Simulating Nuclear Scattering Experiment

Erick Arcos, Suzanne Harmon, Ben Poplin

Department of Physics

Pacific Lutheran University

arcosea@plu.edu

December 7, 2021

Abstract

In Nuclear Scattering Experiments, particles are fired at nuclei in order to gain insight on some of the properties of the nuclei. In this research, we perform an analogous experiment, in which steel ball bearings are fired at a plastic cylindrical target that represents a nucleus. By firing the ball bearings at different impact parameters b (distance away from the center axis through the target), and analyzing where the ball bearings land on shell chamber after collision with the target, we determine the scattering angles θ (from the center axis). By plotting $cos(\frac{\theta}{2})$ against its corresponding impact parameter b, we found that the reciprocal of the sum of radius of the nucleus R and that of ball bearings (r) was: $\frac{1}{R+r} = (0.372 \pm 0.007)\ cm^{-1}$. This meant that the radius of the target was (2.47 ± 0.05) cm. We also directly measured the radius of the target to be (2.583 ± 0.005) cm, yielding a 4.3 % difference. In addition to finding the radius, we also relate and model the number of particles Δn that will scatter into angles between θ and $\theta + \Delta\theta$ in the shell chamber.

Introduction & Apparatus

Physics at the atomic and molecular level can be a difficult field of study. In order to understand more about the properties and structure of atoms, nucleus, and molecules, one method, known as *Nuclear Scattering*, is to simply bombard them with subatomic particles such as electrons, protons, neutrons, and photons. By analyzing and observing the scattering of the subatomic particles on some shell chamber (outer ring or shell) at different scattering angles θ; qualitative observations and conclusions can be made about the structure of the atoms and/or molecules that could not be done otherwise. In this experiment, we consider it as a 2D mechanical problem; that means we do not keep track of the vertical angle and/or height change from where the particles hit the nuclei target and the height of where it lands on the shell.

Following the Pacific Lutheran University 499A *PLU Nuclear Scattering Simulation Handout*, we simulate this experiment on a more macroscopic level. Instead of shooting subatomic particles, we will be firing ~44mm steel balls from a bulb operated air gun into a Lucite target that represents the atom or molecule we may want to study. By moving the impact parameter b (which moves the gun barrel) across a plane perpendicular to the axis of the target, we can fire steel balls and analyze their scattering angle.

By studying the scattering angles and the ball distribution, we aim to indirectly determine the size (radius) of this target. However, in actual nuclear experiments, oftentimes, there is no control of the impact parameter. Instead, a beam of particles of some range that is covered by the total width of the impact parameter is used instead to study the distribution of the scattering angles that were observed. This method of studying at "nuclear" level is also done in this experiment.

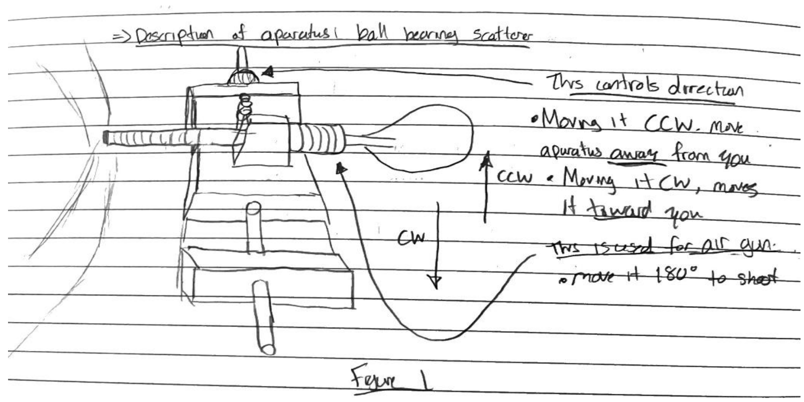


Diagram 1: Sketch of the Apparatus and Description of How It Operates.

In order to better understand the apparatus, consider the sketch above. As we can see from **Diagram 1**, By rotating a screw either CCW or CW, the apparatus moves sideways perpendicular to the center axis. The impact parameter b is defined as the distance from the zero/center axis (PA axis on **Diagram 2**). Using the fact that the impact parameter b changes by $0.14\,\frac{cm}{rev}$, we can determine the perpendicular distance from the center axis (impact parameter b) if the number of revolutions conducted is kept on record. One key note about this apparatus is that in order to avoid backlash from the gears, when firing the balls, the screw should and is only moved/spun in one direction.

