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Mathematical modeling of flocking behavior

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Abstract

In this thesis, the flocking behaviour of prey when threatened by a group of predators, is investigated using dynamical systems. By implementing the unicycle model, a simulation is created using Simulink and Matlab. A set of forces are set up to describe the state of the prey, that in turn determines their behaviour in different scenarios. An effective strategy is found so all members of the flock can survive the predator attack, taking into account the advantages of the predator's greater translational velocity and the prey's higher angular velocity. Multiple obstacles and an energy constraint are added to make the model more realistic. The objective of this thesis is to develop a strategy that maximizes the chance of survival of each flock member by not only staying together in a group but also making use of environmental advantages.

Contents

Nomenclature

ω Angular velocity

ω_{ca} Degree of decay constant for collision avoidance force

ω_{max} Maximum angular velocity

ω_{oa} Degree of decay constant for obstacle avoidance force

ω_{pa} Degree of decay constant for predator avoidance force

ω_{sat} Saturated angular velocity

θ Agent's orientation in the fixed frame

$\Delta\theta$ Rotation angle of the rotated reference system

A Matrix to generate head velocities

c_{ca} Weighting constant for collision avoidance force

c_E Constant for energy dependence

c_{fl} Weighting constant for flocking force

c_g Weighting constant for grouping force

c_{la} Weighting constant for line avoidance force

c_{oa} Weighting constant for obstacle avoidance force

c_{pa} Weighting constant for predator avoidance force

E_0 Initial energy reserve of the agent

E Energy reserve of the agent

$\mathbf{f_{ca}}$ Collision avoidance force

$\mathbf{f_{fl}}$ Flocking force

$\mathbf{f_g}$ Grouping force

$\mathbf{f_{la}}$ Line avoidance force

$\mathbf{f_o}$ Obstacle force

$\mathbf{f_{oa}}$ Obstacle avoidance force

$\mathbf{f_{pa}}$ Predator avoidance force

L Length of prey agents

$\mathbf{L_{mz}}$ Vector from predator center of mass to prey

$\mathbf{L_z}$ Vector from closest predator to prey

m Set of obstacles

n Set of prey

$\mathbf{o}$ Obstacle's position vector

$\mathbf{p}$ Predator's position vector

$\mathbf{p_{close}}$ Predator closest to any prey

q Set of predators

R Rotational matrix used for the survival tactic

R_d Perception radius for prey

R_n Radius defining neighbours

R_{oa} Radius the agent avoids the obstacle with

$R_{oa,in}$ Inner radius for obstacle (radius for obstacle edge)

$R_{oa,out}$ Outer radius for obstacle (spotting radius)

R_{pa} Radius for predator avoidance force

R_{pc} Radius for defining a predator being close

R_{ti} Inner survival tactic radius

R_{to}	Outer survival tactic radius
t	Time
T	Linear transformation matrix to another reference system
$\mathbf{u}$	Sum of all forces, control function
v	Translational velocity
v_{max}	Maximum translational velocity
v_{sat}	Saturated translational velocity
x, y	Position in cartesian coordinates
$\tilde{x}, \tilde{y}$	Coordinates in a new reference system
x_h, y_h	Head position in cartesian coordinates
$\tilde{x}_h, \tilde{y}_h$	Head coordinates in a new reference system
$\mathbf{z}$	Prey's position vector
$\mathbf{z_{mc}}$	Prey's center of mass

1 Introduction

1.1 Grouping and animal behaviour

Animal behaviour has been subject to strong scientific interest for many decades and is today a large area of research. These behaviours we can see today have evolved during many thousands of years to be optimally suited and equipped for the prevailing conditions and environment. This makes it a very interesting field of study, which can tell us a great deal of how and why we humans interact.

Grouping behaviour is apparent for many animal species. It is defined as a permanent or temporary collection of individuals in a chosen area. Grouping is advantageous for survival due to greater possibility to detect danger, mating efficiency and gathering and finding food [12]. Several theories also state that grouping improves the learning from external stimuli and reduces the overall aggression of the individuals [1].

Mathematical modelling of animal behaviour is a fairly new area of research. Behaviours of animals are in many cases complex and difficult to model. An area where progress has been made during the past years is within the mathematical and physical study of flocking behaviour. A flock, which is an interaction with lower theoretical complexity, is defined as the collective motion of individual agents [2], which is in many cases attained without external stimuli or controller [1].

This thesis purposes to explain the theory behind the modelling of flocking behaviour and present and discuss a strategy and model for a flock with individual agents, both unthreatened and threatened by predator agents. As animal behaviour is difficult to model there are many diverse theories and examples of methods for modelling flocking behaviour. To be able to make associations with what actually happens in nature, the prey and predator agents have been chosen to represent similar profiles to zebras and lions in the wild.

1.2 Biological background

There is a predator-prey relationship between zebras and lions: the zebra wants to survive and the lion wants to eat. To increase their success rate, the lions use group tactics to make a successful hunt, in which they are successful about 3 times out of 10 [11]. The zebras use zig-zagging as a tactic to maximize their survival. They have higher stamina but lower maximum velocity (64 km/h) than lions (80 km/h) that use smart tactics to compensate for their lower stamina to get the edge.

Zebras caught are in most cases either very young (younger than 1 year) or have health problems that makes them more vulnerable for an attack. It is important to point out that even if the lion catches a zebra, it is not certain that it will be eaten. There are documented cases [13] where the zebra can escape after being bitten by one lion therefore a group attack is almost essential for predator success.

1.3 Reynolds' theory

Regarding the study of flocking behaviour, one of the first to start exploring the potential and possibilities of this topic was Craig W. Reynolds who, in 1987, published his paper on "Flocks, Herds, and Schools; A distributed Behavioural Model". In this paper, Reynolds defined fundamental aspects still referred to in more recent research [2]. Reynolds defined three empirical rules:

1. Flock centering: The desire for agents to stay together with and nearby other agents.
2. Collision avoidance: Avoid collision when agents are close to each other.
3. Velocity matching: Attempt to keep similar velocity and direction as other agents.

The behaviours that make up Reynolds flocking model are based on the principle that each agent relates to its neighbouring agents [2][3]. This has an interesting impact when, for example, encountering an obstacle. Each agent wants to stay close to his neighbours and when faced with an obstacle, in the way of a desired destination, the flock as a whole does not have a difficulty splitting into smaller groups and to later rejoin behind the obstacle [2].

Reynolds makes a comparison to a particle system, meaning that many features are similar. One vast difference, that gives the modelling of flocking behaviour an additionally complex geometrical state, is the fact that each and every agent has a direction [3]. Reynolds also states that it is difficult to point out further differences between a particle system and a model of flocking behaviour, apart from the knowledge that animal behaviour is a complex matter, which is hard to understand and fully describe [3].

1.4 Subsequent studies

Reynolds' work is sometimes seen as the starting point of further studies of modelling of flocking behaviour [2]. The modelling is often divided into two principal frameworks: individual and population. The individual based model, IBM, is recognized as when each agent acts on individual decisions leading to collective behaviour whereas in a population based model, individuals make decisions taking the overall flock movement into account, the density of the flock for example [1][10]. The most common field of study is the individual based model [1].

When modelling with IBM, there has been a strong interest for the consensus problem [2]; the ability for each agent to refer to other agents velocity and adjust to a coherent velocity [2][3]. Consensus problems are frequently encountered within studies of dynamic systems and often the consensus is chosen as a general function on which all agents have a general agreement. When working with an individual based flocking model, this is not the case since there is no limitation or agreement on a coherent movement [2].

Modelling of interaction between flocking prey and predators is a new and interesting aspect within the study of modelling of flocking behaviour. It brings a new dimension to the model where rules of behaviour need to be defined taking an agent's chances for survival into account [1]. In 2005 Lee et al. published a paper on "Dynamics of prey-flock behaviour in response to predators attack" [1] and it is frequently referenced in subsequent studies within this field of study. Lee et al. discuss a model for behaviour of two species interacting in a prey-predator manner. A molecular dynamics simulation was carried out where particles were used to represent prey and a predator. Each prey agent was subject to a set of forces originating from Reynolds' theory of flocking with an addition of an attacking avoidance force. The predator was set to run to the centre of the flocking prey, and by this, disturbing the flocking movement, which was then studied [1]. Several interesting conclusions were made of which some are to be further considered in this thesis.

1.5 Realisation of theoretical biology

By setting up an autonomous system it has been possible to make an approximate model of the flock's behaviour. If we consider animals in flock, we can approximate them by a particle system with direction and by adding predators to the existing system, it is possible to model the flock reaction using an attraction-repulsion model. To make the simulation more realistic, some static obstacles were added (i.e. trees, stones etc.) which results in greater range of motion for the prey. Constraints were also added to limit the speed and the energy of the animals. These constraints are simple and follow basic laws used in physics (e.g. kinetic energy). In biology more complex relationships exist, which our configuration does not account for. Our model builds partly on already existing mathematical models describing flocking behaviour and uses effective strategies to escape the group of attacking predators and saving all the members of the flock.

2 Theoretical behaviour model

In the beginning of this chapter the underlying mathematical model is presented after identifying the problem described in Chapter 1. Subsequently, "imaginary" forces, that regulate the agents' kinematics, are introduced and explained, followed by the description of the prey tactics used in the model.

2.1 Identifying the problem

We want to find the agents' velocities, which have coordinates (x, y, θ) in a fixed cartesian coordinate system.The velocities in different directions can be formulated as a time dependent equation system:

$$\begin{cases} \dot{x} = v \cdot \cos\theta \\ \dot{y} = v \cdot \sin\theta \\ \dot{\theta} = \omega \end{cases} \tag{2.1.1}$$

where v is the magnitude of the translational velocity and ω is the angular velocity. Since a point has no direction, θ can not be found with the information presented above. However, when using the unicycle model, it is possible to find θ through setting an arbitrary point and find the orientation with reference to that point.

2.2 Saturation function

Since there is an upper (and lower) limit for the discussed velocities, we have to find a way to limit those. It can be done with the help of a saturation function, that sets an interval for the allowed values.

The saturation function for the translational velocity is given by:

$$sat(v) = \begin{cases} v_{max} & \text{if } v > v_{max} \\ v & \text{if } 0 \leq v \leq v_{max} \\ 0 & \text{if } v < 0 \end{cases} \tag{2.2.1}$$

where v_{max} is the agents' maximum translational velocity.

In the same way the angular velocity's saturation function is expressed as:

$$sat(\omega) = \begin{cases} \omega_{max} & \text{if } \omega > \omega_{max} \\ \omega & \text{if } -\omega_{max} \leq \omega \leq \omega_{max} \\ -\omega_{max} & \text{if } \omega < -\omega_{max} \end{cases} \tag{2.2.2}$$

where ω_{max} is the magnitude of the maximum angular velocity.

2.3 Unicycle model

The unicycle model is usually used in control theory when an object's orientation is to be determined [4]. It is achieved by setting a base point (x, y) and an arbitrary point (x_h, y_h) relative to the object to define its orientation by an angle, in this case θ. This is done in a two dimensional inertial frame of reference. The simplest point to choose is the animal's head, therefore the distance between the two points is constant and can be defined as the animal's length, as shown in Figure 1 below.

