A. $-\sin x$
B. $\cos x$
C. $-\cos x$
D. 0

Solution:

$$\begin{aligned}
&a=x+\frac{d}{dx}\\
&[a,\cos x]=\left[x+\frac{d}{dx},\cos x\right]=[x,\cos x]+\left[\frac{d}{dx},\cos x\right]=0+\left[\frac{d}{dx},\cos x\right]\\
&\left[\frac{d}{dx},\cos x\right]\psi(x)=\frac{d}{dx}\cos x\psi(x)-\cos x\frac{d\psi}{dx}\\
&=\cos x\frac{d\psi}{dx}+(-\sin x)\psi-\frac{\cos x d\psi}{dx}=-\sin x\psi\\
&[a,\cos x]\psi(x)=-\sin x\psi\\
&[a,\cos x]=-\sin x
\end{aligned}$$

The correct option is **(a)**

14. Consider the operator $\vec{\pi}=\vec{p}-q\vec{A}$, where $\vec{p}$ is the momentum operator, $\vec{A}=(A_x,A_y,A_z)$ is the vector potential and q denotes the electric charge. If $\vec{B}=(B_x,B_y,B_z)$ denotes the magnetic field, the z-component of the vector operator $\vec{\pi}\times\vec{\pi}$ is
[NET DEC 2016]

A. $iq\hbar B_z+q(A_xp_y-A_yp_x)$
B. $-iq\hbar B_z-q(A_xp_y-A_yp_x)$
C. $-iq\hbar B_2$
D. $iq\hbar B_z$

Solution: $\vec{\pi}=\vec{p}-q\vec{A}$

$$\begin{aligned}
&(\vec{\pi}\times\vec{\pi})\psi=(\vec{p}-q\vec{A})\times(\vec{p}-q\vec{A})\psi=\vec{p}\times\vec{p}\psi-q\vec{p}\times\vec{A}\psi-q\vec{A}\times\vec{p}\psi+q^2\vec{A}\times\vec{A}\psi\\
&\vec{p}\times\vec{p}\psi=0\\
&-q\vec{p}\times\vec{A}\psi=-q(-i\hbar\vec{\nabla}\times\vec{A})\psi=qi\hbar\vec{B}\psi\\
&q\vec{A}\times\vec{p}\psi=q(\vec{A}(-i\hbar\vec{\nabla}))\psi=0\\
&q^2\vec{A}\times\vec{A}\psi=0\\
&\vec{\pi}\times\vec{\pi}=qi\hbar\vec{B}
\end{aligned}$$

So, z component is given by $qi\hbar B_z$

The correct option is **(d)**

15. The two vectors $\begin{pmatrix} a \\ 0 \end{pmatrix}$ and $\begin{pmatrix} b \\ c \end{pmatrix}$ are orthonormal if

[NET JUNE 2017]

A. $a=\pm 1, b=\pm 1/\sqrt{2}, c=\pm 1/\sqrt{2}$

B. $a=\pm 1, b=\pm 1, c=0$

C. $a=\pm 1, b=0, c=\pm 1$

D. $a=\pm 1, b=\pm 1/2, c=1/2$

Solution: $|\phi_1\rangle=\begin{pmatrix} a \\ 0 \end{pmatrix}, |\phi_2\rangle=\begin{pmatrix} b \\ c \end{pmatrix}$

$\langle\phi_1 \mid \phi_1\rangle=1 \quad \Rightarrow a=\pm 1$

$\langle\phi_2 \mid \phi_2\rangle=1 \quad \Rightarrow |b|^2+|c|^2=1$

$\langle\phi_1 \mid \phi_2\rangle=0 \quad \Rightarrow \begin{pmatrix} a & 0 \end{pmatrix}\begin{pmatrix} b \\ c \end{pmatrix}=0$

