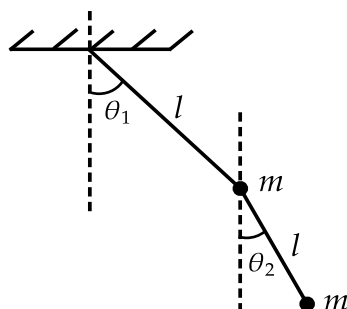


Practice set-1

1. A double pendulum consists of two point masses m attached by strings of length l as shown in the figure:
The kinetic energy of the pendulum is

[NET/JRF(DEC-2011)]



- A. $\frac{1}{2}ml^2 [\dot{\theta}_1^2 + \dot{\theta}_2^2]$
 B. $\frac{1}{2}ml^2 [2\dot{\theta}_1^2 + \dot{\theta}_2^2 + 2\dot{\theta}_1\dot{\theta}_2 \cos(\theta_1 - \theta_2)]$
 C. $\frac{1}{2}ml^2 [\dot{\theta}_1^2 + 2\dot{\theta}_2^2 + 2\dot{\theta}_1\dot{\theta}_2 \cos(\theta_1 - \theta_2)]$
 D. $\frac{1}{2}ml^2 [2\dot{\theta}_1^2 + \dot{\theta}_2^2 + 2\dot{\theta}_1\dot{\theta}_2 \cos(\theta_1 + \theta_2)]$

Solution:

Let co-ordinate (x_1, y_1) and (x_2, y_2) .

$$K.E. = \frac{1}{2}m(\dot{x}_1^2 + \dot{y}_1^2) + \frac{1}{2}m(\dot{x}_2^2 + \dot{y}_2^2)$$

$$x_1 = l \sin \theta_1, y_1 = l \cos \theta_1 \Rightarrow \dot{x}_1$$

$$= l \cos \theta_1 \dot{\theta}_1, \dot{y}_1 = -l \sin \theta_1 \dot{\theta}_1$$

$$x_2 = l \sin \theta_1 + l \sin \theta_2, y_2 = l \cos \theta_1 + l \cos \theta_2$$

$$\Rightarrow \dot{x}_2 = l \cos \theta_1 \dot{\theta}_1 + l \cos \theta_2 \dot{\theta}_2, \dot{y}_2$$

$$= l(-\sin \theta_1 \dot{\theta}_1) + l(-\sin \theta_2) \dot{\theta}_2$$

Put the value of $\dot{x}_1, \dot{y}_1, \dot{x}_2, \dot{y}_2$ in K.E equation, one will get

$$T = \frac{1}{2}ml^2 [2\dot{\theta}_1^2 + \dot{\theta}_2^2 + 2\dot{\theta}_1\dot{\theta}_2 \cos(\theta_1 - \theta_2)]$$

So the correct answer is **Option (B)**

2. A particle of mass m moves inside a bowl. If the surface of the bowl is given by the equation $z = \frac{1}{2}a(x^2 + y^2)$, where a is a constant, the Lagrangian of the particle is

[NET/JRF(DEC-2011)]

- A. $\frac{1}{2}m(\dot{r}^2 + r^2\dot{\phi}^2 - gar^2)$
 B. $\frac{1}{2}m[(1 + a^2r^2)\dot{r}^2 + r^2\dot{\phi}^2]$
 C. $\frac{1}{2}m(\dot{r}^2 + r^2\dot{\theta}^2 + r^2\sin^2\theta\dot{\phi}^2 - gar^2)$
 D. $\frac{1}{2}m[(1 + a^2r^2)\dot{r}^2 + r^2\dot{\phi}^2 - gar^2]$

Solution:

$$L = \frac{1}{2}m(\dot{x}^2 + \dot{y}^2 + \dot{z}^2) - mgz,$$

$$\text{where } z = \frac{1}{2}a(x^2 + y^2)$$

It has cylindrical symmetry. Thus $x = r \cos \phi, y = r \sin \phi, z = \frac{1}{2}a(r^2)$

$$\begin{aligned}\dot{x} &= \dot{r} \cos \phi - r \sin \phi \dot{\phi}, \dot{y} \\ &= \dot{r} \sin \phi + r \cos \phi \dot{\phi} \text{ and } \dot{z} = a(r\dot{r})\end{aligned}$$

$$\text{So, } L = \frac{1}{2}m[(1 + a^2r^2)\dot{r}^2 + r^2\dot{\phi}^2 - gar^2]$$

So the correct answer is **Option (D)**

3. The Lagrangian of a particle of mass m moving in one dimension is given by

$$L = \frac{1}{2}m\dot{x}^2 - bx$$

where b is a positive constant. The coordinate of the particle $x(t)$ at time t is given by: (in following c_1 and c_2 are constants)

[NET/JRF(JUNE-2013)]

A. $-\frac{b}{2m}t^2 + c_1t + c_2$

B. $c_1t + c_2$

C. $c_1 \cos\left(\frac{bt}{m}\right) + c_2 \sin\left(\frac{bt}{m}\right)$

D. $c_1 \cosh\left(\frac{bt}{m}\right) + c_2 \sinh\left(\frac{bt}{m}\right)$

Solution:

$$\begin{aligned}\text{Equation of motion } \frac{d}{dt}\left(\frac{\partial L}{\partial \dot{x}}\right) - \frac{\partial L}{\partial x} &= 0 \Rightarrow \frac{d}{dt}(m\dot{x}) + b \\ &= 0 \Rightarrow m\ddot{x} + b = 0 \Rightarrow m\ddot{x} = -b \\ \frac{d^2x}{dt^2} &= -\frac{b}{m} \Rightarrow \frac{dx}{dt} = -\frac{b}{m}t + c_1 \Rightarrow x \\ &= -\frac{b}{m}\frac{t^2}{2} + c_1t + c_2\end{aligned}$$

So the correct answer is **Option (A)**

4. A particle moves in a potential $V = x^2 + y^2 + \frac{z^2}{2}$. Which component(s) of the angular momentum is/are constant(s) of motion?

[NET/JRF(DEC-2013)]

A. None

B. L_x, L_y and L_z

C. only L_x and L_y

D. only L_z

Solution:

A particle moves in a potential $V = x^2 + y^2 + \frac{z^2}{2}$

$$V(r, \theta, \phi) = r^2 \sin^2 \theta \cos^2 \phi + r^2 \sin^2 \theta \sin^2 \phi + \frac{r^2}{2} \cos^2 \theta$$

$$V(r, \theta, \phi) = r^2 \sin^2 \theta + \frac{r^2}{2} \cos^2 \theta$$

Now ϕ is cyclic-co-ordinate (p_ϕ) i.e L_z is constant of motion.

So the correct answer is **Option (D)**

5. A pendulum consists of a ring of mass M and radius R suspended by a massless rigid rod of length l attached to its rim. When the pendulum oscillates in the plane of the ring, the time period of oscillation is

[NET/JRF(DEC-2013)]

A. $2\pi\sqrt{\frac{l+R}{g}}$

B. $\frac{2\pi}{\sqrt{g}}(l^2 + R^2)^{1/4}$

C. $2\pi\sqrt{\frac{2R^2 + 2Rl + l^2}{g(R+l)}}$

D. $\frac{2\pi}{\sqrt{g}}(2R^2 + 2Rl + l^2)^{1/4}$

Solution:

The moment of inertia about pivotal point is given by

$$I = I_{c,m} + Md^2 = MR^2 + M(l+R)^2$$

If ring is displaced by angle θ then potential energy is $-Mg(l+R)\cos\theta$.

