



1. Condensed Matter Physics

1.1 What Is Condensed Matter Physics?

“Condensed matter physics is the field of physics that deals with the macroscopic and microscopic physical properties of matter. In particular, it is concerned with the "condensed" phases that appear whenever the number of constituents in a system is extremely large and the interactions between the constituents are strong. The most familiar examples of condensed phases are solids and liquids, which arise from the electromagnetic forces between atoms.”

1.2 Lattice Vibrations

Lattice is defined as the periodic arrangement of atoms connected through elastic springs.
 at $T = 0K$, the energy of lattice is minimum and it will execute Simple Harmonic Motion(SHM)
 at $T \neq 0K$: as temperature increases particles start vibration and lattice energy increases oscillations will become anharmonic

Vibration of 1-Dimensional monoatomic lattice:

- Number of atoms in primitive unit cell, $P=1$
- Only nearest neighbours interaction is considered.
- All atoms perform Simple Harmonic Motion
- $\omega^2 = \frac{2c}{m}(1 - \cos ka) = \frac{4c}{m} \sin^2\left(\frac{ka}{2}\right)$ (Dispersion Relation)
- Energy quanta of lattice vibration (elastic waves) is phonon.

If K is small (λ large):

$$k \rightarrow 0 \quad \sin\left(\frac{ka}{2}\right) \simeq \frac{ka}{2}$$

$$\omega = \sqrt{\frac{c}{m}}ka \Rightarrow \omega \propto k$$

$$\text{Group velocity, } V_g = \frac{d\omega}{dk} = \sqrt{\frac{c}{m}}a$$

$$\text{Phase velocity, } V_p = \frac{\omega}{k} = \sqrt{\frac{c}{m}} a$$

$$V_g = V_p \text{ and independent of wave vector, } K$$

and at this wavelength, this medium behaves as non-dispersive medium

If K is large ($\lambda \rightarrow$ small):

at the Brillouin zone boundary, $v_g = 0$

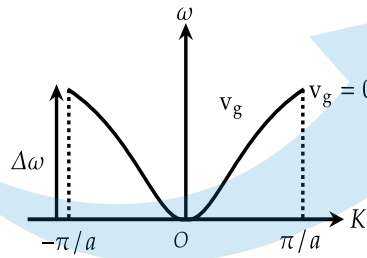
$$v_g = \frac{d\omega}{dk} = \sqrt{\frac{c}{m}} a \left| \cos\left(\frac{ka}{2}\right) \right| = 0$$

$$\cos \frac{ka}{2} = \cos \frac{\pi}{2}$$

$$\Rightarrow K = \pm \pi/a \rightarrow \text{Boundary of Brillouin zone}$$

$V_p \neq V_g$ hence it is, Dispersive medium.

$$\omega_{\max} = \sqrt{\frac{4c}{m}} \sin^2\left(\frac{ka}{2}\right) = \sqrt{\frac{4c}{m}}$$



each Brillouin zone represents a Band, so band width

$$\Delta\omega = \omega_{\text{top}} - \omega_{\text{bottom}}$$

$$= \sqrt{\frac{4c}{m}} - 0 \Rightarrow \sqrt{\frac{4c}{m}}$$

1.2.1 Lattice Vibrations in 1-D diatomic lattice

Dispersion Relation

$$\omega_{\pm}^2 = c \left(\frac{1}{m_1} + \frac{1}{m_2} \right) \pm c \sqrt{\left(\frac{1}{m_1} + \frac{1}{m_2} \right)^2 - \frac{4 \sin^2\left(\frac{ka}{2}\right)}{m_1 m_2}}$$

(a) Optical Mode

Optical Branch:

i) at $K \rightarrow 0$ ($\lambda \rightarrow$ long) : ($m_1 < m_2$)

$$\sin\left(\frac{ka}{2}\right) \simeq 0$$

$$\omega_{+}^2 = 2c \left(\frac{1}{m_1} + \frac{1}{m_2} \right) \Rightarrow \omega_{+} = \sqrt{2c \left(\frac{1}{m_1} + \frac{1}{m_2} \right)}$$

ii) at $K = \pi/a$ ($\lambda \rightarrow$ Short)

$$\sin\left(\frac{ka}{2}\right) \approx 1 \Rightarrow \omega_{+} = \sqrt{\frac{2c}{m_1}}$$

- At zone boundary ($K = \pm \pi/a$), frequency of optical branch depends upon lighter mass.

- Near centre of BZ ($K \approx 0$), frequency of optical branch depends upon both masses.

(b) Acoustical Branch

$$\omega_{\pm}^2 = c \left(\frac{1}{m_1} + \frac{1}{m_2} \right) - c \sqrt{\left(\frac{1}{m_1} + \frac{1}{m_2} \right)^2 - \frac{4 \sin^2(ka/2)}{m_1 m_2}}$$

i) at $k = 0$ (centre of Brillouin zone):

$\omega_{-} = 0$, Frequency of acoustical branch will be zero

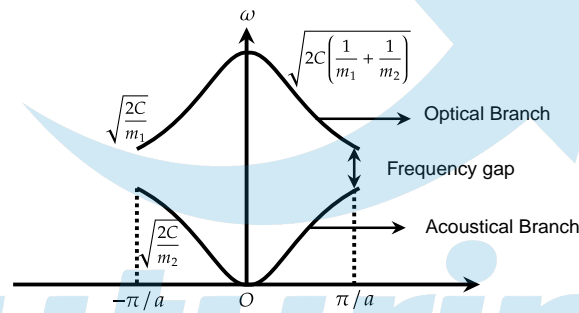
ii) at $k \rightarrow 0$ (long wave length):

$$\sin^2 \left(\frac{ka}{2} \right) \simeq \frac{k^2 a^2}{4} \Rightarrow \omega_{-} = \sqrt{\frac{c}{2(m_1 + m_2)}} ka$$

$$v_g = \frac{d\omega}{dk} = \sqrt{\frac{c}{2(m_1 + m_2)}} a$$

iii) at Boundary ($k \rightarrow \pi/a$):

$$\omega_{-}^2 = \frac{2c}{m_2} \Rightarrow \omega_{-} = \sqrt{\frac{2c}{m_2}}$$



- Frequency gap = $\sqrt{\frac{2c}{m_1}} - \sqrt{\frac{2c}{m_2}} \Rightarrow \propto \left(\sqrt{\frac{m_2}{m_1}} \right)^2$
- optical Branch: electro magnetic wave propagates in solid.
Acoustical: sound wave propagates in solid.
- Band width ($\Delta\omega$)
for optical is $\sqrt{2c} \left(\frac{1}{m_1} + \frac{1}{m_2} \right) - \sqrt{\frac{2c}{m_1}}$ and for acoustical : $\sqrt{\frac{2c}{m_2}}$

	1-Dimensional	2-Dimensional	3-Dimensional
no. of Branches	p	$2p$	$3p$
acoustical	1	2	3
optical	$p - 1$	$p - 2$	$p - 3$

1.3 Thermal Properties of Solids

Classical Theory (Dulong-Peppits theory)

- Each atom is considered as 3-Dimensional Harmonic oscillator
- Internal Energy, $U = 3NK_B T = 3RT$
- $C_V = \frac{\partial U}{\partial T} = 3R$ (constant)
- Valid for high temperature

- Fails at low temperature

Quantum theory

Each atom is considered as Quantum Harmonic Oscillator

a) Einstein's Theory :

$$C_V = 3R \left(\frac{\theta_E}{T} \right)^2 \frac{e^{\theta_E/T}}{(e^{\theta_E/T} - 1)^2}$$

at high temperature, $C_V \approx 3R$ (valid)

at low temperature, $C_V = 3R \left(\frac{\theta_E}{T} \right)^2 e^{-\theta_E/T}$ (invalid)

b) Debye Theory:

- Oscillators are now coupled and frequency of vibration depends on density of modes.
- C_V at low temperature $= \frac{12}{5} \pi^2 R \left(\frac{T}{\theta_D} \right)^3$ and $\theta_D = \frac{\hbar \omega_D}{K_B}$ (Debye temperature), Hence $C_V \propto T^3$ and it is valid with the experimentally observed results
- C_V at High temperature $= 3R$ (valid)

1.4 The classical Free Electron Theory of Metals (Drude-Lorentz Theory of Metals)

Drude and Lorentz proposed this theory in 1900. According to this theory, the metals containing the free electrons obey the laws of classical mechanics.

Assumptions (salient features) in classical free electron theory

The classical free electron theory is based on the following postulates.

1. The valence electrons of atoms are free to move about the whole volume of the metal, like the molecules of a perfect gas in a container.
2. The free electrons move in random direction and collide with either positive ions fixed to the lattice or the other free electrons. All the collisions are elastic in nature i.e., there is no loss of energy.
3. The momentum of free electrons obeys the laws of the classical kinetic theory of gases.
4. The electron velocities in a metal obey classical Maxwell-Boltzman distribution of velocities.
5. When the electric field is applied to the metal, the free electrons are accelerated in the direction opposite to the direction of applied electric field.
6. The mutual repulsion among the electrons is ignored, so that they move in all the directions with all possible velocities.
7. In the absence of the field, the energy associated with an electron at temperature T is given by $\frac{3}{2}kT$. It is related to the kinetic energy equation $\frac{3}{2}kT = \frac{1}{2}mv_{th}^2$. Here v_{th} represents the thermal velocity.

1.4.1 Classical free electron theory-electrical conductivity

The classical free electron theory was proposed by Drude and Lorentz. According to this theory the electrons are moving freely and randomly moving in the entire volume of the metal like gas atoms in the gas container. When an electric field is applied the free electrons get accelerated. When an electric field E is applied between the two ends of a metal of area of cross section A

Force acting on the electron in the electric field $= eE$

From Newton's second law $F = ma$

$$\text{The acceleration of electron } a = \frac{F}{m} = \frac{eE}{m}$$

The average velocity acquired (i.e. drift velocity) by the electrons by the application of electric field is

$$v_d = a\tau = \frac{eE\tau}{m}$$

$$v_d = a\tau = \frac{eE\tau}{m} = \frac{eE}{m} \frac{\lambda}{v_r}$$

Where v_r = RMS velocity.

The relation between current and drift velocity is

$$i = neAv_d$$

The current density is $J = nev_d$

$$\text{Substituting the value of } v_d \text{ we get } J = \frac{ne^2E\tau}{m}$$

$$\text{Conductivity } \sigma = \frac{J}{E} = \frac{ne^2\tau}{m}$$

$$\text{Resistivity } \rho = \frac{1}{\sigma} = \frac{m}{ne^2\tau}$$

$$\text{Mobility } \mu = \frac{v_d}{E} = \frac{e\tau}{m}$$

1.4.2 Success of classical free electron theory

1. It verifies ohm's law
2. It explains electrical conductivity of metals.
3. It explains thermal conductivity of metals.
4. It derives Widemann - Franz law. (i.e. the relation between electrical and thermal conductivity).

1.4.3 Draw backs of classical free electron theory.

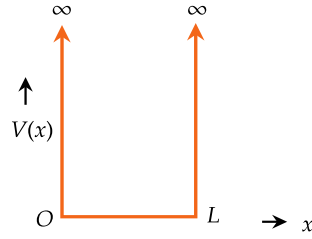
1. It could not explain the photoelectric effect, Compton Effect and black body radiation.
2. Electrical conductivity of semiconductors and insulators could not be explained.
3. Widemann - Franz law ($\frac{K}{\sigma T} = \text{constant}$) is not applicable at lower temperatures
4. Ferromagnetism could not be explained by this theory. The theoretical value of paramagnetic susceptibility is greater than the experimental value.
5. According to classical free electron theory the specific heat of metals is given by $4.5R$, where as the experimental value is given by $3R$
6. According to classical free electron the electronic specific heat is equal to $\frac{3}{2}R$ while the actual value is $0.01R$

1.4.4 Sommerfeld's Quantum Theory for Electronic Heat Capacity

Up to this point, we have only considered contributions to the heat capacity from vibrations within the solid. In metals, the free conduction electrons also contribute to the heat capacity. In the free electron model of metals, the conduction electrons are treated as a perfect gas obeying Fermi-Dirac statistic. Interactions of the electrons with the positively charged atomic ions and with the other electrons are neglected. This is not such a bad approximation as it may appear at first: the ions provide a positively charged background that partly screens the electrons from each other; and the residual collisions are often relatively unimportant - the energetically accessible final states are often already occupied, making any collisional excitation process forbidden by the Pauli exclusion principle. The first step in deriving the heat capacity is to determine the density of states. We will first do this in momentum space, and then transform the result into an expression describing the density of states per unit energy.

