Kater's Reversible Pendulum

James Atkisson and Kevin Chen
Department of Physics and Astronomy, The University of North Carolina at Chapel Hill
(Physics 281, Group δ)
(Dated: September 14, 2025)

This experiment aims to determine an accurate reading on the acceleration of gravity at its location on Earth, utilizing the Kater's pendulum. The Kater's pendulum, is a reversible pendulum that allows physicists to ignore hard to measure variables such as moment of inertia or length from center of mass to the pivot. By measuring multiple distances, we recorded the oscillation period of the pendulum, and found distances which had the same period. With this knowledge of the period and distance, we would result in a value of $g = 9.685 \pm 0.102\text{m/s}^2$.

I. INTRODUCTION

On Earth, gravity is the defining force that literally holds us together. Being a fundamental force that influences all objects on the planet, understanding and measuring its value precisely is a crucial detail that physicists have dealt with for years. Gravity is a force present everywhere, from a star placed light years away to an apple falling from a tree. Due to its presence everywhere, it in turn affects almost every calculation involving mechanical physics. Therefore, it is essential we find as accurate of a value as possible, for calculations, simulations, and engineering all rely on this. For this lab, we used a Kater's pendulum, a reversible pendulum, to find the value of g throughout its period.

During the 18th century, scientists had the goal of determining the shape of the Earth and the properties of gravity of how it weakens at different locations. To solve this, they would look for an accurate and reliable method to measure g in various locations, and there were many problems with using a regular pendulum. The quantities needed to calculate the value of g were difficult to measure with good precision, such as the distance from the pivot to the center of mass l or the moment of inertia I. To solve this, in 1815 Henry Kater would create a reversible pendulum capable of canceling out these quantities, allowing for accurate measurements.

The Kater pendulum has two masses, one of which is adjustable between two pivot points. With these pivot points, the pendulum is reversible and can be changed from one end to the other. By carefully measuring the oscillation period in both configurations, we adjust one of the masses until we find an equal period between it and its reversed configuration, allowing us to solve for g.

II. THEORETICAL BACKGROUND

For a traditional physical pendulum, it's period of small-angle oscillations is given by:

$$T = 2\pi\sqrt{\frac{I}{mgL}} \tag{1}$$

where I is the moment of inertial, m is the mass, g is the gravitational acceleration, and L is the distance from the pivot to the center of mass. However since this distance cannot be accurately measured due to stretches and vibration in a traditional pendulum string, this is where Kater's Pendulum comes in. Since the distance of the rod is of a fixed length Δx, Kater's pendulum has two periods T_1 and T_2 that correspond to the oscillations about its two pivot points. When both of these periods are equal to each other, it can be shown that that

$$g = 4\pi^2(\frac{\Delta x}{T^2}) \tag{2}$$

This equation allows g to be accurately determined by measuring the period of oscillation at the configuration where $T_1 = T_2$. Moving the adjustable mass along Δx allows us to achieve this condition.

A. Derivation

Period can be related to frequency through the equation:

$$T = \frac{1}{f} \tag{3}$$

We also know that frequency and the angular frequency are related through:

$$\omega = 2\pi f \tag{4}$$

Putting angular frequency in terms of period yields:

$$\omega = \frac{2\pi}{T} \tag{5}$$

Thus if the periods are equal to each other, the angular frequencies are also equal. In this scenario it can be shown that:

$$\omega_1^2 = \omega_2^2 = \frac{g}{\Delta x} \tag{6}$$

Substituting our earlier equation for angular frequency:

$$\frac{4\pi^2}{T^2} = \frac{g}{\Delta x} \tag{7}$$

Rearranging to solve for g yields our final equation:

$$g = 4\pi^2 (\frac{\Delta x}{T^2}) \tag{8}$$

Where Δx is the length of the rod and T is the measured period.

III. METHODS

To begin, we set the pendulum on an even surface to ensure there's no error in the equilibrium point of the pendulum. In terms of orientation, we began with the fixed mass above the upper pivot point of the pendulum leg. We then set the adjustable mass at the furthest distance from the pivot. From there, a detector was set at the point of equilibrium to measure out the length of a period. Recording the distance of the adjustable mass and period, we then shift the adjustable mass by one notch, taking into account the change in distance and repeat. After the adjustable mass reached the shortest distance possible, the leg of the pendulum was reversed and we recorded each notch again, with the fixed mass below the hanging pivot point.

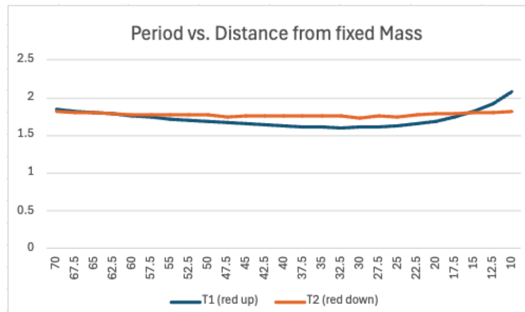
Once we had recorded all period measurements, we lined each measurement of the same distance in a spreadsheet, and found the trials that had the least time difference in periods. After identifying this, we repeated another 3 trials for each of those trials, and then averaged them out, keeping the high and low variations for error calculations.