The Lagrangian is given by

$$\begin{aligned} L &= \frac{1}{2}I\dot{\theta}^2 - V(\theta) \\ &= \frac{1}{2}(MR^2 + M(l+R)^2)\dot{\theta}^2 + Mg(l+R)\cos\theta \\ \frac{d}{dt}\left(\frac{\partial L}{\partial \dot{\theta}}\right) - \left(\frac{\partial L}{\partial \theta}\right) &= 0 \Rightarrow (MR^2 + M(l+R)^2)\ddot{\theta} + Mg(l+R)\sin\theta = 0 \end{aligned}$$

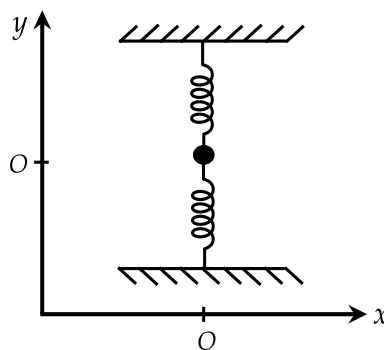
$$\text{For small oscillation } \sin\theta = \theta \Rightarrow (MR^2 + M(l+R)^2)\ddot{\theta} + Mg(l+R)\theta = 0$$

$$\text{Time period is given by } 2\pi\sqrt{\frac{2R^2 + 2Rl + l^2}{g(R+l)}}.$$

So the correct answer is **Option (C)**

6. Consider a particle of mass m attached to two identical springs each of length l and spring constant k (see the figure). The equilibrium configuration is the one where the springs are unstretched. There are no other external forces on the system. If the particle is given a small displacement along the x -axis, which of the following describes the equation of motion for small oscillations?

[NET/JRF(DEC-2013)]



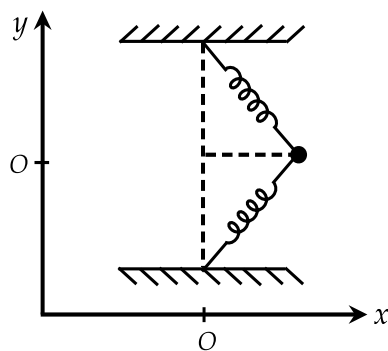
A. $m\ddot{x} + \frac{kx^3}{l^2} = 0$

B. $m\ddot{x} + kx = 0$

C. $m\ddot{x} + 2kx = 0$

D. $m\ddot{x} + \frac{kx^2}{l} = 0$

Solution:



The lagrangian of system is given by

$$L = \frac{1}{2}m\dot{x}^2 - V(x)$$

The potential energy is given by

$$V(x) = \frac{k}{2} \left[(x^2 + l^2)^{\frac{1}{2}} - l \right]^2 + \frac{k}{2} \left[(x^2 + l^2)^{\frac{1}{2}} - l \right]^2$$

$$V(x) = k \left[(x^2 + l^2)^{\frac{1}{2}} - l \right]^2$$

For small oscillation one can approximate potential by Taylor expansion

$$V(x) = kl^2 \left[\left(1 + \frac{x^2}{l^2} \right)^{\frac{1}{2}} - 1 \right]^2 \Rightarrow V(x)$$

$$= kl^2 \left[\left(1 + \frac{1}{2} \frac{x^2}{l^2} - \frac{1}{8} \frac{x^4}{l^4} \right) - 1 \right]^2$$

$$V(x) = \frac{kl^2}{4} \left(\frac{x^2}{l^2} \right)^2 \Rightarrow V(x) = k \left(\frac{x^4}{4l^2} \right)$$

So Lagrangian of system is given by $L = \frac{1}{2}m\dot{x}^2 - k \left(\frac{x^4}{4l^2} \right)$

$$\text{The Lagranges equation of motion } \frac{d}{dt} \left(\frac{\partial L}{\partial \dot{x}} \right) - \left(\frac{\partial L}{\partial x} \right) = 0 \Rightarrow m\ddot{x} + \frac{kx^3}{l^2} = 0$$

So the correct answer is **Option (A)**

7. The equation of motion of a system described by the time-dependent Lagrangian

$$L = e^{\gamma t} \left[\frac{1}{2}m\dot{x}^2 - V(x) \right] \text{ is}$$

[NET/JRF(DEC-2014)]

A. $m\ddot{x} + \gamma m\dot{x} + \frac{dV}{dx} = 0$

B. $m\ddot{x} + \gamma m\dot{x} - \frac{dV}{dx} = 0$

C. $m\ddot{x} - \gamma m\dot{x} + \frac{dV}{dx} = 0$

D. $m\ddot{x} + \frac{dV}{dx} = 0$

Solution:

$$\begin{aligned}
 \because L &= e^{\gamma t} \left[\frac{1}{2} m \dot{x}^2 - V(x) \right] \Rightarrow \frac{\partial L}{\partial \dot{x}} \\
 &= e^{\gamma t} m \dot{x} \text{ and } \frac{\partial L}{\partial x} = -\frac{\partial V}{\partial x} e^{\gamma t} \\
 \therefore \frac{d}{dt} \left(\frac{\partial L}{\partial \dot{x}} \right) - \frac{\partial L}{\partial x} &= 0 \Rightarrow \frac{d}{dt} (e^{\gamma t} m \dot{x}) + \frac{\partial V}{\partial x} e^{\gamma t} \\
 &= m \ddot{x} e^{\gamma t} + m \dot{x} \gamma e^{\gamma t} + \frac{\partial V}{\partial x} e^{\gamma t} = 0 \\
 \left(m \ddot{x} + m \gamma \dot{x} + \frac{\partial V}{\partial x} \right) e^{\gamma t} &= 0 \Rightarrow m \ddot{x} + \gamma m \dot{x} + \frac{\partial V}{\partial x} = 0
 \end{aligned}$$

So the correct answer is **Option (A)**

8. A particle of unit mass moves in the xy -plane in such a way that $\dot{x}(t) = y(t)$ and $\dot{y}(t) = -x(t)$. We can conclude that it is in a conservative force-field which can be derived from the potential

[NET/JRF(JUNE-2015)]

A. $\frac{1}{2} (x^2 + y^2)$

B. $\frac{1}{2} (x^2 - y^2)$

C. $x + y$

D. $x - y$

Solution:

$$\begin{aligned}
 \because \dot{x} &= y \text{ and } \dot{y} = -x \\
 \Rightarrow \ddot{x} &= \dot{y} = -x \text{ and } \ddot{y} = -\dot{x} = -y \\
 \Rightarrow \ddot{x} + x &= 0 \text{ and } \ddot{y} + y = 0 \\
 \text{that is possible for } L &= \frac{1}{2} m \dot{x}^2 + \frac{1}{2} m \dot{y}^2 - \frac{1}{2} (x^2 + y^2) \\
 \Rightarrow V &= \frac{1}{2} (x^2 + y^2)
 \end{aligned}$$