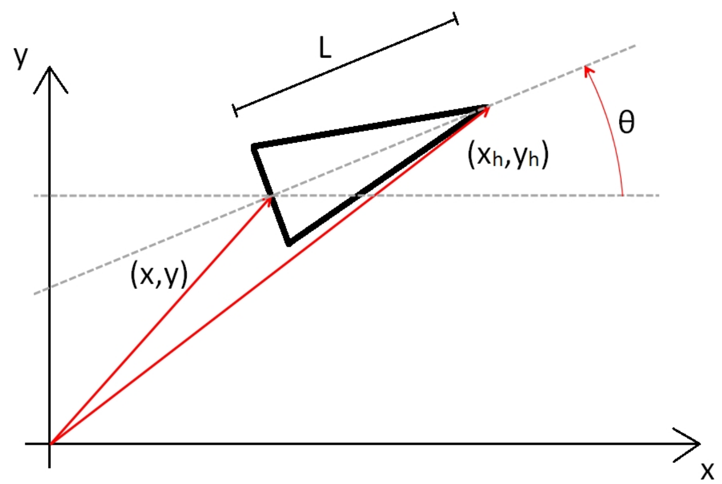


Figure 1: An agent with length L placed in a two dimensional coordinate system

The head coordinates can be defined by the following equation system:

$$\begin{cases} x_h = x + L \cdot \cos\theta \\ y_h = y + L \cdot \sin\theta \end{cases} \tag{2.3.1}$$

where L is the length of the animal and θ is the angle of the head relative to the body. Now that the head's coordinates are defined, its velocities, v and ω can be found. It is done by taking the time derivatives of 2.3.1 and substituting it in 2.1.1 which gives:

$$\begin{cases} \dot{x}_h = \dot{x} + L \cdot (-\sin(\theta)) \cdot \dot{\theta} & = v\cos\theta - \omega \cdot L\sin\theta \\ \dot{y}_h = \dot{y} + L \cdot \cos(\theta) \cdot \dot{\theta} & = v\sin\theta + \omega \cdot L\cos\theta \end{cases} \tag{2.3.2}$$

2.3.2 can also be written as a system of differential equations in the following way:

$$\begin{pmatrix} \dot{x}_h \\ \dot{y}_h \end{pmatrix} = \overbrace{\begin{pmatrix} \cos\theta & -L\sin\theta \\ \sin\theta & L\cos\theta \end{pmatrix}}^{A} \begin{pmatrix} v \\ \omega \end{pmatrix} \tag{2.3.3}$$

A is invertible if and only if det(A) $\neq 0$. If we let the first term of the right hand side in 2.3.3 to be A then its determinant can be calculated in the following way:

$$\begin{vmatrix} \cos\theta & -L\sin\theta \\ \sin\theta & L\cos\theta \end{vmatrix} = \cos\theta \cdot L\cos\theta - L(-L\sin\theta)\cdot\sin\theta = L(\cos^2\theta + \sin^2\theta) \neq 0$$

given that $L \neq 0$ since $(\cos^2\theta + \sin^2\theta) = 1$
As A is invertible, 2.3.3 can be solved for $\begin{pmatrix} v \\ \omega \end{pmatrix}$, that yields:

$$\begin{pmatrix} v \\ \omega \end{pmatrix} = A^{-1} \begin{pmatrix} \dot{x}_h \\ \dot{y}_h \end{pmatrix} \tag{2.3.4}$$

2.3.1 Independence of reference frames

So far in the model it was assumed that a common global frame is used by all agents. It may be questioned how realistic this assumption is. Since the model's purpose is to mimic nature, it can be assumed that a global system does not exist, hence each agent has its own local coordinate system, its body frame, depending on its position. By showing that using a fixed global or local coordinate system as a frame is equivalent, the model can be considerably simplified.

The change of reference frame is the same as changing the basis vectors spanning the initial $\mathbb{R}^2$ space the agents are in. To do that we need a linear transformation matrix that can change the reference frame. Let us define a transformation matrix T as such, then we can write:

$$\begin{pmatrix} \tilde{x} \\ \tilde{y} \end{pmatrix} = T \begin{pmatrix} x \\ y \end{pmatrix} \tag{2.3.5}$$

where $\tilde{x}$ and $\tilde{y}$ are the new x and y coordinates in the transformed frame. For the head coordinates the same transformation is valid thus:

$$\begin{pmatrix} \tilde{x}_h \\ \tilde{y}_h \end{pmatrix} = T \begin{pmatrix} x_h \\ y_h \end{pmatrix} \tag{2.3.6}$$

Since T consists of independent column vectors that are not functions of time, when these transformations are differentiated, T will behave as a constant. It is easy to see that the derivative of the head coordinates becomes the following in the new reference frame:

$$\begin{pmatrix} \dot{\tilde{x}} \\ \dot{\tilde{y}} \end{pmatrix} = T \begin{pmatrix} \dot{x} \\ \dot{y} \end{pmatrix} \tag{2.3.7}$$

$$\begin{pmatrix} \dot{\tilde{x}}_h \\ \dot{\tilde{y}}_h \end{pmatrix} = T \begin{pmatrix} \dot{x}_h \\ \dot{y}_h \end{pmatrix} \tag{2.3.8}$$

According to 2.3.4, the velocities can be acquired in the new frame of reference and by substituting 2.3.8 in that equation we can find the velocities in the new reference system. It can be shown that:

$$\begin{pmatrix} v \\ \omega \end{pmatrix} = A^{-1} \begin{pmatrix} \dot{x}_h \\ \dot{y}_h \end{pmatrix} = A^{-1}T^{-1} \begin{pmatrix} \dot{\tilde{x}}_h \\ \dot{\tilde{y}}_h \end{pmatrix} = (TA)^{-1} \begin{pmatrix} \dot{\tilde{x}}_h \\ \dot{\tilde{y}}_h \end{pmatrix} \tag{2.3.9}$$

We can draw the conclusion that the translational and angular velocities are not dependent on the frame chosen as long as it is fixed in space.

Let us demonstrate this property with a practical example: let us assume that each agent has an own coordinate system with a certain orientation. It is rotated by an angle $\Delta\theta$, shown by Figure 2, with respect to the global frame.

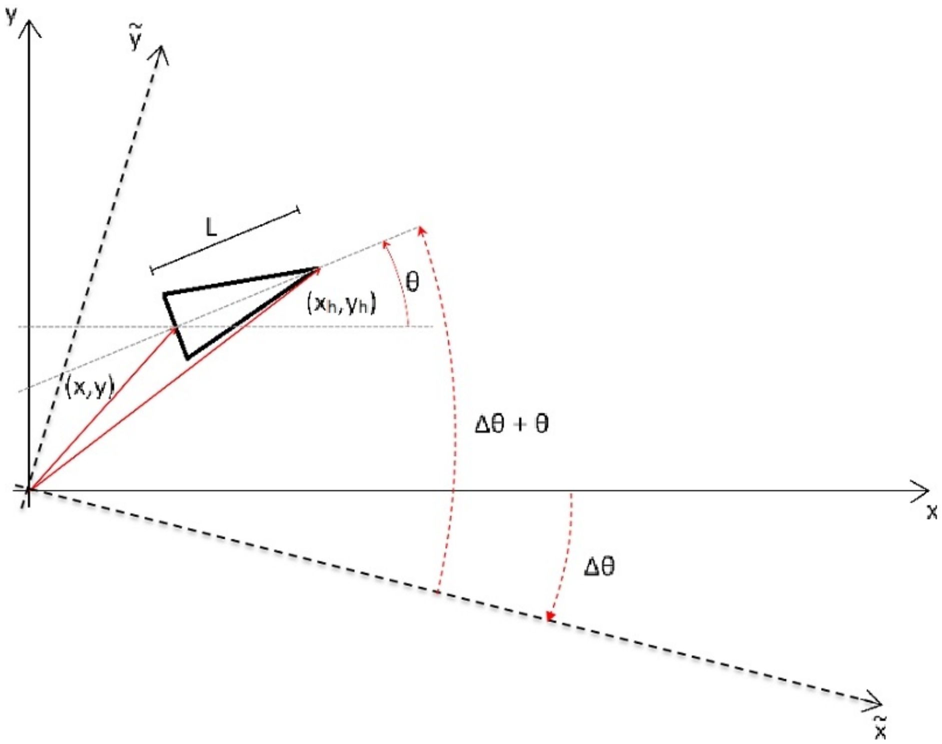


Figure 2: The rotation of the original fixed coordinate system with an angle $\Delta\theta$.

In this case the transformation matrix is a simple rotational matrix acting *clockwise* on the coordinate system, which is equivalent to rotating the position vector *anticlockwise*, so T can be expressed as:

$$T = \begin{pmatrix} \cos\Delta\theta & -\sin\Delta\theta \\ \sin\Delta\theta & \cos\Delta\theta \end{pmatrix} \tag{2.3.10}$$

Now that T is identified, by substituting it in 2.3.9 the following is acquired:

$$\left(\begin{pmatrix} \cos\Delta\theta & -\sin\Delta\theta \\ \sin\Delta\theta & \cos\Delta\theta \end{pmatrix} \begin{pmatrix} \cos\theta & -L\sin\theta \\ \sin\theta & L\cos\theta \end{pmatrix} \right)^{-1} \begin{pmatrix} \dot{\tilde{x}}_h \\ \dot{\tilde{y}}_h \end{pmatrix} = \begin{pmatrix} v \\ \omega \end{pmatrix} \tag{2.3.11}$$

which gives the solution to the problem.

2.3.2 Applied onto the problem

To limit the maximum translational and angular velocities, saturation functions are added, given by 2.2.1 and 2.2.2. After the saturation is applied, they are substituted into equation 2.1.1 to get the desired time derivatives in the x and y directions. The agent's position can be found by integrating these, from which the coordinates of the head can be calculated using 2.3.1. The procedure can be summarized as the following:

$$\begin{pmatrix} \dot{x}_h \\ \dot{y}_h \end{pmatrix} = A \begin{pmatrix} v \\ \omega \end{pmatrix} \rightarrow \begin{pmatrix} v \\ \omega \end{pmatrix} = A^{-1} \begin{pmatrix} \dot{x}_h \\ \dot{y}_h \end{pmatrix} \xrightarrow[\text{function}]{saturation} \begin{pmatrix} v_{sat} \\ \omega_{sat} \end{pmatrix}$$

$$\xrightarrow[\text{into 2.1.1}]{substituted} \begin{pmatrix} \dot{x}_{new} \\ \dot{y}_{new} \end{pmatrix} \xrightarrow{\text{integration}} \begin{pmatrix} x \\ y \end{pmatrix} \rightarrow \begin{pmatrix} x_{h,new} \\ y_{h,new} \end{pmatrix} \tag{2.3.12}$$

If we choose to derive the head positions with respect to time in the next time step we can create a loop, thus creating a process for a chosen amount of time. This algorhitm is run each iteration thus the controller problem is solved. More details about implementing the controller is given in Chapter 3.

2.4 Energy constraint

An energy constraint, $E_i(t)$, is added to the model to make it more realistic, since the agents cannot run for an infinite amount of time due to their physical limitations. The constaint also makes the agent's movement finite, because all forces are switched off when the energy sinks to zero. The expression is defined as:

$$E_i(t) = E_{i,0} - c_E \cdot \int_0^{t_1} v_i(t)^2 \mathrm{d}t \tag{2.4.1}$$

where $E_{i,0}$ is the ith agent's initial energy, c_E is a constant that is related to the agent's mass and $v_i(t)$ is the translational velocity of agent i at time t which lies in the interval $[0, t_1]$ where t_1 is the length of the simulation. This is a result from making the assumption that the energy consumtion is proportional to to the square of the translational velocity.

2.5 Behaviour modelling

In Section 2.3 the unicycle model was introduced but no details were disclosed about what the controller function contains.

Several studies have used a force model to describe flocking behaviour mathematically, for example Lee et. al. and Reza Olfati-Saber [1][2]. The aim with this thesis is to combine the models that the cited papers provide to describe the prey's behaviour that both can use the environment to its advantage to escape a group of predators and also have the ability to flock.

To be able to simulate the prey's behaviour, it is essential to control it. Let us assume that the animal's reaction to external stimulus, which is described by its translational and angular velocity, can be expressed in a dynamical system. The control function, $\mathbf{u}$, is responsible for regulating the velocities, therefore it can be expressed as a vector:

$$\mathbf{u} = \begin{pmatrix} \dot{x}_h \\ \dot{y}_h \end{pmatrix} \tag{2.5.1}$$

To determine the control function $\mathbf{u}$, several forces are used that rule the speed. These forces are not real because in this thesis no acceleration is taken into account for the reason to simplify the simulation, thus the velocity consensus term, suggested by Reynolds, is omitted.

The function controlling agent i's movement is given by:

$$\mathbf{u_i} = \mathbf{f_{i,g}} + \mathbf{f_{i,ca}} + \mathbf{f_{i,fl}} + \mathbf{f_{i,pa}} + \mathbf{f_{i,o}} \tag{2.5.2}$$

where the grouping force is given by $\mathbf{f_{i,g}}$, collision avoidance force is $\mathbf{f_{i,ca}}$, $\mathbf{f_{i,fl}}$ is the flocking force, $\mathbf{f_{i,pa}}$ is the predator avoidance force and the obstacle force is defined as $\mathbf{f_{i,o}}$.

The forces that are presented in the coming section have one property in common: they use the position of the agent that it is acting on which is denoted by $\mathbf{z_i}$. When the force is also related to other agents, that are surrounding agent i, they are then defined as $\mathbf{z_j}$. These forces describe the state of one agent in the flock except if stated otherwise. The control function for each individual in the flock is computed by summing the forces acting on it.

2.5.1 Predator avoidance force

The flocking behaviour is affected when an agent faces danger, in our case; caused by a predator. The members have finite senses however, which means in practice, if the predator is closer than the predator detection distance R_d, as shown in Figure 3, only then comes this force into effect. The most effective strategy to avoid predators was found to be the following equation, based on biological observations [1], which takes all attacking predators, $\mathbf{p_k}$, into account :

$$\mathbf{f_{i,pa}} = \sum_{k:|\mathbf{z_i}-\mathbf{p_k}|\leq R_d} c_{pa} \cdot \left(\frac{1}{1+\exp(\omega_{pa}(|\mathbf{z_i}-\mathbf{p_k}|-R_{pa}))} + 0,3 \right) \cdot \frac{\mathbf{z_i}-\mathbf{p_k}}{|\mathbf{z_i}-\mathbf{p_k}|} \tag{2.5.3}$$

where c_{pa} and ω_{pa} govern the force's magnitude and gradient respectively, as shown in Figure 4. To avoid confusion in behaviour, when several predators are attacking from opposite direction, the function increases rapidly when the attackers get closer than a certain distance, R_{pa}.

