Measuring Propeller's Lift Coefficient

Erick Arcos

Department of Physics

Pacific Lutheran University

arcosea@plu.edu

May 18, 2022

Abstract

Lift, one of the four major forces involved in aerodynamics, is a force that combats the force due to gravity for any mass object. It is often induced due to differences in pressure from a fluid flowing around an object. In this project, a 1000 kV brushless DC motor is used to spin a 2-bladed propeller at increasing, constant angular velocities ω stationed on a weighing scale. There were two distinct experiments conducted: one where the propeller is spun counterclockwise and the other where it is spun clockwise, as seen from a birds-eye view. When the propeller is spun counterclockwise, creating an upward pointing lift force, it was found that the lift coefficient was 0.405. However, when the propeller was spun clockwise, a downward pointing "lift" force was induced and yielded a 0.304 lift coefficient. The differences were largely due to the assumptions made in our theoretical analysis of the lift produced from a rotating propeller. Aside from the experiments, an additional part of this project was to verify the motors kV rating. The data collected showed that the brushless DC motor used had a kV rating within 1.43% of the expected 1000 $\frac{rpm}{V}$.

Introduction & Theory

The purpose of this project is to attempt to model the lift generated from a rotating propeller. In order to do so, we should first consider the most general Lift Equation for an airfoil which can be described with the following:

$$F_L = \frac{1}{2} \cdot C_l \cdot \rho_{ext} \cdot v^2 \cdot A \tag{1}$$

where ρ_{ext} is the gas density $(\frac{kg}{m^3})$ of the fluid that an object (airfoil) is subject to (typically air), v is the speed $(\frac{m}{s})$ of the fluid flow relative to the object, A is the surface area (m^2) of the object over which the fluid/gas flows over, and C_l is the lift coefficient which is highly dependent on the shape of the airfoil and determined by the angle of attack. The angle of attack is the angle between a body's (e.g. airfoils) reference/direction line and that of incoming air flow. The term $\frac{\rho \cdot v^2}{2}$ in the lift equation is often referred to as dynamic pressure (which appears in Bernoulli's pressur equation). The dynamic pressure is essentially the reason there is a force exerted on the moving body.

While the lift equation is excellent for describing the lift produced on a moving body, there is a problem with attempting to use it to describe a rotating dependent aircraft. In order to explain the problem, consider **Figure 1 .**

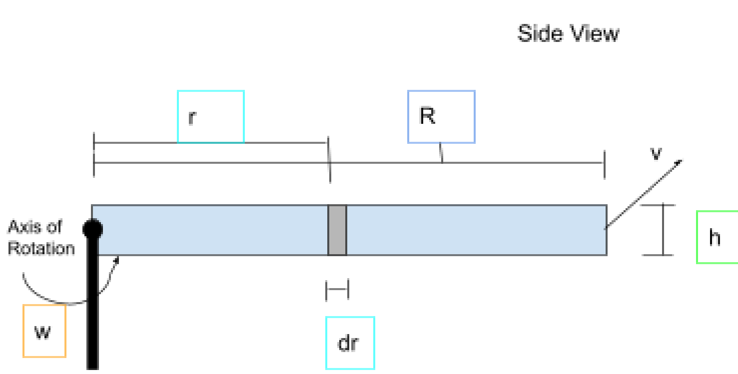


Figure 1. Side-View of Moving Blade of Radius R Relative to the Axis of Rotation.

The problem is that parts of the rotating wings (propeller blades) are moving faster than other parts relative to the center axis of rotation. If the blade (of length R) as a whole solid body, is rotating at a constant relative angular velocity ω, then the velocity at any point r away from the center axis is described using the following:

$$v(r) = \omega \cdot r \tag{2}$$

We expect there to be a higher velocity towards the edge of the blade (distance R) than near the center of axis of rotation since the blade is moving a greater distance at any given moment for the same angular velocity ω. Therefore, we could assume that a rotating blade has a linear velocity on some segment dr. With this assumption, we can convert the general lift differential equation:

$$dF_l = \frac{1}{2} \cdot C_l \cdot p_{ext} \cdot v(r)^2 \cdot h \cdot dr \tag{3}$$

