Practice set-1

1. A uniform magnetic field in the positive z-direction passes through a circular wire loop of radius 1 cm and resistance 1Ω lying in the xy-plane. The field strength is reduced from 10 tesla to 9 tesla in $1s$. The charge transferred across any point in the wire is approximately

[NET JUNE 2015]

A. 3.1×10^{-4} coulomb **B.** 3.4×10^{-4} coulomb

C. 4.2×10^{-4} coulomb **D.** 5.2×10^{-4} coulomb

Solution:

$$\varepsilon=-\frac{d\phi}{dt}\Rightarrow I=\frac{dq}{dt}=\frac{\varepsilon}{R}=-\frac{1}{R}\frac{d\phi}{dt}$$
$$dq=-\frac{A}{R}dB=\frac{-\pi r^2}{R}dB$$
$$dq=\frac{-3.14\times\left(10^{-2}\right)^2}{1}\times1=3.14\times10^{-4}\text{ coulomb}$$

The correct option is **(a)**

2. A magnetic field B is $B\hat{z}$ in the region $x>0$ and zero elsewhere. A rectangular loop, in the xy-plane, of sides l (along the x-direction) and h (along the y - direction) is inserted into the $x>0$ region from the $x<0$ region at constant velocity $v=v\hat{x}$. Which of the following values of l and h will generate the largest EMF?

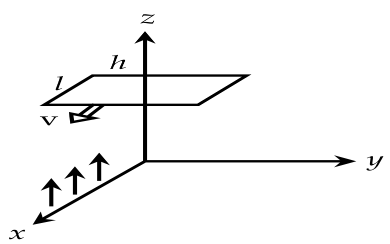
[NET JUNE 2016]

A. $l=8,h=3$ **B.** $l=4,h=6$

C. $l=6,h=4$ **D.** $l=12,h=2$

Solution: $\phi_m\propto Bhx$

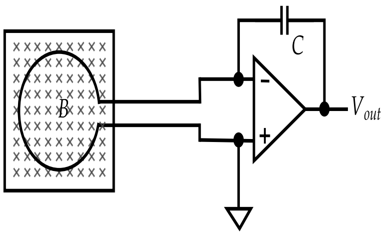
$$\varepsilon\propto\frac{-d\phi_m}{dt}\propto Bvh\propto h$$



The correct option is **(b)**

Practice set-2

1. Consider a conducting loop of radius a and total loop resistance R placed in a region with a magnetic field B thereby enclosing a flux ϕ_0. The loop is connected to an electronic circuit as shown, the capacitor being initially uncharged



If the loop is pulled out of the region of the magnetic field at a constant speed u, the final output voltage V_{out} is independent of

[GATE 2010]

A. ϕ_0 **B.** u **C.** R **D.** C

Solution: The correct option is **(a)**

2. A circular loop made of a thin wire has radius 2 cm and resistance 2Ω. It is placed perpendicular to a uniform magnetic field of magnitude $\left|\vec{B}_0\right| = 0.01$ Tesla. At time $t = 0$ the field starts decaying as $\vec{B} = \vec{B}_0 e^{-t/t_0}$, where $t_0 = 1s$. The total charge that passes through a cross section of the wire during the decay is Q. The value of Q in μC (rounded off to two decimal places) is

[GATE 2019]

Solution:

$$\varepsilon = -\frac{d\phi}{dt} = -\frac{AdB}{dt}, I = \frac{\varepsilon}{R} = -\frac{d\phi}{dt}\frac{1}{R}$$

$$-\frac{d\phi}{dt} = -\pi r^2 \frac{d}{dt}\left(B_0 e^{-t/t_0}\right) = \pi r^2 B_0 e^{-t}\,(t_0 = 1)$$

$$Q = \int_0^\infty I(t)dt = \int_0^\infty \frac{\pi r^2}{R} B_0 e^{-t} dt = \frac{\pi r^2 B_0}{R}\left|\frac{e^{-t}}{-1}\right|_0^\infty$$

$$= 3.14 \times \left(2\times 10^{-2}\right)^2 \times 0.01 = 6.28\mu C$$

3. A circular conducting ring of radius R rotates with constant angular velocity ω about its diameter placed along the x-axis. A uniform magnetic field B is applied along the y-axis. If at time $t = 0$ the ring is entirely in the xy-plane, the emf induced in the ring at time $t > 0$ is

[JEST 2012]

A. $B\omega^2\pi R^2 t$ **B.** $B\omega\pi R^2\tan(\omega t)$

C. $B\omega\pi R^2\sin(\omega t)$ **D.** $B\omega\pi R^2\cos(\omega t)$

Solution:

$$\phi_m = \vec{B}\cdot\vec{A} = BA\cos(90-\theta) = BA\sin\omega t$$

$$\varepsilon = -\frac{d\phi_m}{dt} = -\frac{d}{dt}(\vec{B}\cdot\vec{A})$$

$$= -\frac{d}{dt}[BA\sin\omega t] = -BA(\cos\omega t)\omega$$
$$\varepsilon = -B\pi R^2\omega\cos\omega t$$
$$\varepsilon = B\omega\pi R^2\cos\omega t$$

The correct option is **(d)**

4. Two parallel rails of a railroad track are insulated from each other and from the ground. The distance between the rails is 1 meter. A voltmeter is electrically connected between the rails. Assume the vertical component of the earth's magnetic field to the 0.2 gauss. What is the voltage developed between the rails when a train travels at a speed of 180 km/h along the track? Give the answer in milli-volts.

[JEST 2018]

Solution:

$$\text{Induced emf } \varepsilon = Blv$$
$$= \left(0.2\times 10^{-4}\right)\times 1\text{ m}\times 180\times\frac{10}{60\times 60}$$
$$= 10^{-3}\text{ volts } = 1\text{mV}$$

5. A circular metal loop of radius $a = 1$ m spins with a constant angular velocity $\omega = 20\pi\text{rad/s}$ in a magnetic field $B = 3$ Tesla, as shown in the figure. The resistance of the loop is 10 ohms. Let P be the power dissipated in one complete cycle. What is the value of $\frac{P}{\pi^4}$ in Watts?

[JEST 2019]

Solution:

Magnetic flux through the loop is

$$\phi_m = \int_S \vec{B}d\vec{a} = B\times\pi a^2\times\cos\omega t$$

Induced e.m.f

$$\varepsilon = -\frac{d\phi_m}{dt} = \omega B\times\pi a^2\times\sin\omega t$$

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Power dissipated

$$p = \frac{\varepsilon^2}{R} = \frac{\omega^2 B^2\pi^2 a^4\sin^2\omega t}{R}$$

Power dissipated in one complete cycle

$$P = \langle p\rangle = \frac{\omega^2 B^2\pi^2 a^4}{2R} \quad \because \langle\sin^2\omega t\rangle = \frac{1}{2}$$
$$\frac{P}{\pi^4} = \frac{\omega^2 B^2 a^4}{2\pi^2 R} \Rightarrow P = \frac{(20\pi)^2(3)^2(1)^4}{2(10)(10)} = 18$$

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