Notes on Combinatorics

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Text -

Chapter 1

Mappings and The Integers

1.1 Mappings

One topic of great importance in the whole of mathematics, especially in tat of Algebra, Calculus, Topology; is the notion of a "mapping" or "function". A mapping can informally be thought of as being a rule that takes an element from a specified set S and sends it to a set T. We will however, require a formal definition of such an object if we want to be able to do mathematics with it.

Definition 1.1.1. Let $S \neq \emptyset$ and $T \neq \emptyset$ be sets. A *Mapping*, $M : S \to T$ is the set $M \subseteq S \times T$; such that $\forall s \in S$, there exists a unique $t \in T$ shuch that $(s, t) \in M$

If we let $\sigma : S \to T$ be a mapping, we can also write $S \xrightarrow{\sigma} T$, or if $(s, t) \in \sigma$ is known, we can write $\sigma : s \to t$.We represent the latter by using the notation "$\sigma(s) = t$" (however some algebraists will write $(s)\sigma = t$ or $s\sigma = t$, we however will use $\sigma(s) = t$). In fact, it turns out that, though precise, our current definition is a bit clunky; so we can introduce an equivalent definition which allows use to make use of our prefered notation:

Definition 1.1.2. Let $S \neq \emptyset$ and $T \neq \emptyset$ be sets. A *Mapping*, $M : S \to T$ is the set $M \subseteq S \times T$; such that:

1. If $s \in S$, then $\exists t \in T$ such that $M(s) = t$
2. If $s \in S$ and $t_1, t_2 \in T$ $M(s) = t_1$ and $M(s) = t_2$ then $t_1 = t_2$

Definition 1.1.3. Let $M : S \to T$ be a mapping. The *Image* of S under M is the set

$$M(S) = \{M(s) : s \in S\} \tag{1.1}$$

Using the definition of an image, we can further define for $s \in S$, $M(s) = t$ to be the *image* of s under M