Free electron gas in one dimensional box:

We consider first a free electron gas in one dimension. We assume that an electron of mass m is confined to a length L by infinite potential barriers.



$$V(x) = 0 \text{ for } 0 < x < L$$

$$= \infty \text{ for } x \leq 0 \text{ and } x \geq L$$

The wave-function ψ_n of the electron occupying the n^{th} state is determined from the solution of the Schrodinger equation i.e.

$$\frac{d^2\psi_n}{dx^2} + \frac{2m}{\hbar^2} (E_n - V) \psi_n = 0 \Rightarrow \frac{d^2\psi_n}{dx^2} + \frac{2m}{\hbar^2} E_n \psi_n = 0$$

Note that this is a one-electron equation, which means that we neglect the electron-electron interactions. We use the term orbital to describe the solution of this equation.

The general solution to this equation is

$$\psi_n(x) = A \sin Kx + B \cos Kx \quad \text{where } K = \sqrt{\frac{2Em}{\hbar^2}}$$

Boundary condition $\psi_n(0) = 0$ and $\psi_n(L) = 0$ i.e. finally

Wave-function is $\psi_n(x) = A \sin\left(\frac{n\pi}{L}x\right)$;

where A is a constant and n is an integer. From here, we obtain for the eigenvalues

$$E_n = \frac{n^2 \hbar^2 \pi^2}{8mL^2} \quad \text{where } A = \sqrt{\frac{2}{L}}$$

These solutions correspond to standing waves with a different number of nodes within the potential well. Now we need to accommodate N valence electrons in these quantum states. According to the Pauli exclusion principle no two electrons can have their quantum number identical. That is, each electronic quantum state can be occupied by at most one electron. The electronic state in a 1D solid is characterized by two quantum numbers that are n and m_s , where n describes the orbital $\psi_n(x)$, and m_s describes the projection of the spin momentum on a quantization axis. Electron spin is equal to $S = 1/2$, so that there $(2S + 1) = 2$ possible spin states with $m_s = \pm \frac{1}{2}$. Therefore, each orbital labelled by the quantum number n can accommodate two electrons, one with spin up and one with spin down orientation.

Let n_F denote the highest filled energy level, where we start filling the levels from the bottom ($n = 1$) and continue filling higher levels with electrons until all N electrons are accommodated. It is convenient to suppose that N is an even number. The condition $2n_F = N$ determines n_F , the value of n for the uppermost filled level. The energy of the highest occupied level is called the Fermi energy E_F , which will be solved in the next section. All the electronic levels are filled up to the Fermi energy. All the levels above are empty.

Density of state

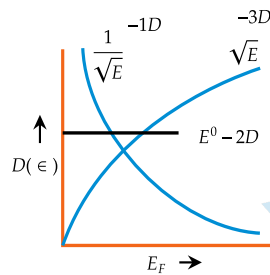
The Density of state is defined as no of electronic state present in a unit energy range, it is denoted by $D(\epsilon)$ and is given by $D(\epsilon) = \left(\frac{dn}{dE}\right)$

Where dn represents the no of electronic quantum states present in the energy interval E and $E + dE$, for a free electron gas, since each energy level contains two electronic states, one with spin up and the other with spin down. The value given by

$$D(E) = 2 \left(\frac{dn}{dE} \right) \Rightarrow \frac{dE}{dn} = \frac{\hbar^2}{2m} \left(\frac{\pi}{L} \right)^2 2n = \frac{\hbar^2 n}{4mL^2} \Rightarrow D(E) = 2 \times \left(\frac{4mL^2}{nh^2} \right) = \frac{8mL^2}{h^2} \times \frac{L}{n}$$

$$\frac{1}{n} = \left(\frac{h^2}{8mL^2 E} \right)^{1/2}$$

$$\text{Hence } D(E) = \left(\frac{8mL^2}{h^2 E} \right)^{1/2} = \frac{4L}{h} \left(\frac{m}{2E} \right)^{1/2}$$



$$D(E) \propto (E)^{-1/2} \quad \dots\dots 1D$$

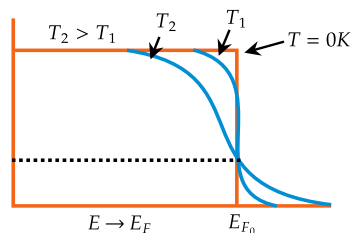
Note $D(E) \propto E^0 \quad \dots 2D$

$$D(E) \propto \sqrt{E} \quad \dots 3D$$

Filling of Energy level: Fermi-Dirac distribution function

The Density of States tells us what states are available. We now wish to know the occupancy of these states. Electrons obey the Pauli exclusion principle. So, we may only have two electrons (one spin-up and one spin-down) in any quantum state. The distribution of electrons in the allowed energy level follows the Pauli's exclusion principle each energy level is doubly degenerate thus a total of N non-interacting electrons at $0K$ can be filled in $N/2$ energy levels, the top most filled level is the $(N/2)$ th level and the level lying above it are empty. This level is therefore, the Fermi level as it divides the filled and empty levels at $0K$.

These electrons are rather distributed among the discrete energy level having energies ranging from 0 to E_{F0} .



The Fermi distribution function is $f(E) = \frac{1}{e^{E-E_F/kT} + 1}$ for occupied. At absolute zero the Equation

$$F(E) = 1 \quad \text{for } E \leq E_{F0}$$

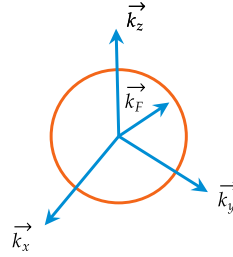
$$= 0 \quad \text{for } E > E_{F0}$$

The Fermi distribution function is a step function, at 0 K. For temperature, greater than 0 K but less than the melting point of the metal such that $k_B T \ll E_F$ the distribution function loses its step character.

The probability of occupation $f(E)$ decreases gradually from 1 to 0 near E_F this indicates that some of the state below E_F are empty while some others above it are filled. This is because some of the electrons from the energy state below E_F gain thermal energy and get excited to the state above E_F at $E = E_F$

$$f(E_F) = \frac{1}{2}$$

Density of State in Three Dimensions:



Fermi energy is given as $E_F = \frac{p^2}{2m} = \frac{\hbar^2 K_F^2}{2m}$

$$E_F = \frac{\hbar^2}{2m} K_F^2 \text{ where } K_F \text{ is Fermi radius } K_F = \left(\frac{3\pi^2 N}{V} \right)^{1/3}$$

$$E_F = \frac{\hbar^2}{2m} \left(\frac{3\pi^2 N}{V} \right)^{2/3} = E_F = 3.65 \times 10^{-19} n^{2/3} \text{ eV}$$

i.e. E_F depends on both the Electronic concentration and mass

$$\text{The Total no. of electron is Therefore } N = \frac{V}{3\pi^2} K_F^3 = \frac{V}{3\pi^2} \left(\frac{2mE_F}{\hbar^2} \right)^{3/2}$$

\therefore Total No of state is equal to the total No of electron is i.e

$$\int_0^{E_F} D(E) dE = N = \frac{V}{3\pi^2} \left(\frac{2mE_F}{\hbar^2} \right)^{3/2}$$

$$\int D(\epsilon) dE = \int \frac{V}{2\pi^2} \left(\frac{2m}{\hbar^2} \right)^{3/2} E^{1/2} dE$$

$$D(\epsilon) = \frac{V}{2\pi^2} \left(\frac{2m}{\hbar^2} \right)^{3/2} E^{1/2} \Rightarrow D(\epsilon) \propto \sqrt{E} - 3D$$

This result may also be obtained and expressed in a simple way by taking the natural logarithm of expression

$$\ln N = 3/2 \ln E_F + \text{constant}$$

$$\frac{dN}{N} = \frac{3}{2} \frac{dE_F}{E_F} \quad \text{or} \quad \frac{dN}{dE_F} = 3/2 \left(\frac{N}{E_F} \right) \quad \text{at} \quad E = E_F$$

The Density of filled electronic state $N(E)$, at any temperature T is obtain by multiplying the density of state $D(E)$ at 0 K by probability occupation $f(E)$ of the quantum state E at that temperature.

$$N(\epsilon) = D(\epsilon) f(E)$$

$$N(\epsilon) dE = D(\epsilon) f(E) dE$$

Fermi Energy (E_{F0})

Fermi energy is an important parameter, which can be determined by the following method. If the total number of free electrons in a metal is N , then we have

$$\begin{aligned}
 N &= \int_0^\infty N(\epsilon) dE = \int_0^\infty D(\epsilon) f(E) dE \\
 &= \frac{V}{2\pi^2} \left(\frac{2m}{\hbar^2} \right)^{3/2} \int_0^\infty E^{1/2} f(E) dE \\
 &= \frac{V}{2\pi^2} \left(\frac{2m}{\hbar^2} \right)^{3/2} \left[\int_0^{E_{F0}} E^{1/2} f(E) dE + \int_{E_{F0}}^\infty E^{1/2} f(E) dE \right] \\
 \epsilon &< E_F \\
 &= \frac{V}{2\pi^2} \left(\frac{2m}{\hbar^2} \right)^{3/2} \int_0^{E_{F0}} E^{1/2} dE \\
 E_{F0} &= \frac{\hbar^2}{2m} \left(\frac{3\pi^2 N}{V} \right)^{2/3} = \frac{\hbar^2}{2m} (3\pi^2 n)^{2/3} \Rightarrow E_F \propto n^{2/3}
 \end{aligned}$$

Where $n = \frac{N}{V}$ is the number density of electrons (number of electrons per unit volume).

Average Kinetic Energy \bar{E}

$$\begin{aligned}
 \bar{E}_0 &= \frac{1}{N} \int_0^\infty E D(E) f(E) dE \\
 \bar{E}_0 &= \frac{V}{2\pi^2} \left(\frac{2m}{\hbar^2} \right)^{3/2} \frac{1}{N} \left[\int_0^{E_{F0}} E E^{1/2} f(E) dE + \int_{E_F}^\infty E E^{1/2} f(E) dE \right] \\
 &= \frac{V}{2\pi^2} \left(\frac{2m}{\hbar^2} \right)^{3/2} \frac{1}{N} \int_0^{E_{F0}} E^{3/2} dE = \frac{2}{5} \frac{V}{2\pi^2} \left(\frac{2m}{\hbar^2} \right)^{3/2} \frac{1}{N} E_F^{5/2} \\
 \bar{E}_0 &= \frac{3}{5} E_{F0} \quad \text{--- 3D}
 \end{aligned}$$

Note

Average energy in 1-dimension and 2-dimensions are also written as

$$\left. \begin{aligned} \bar{E} &= \frac{1}{3} E_F \text{ --- 1D} \\ \bar{E} &= \frac{1}{2} E_F \text{ --- 2D} \end{aligned} \right\}$$

Electron Specific heat:

The Free electron gas model explains the electronic to specific heat of metals by restricting the No of electrons contributing to specific heat to $N(K_B T/E_F)$. It yielded the expression for specific heat as

$$C_V = \frac{3}{2} N K_B \left(\frac{K_B T}{E_F} \right) \Rightarrow C_V \approx \frac{3}{2} N K_B \left(\frac{T}{T_F} \right), \quad C_V \propto \left(\frac{T}{T_F} \right)$$

1.5 Band Theory of Solid

The failure of the free electron model is due to the over simplified assumption that a conduction electron in a metal experiences a constant or zero potential due to the ion cores and hence is free to move about crystal. Now the periodic potential described above forms the basis of the band Theory of solids. The behavior of an electron in this potential is describe by constructing the electron wave functions using One-electron approximates. As we shall discuss later, the motion of an electron in a periodic potential yields the following results.

