WITS UNIVERSITY

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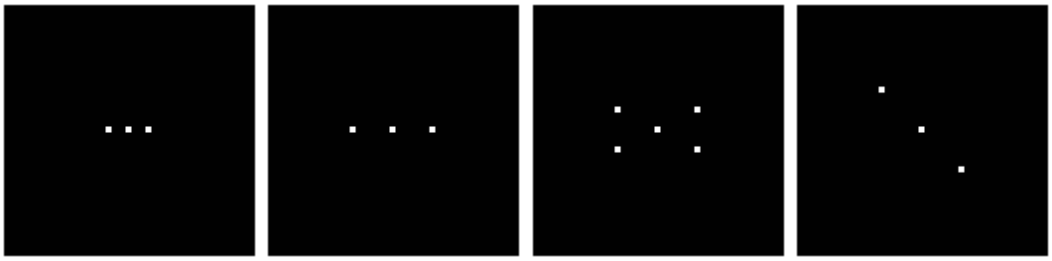
APPM4058A & COMS7238A: Digital Image Processing
Exercise 4

2019-2-27

1 Problems



(a) (b) (c) (d)

Figure 1: Question 1

1. Consider the images in Figure 1. Each is the Fourier Transform of an image. Roughly sketch the original image.
2. Why is there a need for image padding when filtering in the frequency domain? How do you implement the padding?
3. What will be the effects on an image of removing all the data in the frequency domain outside a small central disc?
4. What would be the effect of removing the central disc instead?
5. Give a proof of the convolution theorem.
6. Let $F(u,v)$ denote the Fourier transform of an image $f(x,y)$. By convolution theorem we know that multiplying $F(u,v)$ by a filter function $H(u,v)$ and taking the inverse of Fourier transform will alter the appearance of the image depending on the nature of filter. What is the filtering effect if $H(u,v) = A$, where A is a positive constant? Explain.
7.
   - What does the following three filters do when applied to an image?
   - Find their Fourier transforms, and display the magnitudes of these transforms respectively.

$$h_1 = \frac{1}{16}\begin{bmatrix} 1 & 2 & 1 \\ 2 & 4 & 2 \\ 1 & 2 & 1 \end{bmatrix} \quad h_2 = \begin{bmatrix} -1 & -1 & -1 \\ -1 & 8 & -1 \\ -1 & -1 & -1 \end{bmatrix} \quad h_3 = \begin{bmatrix} 0 & -1 & 0 \\ -1 & 5 & -1 \\ 0 & -1 & 0 \end{bmatrix} \tag{1}$$

2 Lab exercise

1. Create simple images such as those given in Figure 1 and simple sin and cos waves in 2D, then find their Fourier transforms and display the spectrum as images.

2. Given a 5×5 spatial box filter, h, find the corresponding filter in frequency domain, H. Apply H to image 'building'. Show original image and filtered image in both spatial and frequency domain.

3. Replace the filter in the previous question with a Sobel vertical mask, and do the same experiment.
4. Given image ‘DIP’, perform the following operations step by step:
   (a) multiply the image on the left by $(-1)^{x+y}$;
   (b) obtain the DFT of resulting image from the previous step;
   (c) take the conjugate of the DFT;
   (d) compute the inverse DFT;
   (e) multiply the real part of the result by $(-1)^{x+y}$.

   Explain your result mathematically.
5. Implement the basic steps of DFT filtering given in the lecture slides.
6. Using the implementation from previous step, perform filtering using the low pass and high pass frequency domain filters for test images ‘lena’ and ‘barbara’.