Theoretical Description

The diagram below, **Diagram 2** demonstrates the trajectory of the balls fired as described by the red line path - fired from point D, makes contact with target at point C and lands on the shell chamber at point B. However, In order to be able to determine the radius of some unknown target from the trajectory in the diagram, the following assumptions are made:

- The impact from the steel balls striking the target is elastic
- The reflection process is operative

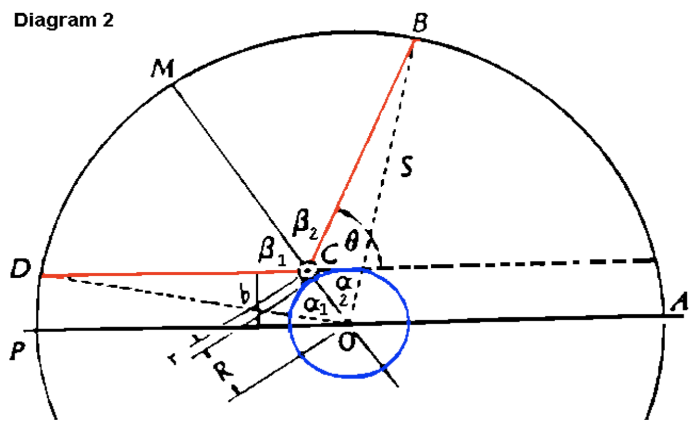


Diagram 2: Trajectory of 'Particle', from NSE Manual

Those assumptions are necessary because if we consider the spin of the particle, the trajectory after collision would no longer be a straight path but rather a curved path. Similarly, if we consider how the surface friction differs for different impact parameters because of the amount of contact made from the 'particles' on the target and so the trajectory would also change. Therefore, we make those assumptions in order to simplify the mechanics of the system.

Under those assumptions and using the diagram above, we can say that angles $\beta_1 = \beta_2$ (call it β). Since triangle ODC is congruent to triangle OBC, $\alpha_1 = \alpha_2$and so arc DM = arc MB. Also note that 5° is the largest angle POD can be; so the above statements and geometric assumptions are only true for a target up to 2.5 inches in diameter. Considering that, scattering angle θ and its complementary angles, we find the following relationships:

$$\theta = \pi - 2\beta_1 \quad (1)$$

$$sin(\beta) = cos(\frac{\theta}{2}) = \frac{b}{R+r} \quad (2)$$

However, since the scattering angle θ cannot be measured directly, we must first determine β_1. This is done so by taking a ratio of the arc length PM and the radius of the chamber S. In order to obtain the arc length PM, we have to consider half the circumference of the shell chamber, the impact parameter b (distance away from zero/center axis), the arc length AB and radius of the chamber S. The following equations are found:

$$\text{Arc PM} = \frac{\pi S - arcAB + b}{2} \quad (3)$$

$$\beta_1 = \frac{arc\ PM}{S} = \frac{\pi S - arcAB + b}{2S} \quad (4)$$

Therefore, the angle β_1 can be found using equation Eq(3) if the radius of the chamber is measured, and the arc length AB and its corresponding impact parameter b is monitored. The arc length AB is the length from the zero position (point A in diagram) to the location where the ball landed - point B on the diagram and these quantities are measured directly from the chamber. Once we have found β_1 using results from Eq(3) on Eq(4), we can use Eq(1) to determine scattering angle and then using Eq(2) and plotting it as a function of the impact parameter b, we can get the following ratio $\frac{1}{R+r}$ as the slope.

For the second experiment and theory, we should first begin by considering N to be the total number of balls fired and letting W be the width/distance that the gun moved perpendicular to the target from one side to the other until the balls fired no longer hit the target (or approximately $2 \cdot arc\ DP$ on the diagram above. So $\frac{N}{W}$ is the particle density and so the number of balls Δn to hit between impact parameter b and Δb is:

$$\Delta n = \frac{N}{W} \Delta b \quad (5)$$

Using Eq(2), we find that the number of particles Δn that will scatter into angles θ and Δθ is:

$$\Delta n = \frac{N}{W} \Delta b = \frac{N}{W} \frac{db}{d\theta} \Delta\theta \quad (6)$$

Then using the cosine form of Eq(2) and taking its derivative, solving for $\frac{db}{d\theta}$ and substituting it into Eq(6), we attain the following expression:

$$\Delta n = \frac{1}{2}(R + r)\frac{N}{W}sin(\frac{\theta}{2})\Delta\theta \qquad (7)$$

The expression above is a 2-D differential cross section which predicts how many particles will scatter into some detector (shell chamber in our case) at angle θ with width Δθ. By integrating Eq(7) for a total range of 2π representing the total angle of a full cycle around the target, we return with N, the number of particles fired.