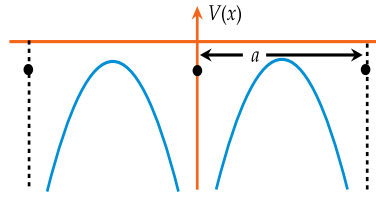


Figure 1.1

- There exists an allowed energy band separated by forbidden region or band gap.
- The electronic energy function $E(K)$ are periodic in the wave vector K
In the free electron Theory E varies with K

$$E_K = \frac{\hbar^2 k^2}{2m}$$

1.5.1 The Bloch Theorem

Bloch's theorem is just a way to describe the wavefunction for periodic solids. Bloch's theorem constrains ψ and thus E for periodic solids. The 1-D Schrödinger equation for an electron moving in a constant potential V_0 is $\frac{d^2\psi}{dx^2} + \frac{2m}{\hbar^2}(E - V_0)\psi = 0$. The solution $\psi(x) = e^{\pm ikx}$. For periodic potential with period equal to the lattice constant a we have

$$\psi(x) = e^{\pm ikx} U_k(x) \quad \text{where } U_k = U_k(x + a)$$

Bloch's theorem in 3-dimensions: Bloch's theorem contains two postulates

- Because we have a solid that is periodic at the atomic scale, we get a traveling wave solution $(e^{i\vec{k}\cdot\vec{r}})$ for ψ that is modulated by the translational symmetry of the lattice $(u_{\vec{k}})$

$$\psi_{\vec{k}}(\vec{r}) = u_{\vec{k}}(\vec{r}) e^{i\vec{k}\cdot\vec{r}} \quad \text{where } u_{\vec{k}}(\vec{r}) = u_{\vec{k}}(\vec{r} + \vec{T})$$

Here, wavefunction reflects the symmetry of the lattice via $u_{\vec{k}}(\vec{r})$ and Traveling wave solution suggests that we must have invoked periodic boundary conditions for the macroscopic solid.

- Alternatively, we can describe the wavefunction a $\psi_{\vec{k}}(\vec{r} + \vec{T}) = c\psi_{\vec{k}}(\vec{r})$. Which is to say, in a slightly different way, the wavefunction at $r + T$ is very related to the wavefunction at r . Here, c is a modulation term, we'll explore in detail below

1.5.2 The Kronig-Penney Model

This model illustrates the behavior of electrons in a periodic potential by assuming a relatively simple one-dimensional model of periodic potential as shown figure.

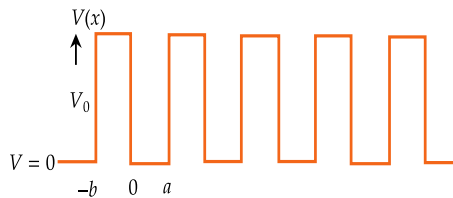


Figure 1.2

For this potential write down the Schrödinger wave equation and its general solution with taking potential constant finally. We get

$$P \cdot \frac{\sin \alpha a}{\alpha a} + \cos \alpha a = \cos ka \quad \text{where } P = \frac{mV_0 b a}{\hbar^2}$$

which is a measure of the area $V_0 b$ of the potential barrier. Thus, increasing P has the physical meaning of bonding an electron more strongly to a particular potential well

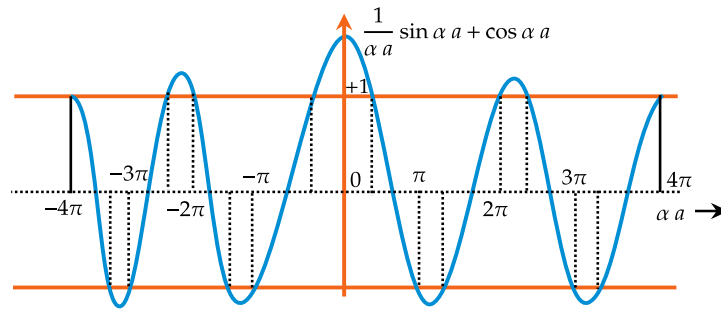


Figure 1.3

$$\frac{1}{\alpha a} \sin \alpha a + \cos \alpha a$$

$$\text{We know that } E = \frac{\alpha^2 \hbar^2}{2m} = \frac{\pi^2 \hbar^2}{2ma^2} n^2$$

It may be noted that since α^2 is proportional to the energy E the abscissa is a measure of the energy. The following conclusion may be drawn from figure.

- i) The energy spectrum of the electrons consists of alternate regions of allowed energy bands (solid lines on abscissa) and forbidden energy band (broken lines)
- ii) The width of the allowed energy bands increases with αa or the energy
- iii) The width of particular allowed energy band decreases with increase in value of P .

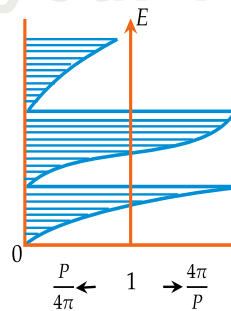


Figure 1.4

Figure- Allowed (shaded) and forbidden (open) energy ranges as a function of P
Energy Verses Wave-Vector relationship

The energy E is also an even periodic function of k with period of $2\pi/a$ i.e $k = \pm n\pi/a$

$$dn = \frac{l}{2\pi} dk$$

Velocity is $v = \frac{1}{\hbar} \frac{dE}{dk}$.

1.5.3 Concept of effective mass electrons and Holes

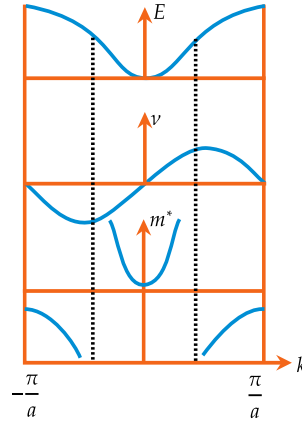


Figure 1.5

In one dimension, an electron with wave-vector k has group velocity

$$v = \frac{d\omega}{dk} = \frac{1}{\hbar} \frac{dE}{dk}$$

If an electric field ε acts on the electron, then in time δt . It will do work

$$\delta E = \text{force times distance} = -e\varepsilon v \delta t$$

$$\text{But } \delta E = \frac{dE}{dk} \delta k = \hbar v \delta t$$

So, comparing equation (ii) with (iii), we have

$$\delta k = -\frac{e\varepsilon}{\hbar} \delta t, \quad \text{or} \quad \hbar \frac{dk}{dt} = -e\varepsilon$$

$$\text{In terms of force } F, \quad \hbar \frac{dk}{dt} = F$$

$$\text{Generalising to three dimensions: } v = \frac{1}{\hbar} \nabla_k E$$

$$\text{where } \nabla_k = \hat{i} \frac{d}{dk_x} + \hat{j} \frac{d}{dk_y} + \hat{k} \frac{d}{dk_z} \quad \text{and} \quad \hbar \frac{dk}{dt} = F$$

$$\text{From equation (i)} \quad v = \frac{d\omega}{dk} = \frac{1}{\hbar} \frac{dE}{dk},$$

$$\text{Differentiating with respect to time } \frac{dv}{dt} = \frac{1}{\hbar} \frac{d^2 E}{dk dt} = \frac{1}{\hbar} \frac{d^2 E}{dk^2} \frac{dk}{dt}$$

$$\text{But from equation (iv), } \hbar \frac{dk}{dt} = F$$

$$\text{So } \frac{dv}{dt} = \frac{1}{\hbar^2} \frac{d^2 E}{dk^2} F$$

$$\text{But from Newton's equation we expect } \frac{dv}{dt} = \frac{1}{m} F$$

$$\text{which leads us to define an effective mass } \frac{1}{m^*} = \frac{1}{\hbar^2} \frac{d^2 E}{dk^2}$$

That is

The dynamics of electrons is modified by the crystal potential;

The effective mass depends on the curvature of the bands;

Flat bands have large effective masses;

Near the bottom of a band, m^* is positive, near the top of a band, m^* is negative.

Hole Concept

Hole wavevector: The total k of a full band is zero: if we remove an electron with wavevector k_e the total k of the band is $k_h + k_e = 0 \Rightarrow k_h = -k_e$

Hole energy: Take the energy zero to be the top of the valence band. The lower the electron energy, the more energy it takes to remove it; thus

$$E_h(k_h) = -E_e(k_e)$$

But bands are usually symmetric, $E(k) = E(-k)$

$$\text{So } E_h(k_h) = E_h(-k_h) = -E_e(k_e)$$

Hole velocity: In three dimensions

$$v_h = \frac{1}{\hbar} \nabla_{k_h} E_h$$

$$\text{But } k_h = -k_e \quad \text{So } \nabla_{k_h} = -\nabla_{k_e}$$

$$\text{and so } v_h = -\frac{1}{\hbar} \nabla_{k_e} (-E_e) = v_e$$

The group velocity of the hole is the same as that of the electron.

Hole effective mass: The curvature of E is just the negative of the curvature of $-E$, So

$$m_h^* = -m_e^*$$

1.6 Semiconductor

According to Band Gap, Semiconductors can be classified into

1. Direct Band gap semiconductor

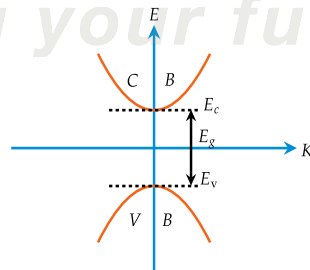


Figure 1.6

- electrons directly jump from top of valence band to bottom of conduction band
- No of energy or momentum of incident photon is transferred to the phonon.
i.e, no phonon involvement
- Semiconductor entire energy is absorbed or emitted in the form of optical energy.
 $\lambda = \frac{hc}{E_g}$ (Used in light-emitting applications)
- example: CdSi, GaAs, CdS etc.

2. Indirect Band Gap Semiconductor

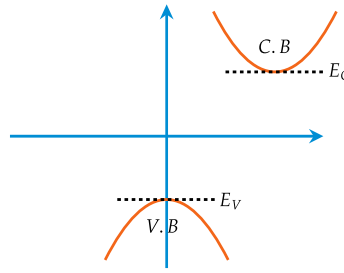


Figure 1.7

- Some energy or momentum of incident photon is transferred to the phonon, to jump electrons from top of valence band to bottom of conduction band. i.e, phonon involvement.
- Here, some part of energy is converted into heat. Hence, they are not used for light emitting applications.
- examples: Si, Ge, SiC etc.

According to Doping Concentration Semiconductors can be classified into

1. Intrinsic Semiconductor (Undoped/Pure)

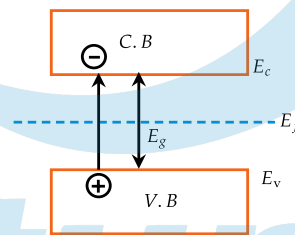


Figure 1.8

- Behaves as insulator at $t = 0K$
- Band gap energy, $E_g = E_C - E_V$
- Fermi energy, $E_F = \frac{E_C + E_V}{2}$ at $T = 0 K$
- Hole is the Vacancy of electrons and it is a fermion.
- electron concentration (n) = $\frac{\text{number of electrons}}{\text{volume}}$
- hole concentration (p) = $\frac{\text{number of holes}}{\text{volume}}$
- Intrinsic carrier concentration, $n_i = p_i = (N_C N_V)^{1/2} e^{-E_g/(2k_B T)}$
- Here Density of states at conduction band edge, $N_C = 2 \left(\frac{2\pi m_e^* K_B T}{h^2} \right)^{3/2}$ and Density of states at valence band edge, $N_V = 2 \left(\frac{2\pi m_h^* K_B T}{h^2} \right)^{3/2}$
- exact position of fermi level at temperature, $T = E_F = \left(\frac{E_C + E_V}{2} \right) + \frac{3}{4} k_B T \ln \left(\frac{m_h^*}{m_e^*} \right)$

2. Extrinsic Semiconductor

1. n-type

- Electrons are majority charge carriers

- Formed by doping the intrinsic Si with pentavalent impurities like P, A, Sb, Bi etc

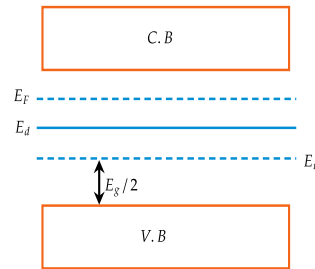


Figure 1.9

- At T increases E_F will moves towards E_i at $T = 0K$
- n -type semiconductor is electrically neutral i.e, net charge = 0
- majority carrier concentration (n) = donor density (N_d)
- $K_{BT} = 25\text{meV}$ at $T = 300\text{ K}$
- as T increases, E_F will moves towards E_i
- N_d remains constant at all temperatures
- Extrinsic semiconductor will become intrinsic semiconductor by raising the temperature and by doping opposite type of impurity.

