Tut 3.

1 a) $4W + 6B = 10$: $P(1W) = \frac{4}{10} = 0{,}4$

b) $P(1B) = \frac{6}{10} = 0{,}6$

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2. $8W + 12B = 20$ Draw 3.

a) $P(3W) = \frac{n(E)}{n(\Omega)} = \frac{\binom{8}{3}}{\binom{20}{3}} = \frac{56}{1140} = \frac{14}{285} = 0.0491\ldots$

OR $P(3W) = \frac{8}{20} \times \frac{7}{19} \times \frac{6}{18} = P(E_1).P(E_2|E_1).P(E_3|E_1E_2)$

$= \frac{336}{6848} = \frac{14}{285} = 0{,}0491$

b) $P(2W + 1B) = \frac{\binom{8}{2}\binom{12}{1}}{\binom{20}{3}} = \frac{336}{1140} = \frac{28}{95} = 0.2947$

OR $P(2W + 1B) = \left(\frac{8}{20} \times \frac{7}{19} \times \frac{12}{18}\right) \times \frac{3!}{2!\,1!} = 0.2947$

c) $P(1W + 2B) = \left(\frac{8}{20} \times \frac{12}{19} \times \frac{11}{18}\right) \times \frac{3!}{2!\,1!} = 0.4632$

$= \frac{\binom{8}{1}\binom{12}{2}}{\binom{20}{3}} = \frac{8 \times 66}{1140} = \frac{44}{95} = 0.4632$

d) $P(3B) = \frac{\binom{12}{3}}{\binom{20}{3}} = \frac{220}{1140} = \frac{11}{57} = 0.1930$

$= \left(\frac{12}{20} \cdot \frac{11}{19} \cdot \frac{10}{18}\right) = 0.1930$

3. $P(A+B \text{ toget}) = \dfrac{2! \times 8! \times 9}{10!} = 0.2 =$

Treat A+B as one object

arranging 2 objects in a row

$$= \frac{9!\,2!}{10!} = 0.2 \qquad \frac{{}^{9}P_{9} \times {}^{2}P_{2}}{{}^{10}P_{10}}$$

$P(A+B$ <u>not</u> $\text{toget}) = 1 - 0.2 = 0.8$

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4. a) 3 even nos. (2, 4, 6) on a die, $\therefore P(\text{even}) = \frac{3}{6} = 0.5$

$P(3\text{even}) = 0.5 \times 0.5 \times 0.5 = 0.125$

b) 1 odd no. $\Rightarrow$ 1 odd + 2 evens

$P(1o + 2e) = {}^{3}C_{1}(0.5)(0.5)^2 = 0.375$

$= \dfrac{3!}{2!\,1!}(0.5)^3 = 0.375$

c) Even sum $\Rightarrow$ 3 evens or 1 even + 2 odds

$\therefore P(\text{even sum}) = 0.125 + 0.375$

$= 0.5$

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5 10 cards : draw 4. Even sum $\Rightarrow$ 4e, 4o, 2e + 2o

$P(e) = P(o) = 0.5$

a) Drawn together $\Rightarrow$ no replacement

$P(\text{even sum}) = P(4e) + P(4o) + P(2e+2o)$

$= 2 \times \left(\frac{5}{10} \times \frac{4}{9} \times \frac{3}{8} \times \frac{2}{7}\right) + {}^{4}C_{2}\left(\frac{5\times4\times5\times4}{10\times9\times8\times7}\right)$

$= \frac{11}{21} = 0.5238$

5(a) cont

OR. Q: the set of all ordered groups of 4 that can be selected from 10 numbers

E_1 : subset of 4 evens

E_2 : subset of 4 odds

E_3 : subsets of 2 even & 2 odd

$$P(E_1 \text{ or } E_2 \text{ or } E_3) = \frac{n(E_1 \text{ or } E_2 \text{ or } E_3)}{n(Q)}$$

$$= \frac{\binom{5}{4} + \binom{5}{4} + \{\binom{5}{2} \times \binom{5}{2}\}}{\binom{10}{4}}$$

$$= \frac{5+5+100}{210} = \frac{11}{21}$$

$$= 0.5238$$

b) with replacement: $P(4e) = (0.5)^4 = 0.0625$

$P(4o) = (0.5)^4 = 0.0625$

$P(2e \& 2o) = {}^4C_2 (0.5)^4 = 0.375$

$\underline{0.5}$

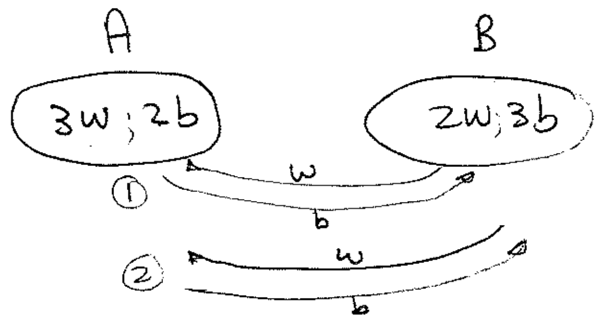
6.

