3.3 Day 1

1. a. $\log_5 x = \dfrac{\log_{10} x}{\log_{10} 5}$

   b. $\log_5 x = \dfrac{\ln x}{\ln 5}$

2. a. $\log_3 x = \dfrac{\log_{10} x}{\log_{10} 3}$

   b. $\log_3 x = \dfrac{\ln x}{\ln 3}$

3. a. $\log_{\frac{1}{5}} x = \dfrac{\log_{10} x}{\log_{10} \frac{1}{5}}$

   b. $\log_{\frac{1}{5}} x = \dfrac{\ln x}{-\ln 5}$

4. a. $\log_{\frac{1}{3}} x = \dfrac{\log_{10} x}{\log_{10} \frac{1}{3}} = \dfrac{-\log_{10} x}{\log_{10} 3}$

   b. $\log_{\frac{1}{3}} x = \dfrac{\ln x}{\ln\left(\frac{1}{3}\right)} = \dfrac{-\ln x}{\ln(3)}$

5. a. $\log_a \frac{3}{10} = \dfrac{\log_{10} \frac{3}{10}}{\log_{10} a}$

   b. $\log_a \frac{3}{10} = \dfrac{\ln(3/10)}{\ln(a)}$

6. a. $\log_a \frac{3}{4} = \dfrac{\log_{10} \frac{3}{4}}{\log_{10} a}$

   b. $\log_a \frac{3}{4} = \dfrac{\ln\left(\frac{3}{4}\right)}{\ln(a)}$

7. a. $\log_{2.6} x = \dfrac{\log_{10} x}{\log_{10} 2.6}$

   b. $\log_{2.6} x = \dfrac{\ln x}{\ln 2.6}$

8. a. $\log_{7.1} x = \dfrac{\log_{10} x}{\log_{10} 7.1}$

   b. $\log_{7.1} x = \dfrac{\ln(x)}{\ln(7.1)}$

12. $\ln(20) = \ln(4 \times 5) = \boxed{\ln 4 + \ln 5}$

18. $\ln(500) = \ln(5^3 \times 4) = \boxed{3\ln 5 + \ln 4}$

19. $\ln \dfrac{5}{64} = \ln 5 - \ln(2^6) = \ln 5 - 6 \ln 2$

20. $\ln \dfrac{2}{5} = \boxed{\ln 2 - \ln 5}$

21. $\log_b 25 = \log_b 5^2 = \boxed{2 \log_b 5}$

22. $\log_b (30) = \log_b (2 \times 3 \times 5) = \boxed{\log_b 2 + \log_b 3 + \log_b 5}$

23. $\log_b \sqrt{3} = \log_b 3^{\frac{1}{2}} = \boxed{\dfrac{1}{2} \log_b 3}$

31. $\log_4 8 = \log_4 2^3 = 3 \log_4 2 = 3 \times \dfrac{1}{2} = \boxed{\dfrac{3}{2}}$

32. $\log_2 (y^2 \times 3^4) = \log_2 y^2 + \log_2 3^4 = \log_2 y^6 + 4 \log_2 3$

$= \boxed{y + 4 \log_2 3}$

33. $\ln(5 e^6) = \ln 5 + \ln e^6 = \ln 5 + 6 \ln 3 = \boxed{\ln 5 + 6}$

34. $\ln \left(\dfrac{b}{e^2} \right) = \ln b - \ln e^2 = \boxed{\ln b - 2}$

35. $\log_5 \dfrac{1}{200} = \log_5 1 - \log_5 (125 \times 2) = \boxed{-3 - \log_5 2}$

36. $-\ln 2y = -\ln(2^3 \times 3) = \boxed{-3 \ln^2 - \ln 3}$

39. $\log_{10} \dfrac{5}{x} = \boxed{\log_{10} 5 - \log_{10} x}$

43. $\ln \sqrt{z} = \ln (z)^{\frac{1}{2}} = \boxed{\dfrac{1}{2} \ln z}$

48. $\log_5 (x^3 y^3 z) = \log_5 x^3 + \log_5 y^3 + \log_5 z$

$= \boxed{3 \log_5 x + 3 \log_5 y + \log_5 z}$