Michelson Interferometer Experiment: Wavelength of HeNe & Airs Index of Refraction

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Abstract

The Michelson Interferometer is a precision instrument in which a beam of light is split into two paths by a partially silvered mirror, striking into a fixed mirror on one path and a movable mirror on the other. After reflection, the mirrors recombine to form a destructive interference pattern. The apparatus is often used in the optical industry to measure index of refraction, measuring the wavelength of light to high precision, and testing lenses and prisms. In this experiment, we used a HeNe laser as the source of light and ran three sets of ten trials. Our best data set collection yielded a HeNe wavelength of (657.2 ± 27.2) nm. Compared to the expected wavelength of HeNe (632.8 nm), we were approximately 3.8% off. In addition, by placing a vacuum chamber in front of the previously movable mirror and observing how the interference pattern changes at different pressures, we calculated the index of refraction of air to be 1.000275.

Theory & Experimental Results

In the first experiment, we placed a diverging lens of ~ 22mm and then aligned and calibrated the Michelson Interferometer so that the beam of lights created a bullseye (destructive interference) pattern on the screen. By turning the knob on the micrometer reading and counting the number of fringes that pass by, we were able to determine the wavelength of the the HeNe laser using the following relationship:

$$\lambda = \frac{2\Delta d}{N} \tag{1}$$

Where Δd is the distance the movable mirror moved and N is the number of fringes that pass by; taking into account that the micrometer reading to distance the mirror moved is a 5:1 ratio. For consistency, we only spun the knob on the micrometer roughly(0.15 ± 0.005) mm each trial, implying that the mirror moved an average of 0.03 mm. We performed three sets of ten trials and found the following results:

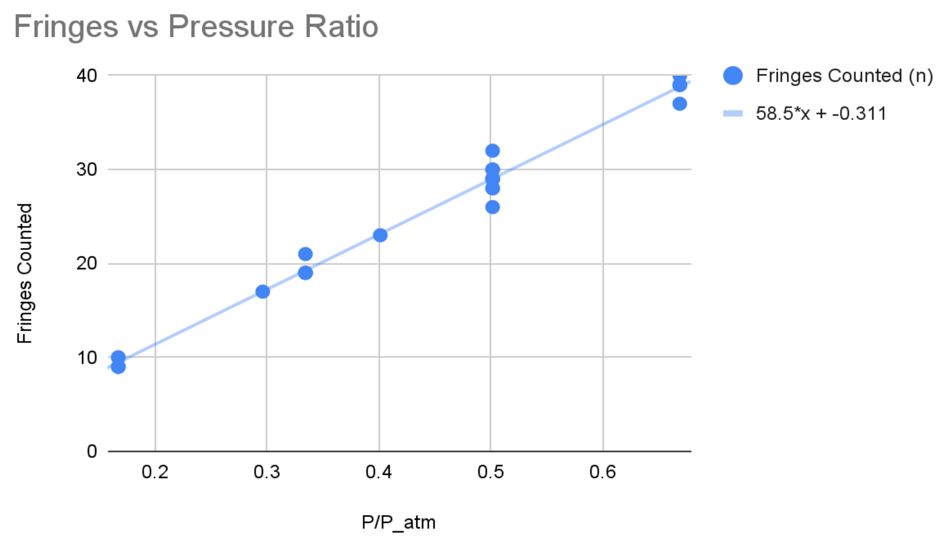
Table 1: Summary of Results From Two Sets

Trial Set	Distance Mirror Moved (mm)	Total Fringes Counted	Wavelength (nm)	Uncertainty (nm)	Percent Difference
1	0.03	891	674.4	36.1	6.5
2	0.03	881.5	683	47.9	7.9
2	0.03	913	657.2	27.3	3.8

Considering that the 'actual' wavelength of HeNe is about 632.82nm, our closest data set yielded a percent difference of 3.8 %, whereas the worst set was a high 7.9 % (sources of error explained in the following section).

In the second experiment, we fixed the movable mirror and placed a vacuum chamber in front of the mirror. By observing how the interference pattern changes as different pressures P are released from the chamber, we can plot the number of fringes counted versus the ratio of $\frac{P}{Patm}$ to obtain the following:

Figure 1: Plot of The Fringes Counted with Respect to the P/P_atm Ratio



Using the plot above, we find the following relationship:

$$Slope\ s\ =\ \frac{N}{\frac{P}{Patm}} = (n_{air,\,atm} - n_{vacuum})\, 2\frac{L}{\lambda} \qquad (2)$$

Where the index of refraction in vacuum n_{vacuum} = 1.00, λ is the wavelength of HeNe and L is (6.99 ± 0.005) cm which symbolizes the length of the vacuum chamber that only encloses air. Using a linear regression on the data, we found that the slope of the plot $s\ =\ \frac{N}{\frac{P}{Patm}} =$ $58.50\ \pm\ 1.30$. Using that information and rearranging Eqn (2) for $n_{air,\,atm}$, we find the following results:

Table 2: Compares the Index of Refraction of Air Using HeNe Wavelength of 632.8 nm on Ideal Experimental to Actual HeNe Wavelength Found in Experiment 01 of 657.2 nm to the Actual Index for Air

	Index of Refraction for Air (n_air)	Uncertainty
Theoretical (Actual)	1.000293	N/A
Ideal Experimental	1.000265	0.000006
Actual Experimental	1.000275	0.000013

As we can see from the table above, using the best wavelength result HeNe found from experiment I, and comparing it to the actual index of refraction of air (1.000293), we came within a few hundred-thousandths of the theoretical value.