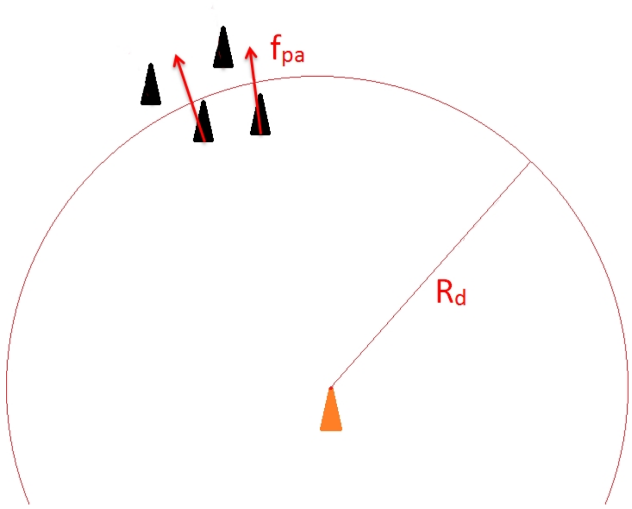


Figure 3: Predator avoidance force is activated when the prey is inside the R_d, the detection radius

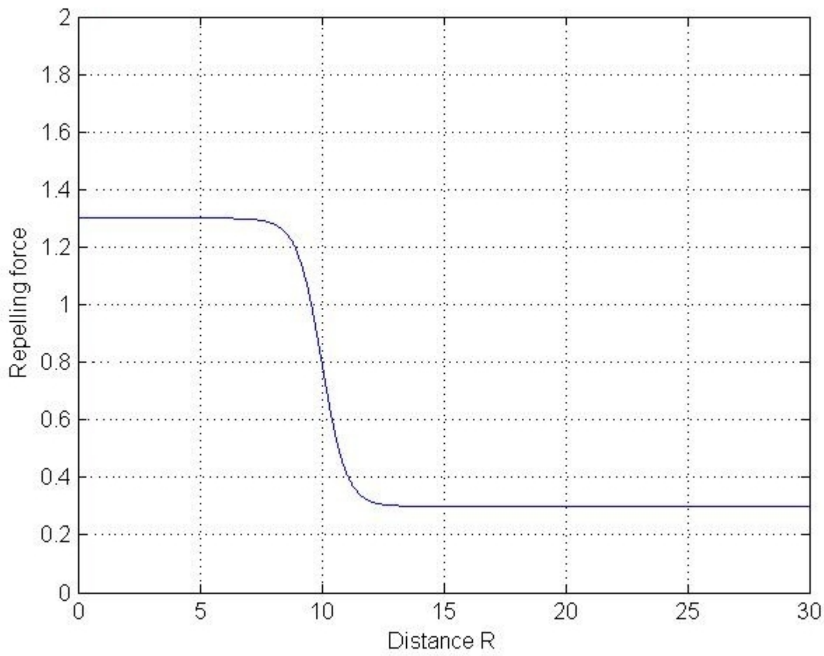


Figure 4: Predator avoidance force increases rapidly as the predator gets closer than 10 length units

2.5.2 Obstacle force

The obstacle force is a combination of forces that give a significant advantage over the attacking predators if obstacles are present, while it can also be used in unthreatened situations: going through a forest for example, the flock members can successfully manouver among the trees. The obstacle force is given by the following equation:

$$\mathbf{f_{i,o}} = \mathbf{f_{i,oa}} + \mathbf{f_{i,la}} \tag{2.5.4}$$

where $\mathbf{f_{i,oa}}$ is the obstacle avoidance force and $\mathbf{f_{i,la}}$ is the line avoidance force.

Obstacle avoidance force was developed to account for obstacles that limit the free movement of flock agents. The obstacles were approximated by circles to simplify the force. It is important to develop a strategy to avoid those hindrances in an effective way not only to save energy but to escape from the attacking agents. To make the force fluent when applied, an outer radius $R_{oa,m,out}$ was introduced for a chosen obstacle m. $R_{oa,m,out}$ is the distance where an agent notices the obstacle, implying that the obstacle forces are activated when inside this radius. The equation, based on biological research [1], that describes this force for agent i and obstacle m inside $R_{oa,m,out}$ is given by:

$$\mathbf{f_{i,oa}} = \sum_{m:|\mathbf{z_i}-\mathbf{o_m}|\leq R_{oa,m,out}} c_{oa} \cdot \frac{1}{1+\exp(\omega_{oa}(|\mathbf{z_i}-\mathbf{o_m}|-R_{oa,m}))} \cdot \frac{\mathbf{z_i}-\mathbf{o_m}}{|\mathbf{z_i}-\mathbf{o_m}|} \tag{2.5.5}$$

where c_{oa} is constant steering the magnitude and ω_{oa} steers the degree of decay in the obstacle avoidance force. $\mathbf{o_m}$ is the position vector of the mth obstacle inside $R_{oa,m,out}$ and $R_{oa,m,in}$ is the obstacle m's radius which determines the radius $R_{oa,m}$, the path that the agent takes.

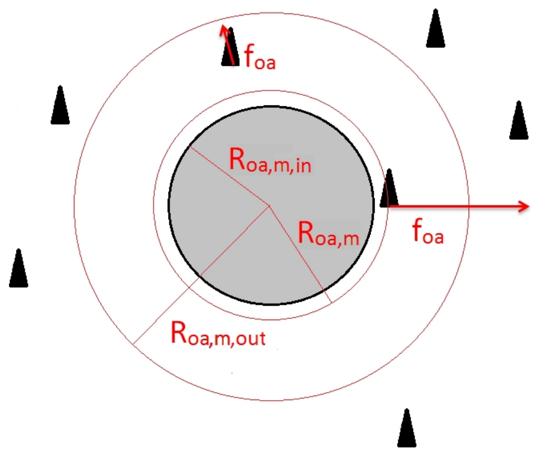


Figure 5: The effect of the obstacle avoidance force at different distances from the center of the obstacle

Since obstacles most likely cannot be moved, it can be safely assumed that this force is the greatest among the forces acting on the prey. A three dimensional plot (Figure 6) demonstrates the force depending on the prey's distance from the obstacle and that a large potential "wall" is felt when the agent is close to $R_{oa,m}$, shown in Figure 7.

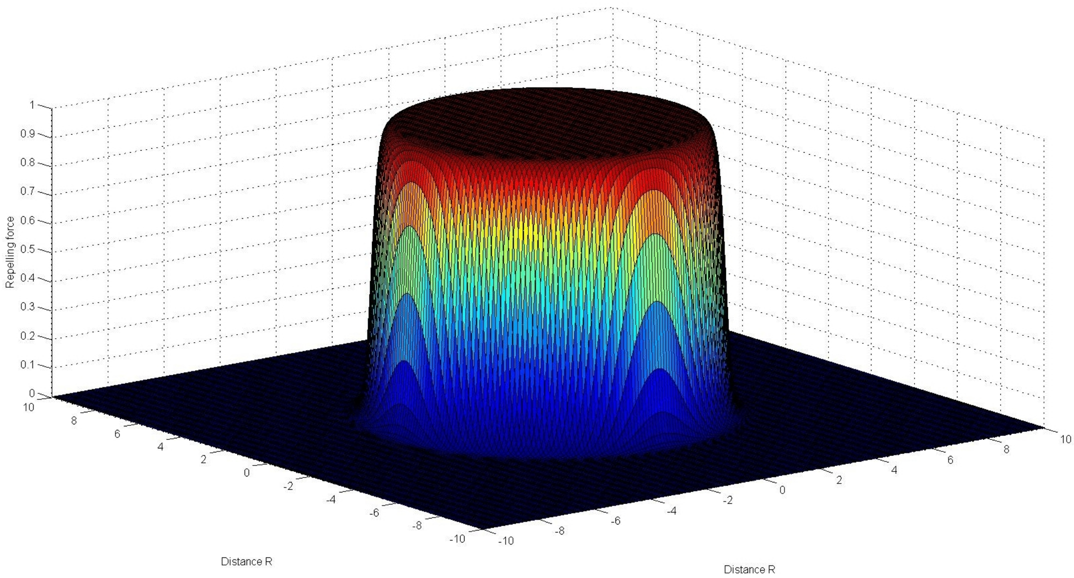


Figure 6: Obstacle avoidance force showing the distance dependence in 3D. The force's magnitude increases exponentially as the agent is getting closer to the center; while outside $R_{oa,m,out}$, it is non existent.

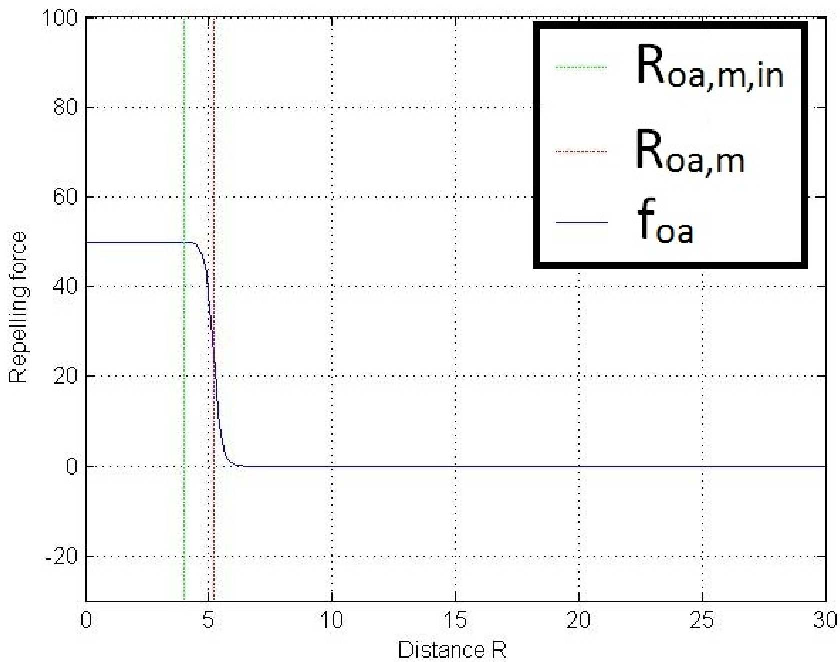


Figure 7: Obstacle avoidance force. The magnitude increases exponentially as the agent is getting closer to the center while outside $R_{oa,m,in}$.

Line avoidance force is activated when a predator is chasing the prey while it is near an obstacle i.e. inside $R_{oa,m,out}$. It stops the predator from making a shortcut during the chase thus increasing the prey's chances for survival. If an obstacle were to be avoided without the line avoidance force whilst under attack, the agents would spread like a V around the obstacle.

Introducing this force will make the agents "hide" behind the obstacle, hence forcing the predators to take the same or a longer way to catch up, as well as make the obstacle avoidance smoother. To do that, the distance from the predator to the center of the obstacle is measured, given by $|\mathbf{p_{k,close}} - \mathbf{o_m}|$. Depending on the prey's position relative to the predator, the prey is either repulsed from or attracted towards the elongated line between the predator and the obstacle's midpoint as displayed in Figure 8. The force can be summarized by the following equations:

$$\mathbf{f_{i,la}} = \begin{cases} \sum\limits_{m \in N_a} c_{la} \cdot \mathrm{proj}_{\mathbf{c}} \mathbf{a} \\ N_a : |\mathbf{z_i} - \mathbf{p_{k,close}}| \leq R_d \land |\mathbf{z_i} - \mathbf{o_m}| \leq R_{oa,m,out} \\ \land |\mathbf{z_i} - \mathbf{p_{k,close}}| \leq |\mathbf{p_{k,close}} - \mathbf{o_m}| \\ \sum\limits_{m \in N_b} -c_{la} \cdot \mathrm{proj}_{\mathbf{c}} \mathbf{b} \\ N_b : |\mathbf{z_i} - \mathbf{p_{k,close}}| \leq R_d \land |\mathbf{z_i} - \mathbf{o_m}| \leq R_{oa,m,out} \\ \land |\mathbf{z_i} - \mathbf{p_{k,close}}| \geq |\mathbf{p_{k,close}} - \mathbf{o_m}| \\ \mathbf{a} = \dfrac{\mathbf{z_i} - \mathbf{p_k}}{|\mathbf{z_i} - \mathbf{p_k}|^2} \\ \mathbf{b} = \mathbf{z_i} - \mathbf{p_k} \\ \mathbf{c} = \begin{bmatrix} 0 & -1 \\ 1 & 0 \end{bmatrix} \cdot \dfrac{\mathbf{o_m} - \mathbf{p_k}}{|\mathbf{o_m} - \mathbf{p_k}|} \\ \mathbf{p_{k,close}} = p_k \in \min\limits_{\{i \in n | k \in q\}} |\mathbf{z_i} - \mathbf{p_k}| \end{cases} \tag{2.5.6}$$

where c_{la} is a constant regulating the force's magnitude while the sign in front of it determines whether the force is attractive or repulsive towards the center line connecting the predator and the obstacle's center. $|\mathbf{z_i} - \mathbf{p_{k,close}}|$ is the distance between agent i and the closest predator to it. $\mathbf{b}$ is the vector from predator k till agent i which is active when the agent is behind the obstacle. $\mathbf{a}$ is the same vector with inverted length, which is active between the obstacle and predator and thus its value increases as it gets closer to the obstacle. $\mathbf{c}$ is the normal vector to the center line. n is the set of prey and q is the set of predators.