$a.b+0 \cdot c=0 \Rightarrow a \cdot b=0$

$|c|^2=1, \quad c=\pm 1 \quad$ so $b=0$

$a=\pm 1, \quad b=0, \quad c=\pm 1$

The correct option is **(c)**

16. Let x denote the position operator and p the canonically conjugate momentum operator of a particle. The commutator

$$\left[\frac{1}{2m}p^2+\beta x^2, \frac{1}{m}p^2+\gamma x^2\right]$$

where β and γ are constants, is zero if

[NET DEC 2017]

A. $\gamma=\beta$

B. $\gamma=2\beta$

C. $\gamma=\sqrt{2}\beta$

D. $2\gamma=\beta$

Solution:

$$\left[\frac{1}{2m}p^2+\beta x^2, \frac{1}{m}p^2+\gamma x^2\right]=0 \Rightarrow \frac{1}{2m}\gamma\left[p^2, x^2\right]+\frac{\beta}{m}\left[x^2, p^2\right]=0$$

$$-\frac{\gamma}{2m}\left[x^2, p^2\right]+\frac{\beta}{m}\left[x^2, p^2\right]=0 \Rightarrow \frac{1}{m}\left[x^2, p^2\right]\left[\frac{-\gamma}{2}+\beta\right]=0 \Rightarrow \gamma=2\beta$$

The correct option is **(b)**

17. Consider the operator $A_x=L_y p_z-L_z p_y$, where L_i and p_i denote, respectively, the components of the angular momentum and momentum operators. The commutator $[A_x, x]$ where x is the x - component of the position operator, is

[NET DEC 2018]

A. $-i\hbar(zp_z+yp_y)$

B. $-i\hbar(zp_z-yp_y)$

C. $i\hbar(zp_z+yp_y)$

D. $i\hbar(zp_z-yp_y)$

Solution: $A_x=L_y p_z-L_z p_y, L_y=zp_x-xp_z, L_z=xp_y-yp_x$

$$\begin{aligned}
[A_x, x] &= [L_y p_z, x]-[L_z p_y, x]=[L_y, x]p_z-[L_z, x]p_y \\
&= [zp_x, x]p_z+[yp_x, x]p_y=z[p_x, x]p_z+y[p_x, x]p_y \\
&= (-i\hbar z p_z)+(-i\hbar y p_y)=-i\hbar(zp_z+yp_y)
\end{aligned}$$

The correct option is **(a)**

Practice Set- 2

1. The quantum mechanical operator for the momentum of a particle moving in one dimension is given by **[GATE 2011]**

A. $i\hbar \frac{d}{dx}$ **B.** $-i\hbar \frac{d}{dx}$
C. $i\hbar \frac{\partial}{\partial t}$ **D.** $-\frac{\hbar^2}{2m} \frac{d^2}{dx^2}$

Solution: The correct option **(b)**

2. If L_x, L_y and L_z are respectively the x, y and z components of angular momentum operator L. The commutator $[L_x L_y, L_z]$ is equal to **[GATE 2011]**

A. $i\hbar\left(L_x^2+L_y^2\right)$ **B.** $2i\hbar L_z$
C. $i\hbar\left(L_x^2-L_y^2\right)$ **D.** 0

Solution: $\lfloor L_x L_y, L_z\rfloor = L_x\left[L_y L_z\right]+\left[L_x, L_z\right] L_y = i\hbar\left(L_x^2-L_y^2\right)$
The correct option is **(c)**

common data questions 3 and 4
In a one-dimensional harmonic oscillator, φ_0, φ_1 and φ_2 are respectively the ground, first and the second excited states. These three states are normalized and are orthogonal to one another ψ_1 and ψ_2 are two states defined by

$$\psi_1=\varphi_0-2\varphi_1+3\varphi_2, \psi_2=\varphi_0-\varphi_1+\alpha\varphi_2, \psi_2=\varphi_0-\varphi_1+\alpha\varphi_2$$

where α is a constant

3. The value of α which ψ_2 is orthogonal to ψ_1 is **[GATE 2011]**

A. 2 **B.** 1
C. -1 **D.** -2

Solution: For orthogonal condition scalar product $\left(\psi_2, \psi_1\right)=0$, so $1+2+3\alpha=0 \Rightarrow \alpha=-1$
The correct option is **(c)**