Error Analysis

In the Experiment, we faced many difficulties with determining the wavelength of HeNe in the first two sets of data collections. It was hard to keep track of counts as an observer and difficult spinning the micrometer reader at an even and slow enough pace. What made this process more difficult is needing ~100 counts of fringes each trial in order to get accurate results. One solution was to position ourselves near the screen so that we could video record the fringes and then count the fringes on the video frame by frame between two observers (for the second set of trials and took the average of fringes counted) in order to reduce the number of fringes that could have been missed. However, the first two data sets demonstrate that we were fairly off from the expected wavelength of HeNe and inconsistent as shown with the high percent difference. My colleagues and I came to the conclusion that the main reason is that we were not actually moving the micrometer reading 0.15 mm and that our presence near the screen made the waves from the light source slightly fragmented, since this optical apparatus is highly sensitive to both sound and light. Not to mention that in the second data set we had run the experiment with lights off in order to make the screen more visible but made it difficult for the person on the micrometer reader to perceive when we had gone 0.15mm. All these factors contributed to a reasonably high percent difference from the expected HeNe wavelength.

On the final data set collection for the wavelength, we recorded the fringes and confirmed results with two observers and conducted the experiment with lights on and no one near the screen. Doing so yielded the best results being less than 4% within the expected wavelength. Using the division rule of propagation, we found that the uncertainty of the wavelength for data set 03 is: $\Delta\lambda = |\lambda| \cdot \sqrt{(\frac{\Delta d}{d})^2 + (\frac{\Delta N}{N})^2} = 27.3$ nm where d is the average micrometer reading (0.15mm), Δd is its uncertainty of 0.005mm, and ΔN is the uncertainty for the number of fringes counted using standard deviation and N is the average number of fringes. Therefore, we find that the wavelength of HeNe in this experiment is λ = (657.2 $\pm$ 27.2) nm.

In the second experiment, we kept track of the pressure released from the vacuum tube in In-Hg for each trial. Using the 1 in-Hg: 3.38639 kPa pressure conversion and the atmospheric pressure, we determined the ratio of pressure P/P_atm and then plotted the number of fringes counted with respect to it. We used the pressure conversion instead of observing it from the pressure reader in order to reduce the uncertainty. Using the plot we ran a linear regression and found the slope to be $\frac{N}{\frac{P}{Patm}} = 58.50 \pm 1.30$. Using this and rearranging Eq(2) we were able to determine that the index of refraction for air using the wavelength from experiment I to be n_air = 1.000275.

In order to determine the uncertainty of the index of refraction for air, we had to consider the use of the division rule of propagation for the equation: $n_{air} = \frac{\lambda \cdot s}{2L} + n_{vacuum}$ where $s = \frac{N}{\frac{P}{Patm}}$. Therefore the uncertainties for the index of refraction become:

Using Ideal $\lambda = 632.8$**:**

$$\Delta n_{air} = \sqrt{\left(\frac{\partial n_{air}}{\partial s} \cdot \delta s\right)^2 + \left(\frac{\partial n_{air}}{\partial L} \cdot \delta L\right)^2} = 0.0000058$$

Using Experimental $\lambda = 632.8$**:**

$$\Delta n_{air} = \sqrt{\left(\frac{\partial n_{air}}{\partial s} \cdot \delta s\right)^2 + \left(\frac{\partial n_{air}}{\partial L} \cdot \delta L\right)^2 + \left(\frac{\partial n_{air}}{\partial \lambda} \cdot \delta \lambda\right)^2} = 0.0000128$$

Where the partials in variable form are: $\frac{\partial n_{air}}{\partial s} = \frac{\lambda}{2L}$, $\frac{\partial n_{air}}{\partial L} = \frac{-\lambda s}{2L^2}$, $\frac{\partial n_{air}}{\partial \lambda} = \frac{s}{2L}$. The variables and their corresponding uncertainty are as follow: $s \pm \delta s = 58.5 \pm 1.3$, $L \pm \delta L = (6.99 \pm 0.005)\ cm$ and $\lambda \pm \delta\lambda = (657.2 \pm 27.3)\ nm$. Note that, unlike the wavelength measured in the first part of this experiment, the ideal/theoretical wavelength of HeNe (~632.8 nm) does not contain measurable uncertainty from our experiment and so we do not have a contributing term for uncertainty when using the rule of propagation.