Law of Mass Action

By this law we can calculate majority and minority carrier concentration.

majority carrier concentration (n) = donor concentration (N_d)

minority concentration = p

$$np = n_i p_i$$

$$np = n_i^2$$

Conductivity :

$$\sigma = ne\mu_e + pe\mu_h$$

$$\sigma \approx ne\mu_e$$

2. p-type

- holes are majority charge carriers
- formed by doping the intrinsic Si with trivalent impurity like B, Ga, In, Al etc.

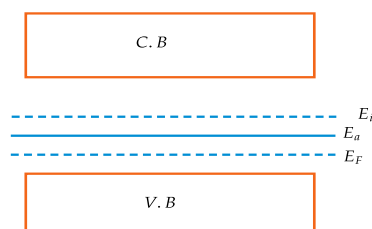


Figure 1.10

- As temperature increases E_F will move towards E_i i.e., extrinsic semiconductor becomes intrinsic semiconductor at $T = 0K$
- p-type semiconductor is electrically neutral.
- majority carrier concentration(P)=acceptor concentration(Na)

law of mass action

$$np = n_i^2 = p_i^2$$

$$n_i = (N_C N_V)^{1/2} e^{-E_g/2K_B T}$$

$$\sigma \approx pe\mu_h$$

Exercise 1.1 Calculate the expression of minimum conductivity in a doped semiconductor? ■

Solution:

$$\sigma = ne\mu_e + pe\mu_h$$

$$np = n_i^2 \Rightarrow p = \frac{n_i^2}{n}$$

$$\sigma = ne\mu_e + \frac{n_i^2}{n}e\mu_h$$

For minimum conductivity $\frac{d\sigma}{dn} = 0$

$$e\mu_e - \frac{n_i^2}{n^2}e\mu_h = 0$$

$$n = n_i \sqrt{\frac{\mu_h}{\mu_e}} \quad \text{similarly } p = n_i \sqrt{\frac{\mu_e}{\mu_h}}$$

$$\sigma_{\min} = n_i \sqrt{\frac{\mu_h}{\mu_e}} e\mu_e + n_i \sqrt{\frac{\mu_e}{\mu_h}} e\mu_h$$

$$\sigma_{\min} = 2n_i e \sqrt{\mu_e \mu_h}$$

1.7 Hall Effect

The production of voltage difference (Hall voltage) across an electrical conductor, transverse to an electric current in the conductor and to an applied magnetic field perpendicular to the current.

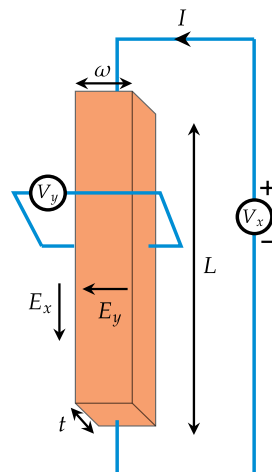


Figure 1.11

Hall effect in semiconductor

$$\text{Hall coefficient, } R_H = \frac{p\mu_h^2 - n\mu_e^2}{e(p\mu_h + n\mu_e)^2}$$

μ_h = hole mobility, μ_e = electron mobility, e = charge of electron

1. Intrinsic semiconductor

$$R_H = \frac{\mu_h - \mu_e}{en_i(\mu_h + \mu_e)}$$

R_H will be negative since $\mu_h < \mu_e$

2. n-type semiconductor

$$n\mu_e \gg p\mu_h$$

$$R_H = -\frac{1}{ne}$$

$$(n = N_d)$$

3. p-type semiconductor

$$p\mu_h \gg n\mu_e$$

$$R_H = +\frac{1}{pe}$$

$$(p = N_a)$$

Hall Effect in Metals

$$R_H = -\frac{1}{ne}$$

Because metals have only electrons as charge carriers.

Hall Coefficient

$$R_H = \frac{E_H}{J_x B_z}$$

$$V_H = -\frac{I_x B_z}{net}$$

Here V_H is Hall Voltage, E_H is the Electric field due to Hall effect

J_x is the current density due to the current I_x and B_z is the applied magnetic field

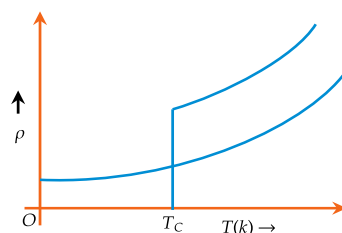
1.8 Superconductivity**1.8.1 Properties of Superconductors**

Figure 1.12

1. At room temperature, the resistivity of ρ of super conducting materials is greater than other element shows as

2. All thermoelectric effects disappear in super conducting state.
3. When a sufficient strong magnetic field is applied to super conductor below critical temperature T_C , it super conducting property is destroyed.
4. When current is passed through the super conducting materials, the heating loss (I^2R) is zero.

As resistivity $\rho = \frac{RA}{l}$ $\rho \rightarrow$ very small (zero at T_C)
 $R = 0$ Hence no heating loss.

5. Flux quantization due to ring is $\phi = n \frac{h}{2e}$

1.9 Meissner Effect

Meissner and Ochsenfeld found that if a super conductor is cooled in a magnetic field to below the critical temperature (transition temperature) then at the transition, the lines of induction are pushed out. The expulsion of magnetic flux from the interior of a piece of super-conducting material as the material undergoes the transition to the super-conducting phase is known as Meissner effect. Figure shows the normal sphere at $T > T_C$ and super conducting sphere at $T < T_C$ showing the expulsion of magnetic lines of induction. Meissner effect is reversible when the temperature is raised from below T_C the flux suddenly penetrates the specimen after it reaches T_C and the substance is in the normal state.

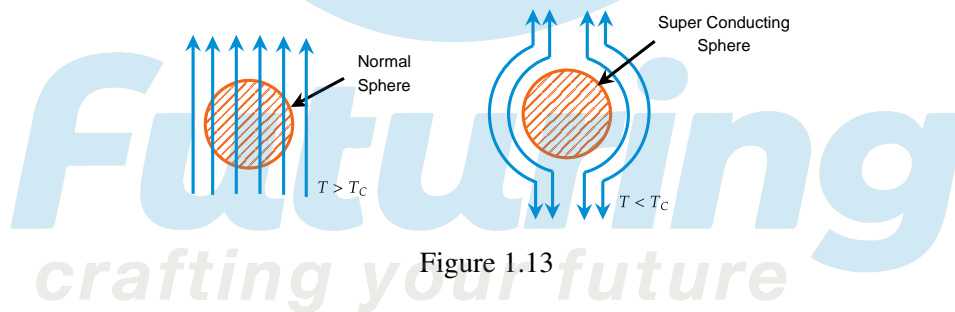


Figure 1.13

As $\vec{B} = \mu_0(\vec{H} + \vec{M})$

$$\vec{B} = \mu_0(1 + \chi)\vec{H} \text{ where } \chi = \frac{\vec{M}}{\vec{H}}$$

Since $\vec{B} = 0$ for super conductor state

$$\chi = -1 \text{ and also } \chi = \mu_r - 1 \text{ hence } \mu_r = 0$$

i.e. a super conductor exhibit perfect diamagnetism. Because of diamagnetic nature, superconducting materials strongly repel external magnet, it leads to a levitation effect.

Properties of Superconductor

Critical Field

The minimum applied magnetic field necessary to destroy super conductivity and restore the normal resistivity is called the critical field H_C , when the magnetic field exceeds the critical value H_C , the super conducting state is destroyed and the material goes into the normal state. Figure shows the critical field H_C as a function of temperature. A specimen is superconducting below the curve and normal above the curve.

For a given substance, value of H_C decrease as temperature increases from $T = 0$ K to T_C (Critical temperature) the curve is nearly parabolic and can be presented as

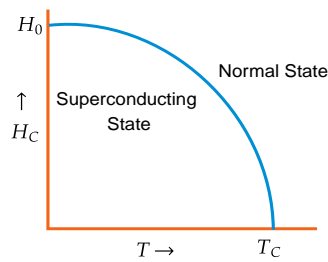


Figure 1.14

$$H_C = H_0 \left[1 - \left(\frac{T}{T_C} \right)^2 \right]$$

where H_0 is the critical field at 0 K.

thus the field has its maximum value H_0 at $T = 0$ K

$$\text{at } T = 0 \quad H_C = H_0 \left[1 - \frac{0}{T_C^2} \right] = H_0$$

$$\text{at } T = T_C \quad H_C = H_0 [1 - 1] = 0$$

Equation is the phase boundary between the normal and super conducting state.

Exercise 1.2 The critical field for niobium is 1×10^5 A/m at 8k and 2×10^5 A/m at 0 K. Calculate the critical temperature of the material. ■

Solution:

$$T_C = \frac{T}{\left[1 - \left(\frac{H_C}{H} \right) \right]^{1/2}} = 11.31 \text{ K}$$

Critical Current Density

The minimum current that can be passed in a sample without destroying its superconductivity is called critical current I_C and its flux will $\phi = \left(\frac{h}{2e} \right)$. If a super conducting material carries a current such that the magnetic field which it produce is equal to H_C , the super conductivity disappears. The current density J at which the super conductivity disappear is called the critical current density J_C for any value of $J < J_C$ the current can sustain itself whereas for values of $J > J_C$ the current cannot sustain itself. This effect is known as Silsbee effect. A super conducting ring of radius r ceases to be a super conductor when the current is

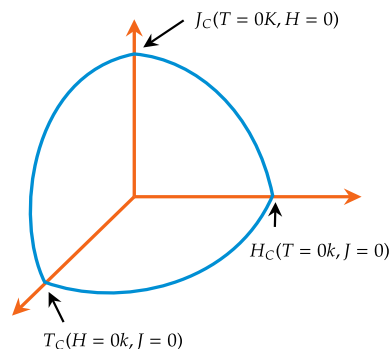


Figure 1.15

$$I_C = 2\pi r H_C$$

$$\text{Critical current density } J_C = \frac{I_C}{\text{Area}} = \frac{2\pi r H_C}{\pi r^2}$$

$$J_C = \frac{2H_C}{r}$$

Dependence of J , H and T is shown in figure as

London Penetration Depth

When a magnetic field is applied to a superconductor, the applied field does not suddenly drop to zero at the surface, instead H decays exponentially according to the formula.

$$H = H_0 e^{-x/\lambda} \quad (1.1)$$

Where H_0 is the applied field on the surface at $x = 0$, x is the distance from the specimen penetration depth λ varies from 300 to about 5000 Å.

Penetration depth λ is defined as the effective depth to which magnetic field penetrates a super conductor.