Procedure I & II

In the first experiment (procedure I) , we aimed to determine the radius of the target. We begin our investigation by determining the radius of the steel balls (r) that were going to be fired. Using a digital caliper, we had measured multiple steel balls and found radius of the steel balls to be an average of (0.2185 ± 0.0025)cm. In addition, we directly measured the radius of the shell chamber (S) to be roughly (33.30 ± 0.05) cm. We also verified using the circumference of the chamber. Using double sided-tape we then place wax-tape on the inside circumference of the chamber covering at minimum ¾ of the circumference (except possibly the area where the slot for the air gun is located). The next step was to locate the zero position, we did this by placing a long rod through the center of the chamber and target. We then rotated the screw until the barrel was right above the rod, and then marked the zero position with a straight line (this is point A) on **Diagram 2**.

We then rotated the screw CW until we got to a point where firing a ball would not hit the target. We then made one full rotation of the screw (screw is marked, so we know when the screw had made a full revolution) until the ball hit the target. For each proceeding full revolution, we shot 10 steel balls and marked their location on the wax tape using colored markers. We alternated colors for each grouping/revolution (color coding scheme is essential). It was also crucial to keep track of the number of rotations that were being made since each revolution of the screw told us that the parameter b changed by 0.1411 cm. We repeated this process until the balls were essentially bouncing straight back. For these turns, since no wax paper was in that region, and the barrel was near the zero axis; it was assumed that all 10 balls for each turn would have bounced back due to the geometry of the angle for which the balls were fired. Another important note was to keep track of when the barrel was right above the rod that went through the center axis of the chamber since this told us the turn/revolution for which the impact parameter was zero (b = 0 cm). We repeated the process until the balls no longer hit the target.

After the firing process was done, the wax paper was removed and taped to a table with a 2meter long stick. For procedure I, we then used the zero position (point A) and measured the arc length to each shot fired for each revolution group to the nearest tenth of a centimeter. The rest of the data analysis was done using computing software as described in the following section.

In the second procedure (II), the wax paper was divided into 20 equal lengths of $\Delta\theta$. We also marked the center of each region $\Delta\theta$ so that we know that the angle at that point is $\theta \pm \frac{\Delta\theta}{2}$. Note that the circumference of the chamber was roughly $2\Delta\theta$ larger than the wax paper we contained because we did not have wax paper in the region for which the balls bounced back because of where the apparatus was located. Therefore, we had approximately 22 intervals of equal arc length to cover the entirety of the circumference of the chamber. This is crucial because the final step before data analysis was to determine the number of balls Δn that landed in each $\Delta\theta$. Granted all that, we can note that $\Delta\theta = \frac{2\pi}{22}$.

Experimental Results & Analysis

It took a total of 39 revolutions to fire bearing balls from one corner of the target to the other. This meant that the impact parameter b changed for a total of width W of $0.1411\frac{cm}{rev} \cdot 39$

rev = 5.50 cm with a total number of balls fired to be 390. Note that the zero position occurred on revolution 21 (b = 0 cm at turn 21). In procedure I, we had measured and recorded the length from each ball fired from the zero position for each revolution. We then ran Q-tests on the minimum and maximum values for each revolution in order to determine if there were any outliers with 95% certainty using $Q_{crit} = 0.466$. We found a total of 3 whose Q values were greater than the critical Q value and thus removed them from the data.

Afterwards we calculated the average arclength for each revolution with their corresponding 9-10 balls fired and its standard deviation. The average arc lengths represent arc AB in Eq(3) and Eq(4). Therefore, using the arc AB averages, the chamber's radius S and the corresponding impact parameter b for each arc AB, we used Eq(3) to determine arc PM. Using these arc lengths, we used Eq(4) to determine the angle β for each revolution. We then used Eq(1) to get the scattering angle θ. The final step was to calculated the ratio $\frac{b}{R+r}$ using the cosine form of Eq(2) with the scattering angle. We then used that ratio for each impact parameter b and plotted it against its corresponding impact parameter. The following graph demonstrates the results:

