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$$l_p = \left(\sum_{i=1}^{D} |x_i - y_i|^p\right)^{1/p} \qquad p = 2 \quad \text{~~gn~~ generally}$$

$$d_2^2(x,y) = \sum_{i=1}^{D} |x_i - y_i|^2 \qquad \text{euclidean distance}$$

$$x = \begin{bmatrix} x_1 \\ x_2 \\ \vdots \\ x_D \end{bmatrix} \qquad y = \begin{bmatrix} y_1 \\ y_2 \\ \vdots \\ y_D \end{bmatrix}$$

$$d_2^2(x,y) = (\underline{x} - \underline{y})^T (\underline{x} - \underline{y})$$

height = x	weight = y		$x_i - \mu_x$	$y_i - \mu_y$
70	40		5	0
67	45	→	2	5
65	50		0	10
58	35		-7	-5
60	30		-5	-10

$$\mu_x = \frac{1}{n} \sum_{i=1}^{n} x_i = \sum_{i=1}^{n} p_i x_i$$

$$\text{var}(x) = \frac{1}{n} \sum_{i=1}^{n} (x_i - \mu_x)^2$$

$$\text{covariance}(x,y) = \frac{1}{n} \sum_{i=1}^{n} (x_i - \mu_x)(y_i - \mu_y)$$

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$$\left.\begin{array}{l}\sum x_i y_i > 0 \\ cov(x,y) > 0\end{array}\right] \; x \text{ increases} \rightarrow y \text{ increases}$$

$$\left.\begin{array}{l}\sum x_i y_i < 0 \\ cov(x,y) < 0\end{array}\right] \rightarrow x \text{ increases} \rightarrow y \text{ decrease}$$

$cov(x,y) = 0 \rightarrow$ cant say anything.

$\rightarrow$ correlation value.

$$\rho(x,y) = \frac{cov(x,y)}{\sqrt{var(x)\, var(y)}}$$

$$-1 \leq \rho(x,y) \leq 1$$

$$\Sigma_{D\times D} = \begin{array}{c|c} & \begin{array}{cccc} f_1 & f_2 & f_3 & \cdots \end{array} \\ \begin{array}{c} f_1 \\ f_2 \\ f_3 \end{array} & \begin{bmatrix} cov(f_1,f_1) & (f_1,f_2) & (f_1,f_3) & \cdots \\ \vdots & & & \\ \vdots & \cdots & & \cdots \end{bmatrix} \end{array}$$

$$cov(f_i,f_j) = cov(f_j,f_i)$$
$$cov(f_i,f_i) = var(f_i)$$

Σ is real, positive, symmetric definite matrix.

positive definite $\rightarrow$ $x' A x > 0$ for any vector x

$\rightarrow$ then $|\Sigma| > 0$ $\quad \Sigma^{-1}$ is also x.

positive def.

$$d_M^2(c_1, x) = (x-\mu_1)' \Sigma^{-1} (x-\mu_1)$$

$$d_M^2(c_1, c_2) = (\mu_2-\mu_1)' \Sigma^{-1} (\mu_2-\mu_1)$$

$$\Sigma = \frac{\Sigma_1 + \Sigma_2}{2}$$

$$\cos\theta = \frac{X \cdot Y}{|X||Y|} \rightarrow \frac{X^T Y}{\sqrt{\sum_{i=1}^{\lambda} x_i^2} \sqrt{\sum_{i=1}^{\lambda} y_i^2}}$$

eigenvalues and eigenvector

$$Ax = \lambda x \qquad A_{D \times D} \quad x_{D \times 1}$$

$$|A - \lambda I| = 0$$

$\lambda_1, \lambda_2 \dots \lambda_D$ → eigenvalue

$v_1, v_2 \dots v_D$ → corresponding eigenvector

1. calculate dispersion matrix Σ:
   a) subtract mean from each data in the datasent
   b) $\Sigma_{D \times D} = X^T X$ $\quad X = \mu$ subtracted dataset
   c) calculate eigenvalues and eigenvectors of Σ

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$\text{trace}(\Sigma) = \sum \lambda_i \qquad \lambda_i \geq 0$

if arrange $\lambda_1 > \lambda_2 \cdots > \lambda_D$

$v_1 \quad v_2 \quad \cdots \quad v_D$

$\xrightarrow[d<D]{}$

$$\frac{\sum_{i=1}^{d} \lambda_i}{\sum_{i=1}^{D} \lambda_i} \geq 97\% \rightarrow$$

not all dimensions are equally important in terms of spread of data.

$$|\Sigma| = \lambda_1 \cdot \lambda_2 \cdot \lambda_3 \cdots \lambda_D$$

$$W_{D\times d} = \begin{bmatrix} v_1 & v_2 & \cdots & v_d \\ & & & \\ & & & \end{bmatrix}$$

$$Y_{d\times N} = W^T_{d\times D} X^T_{D\times N}$$

eg
$$X = \begin{bmatrix} 2 & 1 \\ 3 & 5 \\ 4 & 3 \\ 5 & 6 \\ 6 & 7 \\ 7 & 8 \end{bmatrix} \qquad X' = \begin{bmatrix} -2.5 & -4 \\ -1.5 & 0 \\ -0.5 & -2 \\ 0.5 & 1 \\ 1.5 & 2 \\ 2.5 & 3 \end{bmatrix}$$