The graphical form of equation (1.1) shown as Penetration depth λ depends strongly on temperature and becomes much larger as T approaches T_C . It is related to temperature as

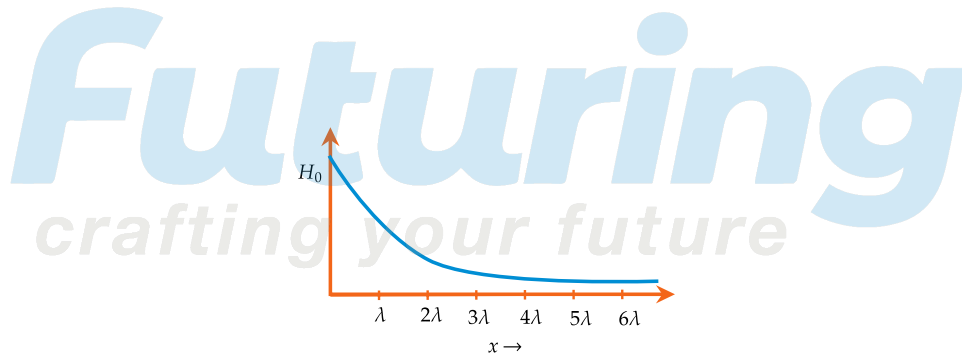


Figure 1.16

$$\left[\frac{\lambda(T)}{\lambda(0)} \right]^2 = \left[1 - \left(\frac{T}{T_C} \right)^4 \right]^{-1} \quad (1.2)$$

Where $\lambda(T)$ and $\lambda(0)$ are the penetration depth at T K and 0 K respectively.

Equation (1.2) implies that super conducting electron density is given as

$$n_s = n_0 \left[1 - \left(\frac{T}{T_C} \right)^4 \right] \quad (1.3)$$

the density of super conducting electron increases from zero to T_C to n_0 at $T = 0$ K as shown in figure

$$(1.4)$$

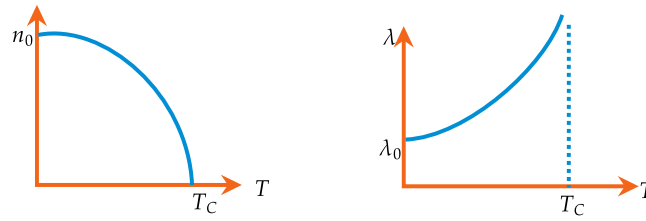


Figure 1.17

from equation (1.2) and (1.3) we get

$$\lambda(T) = \lambda(0) \left(\frac{n_0}{n_s} \right)^{1/2}$$

Note: London equation is defined as

$$\lambda = \left(\frac{m}{\mu_0 n_s e^2} \right)^{1/2} \quad \lambda \propto \sqrt{m}$$

where λ is London penetration depth.

Coherence Length

The coherence length is measure of the distance within which the gap parameter can not change drastically in a spatially varying magnetic field. It is also a measure of the minimum spatial extent of a transition layer between the normal and super conductor. An intrinsic coherent length ξ_0 is given as

$$\xi_0 = \frac{2\hbar V_F}{\pi E_g}$$

Where V_F is electron velocity at the Fermi surface and E_g is energy gap, ξ_0 is characteristic of pure super conductor. In impure materials and alloys the coherence length is shorter than ξ_0 .

Exercise 1.3 Calculate the value of the intrinsic coherence length ξ_0 for pure mercury whose $T_C = 4.15$ K [Given $V_F = 10^6$ m/s]

Solution:

$$\xi_0 = \frac{2\hbar V_F}{\pi E_g}, E_g = 3.53 K_B T_C$$

$$\xi_0 = 331.1 \text{ nm}$$

Isotop Effect

In 1950C A Reynolds and E. M. Maxwell found that the critical temperature T_C varies with the atomic mass M^∞ according to the relation

$$\propto - \frac{\partial \ln T_C}{\partial \ln M} = \frac{1}{2} \quad \Rightarrow \quad T_C M^{1/2} = \text{constant}$$

Thus, the larger the isotropic mass, lower is transition temperature. For example, the transition temperature of mercury changes from 4.185 K to 4.146 K. When its isotopic mass is change from 199.5 to 203.4amu. Now it is known that the mean square amplitude of atomic or lattice vibrations at low temperature is proportional to $\frac{1}{\sqrt{M}}$ and Deye temperature Q_D , of the phonon spectrum is related to M as

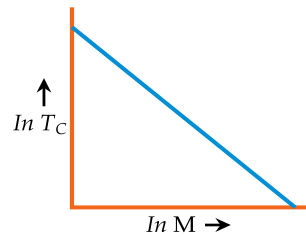


Figure 1.18

$$Q_D \sqrt{M} = \text{constant}$$

From equation (i) and (ii) we get $\frac{T_C}{Q_D} = \text{constant}$

In general, we can write as $T_C \propto Q_D \propto \frac{1}{\sqrt{M}}$

Type-I and Type-II Superconductors

Superconducting materials can be divided into two types, based on their magnetic response. These two types are designated as type I and II.

Type I or the ideal superconductors state are completely diamagnetic, that is when these superconductors are placed in a magnetic field, then all the lines of induction are pushed out from the specimen. These superconductors show Meissner effect. As Magnetic field is increased, the material remains diamagnetic until the critical value H_C is reached. At this point conduction becomes normal and complete magnetic flux penetration takes place.

Type II or hard super conductors are those in which the ideal behavior is seen up to a lower critical field H_{C_1} , beyond which the magnetization gradually changes and attains zero at an upper critical field designated as H_{C_2} . The Meissner effect is incomplete in this region between H_{C_1} and H_{C_2} , this region is known as the Vortex region as shown in figure. The Normal behaviour is observed only beyond H_{C_2} . The lines of induction penetrate gradually from the specimen as the field is increased beyond H_{C_1} and the penetration complete at H_{C_2} . Figures show the behavior of type I and type II super conductor as a function of M and H. It is clear that for type I super conductor, upto H_C , the magnetization of the material grows in proportion to the external magnetic field and then abruptly drops to zero at the transition to the normally conducting state.

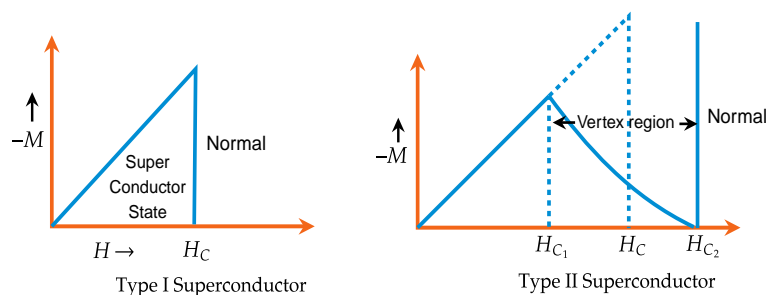


Figure 1.19

Variation of resistivity of a type I super conductor and type II super conductor as a function of applied magnetic field is shown in figure.

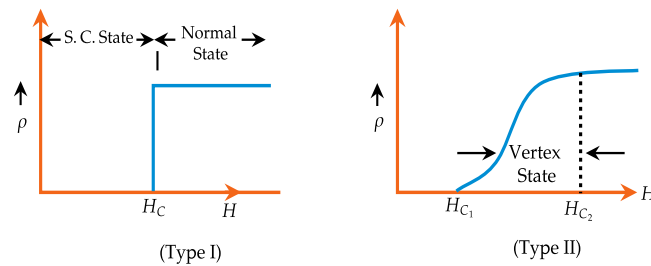


Figure 1.20

BCS Theory

John Bardeen, Lean N. Cooper and John R. Schrieffer developed in 1957 the quantum theory of super conductivity. This theory is based on two experimental results, the isotope effect (as explained earlier) and variation of electronic specific heat with temperature. BCS theory is based on interaction of two electrons through the intermediary of phonons. When an electron approaches an ion in the lattice, there is a coulomb attraction between which causes an increase in the density of ions in the region of distortion. The higher density of ions in the distorted region attracts another electron. Thus a free electron exerts a small attractive force on another electron through phonons with are quanta of lattice vibrations, "A pair of free electrons thus coupled through a phonon is called a cooper pair. Energy of cooper pair is lower than the energy of two individual electrons. The electrons in a cooper pair has opposite spins, so that has a total spin of zero. As a result, the electron pairs in a super conductor are bosons and its radius

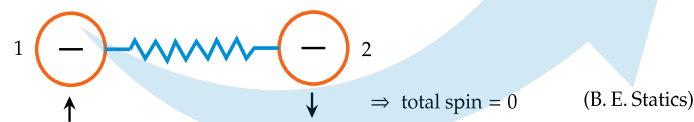


Figure 1.21

$$r_0 = \left(\frac{\hbar v_F}{E_B} \right)$$

\Rightarrow total spin = 0 (B.E.Statics)

When there are non current in super conductor, the linear momentum of the electrons in a cooper pair are equal and opposite for a total of zero. Energy gap E_g of a super conductor at 0 K is given by the formula.

$$E_g(0) = 3.53k_B T_C$$

where k_B is Boltzmann's constant and T_C is the critical temperature of a super conductor. At $T > 0$ K some cooper pairs break up. The resulting individual electrons interact with the remaining cooper pair and reduces the energy gap. At critical temperature T_C , the energy gap disappears, there are no more cooper pairs, and the material is no longer super conducting. The energy gap E_g can be measured by directly microwave radiation of frequency ν at a super conductor E_g can also be measured by utilizing the Josephson effect.

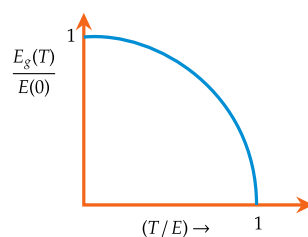


Figure 1.22

- Note** (i) BCS theory valid only for weak coupling superconductor
(ii) BCS theory assumed a spherical FS and isotropic mass.



Practice set-1

1. flux quantum (fluxoid) is approximately equal to 2×10^{-7} gauss-cm². A type II superconductor is placed in a small magnetic field, which is then slowly increased till the field starts penetrating the superconductor. The strength of the field at this point is $\frac{2}{\pi} \times 10^5$ gauss.

A. The penetrating depth of this superconductor is

[NET/JRF(JUNE-2011)]

- A. 100 \AA B. 10 \AA C. 1000 \AA D. 314 \AA

B. The applied field is further increased till superconductivity is completely destroyed. The strength of the field is now $\frac{8}{\pi} \times 10^5$ gauss. The correlation length of the superconductor is

- A. 20 \AA B. 200 \AA C. 628 \AA D. 2000 \AA

2. The potential of a diatomic molecule as a function of the distance r between the atoms is given by $V(r) = -\frac{a}{r^6} + \frac{b}{r^{12}}$. The value of the potential at equilibrium separation between the atoms is:

[NET/JRF(DEC-2011)]

- A. $-4a^2/b$ B. $-2a^2/b$ C. $-a^2/2b$ D. $-a^2/4b$

3. If the number density of a free electron gas in three dimensions is increased eight times, its Fermi temperature will

[NET/JRF(DEC-2011)]

- A. Increase by a factor of 4 B. Decrease by a factor of 4
C. Increase by a factor of 8 D. Decrease by a factor of 8

4. The excitations of a three-dimensional solid are bosonic in nature with their frequency ω and wave-number k are related by $\omega \propto k^2$ in the large wavelength limit. If the chemical potential is zero, the behaviour of the specific heat of the system at low temperature is proportional to

[NET/JRF(DEC-2011)]

- A. $T^{1/2}$ B. T C. $T^{3/2}$ D. T^3

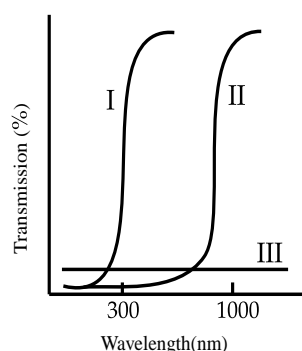
5. Consider a system of non-interacting particles in d dimensional obeying the dispersion relation $\varepsilon = Ak^s$, where ε is the energy, k is the wavevector, s is an integer and A is constant. The density of states, $N(\varepsilon)$, is proportional to