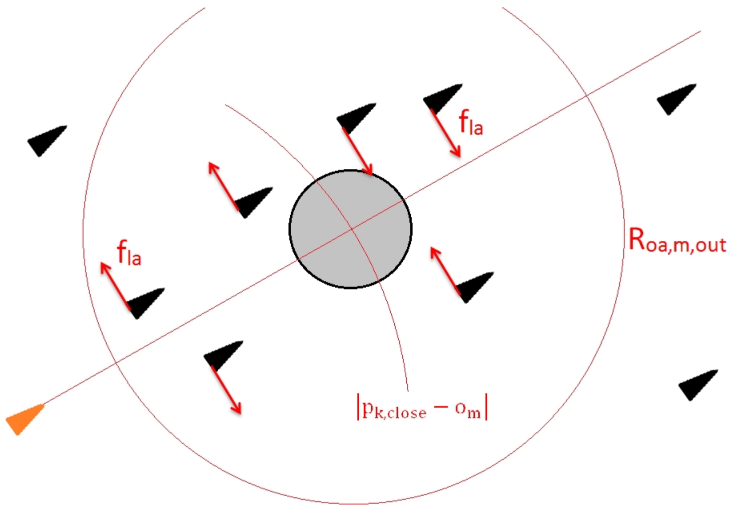


Figure 8: Line avoidance force showing the different scenarios depending if the agent is closer to the predator than the midpoint of the obstacle or not. If the prey is closer than $|\mathbf{p_{k,close}} - \mathbf{o_m}|$ the force is repulsive while if it is further away it becomes attractive

2.5.3 Flocking force

For the flock to stay together there is a force needed to hold the agents together. This is done by taking the position of the agents' center of mass, $\mathbf{z_{mc}}$ and have an attractive force pointing towards it thus causing flocking, see Figure 9. The force is relatively weak and it is only present when the prey does not sense any danger.

$$\begin{cases} \mathbf{f_{i,fl}} = c_{fl} \cdot (\mathbf{z_{mc}} - \mathbf{z_i}) \\ \mathbf{z_{mc}} = \sum_{i=1}^{n} \frac{\mathbf{z_i}}{n} \end{cases} \tag{2.5.7}$$

where c_{fl} governs the overall strength of the force. Due to positive linearity the agents further away are more attracted than those near the center of mass, as displayed in Figure 10.

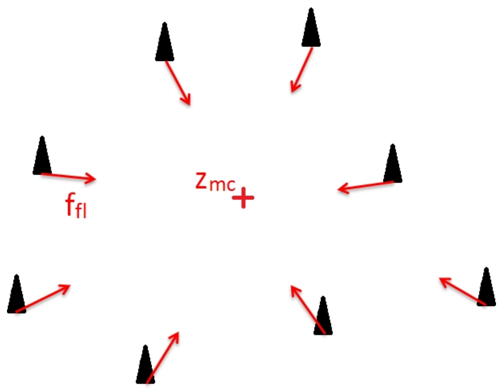


Figure 9: Flocking force is acting on each agent pointing towards a common gathering point, the agents' center of mass.

It is worth pointing out that by choosing the center of mass as a flocking point the total energy of the flock is most optimally used although some agents have to use much more energy to get to the global gathering point, $\mathbf{z_{mc}}$. By having a relatively small force, the agent uses a lower speed to get to the flocking point although it is optimized to get to the same flocking area as soon as possible due to the function's linear property. An other alternative would be to use a point that is almost equally distant from each member but it would be more complex to simulate and less optimal from the flock's point of view concerning energy use.

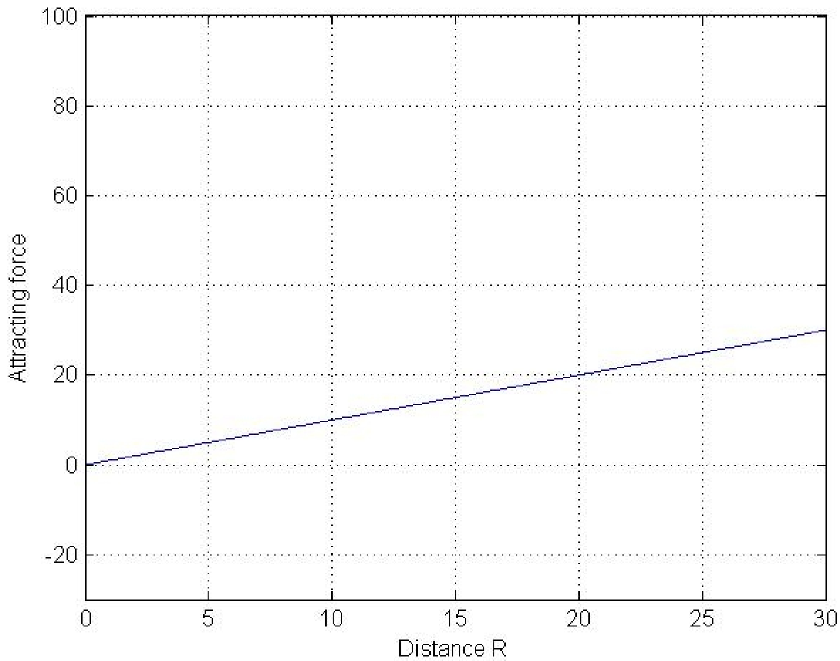


Figure 10: Flocking force increases as the agent gets further away from the flock.

2.5.4 Collision avoidance force

Collision avoidance is always necessary, if there is a group of agents, to avoid accidents and to have a peaceful group dynamic [8]. To model the agent's protection of its personal space, a function rapidly decreasing with distance is desired, therefore the exponential function was used as shown below:

$$\mathbf{f_{i,ca}} = \sum_{\substack{j:|\mathbf{z_i}-\mathbf{z_j}|\leq R_n \\ j\neq i}} c_{ca} \cdot \frac{1}{1+\exp(\omega_{ca}(|\mathbf{z_i}-\mathbf{z_j}|-R_{ca}))} \cdot \frac{\mathbf{z_i}-\mathbf{z_j}}{|\mathbf{z_i}-\mathbf{z_j}|} \tag{2.5.8}$$

where c_{ca} is a constant, steering the magnitude of the force, and ω_{ca} is a constant responsible for the degree of decay. The force is only activated when the agents are closer than the radius for neighbour detection, R_n, displayed in Figure 11. The trivial case, when the agent has to avoid itself, is omitted. The force can increase fast depending on the agent's temper (governed by ω_{ca}) and aggression (dependent on the parameter c_{ca}) of the agent. An example for a typical agent is shown by Figure 12. The assumption was made that no dominant agents are in the flock thus everyone has the same ω_{ca} and c_{ca}.

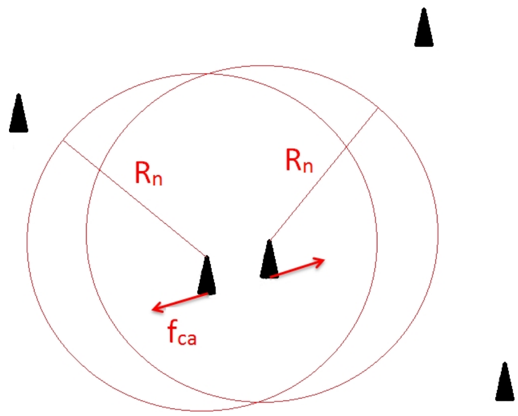


Figure 11: Collision avoidance force is only felt while inside the neigbour detecting radius, R_n.

Collision avoidance is important but it can happen that animals bump into each other when a stronger forces are present, like threat from predators or presence of obstacles. Therefore, this repulsive force is weaker than the predator avoidance and obstacle forces. The direction vectors are normalized to enhance the function's controllability.

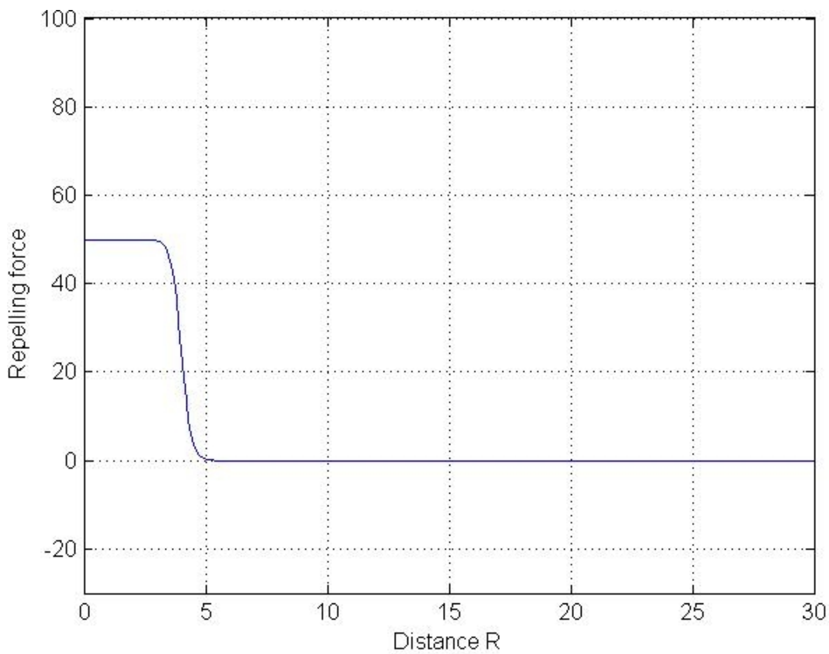


Figure 12: Collision avoidance force increases fast in a short distance interval thus almost having linear characteristics

2.5.5 Grouping force

The most important aspect of flock behaviour is the cooperation between the agents. It is demonstrated by acting and staying together for the reasons mentioned in Chapter 1. To achieve this, each flock member needs to have information about its closest neigbours' locations. The force affects the agents in a local area which is defined by a distance,R_n, within which they consider surrounding flock members to be neigbours, as shown in Figure 13.

$$\mathbf{f_{i,g}} = \sum_{\substack{j:|\mathbf{z_j}-\mathbf{z_i}|\leq R_n \\ j\neq i}} c_g \cdot (\mathbf{z_j} - \mathbf{z_i}) \tag{2.5.9}$$

where c_g is a constant determining the force's magnitude. This force encourages local groups to form after recovering from an attack, so if the rest of the group is lost, flocking can still be performed locally. The force is relatively weak when compared to obstacle or predator avoidance forces. This force also creates a distance between the agents by opposing the collision avoidance force, as seen in Figure 14.

