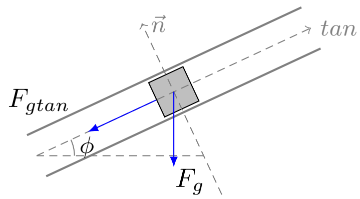
2a)



$m = 0,5kg \quad \phi = 25°$
$F_g = mg$
$F_{gtan} = F_g \sin(\phi) = mg\sin(\phi)$

$\Delta x \sin(\phi) = h$
$\Delta x = \dfrac{h}{\sin(\phi)}$

$$F = ma \Leftrightarrow a = \frac{F}{m} = g\sin(\phi)$$

$$x = \frac{1}{2}at^2$$

$$x = \frac{1}{2}g\sin(\phi)t^2 \Leftrightarrow \frac{h}{\sin(\phi)} = \frac{1}{2}g\sin(\phi)t^2$$

$$\Leftrightarrow \boxed{t = \sqrt{\frac{2h}{g\sin^2(\phi)}} = 0.585s}$$

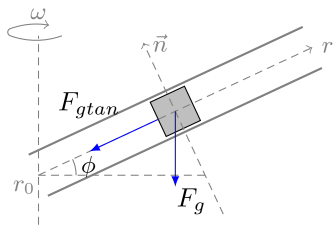
2b)

$F_{atr} = \mu F_n$
$\ddot{r}_{tan} = 0 \Leftrightarrow F_{atr} = F_{gtan}$

$F_n = mg\cos(\phi) \quad F_{gtan} = mg\sin(\phi)$

$$\boxed{\mu = \frac{\sin(\phi)}{\cos(\phi)} = tg(\phi) = 0.466}$$

2c)



$1 g.l. : r$
$\phi = 25°$

$$v_k = \frac{\partial r_k}{\partial q_j}\dot{q}_j + \frac{\partial r_k}{\partial t}$$

$x = r\cos(\omega t)\cos(\theta)$ $\qquad v_x = \dot{r}\cos(\omega t)\cos(\theta) - r\omega\sin(\omega t)\cos(\theta)$
$y = r\sin(\omega t)\cos(\theta)$ $\qquad v_y = \dot{r}\sin(\omega t)\cos(\theta) - r\omega\cos(\omega t)\cos(\theta)$
$z = r\sin(\phi)$ $\qquad v_z = \dot{r}\sin(\phi)$

$v^2 = v.v = \dot{r}^2\cos^2(\omega t)\cos^2(\phi) + r^2\omega^2\sin^2(\omega t)\cos^2(\phi) + \dot{r}^2\sin^2(\omega t)\cos^2(\phi) + r^2\omega^2\cos^2(\omega t)\cos^2(\phi) \cancel{-r\omega\sin(\omega t)\dot{r}\cos(\omega t)\cos^2(\phi)} \cancel{+r\omega\sin(\omega t)\dot{r}\cos(\omega t)\cos^2(\phi)}$

$\Leftrightarrow v^2 = \dot{r}^2\cos^2(\phi)\cancelto{1}{\left(\cos^2(\omega t) + \sin^2(\omega t)\right)} + r^2\omega^2\cos^2(\phi)\cancelto{1}{\left(\cos^2(\omega t) + \sin^2(\omega t)\right)} + \dot{r}^2\sin^2\phi$

$L = T - V$
$V = mgr\sin(\phi) \qquad T = \frac{1}{2}m(\dot{r}^2\cos^2(\phi) + r^2\omega^2\cos^2(\phi) + \dot{r}^2\sin^2\phi)$

$$\frac{\mathrm{d}}{\mathrm{d}t}\left(\frac{\partial L}{\partial \dot{r}}\right) = \frac{\mathrm{d}}{\mathrm{d}t}\left(m\dot{r}\cos^2(\phi) + m\dot{r}\sin^2\phi\right) = \frac{\partial L}{\partial r} = mr\omega^2 - mg\sin\phi$$

$\Leftrightarrow m\ddot{r} = mr\omega^2 - mg\sin\phi$
$\Leftrightarrow \boxed{\ddot{r} = r\omega^2 - g\sin\phi}$