$\mu_1 = 4.5 \quad \mu_2 = 5$

$$\Sigma = \frac{1}{n} X'^T X' = \frac{1}{6}\begin{bmatrix} 17.5 & 22 \\ 22 & 34 \end{bmatrix} = \begin{bmatrix} 2.92 & 3.67 \\ 3.67 & 5.67 \end{bmatrix}$$

$\lambda_1 = 8.22 \qquad \lambda_2 = 0.37$

v_1

$$W_* = \begin{bmatrix} 2.55 \\ 3.67 \end{bmatrix}$$

$$w^T = \begin{bmatrix} 2.55 & 3.67 \end{bmatrix}$$

$$y = w^T X^T = \cancel{\begin{bmatrix} 8.77 \\ 2.6 \\ 21.21 \\ 34.77 \\ 40.92 \\ 47.21 \end{bmatrix}}$$

$$= [8.77 \mid 2.6 \mid 21.21 \mid 34.77 \mid 40.92 \mid 47.21]$$

Intra class distance should be small (within)
Inter class distance should be large (between)

Machine Learning

	Classification	Clustering
1.	Supervised	unsupervised
2.	Given an unknown pattern and assign into predefined classes	Based on similarity measure among patterns, develop an algo to find natural grouping
3.	Generate and train a classification model with the help of training data.	

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Classification

→ Probabilistic (Baye's)

→ Similarity based (MDC, KNN, LDC, QDC)

→ SVM

Bayes' Rule

$$f(x) = \frac{1}{(2\pi)^{d/2} |\Sigma|^{1/2}} \exp\left\{ -\frac{1}{2} (x-\mu)^T \Sigma^{-1} (x-\mu) \right\}$$

multidimensional gaussian function.

Let there be M classes ($M \geq 2$), prior probability be $P_1, P_2 \ldots P_M$. Where $0 \leq P_i \leq 1$ and $\sum_{i=1}^{M} P_i = 1$

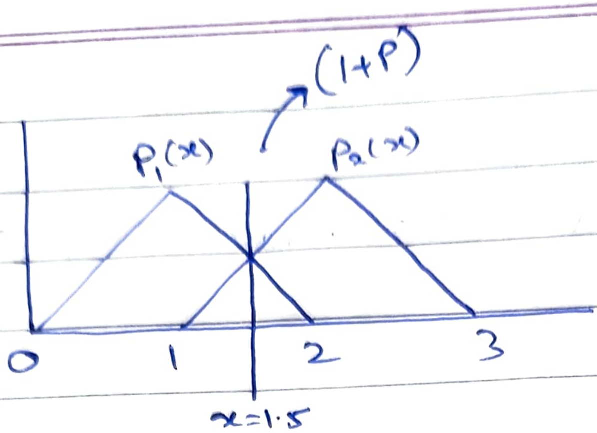
let $p_1(x), p_2(x) \ldots p_M(x)$ be the conditional probability density function.

Given an unknown pattern x_0, x_0 be in class i if $P_i p_i(x_0) \geq P_j p_j(x_0)$ $\forall$ $i \neq j$.

eg let $M=2$ Prior prob $P, (1-P)$

$$p_1(x) = \begin{cases} x & 0 \leq x \leq 1 \\ 2-x & 1 \leq x \leq 2 \\ 0 & \text{otherwise} \end{cases}$$

$$p_2(x) = \begin{cases} x-1 & 1 \leq x \leq 2 \\ 3-x & 2 \leq x \leq 3 \\ 0 & \text{oth.} \end{cases}$$



1.5 ≥ class 1
1.5 ≤ class 2

0 to 1

$P_1 = P$ $\quad$ $P_2 = 1-P$

$p_1(x) = x$ $\quad$ $p_2(x) = 0$

$P_1\, p_1(x) \geq \cancel{P_2\, p_2(x)}^{\,0}$ $\Rightarrow$ x should be in class 1

case 2 $\quad$ 2 to 3

$P_1 = P$ $\quad$ $P_2 = 1-P$

$p_1(x) = 0$ $\quad$ $p_2(x) = 3-x$

$P_2\, p_2(x) \geq \cancel{P_1\, p_1(x)}^{\,0}$ $\Rightarrow$ x should be class 2

case 3 $\quad$ 1 to 2

$P_1 = P$ $\quad$ $P_2 = 1-P$

$p_1(x) = 2-x$ $\quad$ $p_2(x) = x-1$

$$P_1\, p_1(x) \geq P_2\, p_2(x)$$

$$P(2-x) \geq (1-P)(x-1)$$

$$2P - xP \geq x - xP + P - 1$$

$\boxed{x \leq 1+P}$ for class 1

$\boxed{x \geq 1+P}$ for class 2