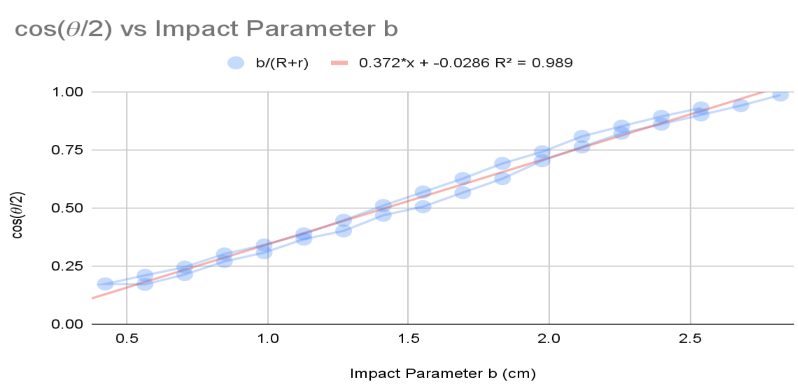


Figure 1: Demonstrates the ratio $\frac{1}{R+r}$ as the slope

The reason that each parameter b in the graph above has approximately 2 points is because we are considering the impact parameter b to be the distance from the zero axis. That implies that there will be a symmetry with the impact parameters since balls were fired on both sides of the target. Using the slope from the graph above, we find the relationship that $\frac{1}{R+r} = 0.372$ cm^{-1} .

To further verify this, we ran a linear regression on Excel and found that:

$$\frac{1}{R+r} = (0.372 \pm 0.007)\, cm^{-1} \text{ or } R + r = (2.686 \pm 0.0188) \text{ cm} \qquad (8)$$

Therefore, using the radius of the ball that we had previously determined and rules of propagation, we find that the radius of the target is approximately (2.47$\pm$ 0.02) cm. We can also compare the actual radius directly measured which ultimately yielded a 4.3 % error difference. The following table demonstrates a summary of the measurements and findings.

Table 1: Summary of Experimental Results in Finding Target Radius

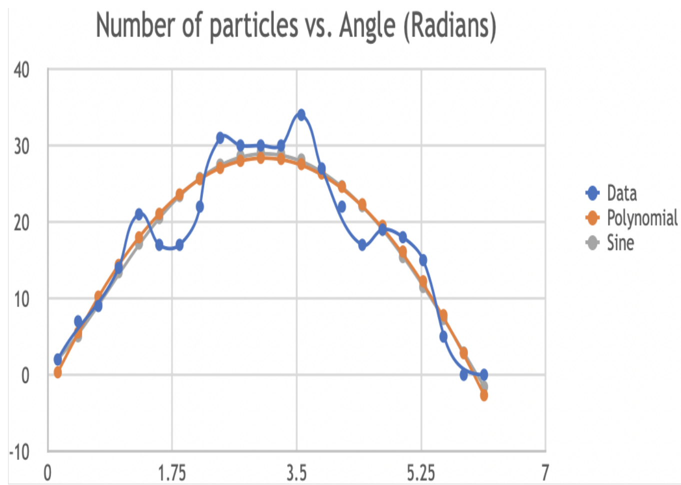
Object	**Measurement**
Radius of Chamber (S)	33.30 $\pm$ 0.05 cm
Radius of Ball Bearings (r)	0.2185 $\pm$ 0.0025 cm
Direct Measurement Target Rad (R)	2.583 $\pm$ 0.005 cm
Experimental Measurement of Target Rad (R)	2.47 $\pm$ 0.05 cm
Percent Error in Target Radius	4.3 %

In the second part of this experiment (procedure II), the goal was to analyze the number of particles that are to hit at any given angle and verify that such a relationship can be modeled using a function that resembles Eq(7) . After having marked the 22 $\Delta\theta$ ranges, we then counted the number of particles Δn that landed in each range and assumed it to have angle $\theta \pm \frac{\Delta\theta}{2}$. The following table and graph demonstrates our results.

Table 2: Demonstrates the Number of particles that landed in angles $\theta \pm \frac{\Delta\theta}{2}$ or arc length $L \pm \frac{\Delta L}{2}$

Arclength	Number of Particles	Angle(Radians)
4.75	2	0.1426426426
14.25	7	0.4279279279
23.75	9	0.7132132132
33.25	14	0.9984984985
42.75	21	1.283783784
52.25	17	1.569069069
61.75	17	1.854354354
71.25	22	2.13963964
80.75	31	2.424924925
90.25	30	2.71021021
99.75	30	2.995495495
109.25	30	3.280780781
118.75	34	3.566066066
128.25	27	3.851351351
137.75	22	4.136636637
147.25	17	4.421921922
156.75	19	4.707207207
166.25	18	4.992492492
175.75	15	5.277777778
185.25	5	5.563063063
194.75	0	5.848348348
204.25	0	6.133633634

Figure 2: Plot of Table 2with Best-Fit Functions



In **Table 2**, we have found the center arc length to each equal length interval and converted such length into an angle using the relationship $\theta = \frac{L}{S}$ where L is the arclength to each midpoint from the zero position and S here is the circumference of the chamber. Note that the grey background arc lengths are the two intervals that could've covered the slot where the air gun was located. Due to the gun being fairly close to the zero axis - within 3 revolutions on both sides of the center axis, it is assumed that all 60 balls would have reflected back.