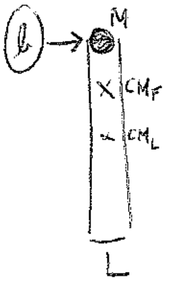
2d)
$\ddot{r} = 0 \Leftrightarrow r\omega^2 - g\sin\phi = 0$

$$\boxed{\omega = \sqrt{\frac{g\sin\phi}{r}}} = 6.436 \text{ rad/s}$$

3a) $F_{ext} = 0 \Rightarrow p = cte \Rightarrow v_{CM_i} = v_{CM_f}$

$$v_{CM_i} = \frac{v_M \times m_M}{m_M + m_L} = 5 \, m/s$$

b)



$$I^* = \overbrace{M\left(\frac{L}{4}\right)^2}^{M} + \left(\overbrace{\frac{1}{12}ML^2}^{L} + \overbrace{M\left(\frac{L}{4}\right)^2}^{\Delta x \, L_{CM}}\right) = \frac{10}{6}$$

$$\vec{L}_i = \vec{L}_f$$

$$\vec{L}_i = \vec{r} \times \vec{p} \Rightarrow L_i = Mv\frac{L}{4}$$

$$L_f = I^* \omega \Rightarrow \frac{10}{6}\omega = \frac{MvL}{4} \Leftrightarrow \omega = 3 \, rad/s$$

c) $E_{m_i} = \frac{1}{2} M v^2 = 25 \, J$

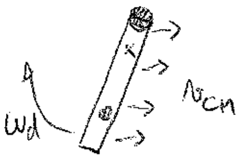
$$E_{m_f} = \frac{1}{2} M_{tot} v_{CM}^2 + \frac{1}{2} I^* \omega^2 = 12{,}5 + 7{,}5 = 20 \, J$$

$$\Delta E_m = \overset{20-25}{\cancel{25-20}} = -5 \, J$$

d) A velocidade imediatamente a seguir à colisão é:

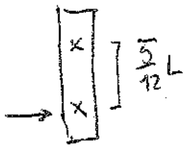
$v = \omega d + v_{CM}$

para um ponto a uma distância d do eixo de rotação ($\cancel{e}$=CM)



O ponto onde se sente o menor impacto é aquele onde v logo após o impacto é zero:

$$v = 0 \Leftrightarrow \omega d = v_{CM} \Leftrightarrow d = \frac{v_{CM}}{\omega} = \frac{20}{12} \cdot \frac{5}{3} = \frac{5}{12}L$$



(4a) $v = \sqrt{\frac{T}{\mu}} = 169 \text{ m/s}$

(b) $\lambda_n = \frac{2L}{n}$ $\quad f_n = \frac{v}{\lambda_n}$

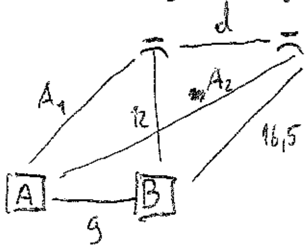
$$f_n = \frac{\sqrt{\frac{T}{\mu}}}{2L} n \sim n\, 110{,}046 \text{ Hz}$$

(c) 4ª Harmónica $= f_4 = 440{,}1845 \text{ Hz}$

Batimento $= f_4 - f = 0{,}1845 \text{ Hz}$

(d) Fundamental : f_1

$$\lambda = \frac{v}{f} = \frac{330}{110} = 3 \text{ m}$$



$d = \sqrt{16{,}5^2 - 12^2} = 11{,}325 \text{ m}$ $\quad A_1 = 15 \text{ m}$

$A_2^2 = 12^2 + (d+9)^2 \Rightarrow A_2 = 23{,}603 \text{ m}$

Para A:

$$\frac{A_1}{\lambda} = \frac{15}{3} = 5$$

$$\frac{A_2}{\lambda} = \frac{23{,}603}{3} = 7{,}86$$

$\left(\frac{A_1}{\lambda} - \frac{A_2}{\lambda}\right)_{mod 1} = 0{,}86 \Rightarrow$ menos mal

Para B:

$$\frac{B_1}{\lambda} = \frac{12}{3} = 4$$

$$\frac{B_2}{\lambda} = \frac{16{,}5}{3} = 5{,}5$$

$\left(\frac{B_1}{\lambda} - \frac{B_2}{\lambda}\right)_{mod 1} = 0{,}5 \Rightarrow$ Os altifalantes estão em completa oposição de fase.

O espectador em A ouve melhor, dado que o espectador em B não ouve de todo.