So the correct answer is **Option (A)**

9. The Lagrangian of a particle moving in a plane is given in Cartesian coordinates as

$$L = \dot{x}\dot{y} - x^2 - y^2$$

In polar coordinates the expression for the canonical momentum p_r (conjugate to the radial coordinate r) is

[NET/JRF(DEC-2015)]

A. $\dot{r} \sin \theta + r \dot{\theta} \cos \theta$

B. $\dot{r} \cos \theta + r \dot{\theta} \sin \theta$

C. $2\dot{r} \cos \theta - r \dot{\theta} \sin 2\theta$

D. $\dot{r} \sin 2\theta + r \dot{\theta} \cos 2\theta$

Solution:

$$\begin{aligned}
 L &= \dot{x}\dot{y} - x^2 - y^2 = \dot{x}\dot{y} - (x^2 + y^2) \\
 x &= r \cos \theta, y = r \sin \theta \Rightarrow \dot{x} = \dot{r} \cos \theta - r \sin \theta \dot{\theta}, \\
 \dot{y} &= \dot{r} \sin \theta + r \cos \theta \dot{\theta} \\
 L &= \dot{r}^2 \sin \theta \cos \theta - r^2 \sin \theta \cos \theta \dot{\theta}^2 + \dot{r} r \cos^2 \theta \dot{\theta} - \dot{r} r \sin^2 \theta \dot{\theta} \\
 P_r &= \frac{\partial L}{\partial \dot{r}} \Rightarrow 2\dot{r} \sin \theta \cos \theta + r \dot{\theta} (\cos^2 \theta - \sin^2 \theta) \\
 \Rightarrow P_r &= \dot{r} \sin 2\theta + r \dot{\theta} \cos 2\theta
 \end{aligned}$$

So the correct answer is **Option (D)**

10. The dynamics of a particle governed by the Lagrangian

$$L = \frac{1}{2}m\dot{x}^2 - \frac{1}{2}kx^2 - kx\dot{x}t \text{ describes}$$

[NET/JRF(DEC-2016)]

- A. An undamped simple harmonic oscillator
- B. A damped harmonic oscillator with a time varying damping factor
- C. An undamped harmonic oscillator with a time dependent frequency
- D. A free particle

Solution:

$$\begin{aligned} L &= \frac{1}{2}m\dot{x}^2 - \frac{1}{2}kx^2 - kx\dot{x}t \\ \frac{\partial L}{\partial \dot{x}} &= m\dot{x} - kxt, \quad \frac{\partial L}{\partial x} = -kx - k\dot{x}t \\ \frac{d}{dt} \left(\frac{\partial L}{\partial \dot{x}} \right) - \frac{\partial L}{\partial x} &= 0 \Rightarrow m\ddot{x} - k\dot{x}t - kx + kx + k\dot{x}t \\ &= 0 \Rightarrow m\ddot{x} = 0 \end{aligned}$$

So motion is equivalent to free particle

So the correct answer is **Option (D)**

11. The parabolic coordinates (ξ, η) are related to the Cartesian coordinates (x, y) by $x = \xi\eta$ and $y = \frac{1}{2}(\xi^2 - \eta^2)$. The Lagrangian of a two-dimensional simple harmonic oscillator of mass m and angular frequency ω is

[NET/JRF(DEC-2016)]

- A. $\frac{1}{2}m \left[\dot{\xi}^2 + \dot{\eta}^2 - \omega^2 (\xi^2 + \eta^2) \right]$
- B. $\frac{1}{2}m (\xi^2 + \eta^2) \left[\left(\dot{\xi}^2 + \dot{\eta}^2 \right) - \frac{1}{4}\omega^2 (\xi^2 + \eta^2) \right]$
- C. $\frac{1}{2}m (\xi^2 + \eta^2) \left[\dot{\xi}^2 + \dot{\eta}^2 - \frac{1}{2}\omega^2 \xi\eta \right]$
- D. $\frac{1}{2}m (\xi^2 + \eta^2) \left[\dot{\xi}^2 + \dot{\eta}^2 - \frac{1}{4}\omega^2 \right]$

Solution:

For two dimensional Harmonic oscillation

$$\begin{aligned} L &= \frac{1}{2}m(\dot{x}^2 + \dot{y}^2) - \frac{1}{2}m\omega^2(x^2 + y^2) \\ x &= \xi\eta, \quad y = \frac{1}{2}(\xi^2 - \eta^2) \\ \dot{x} &= \dot{\xi}\eta + \xi\dot{\eta}, \quad \dot{y} = \xi\dot{\xi} - \eta\dot{\eta} \\ L &= \frac{1}{2}m \left[(\dot{\xi}\eta + \xi\dot{\eta})^2 + (\xi\dot{\xi} - \eta\dot{\eta})^2 \right] - \frac{1}{2}m\omega^2 \left[\xi^2\eta^2 + \frac{1}{4}(\xi^2 - \eta^2)^2 \right] \end{aligned}$$

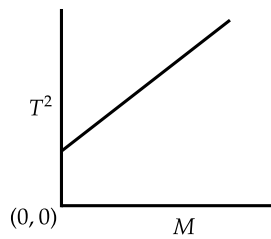
$$\begin{aligned}
 L &= \frac{1}{2}m(\dot{\xi}^2\eta^2 + \xi^2\dot{\eta}^2 + \xi^2\dot{\xi}^2 + \eta^2\dot{\eta}^2) - \frac{1}{8}m\omega^2(\xi^4 + \eta^4 + 2\xi^2\eta^2) \\
 &= \frac{1}{2}m(\xi^2 + \eta^2)(\dot{\eta}^2 + \dot{\xi}^2) - \frac{1}{8}m\omega^2(\xi^2 + \eta^2)^2 \\
 &= \frac{1}{2}m(\xi^2 + \eta^2)\left[\dot{\eta}^2 + \dot{\xi}^2 - \frac{1}{4}\omega^2(\xi^2 + \eta^2)\right]
 \end{aligned}$$

So the correct answer is **Option (B)**

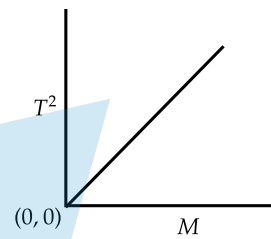
12. The spring constant k of a spring of mass m_s is determined experimentally by loading the spring with mass M and recording the time period T , for a single oscillation. If the experiment is carried out for different masses, then the graph that correctly represents the result is

[NET/JRF(DEC-2017)]

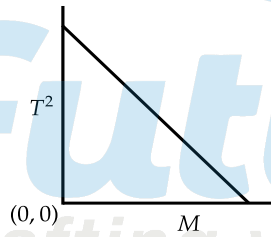
A.



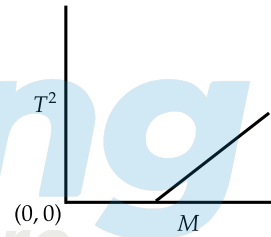
B.



C.



D.