Taking the integral of **Eq (3),** substituting the assumption described by **Eq (2)**, and recognizing that $A = h \cdot R$, where A is just the area of the blade which is coming into contact with the air flow, then the we arrive at the following equation which tells us <u>lift force per blade</u> :

$$F_L = \frac{1}{6} \cdot C_L \cdot \rho_{ext} \cdot A \cdot R^2 \cdot \omega^2 \tag{4}$$

Since the lift force depends on the square of a velocity (relative angular velocity ω), ω is the only independent variable - the force is a function of angular velocity, the rest are just constants. Therefore if we know the radius, area of the propeller and density of the moving gas which can all be measured experimentally; by rotating a blade or propeller at different relative angular velocities ω and measuring the force produced, the lift coefficient C_L can be determined.

Note that the direction of which the blade is rotating actually determines the lift force. Consider the following **Figures 2(a) and 2(b)**:

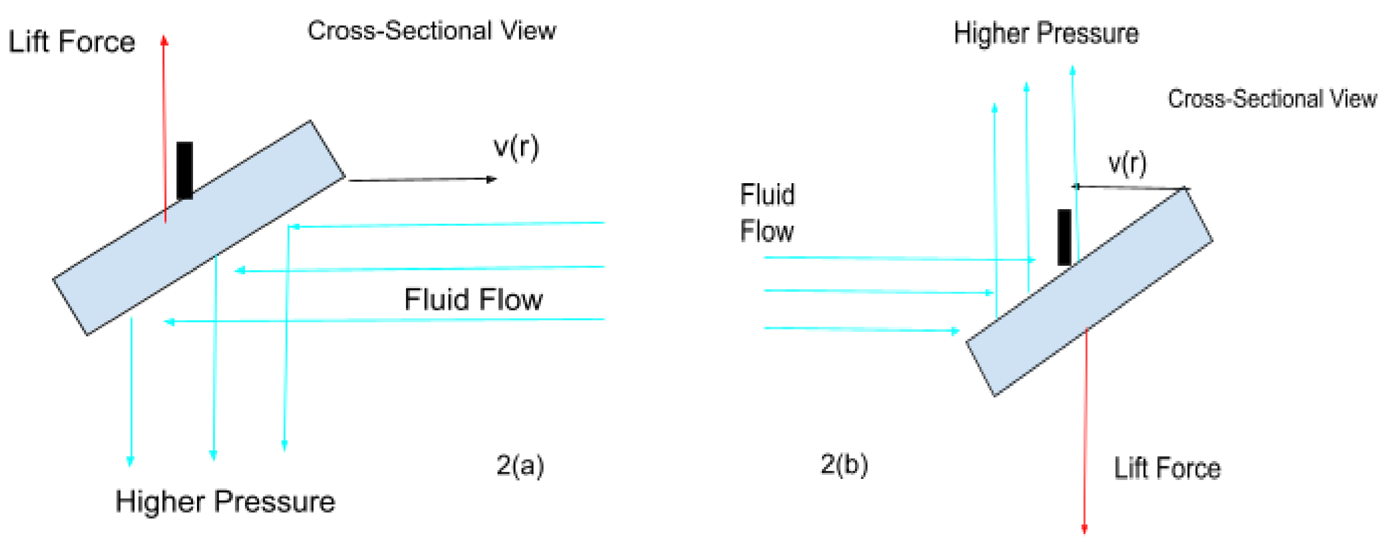


Figure 2 Demonstrates Cross Sectional View of Blade Rotating CounterClockWise (a) and Blade Rotating ClockWise (b) from Birds-Eye View.

On **Figure 2(a)** we see that if the blade is moving to the right (ccw) from birds eye view, then the gas/fluid will be flowing into it from the right side of the propeller and then get redirected downward. This will cause higher pressure below the propeller where the motor (or possibly body of a helicopter) and this increase in pressure causes an upward pressure-gradient force which we consider as lift. Force is called *lift* because it generally points upward relative to a surface and thus combats the force due to gravity. On the other Figure **2(b)**, the blade is moving left (cw) from birds-eye view which induces a fluid flow to the right (left side of blade) which causes higher pressure above the blade/propeller and thus creates a downward force. Granted that the direction of rotation impacts the direction of the force, a goal of this project is to observe and verify that effect.