4. For the value of α determined in Q3, the expectation value of energy of the oscillator in the state ψ_2 is

A. $\hbar\omega$
B. $3\hbar\omega/2$
C. $3\hbar\omega$
D. $9\hbar\omega/2$

Solution: $\psi_2=\phi_0-\phi_1+\alpha\phi_2$ put $\alpha=-1, \langle H\rangle=\frac{\langle\psi_2|H|\psi_2\rangle}{\langle\psi_2|\psi_2\rangle}=\frac{\frac{\hbar\omega}{2}+\frac{3\hbar\omega}{2}+\frac{5\hbar\omega}{2}}{3}=\frac{3}{2}\hbar\omega$
The correct option is **(b)**

5. Which one of the following commutation relations is NOT CORRECT? Here, symbols have their usual meanings. **[GATE 2013]**

A. $\left[L^2, L_z\right]=0$ **B.** $\lfloor L_x, L_y\rfloor=i\hbar L_z$
C. $\left[L_z, L_+\right]=\hbar L_+$ **D.** $\left[L_z, L_-\right]=\hbar L_-$

Solution: The correct option is **(d)**

6. Let $\vec{L}$ and $\vec{p}$ be the angular and linear momentum operators, respectively, for a a particle. The commutator $\lfloor L_x, p_y \rfloor$ gives

[GATE 2015]

A. $-i\hbar p_z$
B. 0
C. $i\hbar p_x$
D. $i\hbar p_z$

Solution:

$$: [L_x, p_y] = [yp_z - zp_y, p_y] = [yp_z, p_y] - [zp_y, p_y] = [y, p_y]\, p_z$$
$$\cdot [p_y, p_y] = 0 \text{ and } [z, p_y] = 0 \Rightarrow [L_x, p_y] = i\hbar p_z \quad \because [y, p_y] = i\hbar$$

The correct option is **(d)**

7. Which of the following operators is Hermitian?

[GATE 2016]

A. $\frac{d}{dx}$
B. $\frac{d^2}{dx^2}$
C. $i\frac{d^2}{dx^2}$
D. $\frac{d^3}{dx^3}$

Solution: The correct option is **(b)**

8. If x and p are the x components of the position and the momentum operators of a particle respectively, the commutator $\left[x^2, p^2\right]$ is

[GATE 2016]

A. $i\hbar(xp - px)$
B. $2i\hbar(xp - px)$
C. $i\hbar(xp + px)$
D. $2i\hbar(xp + px)$

Solution: $\left[x^2, p^2\right] = p\left[x^2, p\right] + \left[x^2 p\right] p = 2i\hbar px + 2i\hbar xp \Rightarrow 2i\hbar(xp + px)$
The correct option is **(d)**

9. For the parity operator P, which of the following statements is NOT true?

[GATE 2016]

A. $P^\dagger = P$
B. $P^2 = -P$
C. $P^2 = I$
D. $P^\dagger = P^{-1}$

Solution: The correct option is **(b)**

10. Which one of the following operators is Hermitian?

[GATE 2017]

A. $i\frac{(p_x x^2 - x^2 p_x)}{2}$
B. $i\frac{(p_x x^2 + x^2 p_x)}{2}$
C. $e^{ip_x a}$
D. $e^{-ip_x a}$

Solution: $A = i\frac{(p_x x^2 - x^2 p_x)}{2}, A^\dagger = -i\frac{\left((p_x x^2)^\dagger - (x^2 p_x)^\dagger\right)}{2} = i\frac{(p_x x^2 - x^2 p_x)}{2}$
The correct option is **(a)**

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