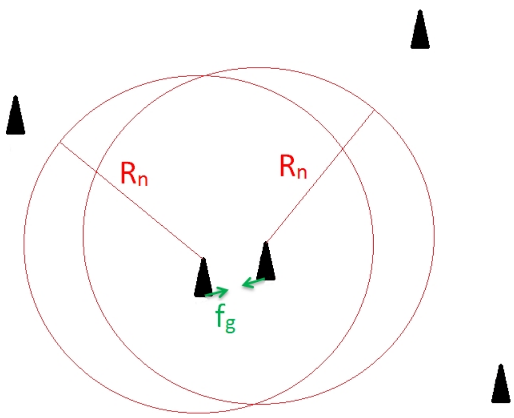


Figure 13: Grouping force acts in a linear fashion on agents that are closer than R_n to each other; agents outside R_n are not considered

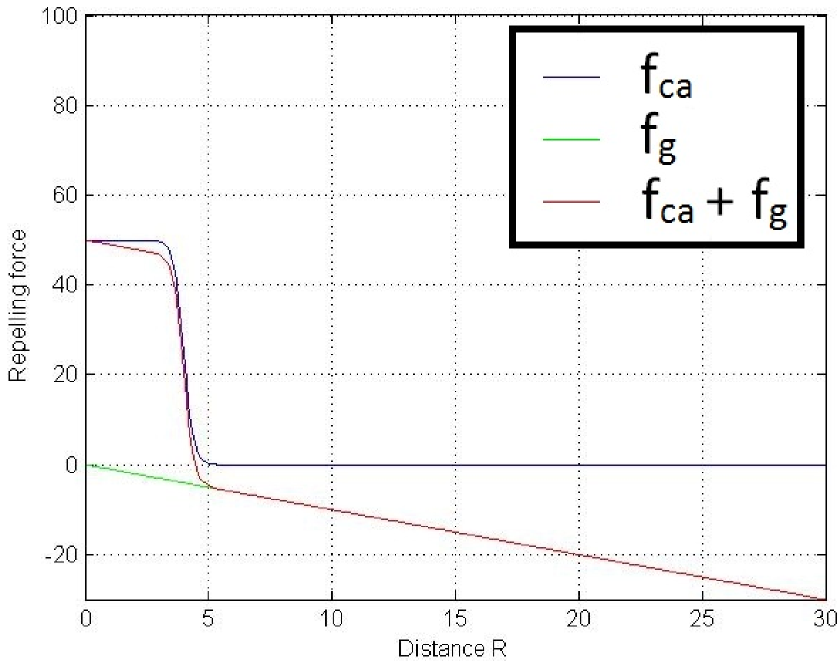


Figure 14: The equilibrium between the collision avoidance and grouping forces is controlled by R_{ca}

2.6 Flock strategies

The agents' behaviour is described with the use of mathematics. Let us represent the change in state with a set of forces that infuence the agent's behaviour. We can distinguish among different strategies used by the flock depending on if facing danger or not, and how great the risk is that one of the flock members is to be captured. It is a valid assumption that the predators are only attempting to catch one agent therefore they will combine forces to maximize their success. Three different strategies are implemented. The default one is when no danger is apparant, another when a danger has appeared and the flock has to escape, and the last one when one agent is jeopardized and wants to survive at all costs.

2.6.1 Casual behaviour

Since there are no predators nearby, there is no predator avoidance force. In this state the flock saves its energy by staying in one area and aligning close to each other to maximize the advantages that come with flocking as described in Chapter 1. The flocking and grouping forces are strong while the collision avoidance force is ten times weaker than in other behaviour forms thus the agents can get closer to each other. A force equilibrium can easily be formed with these parameters which makes the flock stay in one place. Depending on the flock's heading, obstacle force can be activated if the requirements are fulfilled as described in Section 2.5.2.

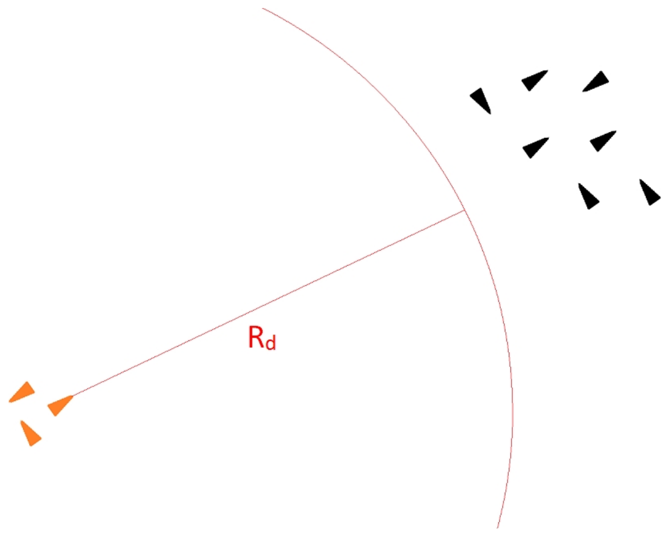


Figure 15: Casual behaviour, showing that the prey are staying in one place due to the fact that a force equilibrium exists between the flocking, grouping and collision avoidance forces, given that the predators are outside R_d

2.6.2 Behaviour when facing danger

When at least one of the attackers is inside the detection radius R_d, the prey can detect them so the fleeing behaviour is triggered. The grouping force is inverted so the prey spread out to make the predators confused. The flocking force is disabled and the collision avoidance force is increased (from 10%) to its original value.

2.6.3 Behaviour for survival

In this behaviour model the agent is ignoring its flock and only concentrates on survival. The consequence is that the grouping and flocking forces are set to zero. When there are only moments left till the prey is caught, it uses a last minute tactic which uses a fast swing to the side that the predator cannot follow because of its lower angular velocity. Activation is according to Figure 17.

The direction of the swing is determined by the predator's position as shown in Figure 16. Taking the cross product between the vectors that are between the closest predator and the prey, $\mathbf{L_z}$, and the center of mass of the chasing predators and the prey, $\mathbf{L_{mz}}$, (in this order) gives the optimal direction for the swing. It is defined by the right hand rule so if the third term of the vectors' cross product, directed out of the plane, is positive the prey turns anticlockwise, and clockwise when it is negative. Although, if inside the red circle, seen in Figure 17, the agent is most likely moving away from the predator, i.e. towards the predators' center of mass, m_p. If it was to turn accordingly to the result from taking the cross product, it would mean turning towards the predators' center of mass. To avoid this, another relation is introduced; if the angle between the vectors is smaller than $\pi/2$, the agent is outside of the circle; if greater, it is inside. When inside the circle, the agent should turn opposite to the direction given by the cross product, hence turning away from the predators' center of mass.

$$L_c = (0,0,1) \cdot \left(\frac{\mathbf{L_z}}{|\mathbf{L_z}|} \times \frac{\mathbf{L_{mz}}}{|\mathbf{L_{mz}}|} \right) \tag{2.6.1}$$

$$L_a = \arccos \left(\frac{\mathbf{L_z}}{|\mathbf{L_z}|} \cdot \frac{\mathbf{L_{mz}}}{|\mathbf{L_{mz}}|} \right) \tag{2.6.2}$$

$$R = \begin{pmatrix} \cos(c_c \cdot c_a \cdot \pi/4) & -\sin\theta(c_c \cdot c_a \cdot \pi/4) \\ \sin(c_c \cdot c_a \cdot \pi/4) & \cos(c_c \cdot c_a \cdot \pi/4) \end{pmatrix} \tag{2.6.3}$$

$$c_c = \begin{cases} 1 & \text{if } L_c \leq 0 \\ -1 & \text{if } L_c > 0 \end{cases} \tag{2.6.4}$$

$$c_a = \begin{cases} 1 & \text{if } L_a \leq \pi/2 \\ -1 & \text{if } L_a > \pi/2 \end{cases} \tag{2.6.5}$$

The rotation is executed by multiplying the predator avoidance vector, $\mathbf{f_{i,pa}}$, for the specified agent with the rotation matrix, R, defined according to 2.6.3. The constant c_c is defined as: if the agent is on the right, 1, or left, -1, side of the predator. c_a is defined as: if the agent is inside, -1, or outside, 1, of the red circle in Figure 16 respectively. They only may take values 1 or -1 according to 2.6.4 and 2.6.5.

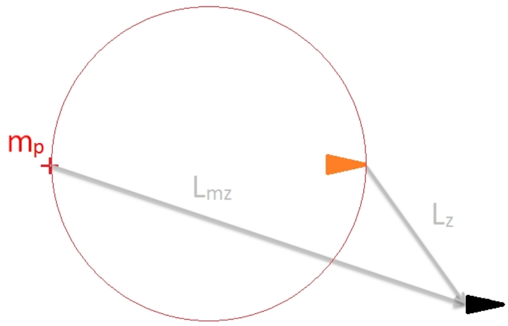
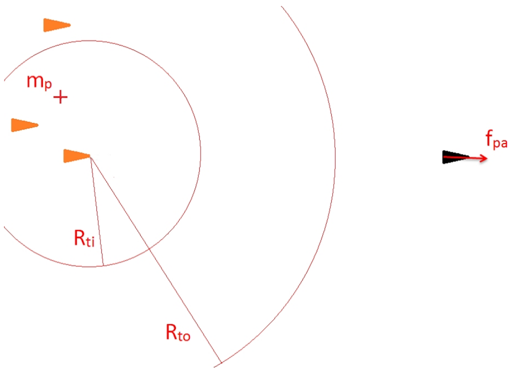
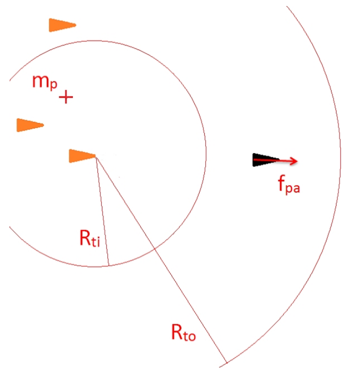


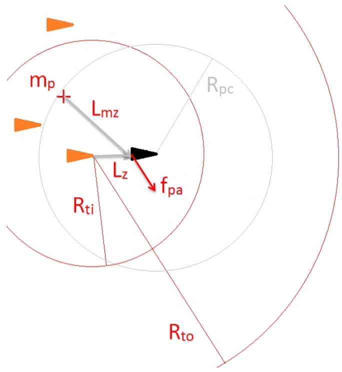
Figure 16: Depending on the predators' center of mass the prey chooses the direction that gives the greatest chance for survival.



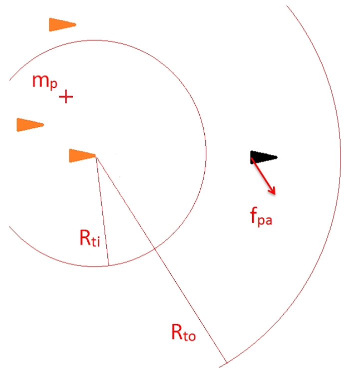
(a) The agent is outside of the outer radius, R_{to}



(b) The agent has entered the outer radius



(c) The agent has entered the inner radius, R_{ti}, and given that only one predator is inside the radius R_{pc}, the survival tactic is activated



(d) The agent is still inside of the outer radius, keeping the survival tactic active until getting outside

Figure 17: The activation process of different tactics

3 Simulation of the model

In this chapter the mathematics presented in Chapter 2 are implemented to be able to carry out the simulation. An introduction is given to the main ideas behind control theory and its implementation in Matlab.

3.1 Control theory

Control theory is the theory behind controlling a system; i.e. controlling the output of an event. Whether it is the temperature of a house, the takeoff of a rocket or in this case the movement of animals, the system needs some kind of controller. They may vary a lot in detail, but the principles are the same.

3.1.1 Fundamentals

There are mainly two types of systems, the open-loop system and the closed-loop system. The open-loop system gives an output regardless of the state of the system. In modeling of flocking behavior this system would give the agents a path regardless of situation, which does not correspond very accurately to reality. The closed-loop system gives an output depending on the state of the system, however. The agents' movement may be controlled differently depending on the agents' positions relative to each other. According to Glad & Ljung [5], the advantages of such a controller are:

- The exact properties of the system being controlled are not needed
- Immeasurable disturbances in the system are accounted for in the feedback, needing no special care
- An unstable system can be controlled using feeback

This system gives a good foundation in flockmodeling, as it is not exact and cannot be investigated upon stability.

3.1.2 Applied to moving objects

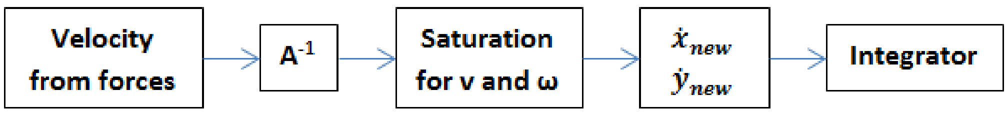
The open-loop system is a loop with different steps, each with its own purpose. It takes an input and transforms it to an output via a controller:

The closed loop is built up with the same steps, but loops back, giving the input feedback about the systems current state.

When applied to movement of objects, the input would be a position, the controller then uses that position to create a velocity vector, which is then integrated into an output, in form of a new position. Using a closed-loop system on moving objects involves solving differential equations, since the movement towards a new position is dependent on the current position. Solving a system of non-linear differential equations this is close to impossible doing analytically, hence a numerical solution using a computer model is introduced in Section 3.2.

3.1.3 Using the Unicycle Model

The unicycle model, described in Section 2.3, is used as a controller in a closed-loop system. The input, as well as output, is the current positions of prey, predators and obstacles, implying this is what the controller uses.



The process can be described as the following:

1. Velocity vectors for the agents are calculated from the forces described in Section 2.5
2. Velocity vectors are transformed into a translational and an angular velocity for each agent using the inverted A-matrix
3. Translational and angular velocities are saturated to the allowed domain
4. Saturated translational and angular velocities are transformed back to velocity vectors
5. Integrates the velocities in x, y and θ to get the new positions and directions

3.2 Simulink

In setting up the block based model, software Matlab was used with the plug-in Simulink, an environment for designing and simulating models for dynamic systems. Providing a graphical workspace to set up blocks with predefined functions, such as an integrator as well as own made blocks with Matlab code.