Equipment & Setup

In order to conduct the certain experiments, various materials were required. The list is as follows: A 1000 Kv Brushless DC Motor, A 2 - Bladed Propeller A 30A Electronic Speed Controller (ESC), A 10kΩ potentiometer (variable resistor), A Tachometer, A Digital Weight Scale, an Arduino Uno Microcontroller and a 3S LiPo Battery and Safety Goggles. The following **Figure 3** shows the connections involved between the electronic components.

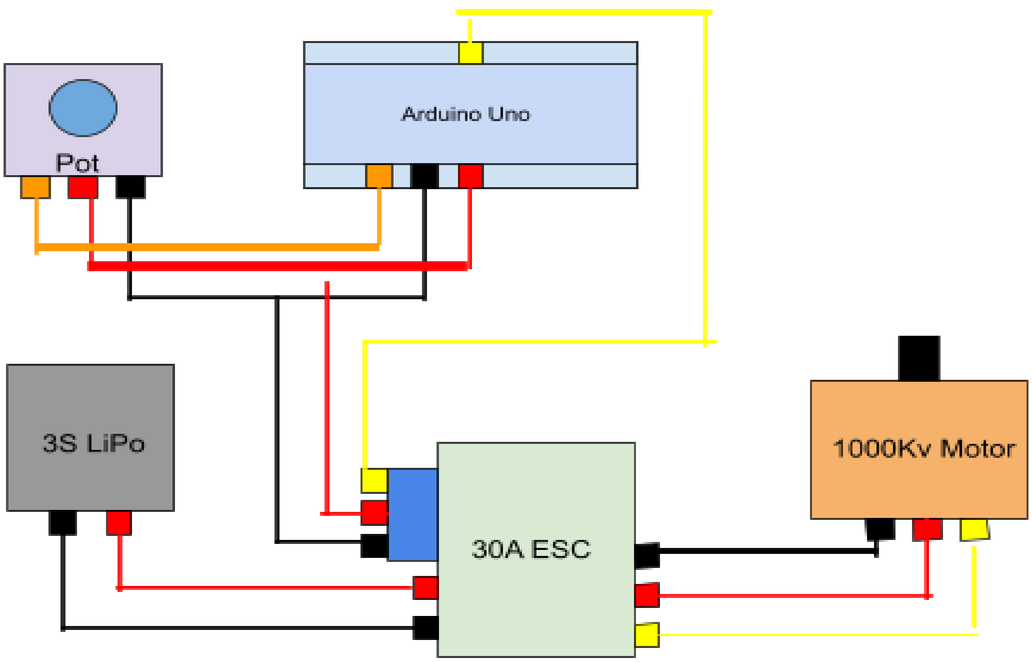


Figure 3: Schematic of the Electronic Components Involved.

Essentially what is going on is that the 3S LiPo battery is supplying power to the ESC and the ESC is supplying power to the Arduino Uno. The potentiometer is then being used a the speed controller (with user interaction) to send analog signals (range: 0 - 1023) which the arduino uno picks up and converts it into a fraction of the maximal signal that the motor can receive and sends that signal to the ESC which controls motor.

In terms of the experimental setup, it's quite simple. The motor is stationed on a platform which is held by a stand. The stand is then placed over a scale. This whole system is sealed in a cardboard box for safety purposes. The tachometer is placed above the propeller pointing downward to the end of the propeller blades. The following Figure shows a sketch of the setup