[NET/JRF(JUNE-2012)]

- A. $\varepsilon^{\frac{s}{d}-1}$ B. $\varepsilon^{\frac{d}{s}-1}$ C. $\varepsilon^{\frac{d}{s}+1}$ D. $\varepsilon^{\frac{s}{d}+1}$

6. The experimentally measured transmission spectra of metal, insulator and semiconductor thin films are shown in the figure. It can be inferred that I, II and III correspond, respectively, to

[NET/JRF(JUNE-2012)]



- A. Insulator, semiconductor and metal
 B. Semiconductor, metal and insulator
 C. Metal, semiconductor and insulator
 D. Insulator, metal and semiconductor

7. The dispersion relation of phonons in a solid is given by

$$\omega^2(k) = \omega_0^2 (3 - \cos k_x a - \cos k_y a - \cos k_z a)$$

The velocity of the phonons at large wavelength is

[NET/JRF(JUNE-2012)]

- A. $\omega_0 a / \sqrt{3}$ B. $\omega_0 a$ C. $\sqrt{3} \omega_0 a$ D. $\omega_0 a / \sqrt{2}$

8. A magnetic field sensor based on the Hall Effect is to be fabricated by implanting As into a Si film of thickness $1 \mu\text{m}$. The specifications require a magnetic field sensitivity of 500 mV/Tesla at an excitation current of 1 mA . The implantation dose is to be adjusted such that the average carrier density, after activation, is

[NET/JRF(DEC-2012)]

- A. $1.25 \times 10^{26} \text{ m}^{-3}$ B. $1.25 \times 10^{22} \text{ m}^{-3}$
 C. $4.1 \times 10^{21} \text{ m}^{-3}$ D. $4.1 \times 10^{20} \text{ m}^{-3}$

9. In a band structure calculation, the dispersion relation for electrons is found to be

$$\epsilon_k = \beta (\cos k_x a + \cos k_y a + \cos k_z a)$$

where β is a constant and a is the lattice constant. The effective mass at the boundary of the first Brillouin zone is

[NET/JRF(DEC-2012)]

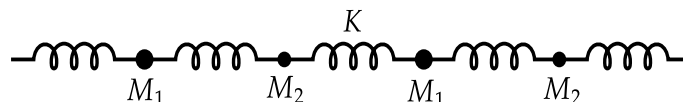
- A. $\frac{2\hbar^2}{5\beta a^2}$ B. $\frac{4\hbar^2}{5\beta a^2}$ C. $\frac{\hbar^2}{2\beta a^2}$ D. $\frac{\hbar^2}{3\beta a^2}$

10. The radius of the Fermi sphere of free electrons in a monovalent metal with an fcc structure, in which the volume of the unit cell is a^3 , is

[NET/JRF(DEC-2012)]

- A. $\left(\frac{12\pi^2}{a^3}\right)^{1/3}$ B. $\left(\frac{3\pi^2}{a^3}\right)^{1/3}$ C. $\left(\frac{\pi^2}{a^3}\right)^{1/3}$ D. $\frac{1}{a}$

11. The phonon dispersion for the following one-dimensional diatomic lattice with masses M_1 and M_2 (as shown in the figure)



is given by

$$\omega^2(q) = K \left(\frac{1}{M_1} + \frac{1}{M_2} \right) \left[1 \pm \sqrt{1 - \frac{4M_1 M_2}{(M_1 + M_2)^2} \sin^2 \left(\frac{qa}{2} \right)} \right]$$

where a is the lattice parameter and K is the spring constant. The velocity of sound is

[NET/JRF(JUNE-2013)]

- A. $\sqrt{\frac{K(M_1+M_2)}{2M_1M_2}} a$ B. $\sqrt{\frac{K}{2(M_1+M_2)}} a$ C. $\sqrt{\frac{K(M_1+M_2)}{M_1M_2}} a$ D. $\sqrt{\frac{KM_1M_2}{2(M_1+M_2)^3}} a$

$$\varepsilon_k = 2\varepsilon_0 \left[\sin^2 \frac{ka}{2} - \frac{1}{6} \sin^2 ka \right]$$

[NET/JRF(JUNE-2013)]

- [NET/JRF(JUNE-2013)]**

- [NET/JRF(DEC-2013)]**

- [NET/JRF(DEC-2013)]**

- [NET/JRF(JUNE-2014)]**

- [NET/JRF(JUNE-2014)]**

- $$\varepsilon(k) = -4\varepsilon_0 \left[\cos \frac{k_x a}{2} \cos \frac{k_y a}{2} + \cos \frac{k_y a}{2} \cos \frac{k_z a}{2} + \cos \frac{k_z a}{2} \cos \frac{k_x a}{2} \right]$$

[NET/JRF(JUNE-2014)]

- A.** $-2\varepsilon_0 a/\hbar$ **B.** $2\varepsilon_0 a/\hbar$ **C.** $-4\varepsilon_0 a/\hbar$ **D.** $4\varepsilon_0 a/\hbar$

19. Consider two crystalline solids, one of which has a simple cubic structure, and the other has a tetragonal structure. The effective spring constant between atoms in the c -direction is half the effective spring constant between atoms in the a and b directions. At low temperatures, the behaviour of the lattice contribution to the specific heat will depend as a function of temperature T as

[NET/JRF(DEC-2014)]

- A. T^2 for the tetragonal solid, but as T^3 for the simple cubic solid
- B. T for the tetragonal solid, and as T^3 for the simple cubic solid
- C. T for both solids
- D. T^3 for both solids

20. A superconducting ring carries a steady current in the presence of a magnetic field \vec{B} normal to the plane of the ring. Identify the INCORRECT statement.

[NET/JRF(DEC-2014)]

- A. The flux passing through the superconductor is quantized in units of hc/e ,
- B. The current and the magnetic field in the superconductor are time independent.
- C. The current density \vec{j} and \vec{B} are related by the equation $\vec{\nabla} \times \vec{j} + \Lambda^2 \vec{B} = 0$, where Λ is a constant
- D. The superconductor shows an energy gap which is proportional to the transition temperature of the superconductor

21. The critical magnetic fields of a superconductor at temperatures $4K$ and $8K$ are 11 mA/m and 5.5 mA/m respectively. The transition temperature is approximately

[NET/JRF(JUNE-2015)]

- A. $8.4K$
- B. $10.6 K$
- C. $12.9 K$
- D. $15.0 K$

22. The low-energy electronic excitations in a two-dimensional sheet of graphene is given by $E(\vec{k}) = \hbar v k$, where v is the velocity of the excitations. The density of states is proportional to

[NET/JRF(JUNE-2015)]

- A. E
- B. $E^{\frac{3}{2}}$
- C. $E^{\frac{1}{2}}$
- D. E^2

23. The dispersion relation of electrons in a 3 -dimensional lattice in the tight binding approximation is given by,

$$\epsilon_k = \alpha \cos k_x a + \beta \cos k_y a + \gamma \cos k_z a$$

where a is the lattice constant and α, β, γ are constants with dimension of energy. The effective mass tensor at the corner of the first Brillouin zone $(\frac{\pi}{a}, \frac{\pi}{a}, \frac{\pi}{a})$ is

[NET/JRF(DEC-2015)]

- A. $\frac{\hbar^2}{a^2} \begin{pmatrix} -1/\alpha & 0 & 0 \\ 0 & -1/\beta & 0 \\ 0 & 0 & 1/\gamma \end{pmatrix}$
- B. $\frac{\hbar^2}{a^2} \begin{pmatrix} -1/\alpha & 0 & 0 \\ 0 & -1/\beta & 0 \\ 0 & 0 & -1/\gamma \end{pmatrix}$
- C. $\frac{\hbar^2}{a^2} \begin{pmatrix} 1/\alpha & 0 & 0 \\ 0 & 1/\beta & 0 \\ 0 & 0 & 1/\gamma \end{pmatrix}$
- D. $\frac{\hbar^2}{a^2} \begin{pmatrix} 1/\alpha & 0 & 0 \\ 0 & 1/\beta & 0 \\ 0 & 0 & -1/\gamma \end{pmatrix}$

24. A thin metal film of dimension $2 \text{ mm} \times 2 \text{ mm}$ contains 4×10^{12} electrons. The magnitude of the Fermi wavevector of the system, in the free electron approximation, is

[NET/JRF(DEC-2015)]

A. $2\sqrt{\pi} \times 10^7 \text{ cm}^{-1}$

B. $\sqrt{2\pi} \times 10^7 \text{ cm}^{-1}$

C. $\sqrt{\pi} \times 10^7 \text{ cm}^{-1}$

D. $2\pi \times 10^7 \text{ cm}^{-1}$

25. For an electron moving through a one-dimensional periodic lattice of periodicity a , which of the following corresponds to an energy eigenfunction consistent with Bloch's theorem?

[NET/JRF(DEC-2015)]

A. $\psi(x) = A \exp\left(i\left[\frac{\pi x}{a} + \cos\left(\frac{\pi x}{2a}\right)\right]\right)$

B. $\psi(x) = A \exp\left(i\left[\frac{\pi x}{a} + \cos\left(\frac{2\pi x}{a}\right)\right]\right)$

C. $\psi(x) = A \exp\left(i\left[\frac{2\pi x}{a} + i \cosh\left(\frac{2\pi x}{a}\right)\right]\right)$

D. $\psi(x) = A \exp\left(i\left[\frac{\pi x}{a} + i\left|\frac{\pi x}{2a}\right|\right]\right)$

26. The band energy of an electron in a crystal for a particular k -direction has the form $\varepsilon(k) = A - B \cos 2ka$, where A and B are positive constants and $0 < ka < \pi$. The electron has a hole-like behaviour over the following range of k :

[NET/JRF(JUNE-2016)]

A. $\frac{\pi}{4} < ka < \frac{3\pi}{4}$

B. $\frac{\pi}{2} < ka < \pi$

C. $0 < ka < \frac{\pi}{4}$

D. $\frac{\pi}{2} < ka < \frac{3\pi}{4}$

27. Consider a one-dimensional chain of atoms with lattice constant a . The energy of an electron with wave-vector k is $\varepsilon(k) = \mu - \gamma \cos(ka)$, where μ and γ are constants. If an electric field E is applied in the positive x -direction, the time dependent velocity of an electron is (In the following B is the constant)

[NET/JRF(DEC-2016)]

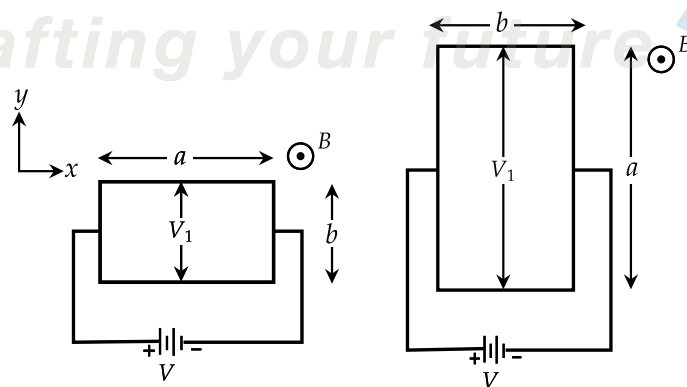
A. Proportional to $\cos\left(B - \frac{eE}{\hbar}at\right)$

B. Proportional to E

C. Independent of E

D. Proportional to $\sin\left(B - \frac{eE}{\hbar}at\right)$

28. A thin rectangular conducting plate of length a and width b is placed in the xy -plane in two different orientations as shown in the figures below. In both cases a magnetic field B is applied in the z -direction and a current flows in the x direction due to the applied voltage V .



