1. Radioactive Decay Solutions

Practice Set-1

1. A radioactive element X decays to Y, which in turn decays to a stable element Z. The decay constant from X to Y is λ_1, and that from Y to Z is λ_2. If, to begin with, there are only N_0 atoms of X, at short times $\left(t \ll \frac{1}{\lambda_1}\right.$ as well as $\left.\frac{1}{\lambda_2}\right)$ the number of atoms of Z will be

[NET/JRF (JUNE-2016)]

a. $\frac{1}{2}\lambda_1\lambda_2 N_0 t^2$ **b.** $\frac{\lambda_1\lambda_2}{2(\lambda_1+\lambda_2)}N_0 t$

c. $(\lambda_1+\lambda_2)^2 N_0 t^2$ **d.** $(\lambda_1+\lambda_2) N_0 t$

Solution:

$$\begin{array}{cccc} & X \xrightarrow{\lambda_1} & Y \xrightarrow{\lambda_2} & Z \\ t=0 & N_0 & O & O \\ t & N_1 & N_2 & N_3 \end{array}$$

Rate equations $N_1 = N_0 e^{-\lambda_1 t}, \dfrac{dN_2}{dt} = \lambda_1 N_1 - \lambda_2 N_2, \dfrac{dN_3}{dt} = \lambda_2 N_2$

$$N_3 = N_0\left[1+\frac{\lambda_1 e^{-\lambda_2 t}}{(\lambda_2-\lambda_1)}-\frac{\lambda_2 e^{-\lambda_1 t}}{(\lambda_2-\lambda_1)}\right]$$

$$= N_0\left[1+\frac{\lambda_1}{(\lambda_2-\lambda_1)}\left(1-\lambda_2 t+\frac{\lambda_2^2 t^2}{2}\right)-\frac{\lambda_2}{(\lambda_2-\lambda_1)}\left(1-\lambda_1 t+\frac{\lambda_1^2 t^2}{2}\right)\right]$$

$$= N_0\left[1+\frac{\lambda_1}{(\lambda_2-\lambda_1)}-\frac{\lambda_1\lambda_2 t}{(\lambda_2-\lambda_1)}+\frac{\lambda_1}{(\lambda_2-\lambda_1)}\frac{\lambda_2^2 t^2}{2}-\frac{\lambda_2}{(\lambda_2-\lambda_1)}+\frac{\lambda_2\lambda_1 t}{(\lambda_2-\lambda_1)}-\frac{\lambda_2}{(\lambda_2-\lambda_1)}\frac{\lambda_1^2 t^2}{2}\right]$$

$$= N_0\left[\frac{\lambda_1}{(\lambda_2-\lambda_1)}\times\frac{\lambda_2^2 t^2}{2}-\frac{\lambda_2}{(\lambda_2-\lambda_1)}\times\frac{\lambda_1^2 t^2}{2}\right]=\frac{\lambda_1\lambda_2 t^2}{2}N_0\left[\frac{\lambda_2}{\lambda_2-\lambda_1}-\frac{\lambda_1}{\lambda_2-\lambda_1}\right]=\frac{1}{2}\lambda_1\lambda_2 N_0 t^2$$

So the correct answer is **Option (a)**

2. What should be the minimum energy of a photon for it to split an α-particle at rest into a tritium and a proton? (The masses of ${}_2^4\mathrm{He}, {}_1^3\mathrm{H}$ and ${}_1^1\mathrm{H}$ are 4.0026amu, 3.0161amu and 1.0073amu respectively, and

1amu $\approx$ 938MeV)

[NET/JRF (DEC-2016)]

a. 32.2MeV **b.** 3MeV **c.** 19.3MeV **d.** 931.5MeV

Solution:

$$\text{From conservation of energy}$$
$$E_\alpha + m_\alpha c^2 = m_{1H^3}c^2 + m_{1H^1}c^2$$
$$\text{or } E_\alpha = [m_{1H^3} + m_{1H^1} - m_\alpha] \times 938\text{MeV} = 19.5\text{MeV}$$

So the correct answer is **Option (c)**

3. If in a spontaneous α-decay of ${}^{232}_{92}U$ at rest, the total energy released in the reaction is Q, then the energy carried by the α-particle is

[NET/JRF (JUNE-2017)]

a. $57Q/58$ **b.** $Q/57$ **c.** $Q/58$ **d.** $23Q/58$

Solution:

$$\text{Energy carried by the} \propto - \text{particle is}$$
$$KE_{\propto c} = \left(\frac{A-4}{A}\right)Q = \frac{228}{232}Q = \frac{57}{58}Q$$

So the correct answer is **Option (a)**

4. The reaction ${}^{63}\text{Cu}_{29} + p \to {}^{63}\text{Zn}_{30} + n$ is followed by a prompt β-decay of zinc ${}^{63}\text{Zn}_{30} \to {}^{63}\text{Cu}_{29} + e^+ + \nu_e$. If the maximum energy of the position is 2.4MeV, the Q value of the original reaction in MeV is nearest to [Take the masses of electron, proton and neutron to be $0.5\text{MeV}/\text{c}^2$, $938\text{MeV}/\text{c}^2$ and $939.5\text{MeV}/\text{c}^2$,respectively.]

