cylinder_num

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system

$$\partial_t u = -\nabla \cdot w + f(u)$$

$$w = -d.\nabla u \quad (\text{. is a componentwise multiplication})$$

$$u = \begin{bmatrix} a \\ a^* \\ [aa^*] \\ [na] \\ [na^* \\ [ana^*] \end{bmatrix}$$

nodes

$$\left(z_j = jh_z = \frac{j}{J}Z, r_i = ih_r = \frac{i}{I}R\right)$$

cells

$$(z_{j+0.5} = z_j + 0.5h_z, r_{i+0.5} = r_i + 0.5h_r)$$

faces

$$(z_j, r_{i+0.5})$$

$$(z_{j+0.5}, r_i)$$

reducing to integration

$$\iint\limits_{i,j} \partial_t urdrdz = \iint\limits_{i,j} (-\nabla \cdot w + f(u))\, rdrdz = -\oint\limits_{i,j} wr \cdot dl + \iint\limits_{i,j} f(u)rdrdz$$

numerically ($?_{k+} = ?_{k+0.5}$)

$$\partial_t u_{i+,j+} r_{i+} h_r h_z = -w^r_{i++,j+} r_{i++} h_z - w^z_{i+,j++} r_{i+} h_r + w^r_{i,j+} r_i h_z + w^z_{i+,j} r_{i+} h_r + f(u_{i+,j+}) r_{i+} h_r h_z$$

$$\partial_t u_{i+,j+} = \frac{-w^r_{i++,j+} r_{i++} + w^r_{i,j+} r_i}{r_{i+} h_r} + \frac{-w^z_{i+,j++} + w^z_{i+,j}}{h_z} + f(u_{i+,j+})$$

$$w^r_{i,j+} = -d_{i,j+}.\frac{u_{i+,j+} - u_{i-,j+}}{h_r}$$

$$w^z_{i+,j} = -d_{i+,j}.\frac{u_{i+,j+} - u_{i+,j-}}{h_z}$$

$$d_{i,j+} = \frac{d_{i+,j+} + d_{i-,j+}}{2d_{i+,j+}.d_{i-,j+}}$$

$$d_{i+,j} = \frac{d_{i+,j+} + d_{i+,j-}}{2d_{i+,j+}.d_{i+,j-}}$$

method (implicit Euler)

$$\check{?} = ?_{t--}$$

$$\begin{aligned}\frac{u_{i+,j+} - \check{u}_{i+,j+}}{\delta t} &= \frac{d_{i++,j+}.(u_{i+++,j+} - u_{i+,j+})r_{i++} - d_{i,j+}.(u_{i+,j+} - u_{i-,j+})r_i}{r_{i+}h_r^2} \\ &+ \frac{d_{i+,j++}.(u_{i+,j+++} - u_{i+,j+}) - d_{i+,j}.(u_{i+,j+} - u_{i+,j-})}{h_z^2} \\ &+ f(u_{i+,j+})\end{aligned}$$

1-iter Newton ($u^0 = \check{u}$)

$$\begin{aligned}\frac{u_{i+,j+} - \check{u}_{i+,j+}}{\delta t} &= \frac{d_{i++,j+}.(u_{i+++,j+} - u_{i+,j+})r_{i++} - d_{i,j+}.(u_{i+,j+} - u_{i-,j+})r_i}{r_{i+}h_r^2} \\ &+ \frac{d_{i+,j++}.(u_{i+,j+++} - u_{i+,j+}) - d_{i+,j}.(u_{i+,j+} - u_{i+,j-})}{h_z^2} \\ &+ f(\check{u}_{i+,j+}) + \frac{\partial f}{\partial u}(\check{u}_{i+,j+})(u_{i+,j+} - \check{u}_{i+,j+})\end{aligned}$$

$$\begin{aligned}\delta t\left(f(\check{u}_{i+,j+}) - \frac{\partial f}{\partial u}(\check{u}_{i+,j+})\check{u}_{i+,j+}\right) &= \left(E + \frac{\delta t}{r_{i+}h_r^2}\left[d_{i++,j+}r_{i++} + d_{i,j+}r_i\right]. + \frac{\delta t}{h_z^2}\left[d_{i+,j++} + d_{i+,j}\right]. - \frac{\partial f}{\partial u}(\check{u}_{i+,j+})\right)u_{i+,j+} \\ &- \frac{\delta t}{r_{i+}h_r^2}d_{i++,j+}r_{i++}.u_{i+++,j+} - \frac{\delta t}{r_{i+}h_r^2}d_{i,j+}r_i.u_{i-,j+} \\ &- \frac{\delta t}{h_z^2}d_{i+,j++}.u_{i+,j+++} - \frac{\delta t}{h_z^2}d_{i+,j}.u_{i+,j-}\end{aligned}$$

$$\delta t\phi(\check{u}_{i+,j+}) = B_{ij}u_{i+,j+} - C_{ij}u_{i+++,j+} - A_{ij}u_{i-,j+} - F_{ij}u_{i+,j+++} - G_{ij}u_{i+,j-}$$

k-th iter
$i + j = 0\ mod\ 2$

$$B_{ij}u^{k+1}_{i+,j+} - C_{ij}u^k_{i+++,j+} - A_{ij}u^k_{i-,j+} - F_{ij}u^k_{i+,j+++} - G_{ij}u^k_{i+,j-} = \delta t\phi(\check{u}_{i+,j+})$$

$$u^{k+1}_{i+,j+} = B^{-1}_{ij}\left(\delta t\phi(\check{u}_{i+,j+}) + C_{ij}u^k_{i+++,j+} + A_{ij}u^k_{i-,j+} + F_{ij}u^k_{i+,j+++} + G_{ij}u^k_{i+,j-}\right)$$

$i + j = 1\ mod\ 2$

$$B_{ij}u^{k+1}_{i+,j+} - C_{ij}u^{k+1}_{i+++,j+} - A_{ij}u^{k+1}_{i-,j+} - F_{ij}u^{k+1}_{i+,j+++} - G_{ij}u^{k+1}_{i+,j-} = \delta t\phi(\check{u}_{i+,j+})$$

$$u^{k+1}_{i+,j+} = \left(\delta t\phi(\check{u}_{i+,j+}) + C_{ij}u^{k+1}_{i+++,j+} + A_{ij}u^{k+1}_{i-,j+} + F_{ij}u^{k+1}_{i+,j+++} + G_{ij}u^{k+1}_{i+,j-}\right)$$

gachi-tex ♂♂