A	B			
2	1			
3	1	OR	2	2
end			3	2
			end.	

(No of ways that A can win)

$$\therefore P(A \text{ wins}) = \frac{1}{2} + \frac{1}{2} \cdot \frac{1}{2} = 0.75$$

7.



4w ; 2b   1w ; 3b

4w ; 1b   1w ; 4b

5w ; 1b   0w ; 4b

5w ; 0b   0w ; 5b

P(5w in A after 2 moves) ?

$\frac{2}{5} \times \frac{2}{6} \times \frac{1}{5} \times \frac{1}{6}$

$= \frac{1}{225} = 0.004\dot{4}$

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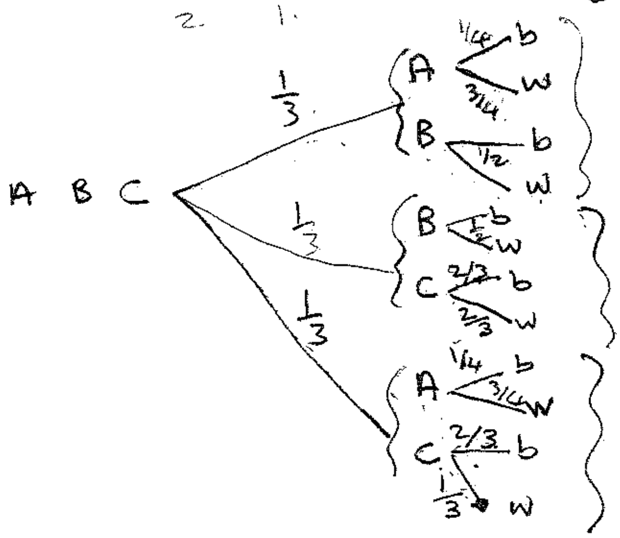
8. 3w 1b (A)   3w 3b (B)   6w 3b (C) $^{3}/_{2} = 3$

2.   1.   62



$P(\text{w from A + w from B}) = \frac{1}{3} \times \frac{3}{4} \times \frac{1}{2}$
$= ^{1}/_{8}$

$P(\text{w from B + w from C}) = \frac{1}{3} \times \frac{1}{2} \times \frac{2}{3}$
$= ^{1}/_{9}$

$P(\text{w from A + w from C}) = \frac{1}{3} \times \frac{3}{4} \times ^{2}/_{3}$
$= ^{1}/_{6}$

a) $\therefore P(ww) = \frac{1}{8} + \frac{1}{9} + ^{1}/_{6} = \frac{29}{72}$
$= 0.402\dot{7}$

b) $P(ww \text{ or } bb) = \frac{29}{72} + P(bb)$.

$P(bb) = \frac{1}{3}\left(\frac{1}{4} \times \frac{1}{2} + \frac{1}{2} \times \frac{1}{3} + ^{1}/_{4} \times \frac{1}{3}\right) = \frac{1}{8} = 0.125$

$\therefore P(ww \text{ or } bb) = 0.52\dot{7} = \frac{19}{36} = \frac{29}{72} + \frac{1}{8}$

9 4 trials as in Q8.

a) $P(2\,ww + 2bb) = \left(\frac{29}{72}\right)^2 \times \left(\frac{1}{8}\right)^2$

$= 0.0025348....$

$\simeq 0.002535$

b) $P(3bb + \overline{bb}) + P(4bb)$

$= P(3bb + bw) + P(3bb + ww) + P(4bb)$

$P(1b,1w) = \frac{1}{3}\left[\frac{1}{3}\cdot\frac{1}{2} + \frac{2}{3}\cdot\frac{1}{2} + \frac{1}{2}\cdot\frac{3}{4} + \frac{1}{2}\cdot\frac{1}{4} + \frac{1}{3}\cdot\frac{3}{4} + \frac{2}{3}\cdot\frac{1}{4}\right]$