[NET/JRF (JUNE-2018)]

a. -4.4 **b.** -2.4 **c.** -4.8 **d.** -3.4

Solution:

$$\text{For } {}^{63}\text{Zn}_{30} \to {}^{63}Cu_{29} + e^+ + \nu_e$$
$$Q_1 = (Zn - 30e) - [Cu - 29e + e] = Zn - Cu - 2e = 2.4\text{MeV}$$
$$\text{For } {}^{63}Cu_{29} + p \to {}^{63}Zn_{30} + n$$
$$Q_0 = [(Cu - 29e) + p] - [(Zn - 30e) + n]$$
$$= Cu - Zn + e + p - n = (-Q_1 - 2e) + e + p - n = -Q_1[e - p + n]$$
$$= -2.4 - (0.5 - 938 + 939.5) = -4.4.\text{MeV}$$

So the correct answer is **Option (a)**

5. A nucleus decays by the emission of a gamma ray from an excited state of spin parity 2^+ to the ground state with spin-parity 0^+what is the type of the corresponding radiation?

[NET/JRF (DEC-2018)]

a. Magnetic dipole **b.** Electric quadrupole

c. Electric dipole **d.** Magnetic quadrupole

Solution:

$$I_i = 2, \quad I_+ = 0$$
$$\Rightarrow L = 2 \text{ and parity change}$$
$$\therefore \text{The transition is of electric quadrupole } (E_2) \text{ nature.}$$

So the correct answer is **Option (b)**

6. The ground state of $^{207}_{12}$ Pb nucleus has spin-parity $J^p = \frac{1^-}{2}$, while the first excited state has $J^p = \frac{5^-}{2}$. The electromagnetic radiation emitted when the nucleus makes a transition from the first excited state to ground state are

[NET/JRF (JUNE-2012)]

a. E2 and E3 **b.** M2 or E3 **c.** E2 or M3 **d.** M2 or M3

Solution:

$$\text{No parity change; } \Delta J = 2, 3$$
$$\text{For } E_l \text{ type, } \Delta\pi = (-1)^l, \text{ (for no parity change } l = 2 \text{)}$$
$$\text{For } M_l \text{ type, } \Delta\pi = (-1)^{l+1}, \text{ (for no parity change } l = 3 \text{)}$$
$$\Delta J = 2, \text{ No parity change} \rightarrow E2; \Delta J = 3, \text{ No parity change} \rightarrow M3$$

So the correct answer is **Option (c)**

Answer key

Q.No.	Answer	Q.No.	Answer
1	**a**	2	**c**
3	**a**	4	**a**
5	**b**	6	**c**
7		8	
9		10	
11		12	
13		14	
15			

Practice Set-2

1. A radioactive element X has a half-life of 30 hours. It decays via alpha, beta and gamma emissions with the branching ratio for beta decay being 0.75. The partial half-life for beta decay in unit of hours is———

[GATE-2019]

Solution: Branching ratio is the fraction of particles (here β) which decays by an individual decay mode with respect to the total number of particles which decays

$$BR = \frac{\left(\frac{dN}{dt}\right)_x}{\left(\frac{dt}{dt}\right)_\beta} = \frac{\left(T_{1/2}\right)_x}{\left(T_{1/2}\right)_\beta} \Rightarrow \left(T_{1/2}\right)_\beta = \frac{\left(T_{1/2}\right)_x}{BR} = \frac{30}{0.75} = 40 \text{ hours}$$

So the correct answer is **40**

2. An α particle is emitted by a $^{230}_{90}$Th nucleus. Assuming the potential to be purely Coulombic beyond the point of separation, the height of the Coulomb barrier is MeV (up to two decimal places). $\left(\frac{e^2}{4\pi\varepsilon_0} = 1.44\text{MeV} - \text{fm}, r_0 = 1.30\text{fm}\right)$

[GATE-2018]

Solution:

The height of coulomb barrier for α particle from

$${}_{90}Th^{230} \rightarrow {}_{88}X^{226} + 2He^4 (\alpha\text{-particle})$$

$$V_C = \frac{1}{4\pi \in_0}\left(\frac{2ze^2}{R}\right)$$

Here, $R_0 = 1.3\text{fm}, \dfrac{e^2}{4\pi\varepsilon_0} = 1.44\text{MeVfm}$

And $R = R_0 A^{1/3}$

Here, we consider pure Columbic interaction

$$A_{Th}^{1/3} = A_X^{1/3} + A_\alpha^{1/3} = (226)^{1/3} + (4)^{1/3} = (6.09 + 1.58) = 7.67, \quad R = R_0 A_{Th}^{1/3} = 1.3(7.67)$$

$$\text{Hence, } V_C = \left(\frac{e^2}{4\pi \in_0}\right)\frac{2\times 90}{1.3(7.67)} = \frac{180\times 1.44}{1.3\times 7.67}\frac{\text{MeV}}{\text{fm}}$$