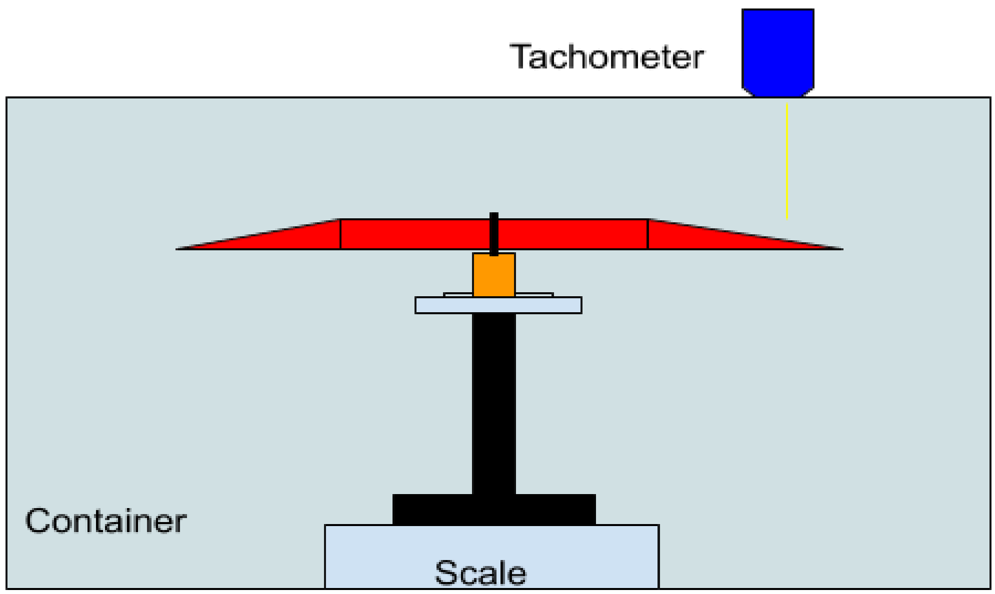


Figure 4: Sketch of the Setup.

Preliminary Data & Results

At the start of this project, the tachometer was not in our possession. Granted that, a task in this project was to verify the brushless DC motor's 1000 kV rating. What kV rating simply tells the RPM per given volt that is supplied to the motor and can be summarized using the following expression:

$$RPM = V_{in} \cdot 1000 \frac{rev}{V \cdot s} \tag{5}$$

In order to verify the kV Rating, a small strip of tape about 2cm long was attached to the rotor of the motor. The rotor was then spun at different angular velocities using the 10kΩ potentiometer as the controller. A sound program known as Audacity was then used to measure the period (time) it took for the tape to make a full revolution. The following **Figure 5** demonstrates a particular trial that was taken.

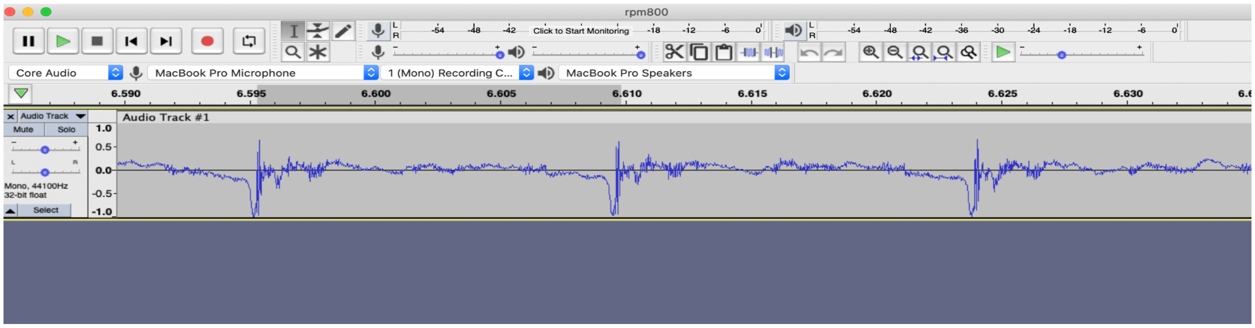
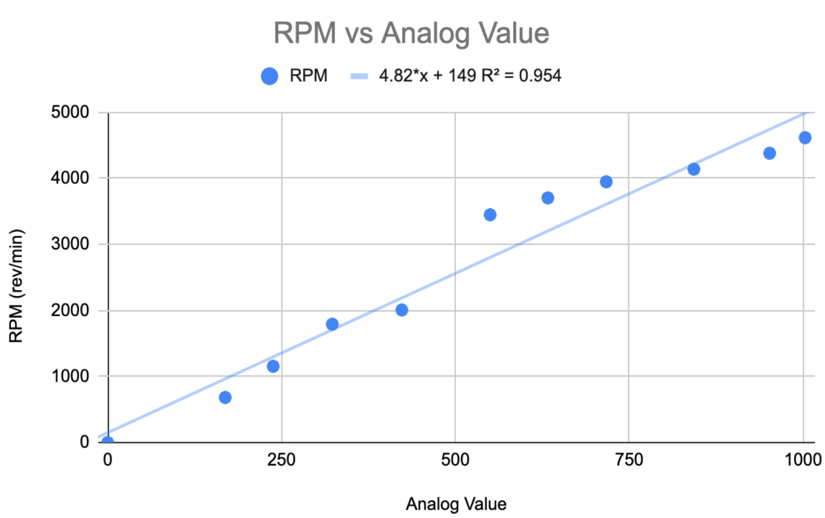


Figure 5: Period of Motor When Spun Using Potentiometer Analog Value of Approximately 800.