Solution:

The Lagrangian of system.

$$\begin{aligned}
 L &= \frac{1}{2} \cdot \frac{m_s}{3} \dot{x}^2 + \frac{1}{2} M \dot{x}^2 - \frac{1}{2} k x^2, \quad \frac{d}{dt} \left(\frac{\partial L}{\partial \dot{x}} \right) - \frac{\partial L}{\partial x} = 0 \\
 \frac{d}{dt} \frac{\partial L}{\partial \dot{x}} &= 0 \Rightarrow \left(\frac{m_s}{3} + M \right) \ddot{x} = -kx \\
 T &= 2\pi \sqrt{\frac{M + \frac{m_s}{3}}{k}} \Rightarrow T^2 = 4\pi^2 \frac{(M + \frac{m_s}{3})}{k}
 \end{aligned}$$

So the correct answer is **Option (A)**

13. The motion of a particle in one dimension is described by the Lagrangian $L = \frac{1}{2} \left(\left(\frac{dx}{dt} \right)^2 - x^2 \right)$ in suitable units. The value of the action along the classical path from $x = 0$ at $t = 0$ to $x = x_0$ at $t = t_0$, is

[NET/JRF(DEC-2018)]

A. $\frac{x_0^2}{2\sin^2 t_0}$

B. $\frac{1}{2}x_0^2 \tan t_0$

C. $\frac{1}{2}x_0^2 \cot t_0$

D. $\frac{x_0^2}{2\cos^2 t_0}$

Solution:

$$L = \frac{1}{2}\dot{x}^2 - \frac{1}{2}x^2$$

From Lagrangian equation of motion, $\frac{d}{dt}\left(\frac{\partial L}{\partial \dot{x}}\right) - \frac{\partial L}{\partial x} = 0$

$$\ddot{x} + x = 0$$

The solution is $x = A \sin t + B \cos t$

$$t = 0, \quad x = 0, \quad B = 0$$

$$x = A \sin t$$

$$t = t_0, \quad x = x_0, \quad A = \frac{x_0}{\sin t_0}$$

$$x = \frac{x_0}{\sin t_0} \sin t, \quad \dot{x} = \frac{x_0}{\sin t_0} \cos t$$

$$\begin{aligned} A &= \int_0^{t_0} L dt = \int_0^{t_0} \frac{1}{2} \dot{x}^2 dt - \int_0^{t_0} \frac{1}{2} x^2 dt \\ &= \frac{1}{2} \frac{x_0^2}{\sin^2 t_0} \int_0^{t_0} \cos^2 t dt - \frac{1}{2} \frac{x_0^2}{\sin^2 t_0} \int_0^{t_0} \sin^2 t dt \\ &= \frac{1}{2} \frac{x_0^2}{\sin^2 t_0} \left[\int_0^{t_0} \cos^2 t dt - \int_0^{t_0} \sin^2 t dt \right] \\ &= \frac{1}{2} \frac{x_0^2}{\sin^2 t_0} \int_0^{t_0} \cos 2t dt = \frac{1}{2} \frac{x_0^2}{\sin^2 t_0} \frac{\sin 2t_0}{2} \Big|_0^{t_0} = \frac{x_0^2}{2} \cot t_0 \end{aligned}$$

So the correct answer is **Option (C)**

14. Two particles of masses m_1 and m_2 are connected by a massless thread of length l as shown in figure below.

The particle of mass m_1 on the plane undergoes a circular motion with radius r_0 and angular momentum L . When a small radial displacement ϵ (where $\epsilon \ll r_0$) is applied, its radial coordinate is found to oscillate about r_0 . The frequency of the oscillations is

[NET/JRF(JUNE-2019)]

A. $\sqrt{\frac{7m_2g}{(m_1 + \frac{m_2}{2})r_0}}$

B. $\sqrt{\frac{7m_2g}{(m_1 + m_2)r_0}}$

C. $\sqrt{\frac{3m_2g}{(m_1 + \frac{m_2}{2})r_0}}$

D. $\sqrt{\frac{3m_2g}{(m_1 + m_2)r_0}}$

Solution:

$$L = \frac{1}{2}(m_1 + m_2)\dot{r}^2 + \frac{1}{2}m_1r^2\dot{\theta}^2 - m_2g(l - r)$$

$$\text{Lagrangian equation of motion; } \frac{d}{dt}\left(\frac{\partial L}{\partial \dot{r}}\right) - \frac{\partial L}{\partial r} = 0$$

$$(m_1 + m_2)\ddot{r} - m_1r\dot{\theta}^2 + m_2g = 0$$

Hence angular momentum is conserved

$$m_1r^2\dot{\theta} = m_1r_0^2\dot{\theta}_0 \Rightarrow \dot{\theta} = \frac{r_0^2\dot{\theta}_0}{r^2}$$

$$\text{For circular motion } m r_0 \dot{\theta}_0^2 = m_2g$$

$$\text{so } r\dot{\theta}^2 = \frac{m_2}{m_1} \left(\frac{r_0}{r}\right)^3 g$$

$$(m_1 + m_2)\ddot{r} - m_2 \left(\frac{r_0}{r}\right)^3 g + m_2 g = 0$$

$$\text{Put } r = r_0 + \epsilon \Rightarrow \ddot{r} = \ddot{\epsilon}$$

$$(m_1 + m_2)\ddot{\epsilon} - m_2 \left(\frac{r_0}{r_0 + \epsilon}\right)^3 g + m_2 g \Rightarrow (m_1 + m_2)\ddot{\epsilon} - m_2 r_0^3 (r_0 + \epsilon)^{-3} g + m_2 g$$

$$(m_1 + m_2)\ddot{\epsilon} - m_2 r_0^3 g r_0^{-3} \left(1 + \frac{\epsilon}{r_0}\right)^{-3} + m_2 g = 0$$

$$(m_1 + m_2)\ddot{\epsilon} + \frac{m_2 3\epsilon}{r_0} = 0 \Rightarrow \omega = \sqrt{\frac{3m_2 g}{(m_1 + m_2) r_0}}$$

So the correct answer is **Option (D)**

15. Which of the following terms, when added to the Lagrangian $L(x, y, \dot{x}, \dot{y})$ of a system with two degrees of freedom will not change the equations of motion?
(check question)

[NET/JRF(DEC-2019)]

A. $x\ddot{x} - y\ddot{y}$

B. $x\ddot{y} - y\ddot{x}$

C. $x\dot{y} - y\dot{x}$

D. $y\dot{x}^2 + x\dot{y}^2$

Solution:

$$\begin{aligned} L(x, y, \dot{x}, \dot{y}) \\ \frac{d}{dt} \left(\frac{\partial L}{\partial \dot{x}} \right) - \frac{\partial L}{\partial x} = 0, \frac{d}{dt} \left(\frac{\partial L}{\partial \dot{y}} \right) - \frac{\partial L}{\partial y} = 0 \\ L' = L(x, y, \dot{x}, \dot{y}) + x\ddot{y} - y\ddot{x} \\ \frac{d'}{dt'} \left(\frac{\partial L'}{\partial \dot{x}} \right) - \frac{\partial L'}{\partial x} = \frac{d}{dt} \left(\frac{\partial L}{\partial \dot{x}} \right) - \frac{\partial L}{\partial x} + \ddot{y} \\ = 0 = 0 + \ddot{y} = 0 \Rightarrow \ddot{y} = c_1 \\ \frac{d}{dt} \left(\frac{\partial L}{\partial \dot{y}} \right) - \frac{\partial L}{\partial y} = \frac{d}{dt} \left(\frac{\partial L}{\partial \dot{y}} \right) - \frac{\partial L}{\partial y} + \ddot{x} \\ = 0 = 0 - \ddot{x} = 0 \Rightarrow \ddot{x} = c_2 \end{aligned}$$