Distane blue from top (cm)	T1 (red up)	T2 (red down)	Period Difference
70	1.8473	1.8119	0.0354
67.5	1.8243	1.8043	0.02
65	1.8025	1.7961	0.0064
62.5	1.7812	1.7909	-0.0097
60	1.761	1.7785	-0.0175
57.5	1.7412	1.7797	-0.0385
55	1.7185	1.7735	-0.055
52.5	1.7028	1.77	-0.0672
50	1.6843	1.7666	-0.0823
47.5	1.667	1.7493	-0.0823
45	1.6507	1.7591	-0.1084
42.5	1.6376	1.7559	-0.1183
40	1.6251	1.7584	-0.1333
37.5	1.6144	1.7581	-0.1437
35	1.607	1.7589	-0.1519
32.5	1.6032	1.7603	-0.1571
30	1.606	1.7302	-0.1242
27.5	1.6108	1.7661	-0.1553
25	1.626	1.7463	-0.1203
22.5	1.649	1.7697	-0.1207
20	1.6888	1.7809	-0.0921
17.5	1.7398	1.7845	-0.0447
15	1.8165	1.7984	0.0181
12.5	1.9162	1.8082	0.108
10	2.0821	1.8187	0.2634

FIG. 1: Shown above is a table comprising of each trial at a different distance of the adjustable center of mass, to the pivot point. Highlighted are the trials at which there was the least time difference in oscillations between the two reversible configurations at the same distance.

IV. RESULTS

Through performing this experiment, our results yielded a value of $g = 9.685 \pm 0.102 \text{m/s}^2$. As per the methods section, this result was calculated by measuring the period about both pivot points, while moving the adjustable mass $2.5cm$ along Δx until we found where both periods of oscillation were equal to one another, $T_1 = T_2$. Utilizing Excel's built-in graphing tools, we plotted our found period values versus each Δx value, or the distance from the fixed mass. We found that there were two inter-



sections were both the periods were equal, at $\Delta x = 65$cm and $\Delta x = 15$cm. We took three additional trials for both Δx values, and took an average for both pivot configurations. We then took an overall average between the 15cm and 65cm trials, resulting in a period value of 1.806. We then took the total length of the rod which was known to be 80cm and used this as our overall Δx value. Both our period and length values were then plugged into Eq. 8 to calculate our g value. The uncertainty of this value was calculated by finding the highest and lowest T values in our 15cm and 65cm trials. We then calculated g using these period values, and took the highest g value. Using this high g value, we subtracted our calculated g value from earlier, and found the difference to be our uncertainty, which was 0.102m/s^2.

V. DISCUSSION AND CONCLUSIONS

The goal of this experiment was to identify a constant for gravity at our location, utilizing the Kater's pendulum to showcase its ability for precision and accuracy. The average value of g that we found was again, $g = 9.685 \pm 0.102 \text{m/s}^2$, closely matching the accepted value of $g = 9.81 \text{m/s}^2$.

Through this data, we can see the Kater's pendulum is fairly accurate within its ability to measure out gravity. However, our calculated value did not precisely match the accepted value, and there are some many possibilities for why this happened. For one, the accepted value of gravity is only accurate in some locations. Earth is not a perfect sphere, as the equator bulges out from the core, resulting in a lower gravity. Due to this, it was found that $g = 9.79 \text{m/s}^2$ at our location. There was also the issue of the pivot points being rods that swing, as this could bring a factor of friction. Along with this, the calculations of the Kater's pendulum relies on small angle approximation. With a large enough angle, it was possible the calculations would contain error within those.

In conclusion, the experiment successfully determined the acceleration due to gravity with an acceptable degree of accuracy. The results not only corroborate the theoretical value of g but also demonstrate the usefulness of the Kater's pendulum in such measurements.

VI. ACKNOWLEDGMENTS

The data for this experiment was collected by James Atkisson, which included utilizing the photogate for data collection, as well as inputting the data into Excel for analysis and calculations. Each trial was conducted by Kevin Chen, which included resetting each trial, moving the adjustable mass, and changing the pivot points. The analysis of the data was performed by both Kevin and James, including relevant excel calculations and error propogation. James wrote the technical background, derivation, results, and acknowledgements. Kevin wrote the abstract, introductions, methods, and discussion.

[1] D. Candela, K. M. Martini, R. V. Krotkov, and K. H. Langley, Bessel's improved Kater pendulum in the teaching lab, Am. J. Phys. 69, 714 (2001).

[2] R. A. Neslon and M. G. Olsson, The pendulum—Rich physics from a simple system, Am. J. Phys. 54, 112 (1986).