$= \frac{17}{36}$

$P(3bb + bw) = \left(\frac{1}{8}\right)^3 \cdot \frac{17}{36} \cdot {}^4C_1 = \frac{17}{18432} \times 4$

$P(3bb + ww) = \left(\frac{1}{8}\right)^3 \cdot \frac{29}{72} \cdot {}^4C_1 = \frac{29}{36864} \times 4$

$\therefore P(3bb + \overline{bb}) = 0.006835....$

$\therefore P(3bb + \overline{bb}) + P(4bb) = 0.007080....$

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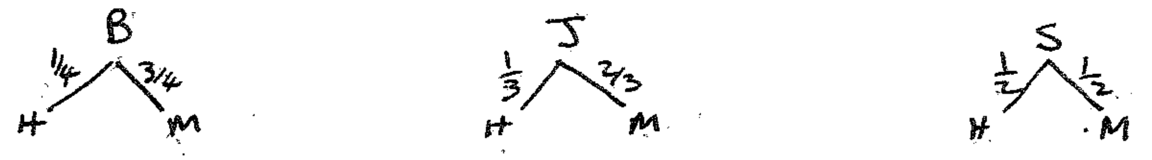
10. $\frac{1}{5} \times \frac{1}{4} \times \frac{1}{3} \times \frac{1}{2} \cdot \frac{1}{1} = \frac{1}{120} = 0.008\dot{3}$

$P(A) \times P(B|A) \times P(C|A+B) \times$

$5!$ $\frac{1 \times 1 \times 1 \times 1 \times 1}{5!}$

11



a) P(each on 1st shot hits 1st)

$P(\text{B hits 1st}) = 1/4 = 0.25$
$P(\text{J hits 1st}) = 3/4 \cdot 1/3 = \frac{1}{4}$
$P(\text{S hits 1st}) = 3/4 \cdot 2/3 \cdot 1/2 = 1/4$

b) $P(\text{no one hits in 1st round}) = 3/4 \cdot 2/3 \times \frac{1}{2} = 1/4$

c) P(J is 1st to hit with his 2nd shot)

$= \left(3/4 \cdot 2/3 \cdot \frac{1}{2}\right) \cdot \left(3/4 \cdot \frac{1}{3}\right)$

$= 1/16 = 0.0625$

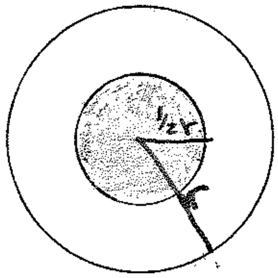
12.

a) $P(X: \text{W L W L W}) = 2/5 \cdot 3/5 \cdot 2/5 \times 3/5 \cdot 2/5 = \left(\frac{2}{5}\right)^3 \left(\frac{3}{5}\right)^2$
$= \frac{72}{3125} = 0.02304$

b) $P(\text{X wins 3 games}) = {}^5C_3 \times \frac{72}{3125} = 0.2304$

c) $P(\text{X wins 3 or 4 or 5 games}) = P(3) + P(4) + P(5)$
$= 0.2304 + {}^5C_4 (2/5)^4 \cdot 3/5 + (2/5)^5$

$= 0.31744$

13. 

Pt. must be in inner $\odot$

area inner $\odot = \pi\left(\frac{r}{2}\right)^2 = \frac{\pi r^2}{4}$

area outer $\odot = \pi r^2$

$\therefore$ P(pt in inner $\odot$) $= \frac{\pi r^2}{4} \Big/ \pi r^2 = 0.25$

14. $P(W) = 0.4$ $\quad$ $P(M) = 0.6$

a) $P(MB) = 0.6 \times 0.09 = 0.054$

$^4P_4$

b) P(4 st. with diff blood grps) $= 4! \times P(O) . P(A) . P(B) . P(AB)$

$= 0.0125$

c) P(4 W with diff bld grps) $= 0.0125$

d) P(4 st are W + bld grps are diff) $= 4! \times 0.4^4 \times P(O) \times P(A) . P(B) . P(AB)$

$= 0.4^4 \times 0.0125$

$= 0.00032$

15. a) 4 questions × 5 marks = 20 (max)