The Arduino Uno microcontroller reads in analog values between 0 to 1023 (binary number) from the potentiometer as a signal. By spinning the motor at different analog values, we can determine the period (s) it took for the rotor to do one complete revolution. Using the period, we can determine the RPM using the equation below. The graph below demonstrates the results of RPM of the motor when spun using different analog values.

$$RPM = 60\frac{sec}{min} \cdot \frac{1\,rev}{period} \qquad (5)$$

Graph 1: Demonstrates Linear Relationship Between RPM of motor and Analog Value From Potentiometer.



As shown in the diagram above, we get a linear relationship which tells us that our experimental data yielded a $(4.82 \ \pm \ 0.01) \frac{RPM}{Analog\ Value}$ ratio. Given that the Arduino Uno can supply a maximum of 5V and that the potentiometer at full throttle has an analog value of 1023, we should have expected a ratio of $4.89 \frac{RPM}{Analog\ Value}$. Therefore, our experimental data yields a percent error of about 1.43 % verifying the motor's linear kV relationship described by **Eq(5)**. Part of the reason is due to the fact that some of the voltage supplied from the arduino goes into the potentiometer to power it in order for it to be able to send signals. Another part is because at low analog values, the arduino uno does not supply enough current to the ESC to be able to control the motor. In addition, each analog value measurement had an average variance value of about 7 due to the potentiometer not actually containing 1023 unique states. Nonetheless, the data gathered showed promising results that the motor does indeed have a kV rating of approximately 1000 RPM.

With regards to the constants involved in the lift generated by a propeller described by Eq (4), the following values were used:

Constants	**Measured Value**
$Air\ Density\ at\ Room\ Temperature\ (\rho)$	$1.204 \frac{kg}{m^3}$
$Height\ (h)$	$(8.40 \ \pm \ 0.05)\ mm$
$Radius\ of\ Propeller\ (R)$	$127.99 \ \pm \ 0.05)\ mm$

Table 1: Summary of Constants

In the table above, the radius R is simply the length of a single blade from the 2-bladed propeller. The height is the average height throughout the face (side view) of the propeller since the height changes with respect to the distance r in the blade with respect to the center axis of rotation. The air density was not measured experimentally but since it was conducted indoors, it was assumed that the density of the fluid (air) was that which has been measured by others to be approximately the provided value. The following Figure demonstrates the different views of the propeller.