By using Simulink, the model could be built in the way described in Section 3.1 giving the possibility of changing individual blocks, rather than the whole code and, at the same time, using the predefined blocks for solving differential equations numerically [6].

When solving differential equations in Matlab, the predefined solvers may be used for highest efficiency of the program as well as providing a solution with easily adjusted accuracy. The predefined solver ode45 was used for all calculations in this thesis. It is based on an explicit Runge-Kutta formula and takes only the immediately following time step into account [7]. The time step was set to be varied and therefore it was adapting to the necessary accuracy of each iteration.

3.3 Plotting methods

The system of non-linear differential equations used in the model of this thesis is so far yet to be solved, making calculations of the model's success impossible. Therefore plots of the agents' progress in time, i.e. simulations, are necessary.

Two types of plots are used in the presentation of the results, Chapter 4: plots of the agents at a fixed time and plots with traces of the agents during a time interval.

4 Simulation results

Below are the results from simulation. Key features have been chosen and specific cases have been selected where certain forces have been isolated. Simulations of longer runs are shown in steps to describe the overall movement of the agents and their interaction with each other.

Constant	Value	Unit
L	2	[length unit]
v_{max}	$0,8 \cdot 25$	[l.u./s]
ω_{max}	3π	[rad/s]
E_0	20 000	[mass.u.$\cdot$(l.u./s)^2]
c_E	1	[m.u./s]
c_{pa}	300	[l.u./s]
ω_{pa}	2	[1/l.u.]
R_{pa}	10	[l.u.]
R_d	30	[l.u.]
c_{oa}	1 200	[l.u./s]
ω_{oa}	8	[1/l.u.]
R_{oa}	$1,3 \cdot R_{oa,in}$	[l.u.]
$R_{oa,out}$	$10 + R_{oa,in}$	[l.u.]
c_{la}	20	[1/s] or [(l.u.)^2/s]
c_{fl}	2	[1/s]
c_{ca}	10	[l.u./s]
ω_{ca}	2	[1/l.u.]
R_{ca}	4	[l.u.]
R_n	10	[l.u.]
c_g	1	[1/s]
R_{to}	2	[l.u.]
R_{ti}	3	[l.u.]
R_{pc}	20	[l.u.]

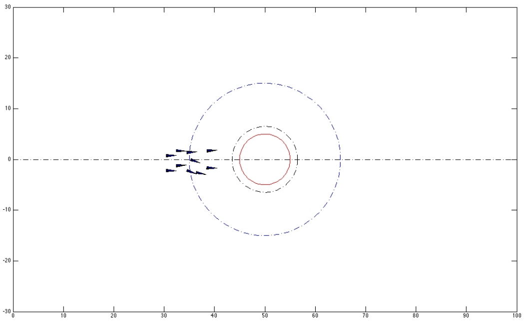
Table 1: Constants and their values used during simulation

4.1 Obstacle avoidance

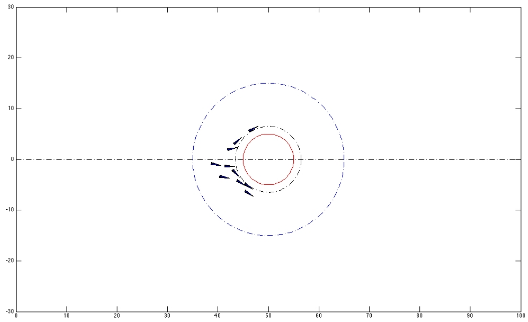
To be able to avoid obstacles is a vital feature that the agents should possess if they are to survive in a harsh environment. Therefore two examples are shown, demonstrating the forces presented in Section 2.5

4.1.1 Single obstacle

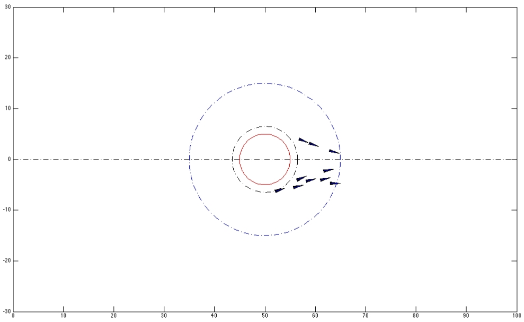
In the simulation a group of agents are faced with an obstacle ahead of them. In Figure 18, a flock being chased by a predator, coming from the left out of the picture, is shown to pass an obstacle smoothly and positioning themselves in such a way that the obstacle is between them and the predator. This is a result partly because of the obstacle avoidance force, keeping the agents from colliding with the obstacle, partly because of the line avoidance force, giving a motion similar to the form of a drop of fluid around a solid body, as seen in Figure 19. As the agents enter the outer radius, their paths are seperated because of the presence of the obstacle avoidance force. When closer to the obstacle no agents are near to the crossing of the center line and the inner radius. Behind the obstacle, the paths are closing in on the center line and the agents form a group.



(a) The agents are approaching the obstacle which is now visible.



(b) The agents have now entered the radius where the line avoidance force is activated.



(c) Once the agents have passed the obstacle they want to close in on the line.

Figure 18: The different steps when passing an obstacle. The red circle defines the obstacle. The outer and inner radius describes the area where the obstacle becomes visible and the area where the obstacle avoidance force has its exponantial increase respectively. The black doted line notes the line of which the agents refer to when chased by a predator agent and faced with an obstacle.

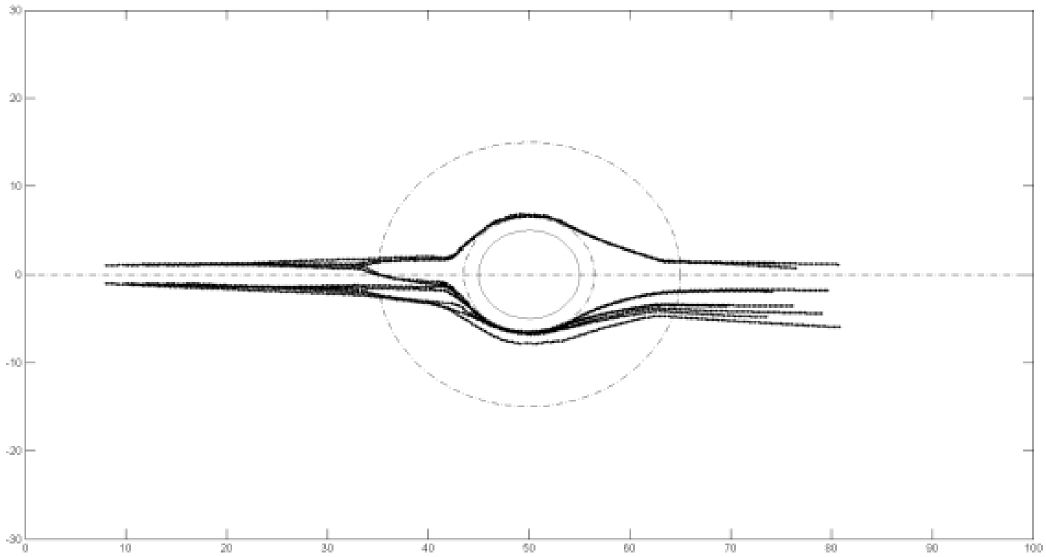
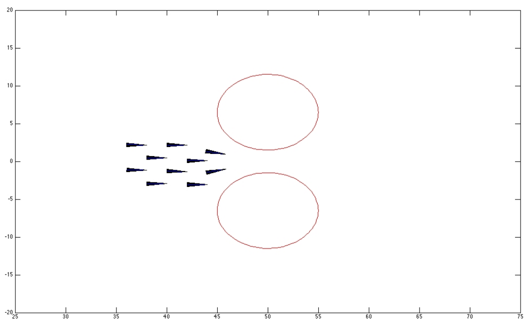


Figure 19: The trajectory of agents' movement when passing an obstacle.

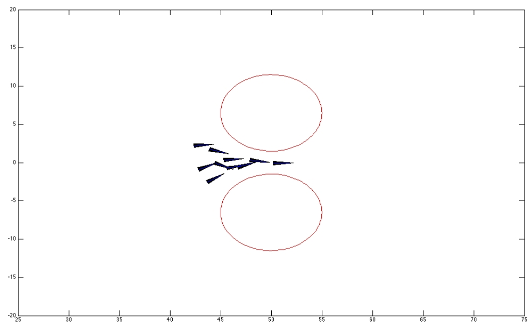
4.1.2 Multiple obstacles

The results of multiple agents facing the narrow path between two obstacles. Showing how the line avoidance force impacts the agents when there are more than two possible routes.

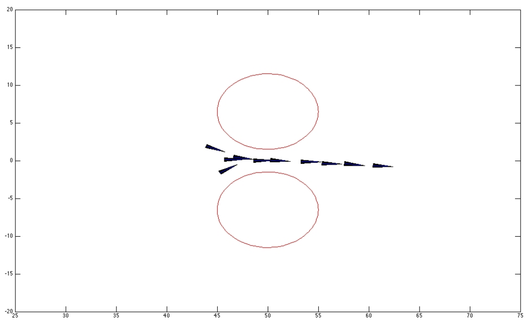
In Figure 20 a flock chased by a predator, out of the picture coming from the left, forced between two obstacles is shown. The obstacle forces push the agents towards the center between the obstacles, while the collision avoidance force positions the agents at a set distance apart. The agents enter the passageway one by one and those not fitting wait rather than turn around because of the danger present behind them. None of the agents are entering either of the obstacles as a result of a very high obstacle avoidance force.



(a) Agents approaching two obstacles



(b) Agents entering passage between obstacles

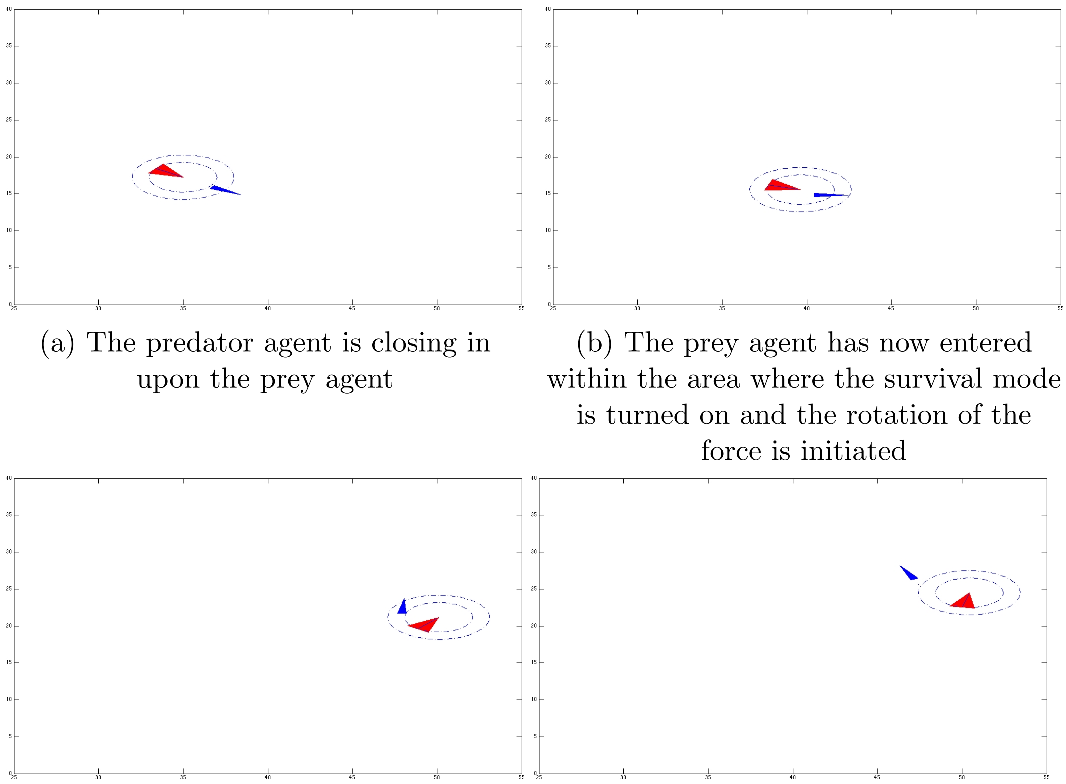


(c) Agent shown at end of passage

Figure 20: The agents' movement when faced with a narrow passage between two obstacles.

4.2 Survival Tactic

The survival tactic was introduced to increase the probability for a prey to avoid a single predator, Figure 21 shows the result when the survival tactic is activated and later deactivated. The outcome is clear; the prey has successfully outmaneuvered the predator and increased the distance between them, thus proving the tactic's efficiency.



(a) The predator agent is closing in upon the prey agent

(b) The prey agent has now entered within the area where the survival mode is turned on and the rotation of the force is initiated

(c) Survival tactic is turned on until the agent has reached the outer radius.

