# Mate 6540

## Tarea 1

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| Problema 1. | Show that if $d$ is a metric for $X$, then $$d'(x, y) = d(x, y)/(1 + d(x, y))$$ is a bounded metric that gives the topology of $X$. [*Hint:* If $f(x) = x/(1+x)$ for $x > 0$, use the mean-value theorem to show that $f(a+b) - f(b) \leq f(a)$.] |
| Problema 2. | If $\{\mathcal{T}_\alpha\}$ is a family of topologies on $X$, show that $\bigcap \mathcal{T}_\alpha$ is a topology on $X$. Is $\bigcup \mathcal{T}_\alpha$ a topology on $X$? |
| Problema 3. | Let $\{\mathcal{T}_\alpha\}$ be a family of topologies on $X$. Show that there is a unique smallest topology on $X$ containing all the collections $\mathcal{T}_\alpha$, and a unique largest topology contained in all $\mathcal{T}_\alpha$. |
| Problema 4. | Show that the collection $$\mathcal{C} = \{[a, b) \mid a < b, a \text{ and } b \text{ rational}\}$$ is a basis that generates a topology different from the lower limit topology on $\mathbb{R}$. |
| Problema 5. | Sea $X = \{f \mid f : [0,1] \to [0,1] \text{ es una función}\}$. Para cada subconjunto $A$ de $[0,1]$, defina $$B_A = \{f \in X \mid f(x) = 0, \quad \forall x \in A\}.$$ Demuestre que $\mathscr{B} = \{B_A \mid A \subseteq [0,1]\}$ es una base para una topología sobre $X$. |