So the correct answer is **Option (B)**

16. A point mass m , is constrained to move on the inner surface of a paraboloid of revolution $x^2 + y^2 = az$ (where $a > 0$ is a constant). When it spirals down the surface, under the influence of gravity (along $-z$ direction), the angular speed about the z -axis is proportional to

[NET/JRF(JUNE-2020)]

A. 1 (independent of z)

B. z

C. z^{-1}

D. z^{-2}

Solution:

Using Lagrangian in cylindrical coordinate

$$L = \frac{1}{2} m (\dot{r}^2 + r^2 \dot{\theta}^2 + \dot{z}^2) - mgz$$

with constraint $x^2 + y^2 = az \Rightarrow r^2 = az \Rightarrow \dot{z} = \frac{2r\dot{r}}{a}$

$$L = \frac{1}{2}m \left(\dot{r}^2 + r^2 \dot{\theta}^2 + \left(\frac{2r\dot{r}}{a} \right)^2 \right) - \frac{mgr^2}{a}$$

θ is cyclic coordinate so $\frac{\partial L}{\partial \theta} = 0 \Rightarrow \frac{\partial L}{\partial \dot{\theta}} = J \Rightarrow mr^2 \dot{\theta} = J \Rightarrow \dot{\theta} \propto \frac{1}{r^2} \propto \frac{1}{z}$

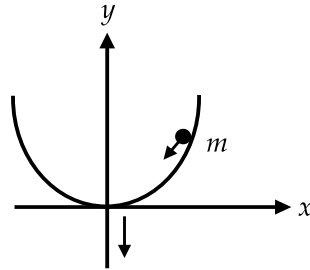
So the correct answer is **Option (C)**

Answer key			
Q.No.	Answer	Q.No.	Answer
1	B	2	D
3	A	4	D
5	C	6	A
7	A	8	A
9	D	10	D
11	B	12	A
13	C	14	D
15	B	16	C

Futuring
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Practice set-2

1. A particle of mass m slides under the gravity without friction along the parabolic path $y = ax^2$, as shown in the figure. Here a is a constant.



The Lagrangian for this particle is given by

[GATE 2012]

A. $L = \frac{1}{2}m\dot{x}^2 - mgax^2$

B. $L = \frac{1}{2}m(1 + 4a^2x^2)\dot{x}^2 - mgax^2$

C. $L = \frac{1}{2}m\dot{x}^2 + mgax^2$

D. $L = \frac{1}{2}m(1 + 4a^2x^2)\dot{x}^2 + mgax^2$

Solution:

Equation of constrain is given by $y = ax^2$, $K.E., T = \frac{1}{2}m(\dot{x}^2 + \dot{y}^2)$

$$\dot{y} = 2ax\dot{x} \Rightarrow T = \frac{1}{2}m(\dot{x}^2 + 4a^2x^2\dot{x}^2)$$

$$= \frac{1}{2}m\dot{x}^2(1 + 4a^2x^2)$$

$$V = mgy = mgax^2$$

$$\therefore L = T - V \Rightarrow L$$

$$= \frac{1}{2}m(1 + 4a^2x^2)\dot{x}^2 - mgax^2$$

So the correct answer is **Option (B)**

2. The Lagrange's equation of motion of the particle for above question is given by

[GATE 2012]

A. $\ddot{x} = 2gax$

B. $m(1 + 4a^2x^2)\ddot{x} = -2mgax - 4ma^2x\dot{x}^2$

C. $m(1 + 4a^2x^2)\ddot{x} = 2mgax + 4ma^2x\dot{x}^2$

D. $\ddot{x} = -2gax$

Solution:

$$\frac{d}{dt} \left(\frac{dL}{dx} \right) = \frac{dL}{dx} \Rightarrow m\ddot{x}(1 + 4a^2x^2)$$

$$= -4ma^2x\dot{x}^2 - 2mgax$$

So the correct answer is **Option (B)**

3. The Lagrangian of a system with one degree of freedom q is given by $L = \alpha \dot{q}^2 + \beta q^2$, where α and β are non-zero constants. If p_q denotes the canonical momentum conjugate to q then which one of the following statements is CORRECT?

[GATE 2013]

- A. $p_q = 2\beta q$ and it is a conserved quantity.
- B. $p_q = 2\beta \dot{q}$ and it is not a conserved quantity.
- C. $p_q = 2\alpha \dot{q}$ and it is a conserved quantity.
- D. $p_q = 2\alpha \dot{q}$ and it is not a conserved quantity.

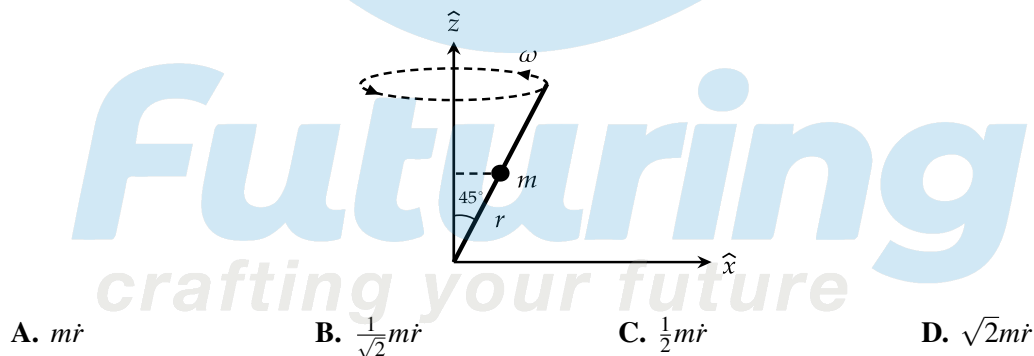
Solution:

$$\text{As, } \frac{\partial L}{\partial \dot{q}} = p_q \text{ but } \frac{\partial L}{\partial q} \neq 0. \text{ Thus, it is not a conserved quantity.}$$

So the correct answer is **Option (D)**

4. A bead of mass m can slide without friction along a massless rod kept at 45° with the vertical as shown in the figure. The rod is rotating about the vertical axis with a constant angular speed ω . At any instant r is the distance of the bead from the origin. The momentum conjugate to r is

[GATE 2014]



- A. $m\dot{r}$
- B. $\frac{1}{\sqrt{2}}m\dot{r}$
- C. $\frac{1}{2}m\dot{r}$
- D. $\sqrt{2}m\dot{r}$

Solution:

$$L = \frac{1}{2}m(\dot{r}^2 + r^2\dot{\theta}^2 + r^2\sin^2\theta\dot{\phi}^2) - mgr\cos\theta$$

Equation of constrain is $\theta = \frac{\pi}{4}$ and it is given $\dot{\phi} = \omega$

$$L = \frac{1}{2}m\left(\dot{r}^2 + \frac{1}{2}r^2\omega^2\right) - \frac{1}{\sqrt{2}}mgr$$

$$\text{Thus the momentum conjugate to } r \text{ is } p_r = \frac{\partial L}{\partial \dot{r}} \Rightarrow p_r = m\dot{r}$$

So the correct answer is **Option (A)**

5. The Lagrangian of a system is given by $L = \frac{1}{2}ml^2[\dot{\theta}^2 + \sin^2\theta\dot{\phi}^2] - mgl\cos\theta$, where m, l and g are constants. Which of the following is conserved?