(d) Survival tactic is turned off

Figure 21: The four key states when the survival tactic is implemented. The inner radius defines the area where the survival tactic is turned on and when passing the outer radius the survival tactic is turned off

4.3 Model's dependability on different environments

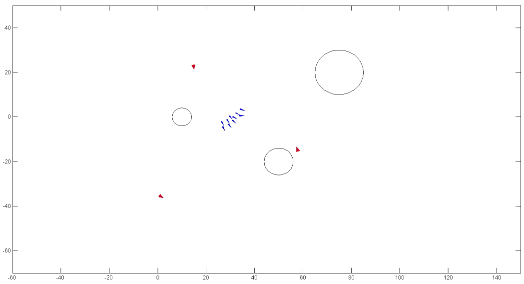
In Figures 22-24, three different environments are created. The first being varied with open areas as well as single obstacles, the second with almost no open areas and all escape routes blocked and the third completely without obstacles. The plots show complete simulations [9] from the start of the attack until the end when the predators run out of energy, all in eight steps.

Figure 22 depicts an open area with some obstacles where the prey are in a compact group (a). When the predators are noticed, the group immediately breaks up and the distance between each agent is increased (b). The predators' first approach is neatly avoided, but then a single prey seems to be focused on (d). It avoids the attacks while the other agents group together (e). As the predators' energy runs out, the prey regroup and stay on a safe distance from the predators (h). Between plots (f) and (g) the single prey uses the obstacle to outmaneuver the following predators.

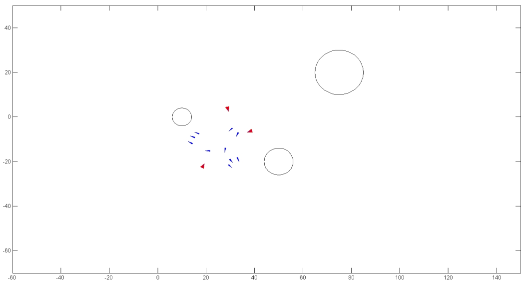
Figure 23 shows the prey in a forest-like environment. They are organized as a dense group (a) and when the predators become visible the group spreads out (b). Similar to Figure 22, the prey avoid the first attack neatly and divide the group into taking different routes between the obstacles (c). The predators target a single prey, but have a hard time keeping up with its agility amongst the obstacles (d), hence making any attempts of encircling the prey useless (e). When the predators' energy run out (f), the prey may regroup yet again as they have succeeded in avoiding the predators (h).

Figure 24 shows an environment much like an open field in nature, with no obstacles. The prey are positioned as a group close together, as the predators circle around (a). When the predators are spotted, the agents instantly move apart, but with no obstacles in the way of the predators they are able to close in rapidly (b). Although, the obstacle free environment allows for the prey to escape as rapidly when the first attack has been avoided (c). The single prey being chased has no advantage of its superior obstacle avoidance or getting the predators out of formation (c)-(e). Once the predators are out of energy, the prey may regroup and keep away from the slowly moving predators (f)-(h).

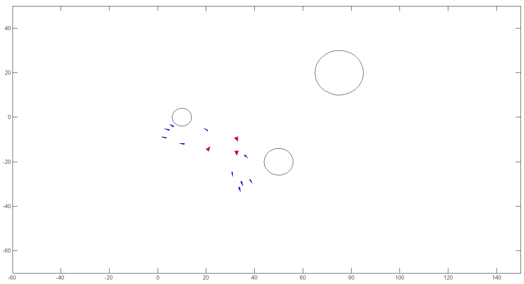
The escaping prey are supposed to see the environment as an advantage rather than a disadvantage; i.e. an obstacle filled environment is in many cases proven to be more advantageous. These plots, Figures 22-24, are just three out of many simulations done, but show the general behavior witnessed in most of the simulations, not all escape attempts were successful however. It can be concluded with great certainty that the majority of the simulations done with the chosen optimal values for the constants gave a successful outcome, i.e. the prey managed to escape.



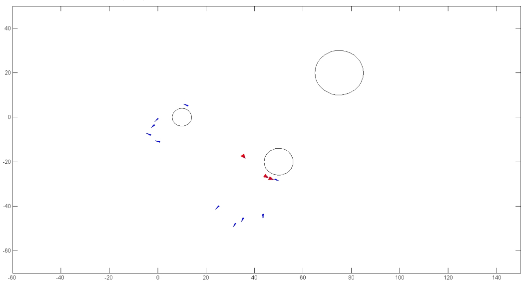
(a) Prey are surrounded



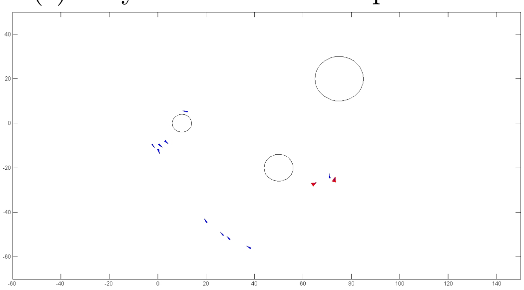
(b) Prey are expanding



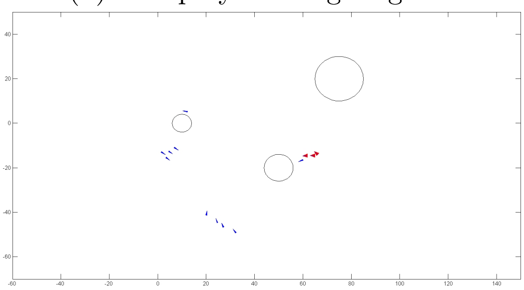
(c) Prey outmanouver the predators



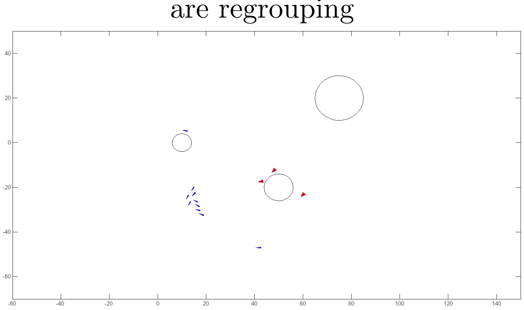
(d) One prey is being targeted



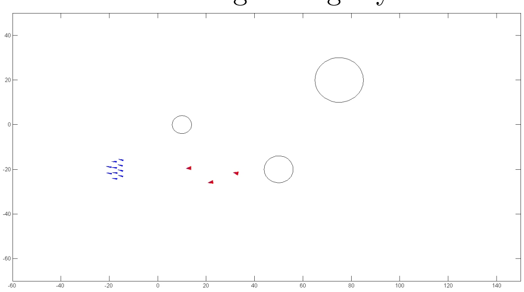
(e) One prey is escaping and the others are regrouping



(f) The single prey is using its advantageous agility

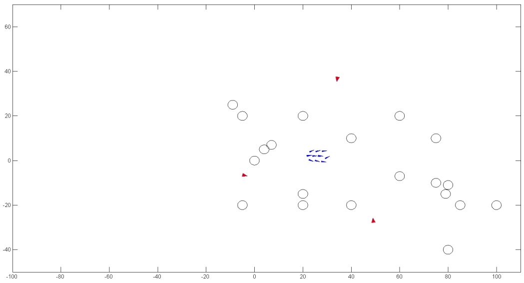


(g) Predators are out of energy

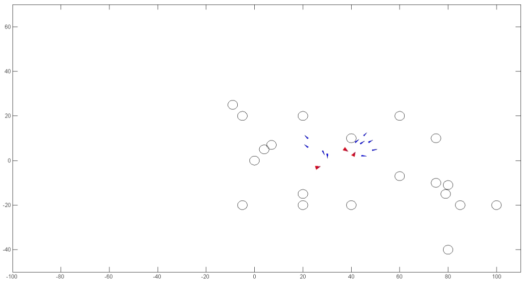


(h) Prey have regrouped

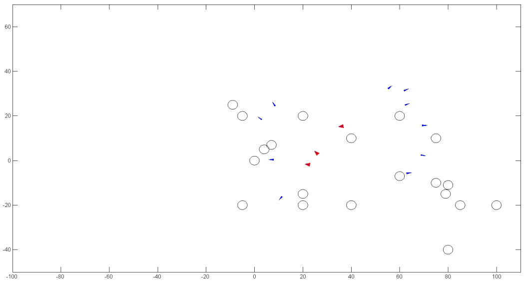
Figure 22: Plots showing ten agents successfully escaping a three predator attack in an environment with multiple obstacles.



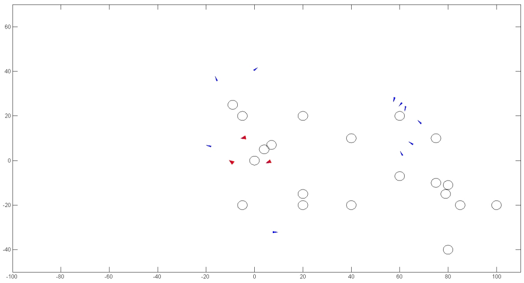
(a) Prey are surrounded



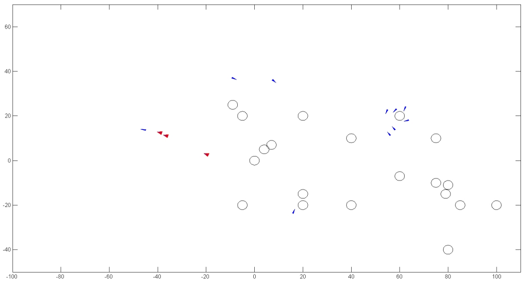
(b) Prey outmanouver the predators



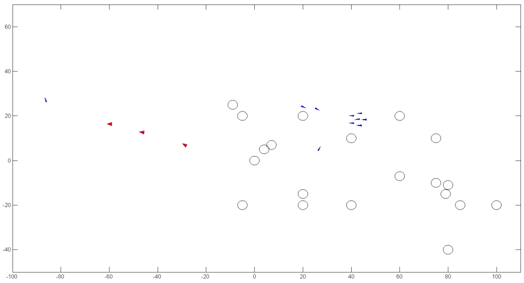
(c) Prey have expanded their area



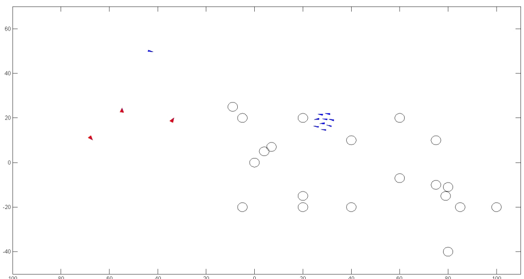
(d) One prey is escaping while the others are regrouping



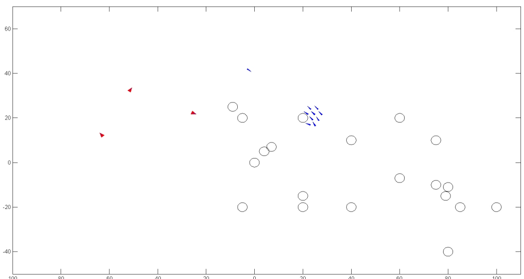
(e) The single prey is avoiding the predators



(f) Predators are out of energy

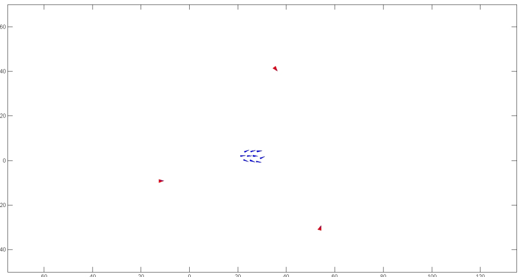


(g) Prey are regrouping

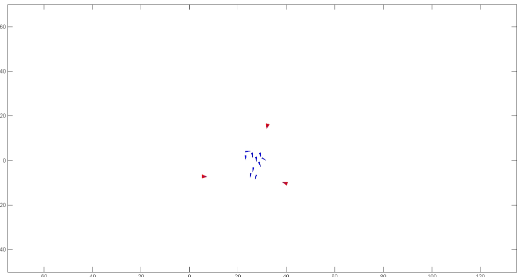


(h) Most of the prey have regrouped

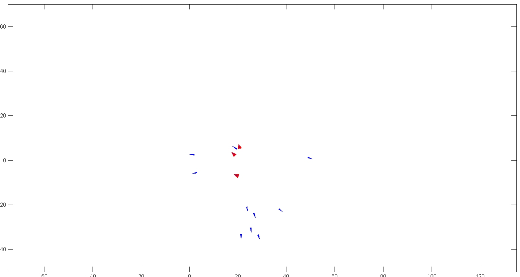
Figure 23: Plots showing ten agents escaping a three predator attack in an obstacle rich environment, with success.