$$V_C = 25.995\text{MeV}$$

So the correct answer is **25.995**

3. Consider the reaction $^{54}_{25}\text{Mn} + e^- \rightarrow ^{54}_{24}\text{Cr} + \text{X}$. The particle X is

[GATE-2016]

a. γ **b.** ν_e **c.** n **d.** π^0

Solution: So the correct answer is **Option (b)**

4. In the nuclear reaction $^{13}C_6 + \nu_e \rightarrow {}^{13}N_7 + X$, the particle X is

[GATE-2017]

a. An electron **b.** An anti-electron

c. A muon **d.** A pion

Solution:

$$^{13}C_6 + \nu_e \rightarrow {}^{13}N_7 + X \Rightarrow {}^{13}C_6 \rightarrow {}^{13}N_7 + X + \bar{\nu}_e$$
$$L_e = 0 \quad 0 + 1 - 1$$
To conserve the Lepton number L_e, X should be e^-

So the correct answer is **Option (a)**

5. In the β-decay of neutron $n \rightarrow p + e - \bar{\nu}_e$, the anti-neutrino $\bar{\nu}_e$, escapes detection. Its existence is inferred from the measurement of

[GATE-2011]

- **a.** Energy distribution of electrons
- **b.** Angular distribution of electrons
- **c.** Helicity distribution of electrons
- **d.** Forward-backward asymmetry of electrons

Solution: So the correct answer is **Option (a)**

6. In the β decay process, the transition $2^+ \rightarrow 3^+$, is

[GATE-2013]

- **a.** Allowed both by Fermi and Gamow-Teller selection rule
- **b.** Allowed by Fermi and but not by Gamow-Teller selection rule
- **c.** Not allowed by Fermi but allowed by Gamow-Teller selection rule
- **d.** Not allowed both by Fermi and Gamow-Teller selection rule

Solution:

According to Fermi Selection Rule:
$\Delta I = 0$, Parity = No Change
According to Gammow-Teller Selection Rule: $\Delta I = 0, \pm 1$, Parity = No Change
In the β decay process, the transition $2^+ \rightarrow 3^+$, $\Delta I = \pm 1$, Parity = No Change.

So the correct answer is **Option (c)**

7. A nucleus X undergoes a first forbidden β-decay to nucleus Y. If the angular momentum (I) and parity (P), denoted by I^P as $\frac{7^-}{2}$ for X, which of the following is a possible I^P value for Y ?

[GATE-2014]

a. $\frac{1^+}{2}$ **b.** $\frac{1^-}{2}$ **c.** $\frac{3^+}{2}$ **d.** $\frac{3^-}{2}$

Solution:

For first forbidden β-decay; $\Delta I = 0, 1$ or 2 and Parity does change.

So the correct answer is **Option (c)**

8. A beam of X - ray of intensity I_0 is incident normally on a metal sheet of thickness 2 mm. The intensity of the transmitted beam is $0.025 I_0$. The linear absorption coefficient of the metal sheet (in m^{-1}) is———- (upto one decimal place)

[GATE-2015]

Solution:

$$I = I_0 e^{-\mu x} \Rightarrow \mu = \frac{1}{x}\ln\left(\frac{I_0}{I}\right) = \frac{1}{2\times10^{-3}}\ln\left(\frac{I_0}{0.025 I_0}\right) = \frac{1}{2\times10^{-3}}\ln(40)$$

$$\Rightarrow \mu = \frac{2.303}{2\times10^{-3}}[\log_{10}40] = 1.151\times10^{3}[2\times0.3010+1] = 1844.4\ \mathrm{m}^{-1}$$

So the correct answer is **1844.4**

9. The atomic masses of ${}^{152}_{63}\mathrm{Eu}, {}^{152}_{62}\mathrm{Sm}, {}^{1}_{1}\mathrm{H}$ and neutron are 151.921749, 151.919756, 1.007825 and 1.008665 in atomic mass units (amu), respectively. Using the above information, the Q-value of the reaction ${}^{152}_{63}Eu + n \rightarrow {}^{152}_{62}Sm + p$ is ———$\times 10^{-3}$ amu (upto three decimal places)
[GATE-2015]

Solution:

$$Q = 152.930414 - (152.927581) = 2.833\times10^{-3}\ \text{a.m.u.}$$

So the correct answer is **2.833**

10. The atomic masses of ${}^{152}_{63}\mathrm{Eu}, {}^{152}_{62}\mathrm{Sm}, {}^{1}_{1}H$ and neutron are 151.921749, 151.919756, 1.007825 and 1.008665 in atomic mass units (amu), respectively. Using the above information, the Q-value of the reaction ${}^{152}_{63}Eu + n \rightarrow {}^{152}_{62}Sm + p$ is ————$\times 10^{-3}$ amu (upto three decimal places)
[GATE-2015]

Solution:

$$Q = 152.930414 - (152.927581) = 2.833\times10^{-3}\ \text{a.m.u.}$$

So the correct answer is **2.833**

Answer key

Q.No.	Answer	Q.No.	Answer
1	**40**	2	**25.995**
3	**b**	4	**a**
5	**a**	6	**c**
7	**c**	8	**1844.4**
9	**2.833**	10	**2.833**
11		12	
13		14	
15			