To pass : need 10 marks or more ⇒ 2 or 3 or 4 correct

Random guessing : $\frac{1}{6}$ chance of being right

$\frac{5}{6}$ chance of being wrong

$$P(\text{at least 2 right}) = P(2) + P(3) + P(4)$$

$$= {}^4C_2\left(\tfrac{1}{6}\right)^2\left(\tfrac{5}{6}\right)^2 + {}^4C_3\left(\tfrac{1}{6}\right)^3\left(\tfrac{5}{6}\right) + {}^4C_4\left(\tfrac{1}{6}\right)^4$$

$$= 0.1319444$$

b) 8 questions ⇒ 4 or more correct

$$P(\text{at least 4 correct}) = 1 - P(\text{at most 3 wrong})$$

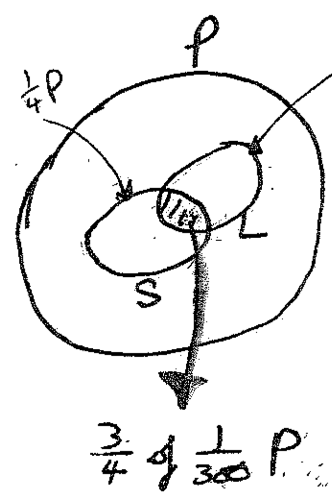
$$= {}^8C_4\left(\tfrac{1}{6}\right)^4\left(\tfrac{5}{6}\right)^4 + {}^8C_5\left(\tfrac{1}{6}\right)^5\left(\tfrac{5}{6}\right)^3 + {}^8C_6\left(\tfrac{1}{6}\right)^6\left(\tfrac{5}{6}\right)^2 + {}^8C_7\left(\tfrac{1}{6}\right)^7\left(\tfrac{5}{6}\right) + {}^8C_8\left(\tfrac{1}{6}\right)^8$$

$$= 0.0306564$$

16. $\frac{1}{4}P$ smoke

$\frac{3}{4}$ of lung cancer patients also smoke

$\frac{1}{300}P$ have lung cancer



a) $P(\text{Smoker has lung cancer}) = P(C|S)$

$$= \frac{n(\text{smokers with cancer})}{n(\text{smokers})}$$

$$= \frac{\frac{3}{4} \times \frac{1}{300}P}{1/4\ P}$$

$$= \frac{1}{100}.$$

b) $n(S) = 1/4\ P \qquad n(L) = \frac{1}{300}P$

$$n(S+L) = \frac{3}{4} \cdot \frac{1}{300}P$$

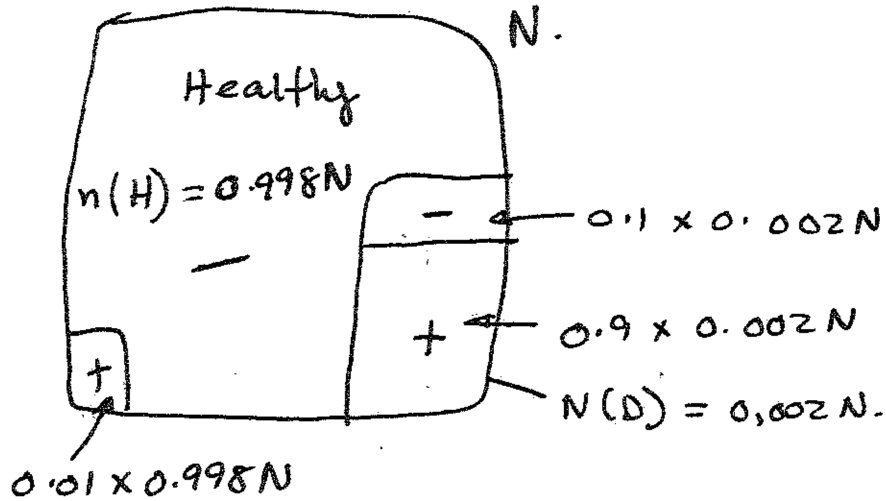
$$P(S \text{ or } L) = \frac{n(S) + n(L) - n(S+L)}{P}$$

$$= \frac{1/4\ P + \frac{1}{300}P - 3/4 \times \frac{1}{300}P}{P}$$

$$= 1/4 + \frac{1}{300} - \frac{1}{400}$$

$$= \frac{301}{1200}$$

17.



$n(+) = 0.01 \times 0.998N + 0.9 \times 0.002N = 0.01178$

$n(D+) = 0.9 \times 0.002N$

$\therefore\ P(D \text{ and } +) = \dfrac{0.9 \times 0.002}{(0.01 \times 0.998) + (0.9 \times 0.002)} = \dfrac{n(D+)}{n(+)}$

$= 0.1528\ldots$

$n(E) = D$ and $+$

$n(Q) = + \Rightarrow H+$ OR $D+$