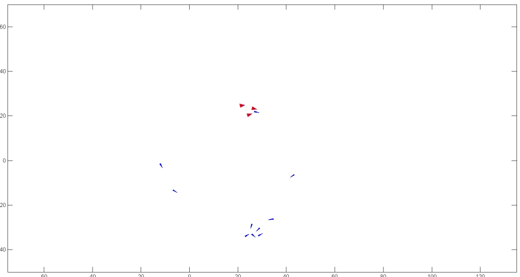
(a) Prey are surrounded



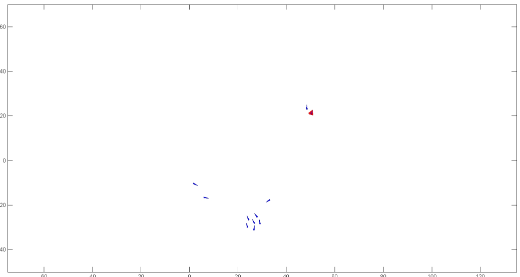
(b) Prey are expanding their area



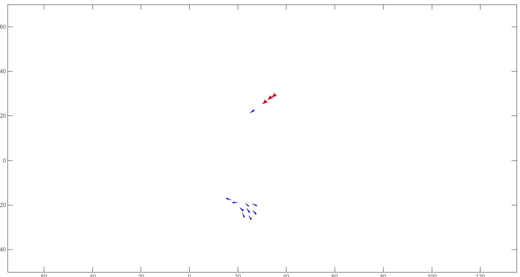
(c) One prey is being targeted



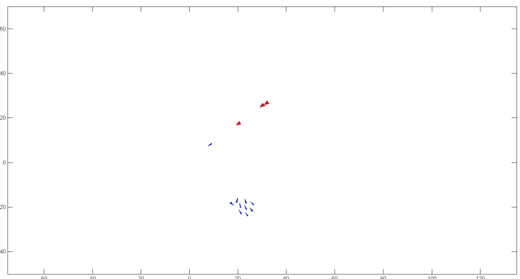
(d) The single prey is using its higher agility



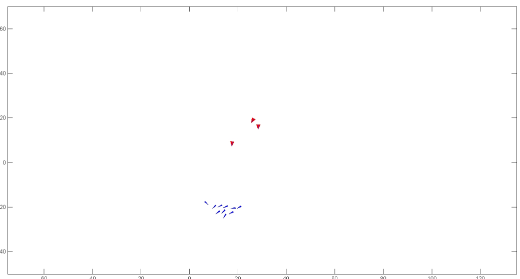
(e) The single prey outmanouvers the predators



(f) Predators are out of energy



(g) Prey are regrouping



(h) Prey have successfuly regrouped

Figure 24: Plots showing ten agents escaping a three predator attack in an obstacle free environment, with success.

5 Discussion

The achieved results correspond to the expected outcome of the model. The individual forces and tactics were designed with a specific task, each doing its part in controlling the agents' movement. It is of interest to examine the dependence of the different variables in each force and thereby draw conclusions to whether the chosen values are the most optimal. In the following section, a selection of the most interesting cases are examined and discussed.

As the model is not analytically solved, the values determined cannot be considered optimal according to the mathematical definition. However they are the values found most efficient after doing numerous simulations in different scenarios with different initial conditions.

During simulations and testing of an added force or a changed value of a constant, variety as well as consistency is of the essence; i.e. to obtain measurable results after a change, the same scenario has to be created. A set of constants valid for one scenario may be invalid in another, variation of scenarios have been examined with both randomness in the initial conditions and by using extreme cases.

5.1 Optimal parameters for successful survival

The values chosen for the different constants, shown in Table 1, are in many cases tested in numerous simulations; however, certain guidelines for their order of magnitude have been derived from reasoning about their effect on the model.

The length of the prey, L, was set to the length of a zebra, which is approximately 2 meters, or in this case 2 length units. The maximum velocities were set to match those of the predator (lion); the translational velocity should be 80% of the predator's and the angular velocity 3π, which is 50% higher than the predator's. If the predator is to be able to run with full speed for approximately 15 seconds it must have an energy reserve of about 9 500 energy units; with twice as much energy the prey would have approximately 20 000 energy units.

The general magnitude of the preceding constants is derived from deciding which forces that should be superior to other forces. To begin with, flocking and grouping are forces describing the desire of the agents; these should be low as they do not regulate any laws in the freedom of movement sense.

Roughly estimating the collision avoidance's constant, c_{ca}, to be able to handle the flocking force of a neighbor at distance 4 length units would give it a value of about 8; 10 was the value chosen through testing. Escaping a single predator in sight should be valued higher than avoiding collision. With c_{pa} set to 1, the value for the predator avoidance force is 0.3 at distance R_d, meaning about 300 times too low to overcome the collision avoidance forces from nine agents, 300 was tested and proven to be working. An obstacle should under any circumstance never be passed through, if the collision avoidance and predator avoidance forces were to take their maximum value and summed up, the number would be just beneath 1 000. To be on the safe side, the value for obstacle avoidance at the edge of an obstacle was set to 1 200, hence c_{oa} is 1 200. The constant for line avoidance, c_{la}, was empirically set to 20.

ω_{pa}, ω_{oa} and ω_{ca} are the constants for the predator avoidance, obstacle avoidance and collision avoidance forces that control the steepness of the edge of each exponential increase, i.e. the derivative is directly proportional to the values of the constants. For both the predator avoidance and collision avoidance forces a medium slope was desired, as seen in Figures 3 and 11, so both ω_{pa} and ω_{ca} were set to 2. For the obstacle avoidance force a greater steep was desired, ω_{oa} set to 8 gives the required properties and maximizes the force outside of the $R_{oa,in}$ radius.

All the different radii were set to reasonable values that were tested empirically, except for R_d, which is the preset sighting distance of the agents, 30 length units.

5.2 Forces' dependabilty on different constants

Changing the constants in the different forces' equations is reflected in the model's behavior. Showing plots of the results when changing the parameters back and forth is not the best way to show this. Since all forces and tactics, thereby also the agents' movement, are dependent on initial conditions as well as the environment, there are endless scenarios.

To begin with; the constants preceding each force's expression will only change the weighting of the forces, i.e. change which force that should be valued higher. Following the argument in Section 5.1, if all these constants were multiplied with the same factor, nothing would be changed with the weighting or valuation of the forces. Nevertheless, doing so changes the total system's response to change the state. When the forces are working in the area where no saturation is needed one might say that the system's response is ideal. Never reaching the outer limits would mean that valuable velocity is unused and always working with saturated velocities would mean that the system is too sensitive. The conclusion is that the preceding constants can be used to manipulate the systems sensitivity.

In the three exponentially dependent forces: predator, obstacle and collision avoidance, there are constants R. These change where the exponential in the denominator becomes most dominant, hence creating the exponential "bump" seen in Figures 4, 7 and 12. This bump's position has different dependence for the different forces; for predator avoidance, it is the distance to a predator at which the instability occurs creating the vast turns, seen in Figures 22e-22f. For obstacle avoidance it would mean the definition of the obstacle's edge radially. Finally, for collision avoidance, it defines the stable distance between the agents. The conclusion regarding these constants would be that they manipulate the reference for where an agent is stable or affected by a significant force.

Changing radii, defining the survival tactic would change the area for when the tactic is active. The inner radius, R_{ti}, controls when the prey starts to turn; if too big, the advantage in agility would be lost in straight paths and if too small, it will not have the space required to perform the turn. The outer radius, R_{to}, would make the agent move in circles if too big and restrict the turn if too small.

Moving the radius for detecting an obstacle, $R_{oa,out}$, further away would make the line avoidance force useless, due to the great number of obstacles taken into account. If closer to the obstacle, the detection would be too late, making the passage unsmooth.

The last manipulable radius is the definition of neighbors, R_n. The effects of altering this would be seen when the flock is disturbed, changing the size of the flock segments.

5.3 Improvement of the model

The model designed and used in this thesis is not finished in terms of describing nature as accurately as possible and the three of us could spend years developing it further. However, improvements are possible.

The forces used and their mathematical functions work very well and should not be the first area to develop further, since this would require all the work done so far to be redone. The different constants should be considered and in some cases be limited or replaced with depending functions. The different detection radii should depend on lighting, environmental differences, age and abilities of the agent and in some cases the randomness in spotting a threat. For longer simulations than 60 to 100 seconds where more than one approach from the predators are made, the prey as well as the predators might be injured; taking this into account when saturating velocities would improve the realism. Longer simulations would require a more developed energy model, where agents may increase their energy reserve. Flocks of more sophisticated agents often have a leader, its impact on the flocks movement and acting would keep the flock as one unit rather than individuals. To sum this up, all agents and scenarios are different; accounting their uniqueness would improve the model.

Reynolds describes a velocity consensus term [2] among the forces, meaning that the agents strive for equality in velocity and direction. Such a force was introduced in an early version of this model. After further studies, it was proved to be inapplicable for a system which is not taking the acceleration into account.

In this thesis three different tactics were developed; a casual, an escape and a survival tactic. These may seem basic, but with further study of their impact on the model one will discover that they are sufficient for general behavior. As seen in Figures 22-24, all scenarios expected in a short simulation are covered. Developing more tactics would be for the more specific cases; preventing an attack, split up the predators or even actively hiding. These are possible to model, but would require design of a simple artificial intelligence. If done; the model could even develop itself, i.e. the agents learn from their mistakes as well as how the system corresponds to certain actions and even remember where they have been before.

The last improvement based on the model in this thesis is the environment. To begin with, the assumption that all obstacles are circles is a simplification. Convex obstacles could be approximated using circles, but as soon as concave obstacles are introduced the approximation becomes invalid; a solution would be dividing each obstacle edge into small parts and then approximate these as convex.

The model in this thesis is based on single points in a two dimensional space, the first improvement could be that each agents has a spread in space making the measure non zero. The next step would be to expand the two dimensional plane into a three dimensional space; the agents could move on a plane with topography, causing change in velocity and energy use depending on the plane's gradient. The assumption that a prey can spot a predator through an obstacle would imply that all obstacles are flat; another dimension could explain different heights of obstacles and their transparency. The final step would be to expand the freedom of movement into the three dimensional space, modeling fish or birds.

5.4 Future for flock modeling

The future for flock modeling is still unknown, although it may be said that this field is expected to develop further. Research done so far, including the one in this thesis, is just the beginning of what can be expected to come. For example, the systems dealt with have not been solved analytically and therefore not been analyzed for stability; progress in this area would increase the ability to control the systems providing more realistic models. Another example would be developing the controller; applying one that takes acceleration into account as well as velocity. As computing speeds increase with new computers, the possibility of making longer yet more detailed simulations in a reasonable amount of time will be enhanced.

As the human population grows, bigger cities become more and more crowded and in central meeting points, such as train stations or airports, humans group together with similarities to animals in nature. In case of an emergency many humans might not act in a rational manner, but rather as a flock of wild animals. Using models such as the one in this thesis could help engineers when constructing emergency routes and dimensioning tight passageways.

Similarly, traffic strain is increasing and planning road repairs is not an easy task. If a flocking model could be used on commuting cars, then simulations of the outcome may be done prior to making temporary occlusions, hence avoiding unnecessary traffic jams.

Lately the development of UAV (Unmanned Aerial Vehicles) has increased; beginning with trajectory guided, single agent operations. When a model similar to the one in this thesis is applied on multiple UAV, usage could be vastly increased and the human factor eliminated.

6 Conclusion

It was to an extent possible to model flocking behavior, not only in the simple case as a moving group, but also in the more advanced case as prey escaping a predator attack. Using a mathematical model such as the unicycle model, combined with forces for desired velocities, was done with good results. Furthermore it was demonstrated to be possible for prey to avoid obstacles as well as use them in an escape tactic. Modeling a single agent's movement and correlation with other agents resulted in group behavior, even though no constraints for the actual group were implemented; showing that the model fits nature where each agent thinks as an individual rather than as a group.

It was shown that a group staying together under attack is at higher risk than if fragmentated, i.e. the predators has an increase in advantage when the density of the group is increasing. On the other hand, when the prey detects a predator, its risk of being targeted is divided with the number of neighbors. Weighting these two probabilities, risk of capture versus chance of survival, thus optimizing the size of each fragment in a flock is a possible development.

According to the results for longer simulations, it was shown that the escaping ratio for prey in an obstacle rich environment was higher than for an obstacle free environment. Hence the conclusion that both the control function, particularly the obstacle avoidance, was successful and that the prey have a higher agility than the predators may be made.

Even though good results were obtained, it must be mentioned that this is a very complex field of study. The total proportion and complicacy of animal behavior and its parameters are much higher than what possibly could be examined in a bachelor's thesis. With that said; the work done in this thesis may be used as a foundation for further development and insight in modeling of flocking behavior.

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