If the Hall voltage across the y -direction in the two cases satisfy $V_2 = 2V_1$ the ratio $a : b$ must be

[NET/JRF(DEC-2016)]

A. $1 : 2$

B. $1 : \sqrt{2}$

C. $2 : 1$

D. $\sqrt{2} : 1$

29. The electrical conductivity of copper is approximately 95% of the electrical conductivity of silver, while the electron density in silver is approximately 70% of the electron density in copper. In Drude's model, the approximate ratio τ_{Cu}/τ_{Ag} of the mean collision time in copper (τ_{Cu}) to the mean collision time in silver (τ_{Ag}) is

[NET/JRF(JUNE-2017)]

A. 0.44

B. 1.50

C. 0.33

D. 0.66

30. The dispersion relation of a gas of spin $\frac{1}{2}$ fermions in two dimensions is $E = \hbar v |\vec{k}|$, where E is the energy, \vec{k} is the wave vector and v is a constant with the dimension of velocity. If the Fermi energy at zero temperature is ϵ_F , the number of particles per unit area is

[NET/JRF(DEC-2017)]

- A. $\frac{\epsilon_F}{(4\pi v \hbar)}$ B. $\frac{\epsilon_F^3}{(6\pi^2 v^3 \hbar^3)}$ C. $\frac{\pi \epsilon_F^{3/2}}{(3v^3 \hbar^3)}$ D. $\frac{\epsilon_F^2}{(2\pi v^2 \hbar^2)}$

31. The dispersion relation for the electrons in the conduction band of a semiconductor is given by $E = E_0 + \alpha k^2$ where α and E_0 are constants. If ω_c is the cyclotron resonance frequency of the conduction band electrons in a magnetic field B , the value of α is

[NET/JRF(JUNE-2018)]

- A. $\frac{\hbar \omega_c}{4eB}$ B. $\frac{2\hbar^2 \omega_c}{eB}$ C. $\frac{\hbar^2 \omega_c}{eB}$ D. $\frac{\hbar^2 \omega_c}{2eB}$

Answer key			
Q.No.	Answer	Q.No.	Answer
1	A	2	D
3	A	4	C
5	B	6	A
7	D	8	B
9	D	10	A
11	B	12	A
13	C	14	D
15	C	16	A
17	B	18	D
19	D	20	A
21	B	22	A
23	C	24	B
25	B	26	A
27	D	28	D
29	D	30	D
31	D		

Practice set-2

1. The valence electrons do not directly determine the following property of a metal

[GATE 2010]

- | | |
|----------------------------|-------------------------|
| A. Electrical conductivity | B. Thermal conductivity |
| C. Shear modulus | D. Metallic luster |

2. The Hall coefficient, R_H , of sodium depends on

[GATE 2010]

- A. The effective charge carrier mass and carrier density
 B. The charge carrier density and relaxation time
 C. The charge carrier density only
 D. The effective charge carrier mass

3. The Bloch theorem states that within a crystal, the wavefunction, $\psi(\vec{r})$, of an electron has the form

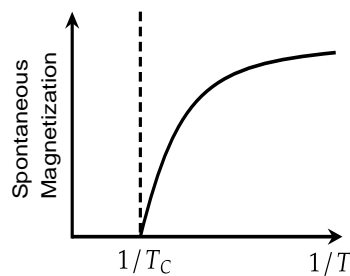
[GATE 2010]

- A. $\psi(\vec{r}) = u(\vec{r})e^{i\vec{k}\cdot\vec{r}}$ where $u(\vec{r})$ is an arbitrary function and \vec{k} is an arbitrary vector
 B. $\psi(\vec{r}) = u(\vec{r})e^{i\vec{G}\cdot\vec{r}}$ where $u(\vec{r})$ is an arbitrary function and \vec{G} is a reciprocal lattice vector
 C. $\psi(\vec{r}) = u(\vec{r})e^{i\vec{G}\cdot\vec{r}}$ where $u(\vec{r}) = u(\vec{r} + \vec{\Lambda})$, $\vec{\Lambda}$ is a lattice vector and \vec{G} is a reciprocal lattice vector
 D. $\psi(\vec{r}) = u(\vec{r})e^{i\vec{k}\cdot\vec{r}}$ where $u(\vec{r}) = u(\vec{r} + \vec{\Lambda})$, $\vec{\Lambda}$ is a lattice vector and \vec{k} is an arbitrary vector

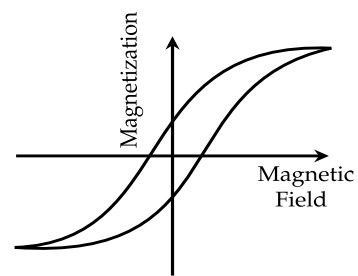
4. In an experiment involving a ferromagnetic medium, the following observations were made. Which one of the plots does NOT correctly represent the property of the medium? (T_C is the Curie temperature)

[GATE 2010]

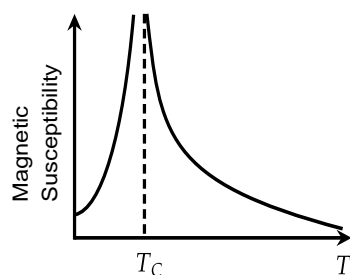
A.



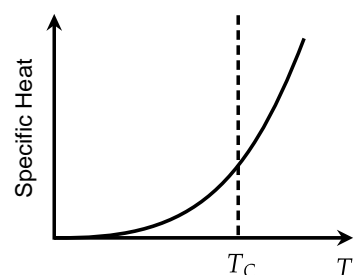
B.



C.



D.



5. For a two-dimensional free electron gas, the electronic density n , and the Fermi energy E_F , are related by [GATE 2010]

A. $n = \frac{(2mE_F)^{3/2}}{3\pi^2\hbar^3}$ B. $n = \frac{mE_F}{\pi\hbar^2}$ C. $n = \frac{mE_F}{2\pi\hbar^2}$ D. $n = \frac{2^{1/3}(mE_F)^{1/3}}{\pi\hbar}$

6. The temperature (T) dependence of magnetic susceptibility (χ) of a ferromagnetic substance with a Curie temperature (T_c) is given by [GATE 2011]

A. $\frac{C}{T-T_c}$, for $T < T_c$
 B. $\frac{C}{T-T_c}$, for $T > T_c$
 C. $\frac{C}{T+T_c}$, for $T > T_c$
 D. $\frac{C}{T+T_c}$, for all temperatures where C is constant.

Common Data for Questions 14 and 15:

The tight binding energy dispersion ($E - k$) relation for electrons in a one-dimensional array of atoms having lattice constant a and total length L is

$$E = E_0 - \beta - 2\gamma\cos(ka)$$

where E_0, β and γ are constants and k is the wave vector.

7. The density of states of electrons (including spin degeneracy) in the band is given by [GATE 2011]

A. $\frac{L}{\pi\gamma a \sin(ka)}$ B. $\frac{L}{2\pi\gamma a \sin(ka)}$ C. $\frac{L}{2\pi\gamma a \cos(ka)}$ D. $\frac{L}{\pi\gamma a \cos(ka)}$

8. The effective mass of electrons in the band is given by [GATE 2011]

A. (a) $\frac{\hbar^2}{\gamma a^2 \cos(ka)}$ B. $\frac{\hbar^2}{2\gamma a^2 \cos(ka)}$ C. $\frac{\hbar^2}{\gamma a^2 \sin(ka)}$ D. $\frac{\hbar^2}{2\gamma a^2 \sin(ka)}$

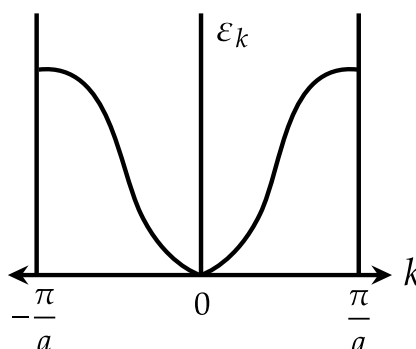
9. Which one of the following CANNOT be explained by considering a harmonic approximation for the lattice vibrations in solids? [GATE 2012]

- A. Debye's T^3 law B. Dulong Petit's law
 C. Optical branches in lattices D. Thermal expansion

10. The group velocity at the boundary of the first Brillouin zone is [GATE 2012]

A. 0 B. 1 C. $\sqrt{\frac{Aa^2}{2}}$ D. $\frac{1}{2}\sqrt{\frac{Aa^2}{2}}$

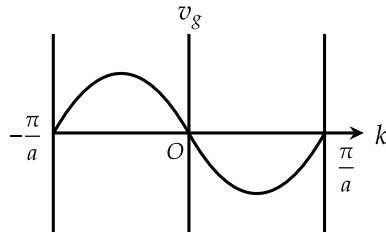
11. The energy ε_k for band electrons as a function of the wave vector k in the first Brillouin zone ($-\frac{\pi}{a} \leq k \leq \frac{\pi}{a}$) of a one dimensional monoatomic lattice is shown as (a is lattice constant)



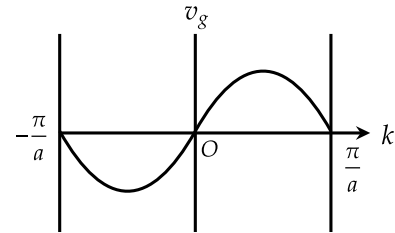
The variation of the group velocity v_g is most appropriately represented by

[GATE 2014]

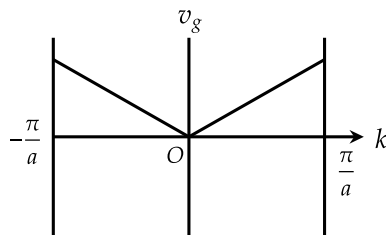
A.



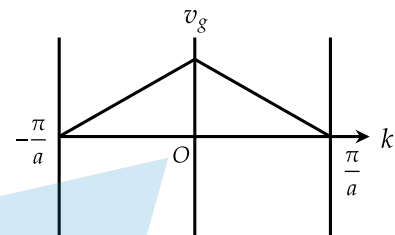
B.



C.



D.



12. The energy dependence of the density of states for a two dimensional non-relativistic electron gas is given by, $g(E) = CE^n$, where C is constant. The value of n is _____

[GATE 2015]

13. The dispersion relation for phonons in a one dimensional monoatomic Bravais lattice with lattice spacing a and consisting of ions of masses M is given by $\omega(k) = \sqrt{\frac{2c}{M}}[1 - \cos(ka)]$, where ω is the frequency of oscillation, k is the wavevector and C is the spring constant. For the long wavelength modes ($\lambda \gg a$), the ratio of the phase velocity to the group velocity is _____

[GATE 2015]

14. In a Hall effect experiment, the hall voltage for an intrinsic semiconductor is negative. This is because (symbols carry usual meaning)

[GATE 2015]

A. $n \approx p$

B. $n > p$

C. $\mu_n > \mu_p$

D. $m_p^* > m_n^*$

15. Consider a metal which obeys the Sommerfield model exactly. If E_F is the Fermi energy of the metal at $T = 0K$ and R_H is its Hall coefficient, which of the following statements is correct?

[GATE 2016]

A. $R_H \propto E_F^{\frac{3}{2}}$

B. $R_H \propto E_F^{\frac{2}{3}}$

C. $R_H \propto E_F^{-\frac{3}{2}}$

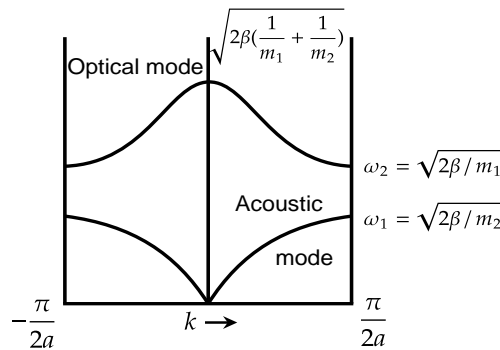
D. R_H is independent of E_F .

16. A one-dimensional linear chain of atoms contains two types of atoms of masses m_1 and m_2 (where $m_2 > m_1$), arranged alternately. The distance between successive atoms is the same. Assume that the harmonic approximation is valid. At the first Brillouin zone boundary, which of the following statements is correct?