[GATE 2016]

- A. $\dot{\phi}\sin^2\theta$
- B. $\dot{\phi}\sin\theta$
- C. $\frac{\dot{\phi}}{\sin\theta}$
- D. $\frac{\dot{\phi}}{\sin^2\theta}$

Solution:

Solution: As ϕ is cyclic coordinate, so $\frac{\partial L}{\partial \phi} = p_\phi = ml^2 \sin^2 \phi$, is a constant since m, l and g are constants. Thus $\dot{\phi} \sin^2 \theta$ is conserved.

So the correct answer is **Option (A)**

6. If the Lagrangian $L_0 = \frac{1}{2}m\left(\frac{dq}{dt}\right)^2 - \frac{1}{2}m\omega^2 q^2$ is modified to $L = L_0 + \alpha q \left(\frac{dq}{dt}\right)$, which one of the following is TRUE?

[GATE 2017]

- A. Both the canonical momentum and equation of motion do not change
- B. Canonical momentum changes, equation of motion does not change
- C. Canonical momentum does not change, equation of motion changes
- D. Both the canonical momentum and equation of motion change

Solution:

For Lagrangian $L_0 = \frac{1}{2}m\left(\frac{dq}{dt}\right)^2 - \frac{1}{2}m\omega^2 q^2$
canonical momentum is $p = m\dot{q}$

and equation of motion is given by

$$\frac{d}{dt} \left(\frac{\partial L}{\partial \dot{q}} \right) - \left(\frac{\partial L}{\partial q} \right) = 0 \Rightarrow m\ddot{q} + m\omega^2 q = 0$$

For Lagrangian $L = L_0 + \alpha q \left(\frac{dq}{dt}\right) \Rightarrow L$

$$= \frac{1}{2}m\left(\frac{dq}{dt}\right)^2 - \frac{1}{2}m\omega^2 q^2 + \alpha q \dot{q}$$

Canonical momentum is $p = m\dot{q} + \alpha q$

Equation of motion is,

$$\frac{d}{dt} \left(\frac{\partial L}{\partial \dot{q}} \right) - \left(\frac{\partial L}{\partial q} \right) = 0 \Rightarrow m\ddot{q} + m\omega^2 q = 0$$

So the correct answer is **Option (B)**

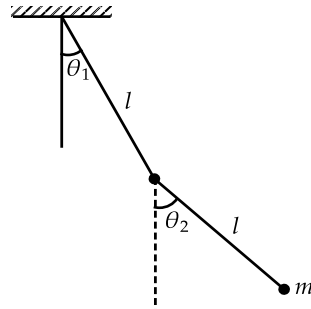
7. A double pendulum consists of two equal masses m suspended by two strings of length l . What is the Lagrangian of this system for oscillations in a plane? Assume the angles θ_1, θ_2 made by the two strings are small (you can use $\cos \theta = 1 - \theta^2/2$).
Note: $\omega_0 = \sqrt{g/l}$.

[JEST 2014]

- A. $L \approx ml^2 \left(\dot{\theta}_1^2 + \frac{1}{2} \dot{\theta}_2^2 - \omega_0^2 \theta_1^2 - \frac{1}{2} \omega_0^2 \theta_2^2 \right)$
- B. $L \approx ml^2 \left(\dot{\theta}_1^2 + \frac{1}{2} \dot{\theta}_2^2 + \dot{\theta}_1 \dot{\theta}_2 - \omega_0^2 \theta_1^2 - \frac{1}{2} \omega_0^2 \theta_2^2 \right)$
- C. $L \approx ml^2 \left(\dot{\theta}_1^2 + \frac{1}{2} \dot{\theta}_2^2 - \dot{\theta}_1 \dot{\theta}_2 - \omega_0^2 \theta_1^2 - \frac{1}{2} \omega_0^2 \theta_2^2 \right)$
- D. $L \approx ml^2 \left(\frac{1}{2} \dot{\theta}_1^2 + \frac{1}{2} \dot{\theta}_2^2 + \dot{\theta}_1 \dot{\theta}_2 - \omega_0^2 \theta_1^2 - \omega_0^2 \theta_2^2 \right)$

Solution:

$$\begin{aligned}
 x_1 &= l \sin \theta_1, \\
 y_1 &= l \cos \theta_1 \\
 x_2 &= x_1 + l \sin \theta_2 \\
 y_2 &= y_1 + l \cos \theta_2 \\
 \dot{x}_2 &= l \cos \theta_1 \dot{\theta}_1 + l \cos \theta_2 \dot{\theta}_2, \\
 \dot{y}_2 &= -l \sin \theta_1 \dot{\theta}_1 - l \sin \theta_2 \dot{\theta}_2 \\
 \dot{x}_2^2 + \dot{y}_2^2 &= l^2 \cos^2 \theta_1 \dot{\theta}_1^2 + l^2 \cos^2 \theta_2 \dot{\theta}_2^2 \\
 &\quad + 2l^2 \cos \theta_1 \dot{\theta}_1 \cos \theta_2 \dot{\theta}_2 + l^2 \sin^2 \theta_1 \dot{\theta}_1^2 \\
 &\quad + l^2 \sin^2 \theta_2 \dot{\theta}_2^2 + 2l^2 \sin \theta_1 \sin \theta_2 \dot{\theta}_1 \dot{\theta}_2 \\
 \Rightarrow \dot{x}_2^2 + \dot{y}_2^2 &= l^2 \dot{\theta}_1^2 + l^2 \dot{\theta}_2^2 + 2l^2 \cos(\theta_1 - \theta_2) \dot{\theta}_1 \dot{\theta}_2 \\
 \text{also } \dot{x}_1^2 + \dot{y}_1^2 &= l^2 \dot{\theta}_1^2
 \end{aligned}$$



$$\begin{aligned}
 L = T - V &= \frac{1}{2}m(\dot{x}_1^2 + \dot{y}_1^2 + \dot{x}_2^2 + \dot{y}_2^2) - mgy_1 - mgy_2 \\
 \Rightarrow L &= \frac{1}{2}m(l^2 \dot{\theta}_1^2 + l^2 \dot{\theta}_1^2 + l^2 \dot{\theta}_2^2 + 2l^2 \cos(\theta_1 - \theta_2) \dot{\theta}_1 \dot{\theta}_2) \\
 &\quad + 2mgl \cos \theta_1 + mgl \cos \theta_2 \\
 \Rightarrow L &= ml^2 \left[\dot{\theta}_1^2 + \frac{1}{2} \dot{\theta}_2^2 + \dot{\theta}_1 \dot{\theta}_2 + \frac{2g}{2l} \left(1 - \frac{\theta_1^2}{2} \right) + \frac{1}{2} \frac{g}{l} \left(1 - \frac{\theta_2^2}{2} \right) \right] \\
 [\because \cos(\theta_1 - \theta_2) &\approx 1] \\
 \Rightarrow L &= ml^2 \left[\dot{\theta}_1^2 + \frac{1}{2} \dot{\theta}_2^2 + \dot{\theta}_1 \dot{\theta}_2 + \frac{g}{l} - \frac{g}{l} \frac{\theta_1^2}{2} + \frac{g}{2l} - \frac{g}{2l} \frac{\theta_2^2}{2} \right]
 \end{aligned}$$

comparing given options, option (B) is correct i.e.