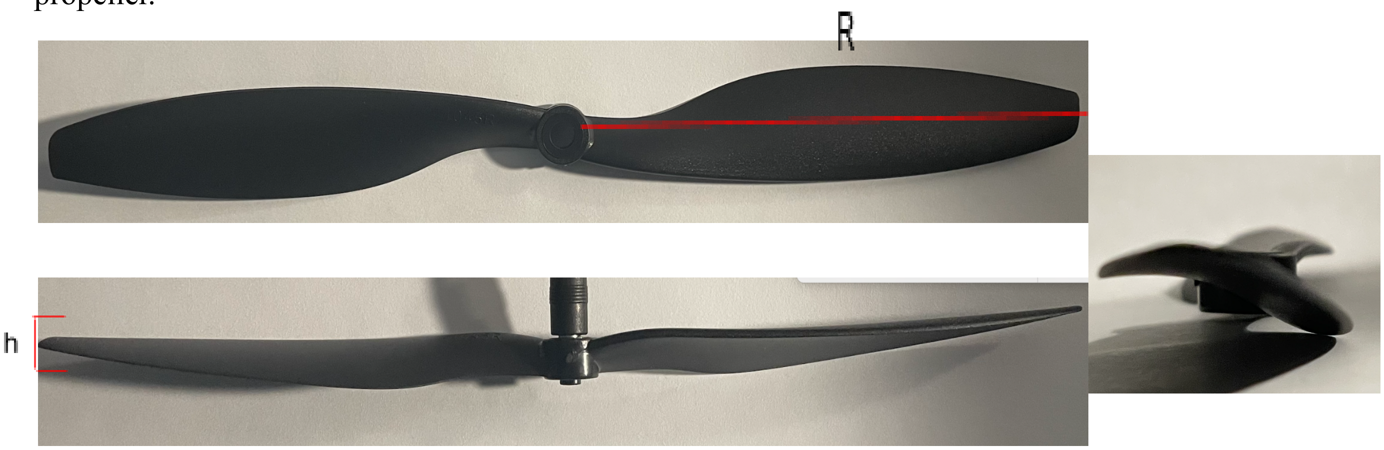


Figure 6: Birds - Eye View (Top Image), Side - View (Bottom Image) & Cross-Section of Propeller

Experimental Results

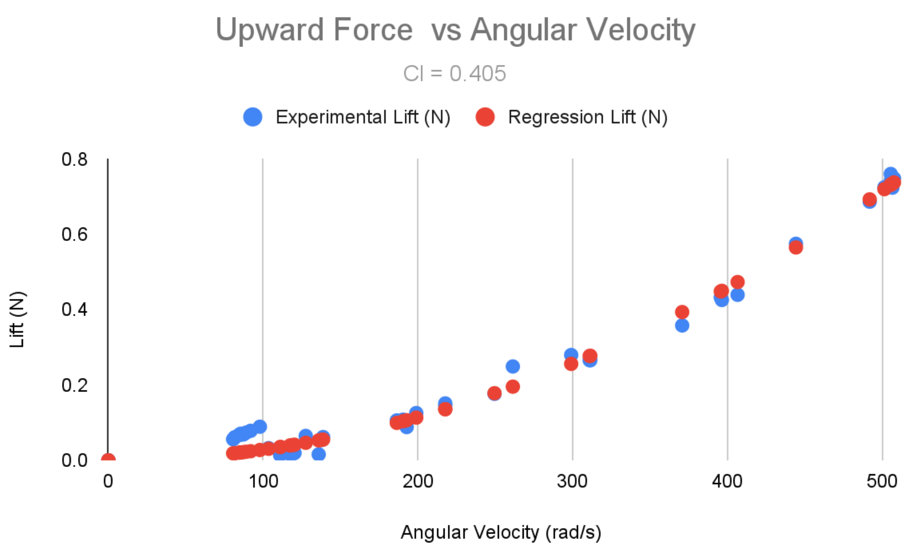
In addition to trying to deduce the lift coefficient experimentally, part of the goal of this project was to simply observe the effect of rotating the propeller in both directions, particularly the direction of the force generated. Therefore two main trials were conducted; one where the propeller was spun counterclockwise as seen from a birds-eye view, and the other where it was spun in the clockwise direction at different “constant” angular velocities ω (which is determined by the RPM of the blade). The lift coefficient was determined using the following strategy. First, we assume that the coefficients in the **Eq (4)** all equate to some other arbitrary constant (k) such that $k = \frac{1}{6} \cdot C_L \cdot \rho_{ext} \cdot A \cdot R^2$. Therefore, by measuring the lift force F_L generated as the angular velocity ω increases, we expect there to be the following relationship:

$$F_l = k \cdot \omega^2 \tag{6}$$

Therefore, by finding a model to fit the data, such as using Excel Solver to do a polynomial regression, we can attain the constant k. Using k, and knowing the other constants (ρ_{ext}, A, R), we can determine the lift coefficient to be the following for a 2-bladed propeller:

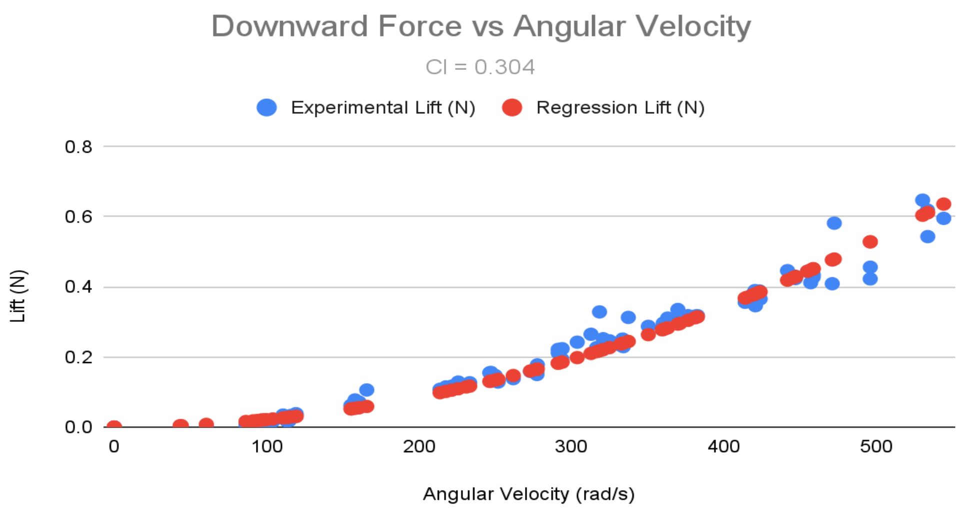
$$C_l = \frac{3 \cdot k}{\rho_{ext} \cdot A \cdot R^2} \qquad (7)$$

In the first trial, the propeller was spun in the counterclockwise direction. As described by **Figure 2(a)**, the theory demonstrates that we should expect an upward force (lift) - and that is exactly what was observed. As the angular velocity ω increased, the force of the system (motor, propeller, stand) on the weighing scale decreased. The following graph shows how the lift force generated varied with respect to angular velocity for the first trial.



Graph 2: Upward Lift Force vs Angular Velocity When Propeller Was Spun CCW

Using the technique described above, the lift coefficient C_l for the propeller was determined to be approximately 0.405. This same process was repeated for the other trial where the propeller was spun in the clockwise direction. The following graph shows the results for the second trial.



Graph 3: Downward Force vs Angular Velocity When Propeller Was Spun CW

In the second trial, since the propeller was spun clockwise, higher pressure was induced above the propeller causing a downward force and thus increasing the force on the weighing scale (as predicted by the theory). By measuring the force difference as a function of angular velocity, we determined that the "lift" coefficient was approximately 0.304. For this second trial, the log of both the lift force F_l and angular velocity ω were taken. The data showed a linear relation between the $log(F_l)$ vs $log(\omega)$ with a slope of 2.23. Granted that the angular velocity is squared in **Eq (4)**, we should have expected a value of 2. This means we got a percent error of about 11.5%. Doing more experiments would have yielded more reliable results.

Analysis & Summary

There are various reasons why the trials yielded quite distinct "lift" coefficients. For starters, only one trial per experiment was conducted and that is not enough data to make any concrete conclusion about what the lift coefficient of the propeller is. In addition, the second trial had nearly double the data points than the first experiment. Conducting more trials would have possibly yielded more reliable results and comparisons. Not to mention that the two experiments conducted as a whole tested fairly different theorems. Lift, by definition, is supposed to be a force that combats the force due to gravity and that is exactly what is observed with trial one. However, in the second experiment, there is an induced downward force, in addition to that due to gravity. If the propeller was spun sideways, the force generated would act more like a thrust force. This implies that the coefficient we found for the second experiment acts more like a thrust coefficient.

There are also the various implications that come with the assumptions that were made. For example, we assumed that there was a linear velocity $v(r)$ for some segment dr, there was always a constant angular velocity, and that the area that the air gas flowed around did not vary. However, comparing **Figure 1** and the side view image in **Figure 6**, we see that that is not the case. Not to mention the cross sectional view in the theory, as shown by **Figure 2(a)** and **2(b)** demonstrate a propeller with an angle of attack to be approximately 45 degrees (relative to the wind). However, as we see in the cross sectional image of the propeller in **Figure 6,** the propeller is fairly curved on one side (not to mention the bending). Therefore, the general equation **Eq (4)** that was used to describe the lift generated from a blade was an extreme simplification of the propeller that was actually experimented.

Despite the oversimplifications that were made in this project in order to analyze our data, there were many key concepts that were thoerized and verified. The idea of how the direction of the force generated by a rotating blade depends on the direction of which the propeller is rotating was observed. The force generated from a rotating blade yields a force that is proportional to the square of the angular velocity which can be modeled with experimental data. In addition, the lift coefficient can be determined using experimental data.

Resources

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