[GATE 2016]

- A. The atoms of mass m_2 are at rest in the optical mode, while they vibrate in the acoustical mode.
 B. The atoms of mass m_1 are at rest in the optical mode, while they vibrate in the acoustical mode.
 C. Both types of atoms vibrate with equal amplitudes in the optical as well as in the acoustical modes.
 D. Both types of atoms vibrate, but with unequal, non-zero amplitudes in the optical as well as in the acoustical modes.

Solution: In optical mode, at Brillouin zone boundary atom of heavier mass (m_2) is at rest, whereas in Acoustic mode, atoms of lighter mass (m_1) is at rest.



So the correct answer is **Option (A)**

17. Consider a 2 - dimensional electron gas with a density of 10^{19} m^{-2} . The Fermi energy of the system is..... eV (up to two decimal places).

$$(m_e = 9.31 \times 10^{-31} \text{ kg}, h = 6.626 \times 10^{-34} \text{ Js}, e = 1.602 \times 10^{-19} \text{ C})$$

[GATE 2017]

18. At low temperatures (T), the specific heat of common metals is described by (with α and β as constants)

[GATE 2018]

- A. $\alpha T + \beta T^3$ B. βT^3 C. $\exp(-\alpha/T)$ D. $\alpha T + \beta T^5$

19. The energy dispersion for electrons in one dimensional lattice with lattice parameter a is given by $E(k) = E_0 - \frac{1}{2}W \cos ka$, where W and E_0 are constants. The effective mass of the electron near the bottom of the band is

[GATE 2018]

- A. $\frac{2\hbar^2}{Wa^2}$ B. $\frac{\hbar^2}{Wa^2}$ C. $\frac{\hbar^2}{2Wa^2}$ D. $\frac{\hbar^2}{4Wa^2}$

20. In a certain two-dimensional lattice, the energy dispersion of the electrons is

$$\varepsilon(\vec{k}) = -2t \left[\cos k_x a + 2 \cos \frac{1}{2} k_x a \cos \frac{\sqrt{3}}{2} k_y a \right]$$

where $\vec{k} = (k_x, k_y)$ denotes the wave vector, a is the lattice constant and t is a constant in units of eV. In this lattice the effective mass tensor m_{ij} of electrons calculated at the center of the Brillouin zone has the

form $m_{ij} = \frac{\hbar^2}{ta^2} \begin{pmatrix} \alpha & 0 \\ 0 & \alpha \end{pmatrix}$. The value of α (rounded off to two decimal places) is_____

[GATE 2019]

21. An ideal gas of non-relativistic fermions in 3-dimensions is at 0 K. When both the number density and mass of the particles are doubled, then the energy per particle is multiplied by a factor

[JEST 2014]

- A. $2^{1/2}$ B. 1 C. $2^{1/3}$ D. $2^{-1/3}$

Answer key			
Q.No.	Answer	Q.No.	Answer
1	C	2	C
3	D	4	C
5	B	6	B
7	A	8	B
9	D	10	A
11	B	12	0
13	1	14	C
15	C	16	A
17	2.34eV	18	A
19	A	20	0.333
21	D		



Futuring
crafting your future

Practice set-3

1. The dispersion relation of photons in a solid is:

$$\omega^2(k) = \omega_0^2 [3 - \cos k_x a - \cos k_y a - \cos k_z a]$$

The velocity of Photons at larger wavelength is:

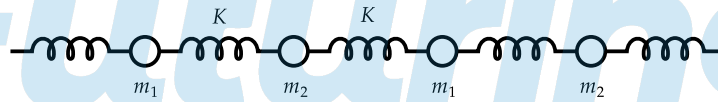
- | | |
|---|--|
| <p>a. $\frac{\omega_0 a}{\sqrt{3}}$</p> <p>c. $\sqrt{3} \omega_0 a$</p> | <p>b. $\omega_0 a$</p> <p>d. $\frac{\omega_0 a}{\sqrt{2}}$</p> |
|---|--|

Solution: for larger wavelength (K should be small)

$$\begin{aligned} \omega^2(k) &= \omega_0^2 \left[3 - \left(1 - \frac{k_x^2 a^2}{2} \right) - \left(1 - \frac{k_y^2 a^2}{2} \right) - \left(1 - \frac{k_z^2 a^2}{2} \right) \right] \\ &= \frac{\omega_0^2 a^2}{2} (k_x^2 + k_y^2 + k_z^2) \\ &= \frac{\omega_0^2 a^2}{2} k^2 \Rightarrow \omega(k) = \frac{\omega_0 a}{2} K \\ V_g &= \frac{d\omega}{dk} = \frac{\omega_0 a}{\sqrt{2}} \end{aligned}$$

So the correct answer is **Option (D)**

2. The phonon dispersion for the following 1 dimensional diatomic lattice with masses M_1 & M_2



is given by:

$$\omega^2(q) = k \left(\frac{1}{m_1} + \frac{1}{m_2} \right) \left[1 \pm \sqrt{1 - \frac{4M_1 M_2}{(M_1 + M_2)^2} \sin^2 \left(\frac{qa}{2} \right)} \right]$$

where a is the lattice parameter and K is the spring constant. The velocity of sound is:

- | | |
|--|---|
| <p>a. $\sqrt{\frac{K(M_1+M_2)}{2M_1M_2}} a$</p> <p>c. $\sqrt{\frac{K(M_1+M_2)}{M_1M_2}} a$</p> | <p>b. $\sqrt{\frac{K}{2(M_1+M_2)}} a$</p> <p>d. done</p> |
|--|---|

Solution: For long wavelength

$$\begin{aligned} \sin^2 \left(\frac{qa}{2} \right) &\approx \frac{q^2 a^2}{4} \\ \omega^2 &= K \left(\frac{1}{M_1} + \frac{1}{M_2} \right) \left[1 - \sqrt{1 - \frac{4M_1 M_2}{(M_1 + M_2)^2} \cdot \frac{q^2 a^2}{4}} \right] \\ &= K \left(\frac{1}{M_1} + \frac{1}{M_2} \right) \left[1 - \left(1 - \frac{1}{2} \cdot \frac{4M_1 M_2}{(M_1 + M_2)^2} \cdot \frac{q^2 a^2}{4} \right) \right] \\ &= k \left(\frac{1}{m_1} + \frac{1}{m_2} \right) \left[\frac{1}{2} \frac{q^2 a^2}{M_1 M_2 \left(\frac{1}{M_1} + \frac{1}{M_2} \right)^2} \right] \end{aligned}$$

$$\omega_-^2 = \frac{1}{2}k \frac{q^2 a^2}{(M_1 + M_2)}$$

$$v_{\text{sound}} = \frac{\omega - 1q}{q} = \sqrt{\frac{K}{2(M_1 + M_2)}}a$$

So the correct answer is **Option (B)**

3. A uniform linear monoatomic chain is modeled by a spring mass system of masses m separated by nearest neighbour distance a and spring constant ω_0^2 . The dispersion relation for this system is:
- a. $\omega(k) = 2\omega_0 \left[1 - \cos \frac{ka}{2}\right]$ b. $\omega(k) = 2\omega \sin^2 \left(\frac{ka}{2}\right)$
 c. $\omega(K) = 2\omega_0 \sin \left(\frac{ka}{2}\right)$ d. $\omega(k) = 2\omega_0 \tan \left(\frac{ka}{2}\right)$

Solution:

$$\omega(k) = \sqrt{\frac{4c}{m}} \sin \left(\frac{ka}{2}\right) \quad c = m\omega_0^2 \text{ (given)}$$

$$= \sqrt{\frac{4cm\omega_0^2}{m}} \sin \left(\frac{ka}{2}\right) = 2\omega_0 \sin \frac{ka}{2}$$

So the correct answer is **Option (C)**

4. A sample of *Si* has electron and hole mobility of $0.13 \& 0.05 \text{ m}^2/\text{vrs}$ respectively at 300K. It is doped with ρ and *Al* with doping densities of $1.5 \times 10^{21} \text{ m}^{-3}$ and $2.5 \times 10^{21} \text{ m}^{-3}$ respectively. The conductivity of the doped *Si* sample at 300k.
- a. $8n^{-1} \text{ m}^{-1}$ b. $32\Omega^{-1} \text{ m}^{-1}$
 c. $20.8\Omega^{-1} \text{ m}^{-1}$ d. $83.2\Omega^{-1} \text{ m}^{-1}$

Solution:

doping of *Al* > doping of ρ
 ie ρ -type semiconductor.

$$\text{majority carrier conc} = (2.5 - 1.5) \times 10^{21} / \text{m}^3$$

$$= 10^{21} / \text{m}^3$$

$$\sigma = pe\mu_h$$

$$\approx 10^{21} \times 1.6 \times 10^{-19} \times 0.05$$

$$= 8\Omega^{-1} \text{ m}^{-1}$$

So the correct answer is **Option (a)**

5. The concentration of electrons n and holes p for an intrinsic semiconductor at temperature T can be explained as

$$n = p = AT^{3/2} \exp(-E_g / 2K_B T)$$

where E_g is the band gap and A is a constant.

If the mobility of both types of carries is proportional to $T^{3/2}$, then the log of the conductivity is a linear function of T^{-1} with slope.

- a. $\frac{E_g}{2k_B}$ b. $\frac{E_g}{k_B}$
 c. $-\frac{E_g}{2k_B}$ d. $-\frac{E_g}{K_B}$

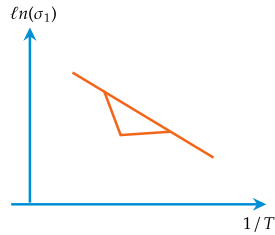
Solution:

Figure 1.23

$$\sigma_i = cT^{3/2} \exp\left(\frac{-E_g}{2k_B T}\right) \cdot eDT^{-3/2}$$

$$\therefore \sigma_i = n_i e (\mu_e + \mu_h)$$

$$\sigma_i = c \exp\left(\frac{-E_g}{2k_B T}\right)$$

$$\ln \sigma_i = \ln c - \frac{E_g}{2k_B T}$$

$$\text{Slope} = -\frac{E_g}{2k_B}$$

So the correct answer is **Option (c)**

6. A junction is made between a metal of work function ω_m , and a doped semiconductor of work function ω_s with $\omega_m > \omega_s$. If the electric field at the interface has been increased by a factor of 3. Then the dopant concentration in Semiconductor would be.
- Increase by a
 - Decrease by 3
 - Increase by 3
 - Decrease by $\sqrt{3}$

Solution:

$$\vec{E} \text{ at metal at the semiconductor interface} \propto (N_d)^2$$

$$\begin{aligned} \vec{E} \text{ increased by} &= (3)^2 \\ &= 9 \text{ times} \end{aligned}$$

So the correct answer is **Option (a)**

7. A magnetic field sensor based on the hall effect is to be fabricated by implanting as into a *si* film of thickness $1\mu\text{m}$. The specifications require a magnetic field sensitivity of $500\text{mV}/\text{T}$ at an excitation current of 1mA . The implantation dose to be adjusted such that the average carrier density, after activation is,
- $1.25 \times 10^{26} \text{ cm}^{-3}$
 - $1.25 \times 10^{22} \text{ cm}^{-3}$
 - $4.1 \times 10^{21} \text{ cm}^{-3}$
 - $4.1 \times 10^{23} \text{ cm}^{-3}$

Solution:

$$\text{magnetic field sensitivity: } \frac{V_H}{B} = 500 \times 10^{-3} \frac{\text{V}}{\text{T}}$$

$$\text{current}(I) = 1\text{mA} = 10^{-3} \text{ A}$$

$$\text{thickness}(t) = 1 \times 10^{-6} \text{ m}$$

$$U_H = \frac{I_x B_2}{n e t} \Rightarrow n = \frac{I_x B_2}{V_H e t}$$
$$n = \frac{10^{-3} \times 1 \text{ T}}{500 \times 10^{-3} \times 1.6 \times 10^{-19} \times 10^{-6}}$$
$$n = 1.25 \times 10^{22} \text{ cm}^{-3}$$

So the correct answer is **Option (b)**





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