$$L = ml^2 \left(\dot{\theta}_1^2 + \frac{1}{2} \dot{\theta}_2^2 + \dot{\theta}_1 \dot{\theta}_2 - \frac{\omega_0^2 \dot{\theta}_1^2}{2} - \frac{1}{4} \omega_0 \dot{\theta}_2^2 \right)$$

So the correct answer is **Option (B)**

8. A bike stuntman rides inside a well of frictionless surface given by $z = a(x^2 + y^2)$, under the action of gravity acting in the negative z direction. $\vec{g} = -g\hat{z}$. What speed should be maintain to be able to ride at a constant height z_0 without falling down?

[JEST 2015]

A. $\sqrt{gz_0}$

B. $\sqrt{3gz_0}$

C. $\sqrt{2gz_0}$

D. The biker will not be able to maintain a constant height, irrespective of speed.

Solution:

$$z = a(x^2 + y^2)$$

Using equation of constrain, we must solve the given system in cylindrical co-ordinate.

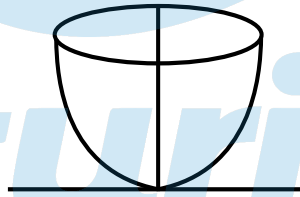
$$\begin{aligned}
 z &= ar^2, \dot{z} = 2ar\dot{r} \Rightarrow L \\
 &= \frac{1}{2}m(\dot{r}^2 + r^2\dot{\theta}^2 + \dot{z}^2) - mgz \\
 \Rightarrow L &= \frac{1}{2}m(\dot{r}^2 + r^2\dot{\theta}^2 + 4a^2r^2\dot{r}^2) - mgar^2 \\
 &= \frac{1}{2}m[\dot{r}^2(1 + 4a^2r^2) + r^2\dot{\theta}^2] - mgar^2
 \end{aligned}$$

Equation of motion

$$\begin{aligned}
 \frac{d}{dt}\left(\frac{\partial L}{\partial \dot{r}}\right) - \frac{\partial L}{\partial r} &= 0 \\
 \Rightarrow m\ddot{r}(1 + 4a^2r^2) + mr^2 4a^2\dot{r} - mr\dot{\theta}^2 + 2mgar &= 0 \\
 \text{At } z = z_0, \dot{r} = 0, \quad r = r_0, \text{ so, } mr_0\dot{\theta}^2 &= 2mgar_0
 \end{aligned}$$

$$\begin{aligned}
 \dot{\theta}^2 &= 2ga \Rightarrow \dot{\theta} = \sqrt{2ga}, \frac{v}{r_0} \\
 &= \sqrt{2ga}, v = \sqrt{2ga} \cdot r_0 \\
 v &= \sqrt{2ga} \cdot \left(\frac{z_0}{a}\right)^{1/2} \\
 &= \sqrt{2gz_0}
 \end{aligned}$$

$$(\because z_0 = ar_0^2)$$



So the correct answer is **Option (C)**

9. The Lagrangian of a particle is given by $L = \dot{q}^2 - q\dot{q}$. Which of the following statements is true?

[JEST 2015]

- A. This is a free particle
- B. The particle is experiencing velocity dependent damping
- C. The particle is executing simple harmonic motion
- D. The particle is under constant acceleration.

Solution:

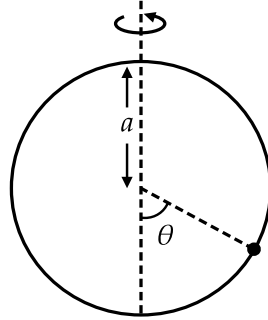
$$\begin{aligned}
 \because L &= \dot{q}^2 - q\dot{q} \Rightarrow \frac{\partial L}{\partial \dot{q}} \\
 &= 2\dot{q} - q \Rightarrow \frac{d}{dt}\left(\frac{\partial L}{\partial \dot{q}}\right) \\
 &= 2\ddot{q} - \dot{q} \\
 \because \frac{d}{dt}\left(\frac{\partial L}{\partial \dot{q}}\right) - \frac{\partial L}{\partial q} &= 0 \Rightarrow 2\ddot{q} - \dot{q} + \dot{q} \\
 &= 0 \Rightarrow 2\ddot{q} = 0 \Rightarrow \frac{d^2q}{dt^2}
 \end{aligned}$$

$$= 0 \Rightarrow \frac{dq}{dt} = C \Rightarrow q = Ct + \alpha$$

So the correct answer is **Option (A)**

10. A hoop of radius a rotates with constant angular velocity ω about the vertical axis as shown in the figure. A bead of mass m can slide on the hoop without friction. If $g < \omega^2 a$ at what angle θ apart from 0 and π is the bead stationary (i.e., $\frac{d\theta}{dt} = \frac{d^2\theta}{dt^2} = 0$)?

[JEST 2016]



A. $\tan \theta = \frac{\pi g}{\omega^2 a}$

B. $\sin \theta = \frac{g}{\omega^2 a}$

C. $\cos \theta = \frac{g}{\omega^2 a}$

D. $\tan \theta = \frac{g}{\pi \omega^2 a}$

Solution:

The Lagrangian of the system is

$$L = \frac{1}{2} m a^2 (\dot{\theta}^2 + \sin^2 \theta \dot{\phi}^2) + m g a \cos \theta$$

The equation of motion is,

$$\frac{d}{dt} \left(\frac{\partial L}{\partial \dot{\theta}} \right) - \left(\frac{\partial L}{\partial \theta} \right) = 0 \Rightarrow m a^2 \ddot{\theta} - m a^2 (\sin \theta \cos \theta \dot{\phi}^2) + m g a \sin \theta = 0$$

When bead is stationary, then

$$\begin{aligned} \frac{d\theta}{dt} &= \frac{d^2\theta}{dt^2} = 0 \\ &\Rightarrow -m a^2 (\sin \theta \cos \theta \dot{\phi}^2) + m g a \sin \theta = 0 \\ &\Rightarrow \dot{\phi} = \omega \text{ and } g < \omega^2 a, \text{ then } \cos \theta = \frac{g}{\omega^2 a} \end{aligned}$$

So the correct answer is **Option (C)**

11. A bead of mass M slides along a parabolic wire described by $z = 2(x^2 + y^2)$. The wire rotates with angular velocity Ω about the z -axis. At what value of Ω does the bead maintain a constant nonzero height under the action of gravity along $-\hat{z}$?