With all that into account, we then plotted number of particles that landed in such $\Delta\theta$ intervals using **Table 2** and got the results in **Figure 2.** In order to determine if a relationship like that in Eq(7) exists, Excel solver was used for different functions in order to find the functions that best relate Δn and θ. We tried a polynomial and sine function (considering phase shifts) and got the following results:

Polynomial: $\Delta n = A\theta^2 + B\theta + C$ where A = - 3.29, B = 20.17, C=-2.48

Sine: $\Delta n = A\,sin(B\theta + \varphi) + C$ where A = 30.80, B = 0.51, φ =0.046, C= -1.65

As we can see from the graph in **Figure 2**, both approximations are accurate. However, using the sine approximation, we can easily relate it to Eq(7). Comparing the sine approximation to Eq(7), we find that the amplitude (coefficient A) is equivalent to $\frac{1}{2}(R + r)\frac{N}{W}\Delta\theta$, the frequency shift B is approximately the $\frac{1}{2}$ that was expected, the phase shift φ is approximately zero which is consistent with Eq(7), and the coefficient C is simply a slight vertical shift in order to better approximate the data.

Discussion and Conclusion

While the linear regression allows us to determine the magnitude of the radius of the target, we should also consider some uncertainty to that value. Based on the linear regression, we found that the slope (s) is related to the radius of the target and ball bearings as such:

$s = \frac{1}{R+r} = 0.372\ cm^{-1}$and so the standard deviation of the slope

$\Delta s = \Delta(R + r) = 0.007\ cm^{-1}$. Therefore, if we take $\frac{\Delta s}{s} = \frac{\Delta(R+r)}{R+r}$, we find that

$\Delta(R + r) = 0.0507\ cm$ and then using rules of propagation, we find that the uncertainty of the radius of the target $\Delta R = \sqrt{(\Delta(R + r))^2 - (\Delta r)^2} \simeq 0.05\ cm$.

With regards to the data from experiment II, there was some discrepancy using solver to determine approximations for our graph. Ideally, if we were to integrate Eq(7) in the range $0 \leq \theta \leq 2\pi$, we should find the result of N which is the total number of particles fired (387). However, when we integrate the sine approximation, we get a value of approximately 110 “particles fired”. The reason for this is because the infinitesimal change in θ (dθ) or rather the ‘

$\Delta\theta$' in Eq(7) is being consumed into the amplitude coefficient A in the sine approximation as a value of $\Delta\theta \simeq \frac{2\pi}{22}$; where the $2\pi$is the total angle angle range and the 22 is the total number of $\Delta\theta$ ranges. This meant we had the width of each bar in the "histogram" graph is be $\frac{2\pi}{22}$ instead of a magnitude of one. However, if we multiply the sine approximation by the reciprocal $(\frac{22}{2\pi})$ to eliminate the unintentional $\Delta\theta$, we get that the integral is approximately 385 (number of particles fired) - which is only a 0.5 % difference from the expected value of 387.

While both experiments had some technical sources of error, we can clearly see that both methods bring some useful insight in the fields of nuclear physics. In the first experiment, we were able to determine properties such as the radius of some unknown 'nucleus' by simply firing 'particles' at the nucleus. One observation to note is that our experiment was based on a cylindrical target. Had the target been a hexagonal cross section, we would have only observed particles landed on special regions in the shell; unlike the cylinder target which had targets land on almost all regions because it was circular. On the second experiment, we were able to find and accurate model the number of particles that were likely to land between some range of $\Delta\theta$.

References

1. D.J Prowse, *Note on the "Welch" Scattering Apparatus* (Los Angeles, California), p854.
2. E.R Wicher, *Elementary Rutherford Scattering Simulator* (Claremont, California 1964), p635-636.
3. Louie, R. PLU *Sakai PHYS 499A PLU Nuclear Scattering Simulation Handout*, Modified 9/03/2021, http://sakai.plu.edu/, Accessed 09/18/2018.
4. Louie, R. PLU *Sakai PHYS 499A Nuclear Scattering Lab Handout Diagrams*, Modified 9/03/2021, http://sakai.plu.edu/, Accessed 09/18/2018