[JEST 2017]

A. $\sqrt{3g}$

B. \sqrt{g}

C. $\sqrt{2g}$

D. $\sqrt{4g}$

Solution:

$$\begin{aligned} L &= \frac{1}{2} m (\dot{r}^2 + r^2 \dot{\theta}^2 + 16 r^2 \dot{\phi}^2) - 2 m g r^2 \Rightarrow L \\ &= \frac{1}{2} m (\dot{r}^2 (1 + 16 r^2) + r^2 \dot{\theta}^2) - 2 m g r^2 \end{aligned}$$

The equation of motion is given by

$$\frac{d}{dt} \left(\frac{\partial L}{\partial \dot{r}} \right) - \frac{\partial L}{\partial r} = 0 \Rightarrow m\ddot{r}(1 + 16r^2) + 16m\dot{r}^2 r - m r \dot{\theta}^2 + 4mgr = 0$$

At equilibrium, $r = r_0$, $\dot{r} = 0$, $\ddot{r} = 0$

$$\text{So, } -mr_0\dot{\theta}^2 + 4mgr_0 = 0 \Rightarrow \dot{\theta} = \Omega = \sqrt{4g}$$

So the correct answer is **Option (D)**

12. A possible Lagrangian for a free particle is

[JEST 2017]

A. $L = \dot{q}^2 - q^2$

B. $L = \dot{q}^2 - q\dot{q}$

C. $L = \dot{q}^2 - q$

D. $L = \dot{q}^2 - \frac{1}{q}$

Solution:

$$\frac{d}{dt} \left(\frac{\partial L}{\partial \dot{q}} \right) - \left(\frac{\partial L}{\partial q} \right) = 0 \Rightarrow 2\ddot{q} - \dot{q} + \dot{q} = 0 \Rightarrow \ddot{q} = 0$$

So the correct answer is **Option (B)**

13. A rod of mass m and length l is suspended from two massless vertical springs with a spring constants k_1 and k_2 . What is the Lagrangian for the system, if x_1 and x_2 be the displacements from equilibrium position of the two ends of the rod?

[JEST 2017]

A. $\frac{m}{8} (\dot{x}_1^2 + 2\dot{x}_1\dot{x}_2 + \dot{x}_2^2) - \frac{1}{2}k_1x_1^2 - \frac{1}{2}k_2x_2^2$

B. $\frac{m}{2} (\dot{x}_1^2 + \dot{x}_1\dot{x}_2 + \dot{x}_2^2) - \frac{1}{4}(k_1 + k_2)(x_1^2 + x_2^2)$

C. $\frac{m}{6} (\dot{x}_1^2 + \dot{x}_1\dot{x}_2 + \dot{x}_2^2) - \frac{1}{2}k_1x_1^2 - \frac{1}{2}k_2x_2^2$

D. $\frac{m}{2} (\dot{x}_1^2 - 2\dot{x}_1\dot{x}_2 + \dot{x}_2^2) - \frac{1}{4}(k_1 - k_2)(x_1^2 + x_2^2)$

Solution:

$$T = \frac{1}{2}MV_{c,m}^2 + \frac{1}{2}I_{c,m}\omega^2 = \frac{1}{2}m \left(\frac{\dot{x}_1 + \dot{x}_2}{2} \right)^2 + \frac{1}{2} \frac{ml^2}{12} \dot{\theta}^2$$

Potential energy is, $V = \frac{1}{2}kx_1^2 + \frac{1}{2}kx_2^2$

$$\sin \theta = \frac{x_2 - x_1}{l} \text{ for small oscillation } \theta$$

$$= \frac{x_2 - x_1}{l} = \dot{\theta}$$

$$= \frac{\dot{x}_2 - \dot{x}_1}{l}$$

$$L = \frac{1}{2}m \left(\frac{\dot{x}_1 + \dot{x}_2}{2} \right)^2 + \frac{1}{2} \frac{ml^2}{12} \left(\frac{\dot{x}_1 - \dot{x}_2}{l} \right)^2 - \frac{1}{2}kx_1^2 - \frac{1}{2}kx_2^2$$

$$= \frac{m}{6} (\dot{x}_1^2 + \dot{x}_1\dot{x}_2 + \dot{x}_2^2) - \frac{1}{2}k_1x_1^2 - \frac{1}{2}k_2x_2^2$$

So the correct answer is **Option (C)**

14. Consider the Lagrangian

$$L = 1 - \sqrt{1 - \dot{q}^2} - \frac{q^2}{2}$$

of a particle executing oscillations whose amplitude is A . If p denotes the momentum of the particle, then $4p^2$ is

[JEST 2015]

- A. $(A^2 - q^2)(4 + A^2 - q^2)$ B. $(A^2 + q^2)(4 + A^2 - q^2)$
 C. $(A^2 - q^2)(4 + A^2 + q^2)$ D. $(A^2 + q^2)(4 + A^2 + q^2)$

Solution: So the correct answer is **Option (A)**

15. Consider the motion of a particle in two dimensions given by the Lagrangian

$$L = \frac{m}{2}(\dot{x}^2 + \dot{y}^2) - \frac{\lambda}{4}(x + y)^2$$

where $\lambda > 0$. The initial conditions are given as $y(0) = 0, x(0) = 42$ meters, $\dot{x}(0) = \dot{y}(0) = 0$. What is the value of $x(t) - y(t)$ at $t = 25$ seconds in meters?

[JEST 2019]

Solution:

$$L = \frac{m}{2}(\dot{x}^2 + \dot{y}^2) - \frac{\lambda}{4}(x + y)^2$$

The equation of motion is

$$\frac{d}{dt} \left(\frac{\partial L}{\partial \dot{x}} \right) - \left(\frac{\partial L}{\partial x} \right) = 0 \Rightarrow m\ddot{x} + \frac{\lambda}{2}x + \frac{\lambda}{2}y = 0 \quad (1)$$

$$\frac{d}{dt} \left(\frac{\partial L}{\partial \dot{y}} \right) - \left(\frac{\partial L}{\partial y} \right) = 0 \Rightarrow m\ddot{y} + \frac{\lambda}{2}y + \frac{\lambda}{2}x = 0 \quad (2)$$

Subtracting equation (1) from (2) gives $m(\ddot{x} - \ddot{y}) = 0 \Rightarrow \ddot{x} - \ddot{y} = 0$

Integrating both sides with t gives

$$\dot{x} - \dot{y} = c_1$$

From the equation $\dot{x}(0) = \dot{y}(0) = 0$, there $c_1 = 0$

$$\text{Hence, } \dot{x} - \dot{y} = 0 \quad (3)$$

Integrating both sides of this equation with t gives

$$x - y = c_2$$

Putting $x(0) = 42, y(0) = 0$ gives

$$42 - 0 = c_2 \Rightarrow 42$$

Therefore, $x - y = 42$

The value of $x - y$ is independent of t .

Therefore, at $t = 25s$

$$x(t) - y(t) = 42$$

Answer key			
Q.No.	Answer	Q.No.	Answer
1	B	2	B
3	D	4	A
5	A	6	B
7	B	8	C
9	A	10	C
11	D	12	B
13	C	14	A
15	42		





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