APPM4058A & COMS7238A: Digital Image Processing

Hairong Wang

School of Computer Science & Applied Mathematics
University of the Witwatersrand, Johannesburg

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Contents

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Outline

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Basics of spatial filtering

- Perform filtering operations on the pixels of an image.
- The process consists of moving the filter from point to point in an image,
- At each point (x, y), the response of the filter at that point is calculated using a predefined relationship.
- Note that a filter is usually a small image. A filter is also termed as mask, kernel, template, or window etc. The values in a filter are referred to as coefficients, rather than pixels.

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Basics of spatial filtering cont.

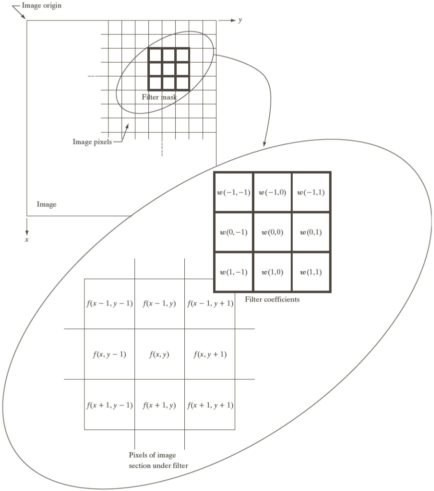

Figure: Linear spatial filtering using a 3 $\times$ 3 mask.

Basics of spatial filtering cont.

- Pointwise operations – each point of old image determines one point of the new image e.g. contrast stretch.
- Local operations – each point of the new image is determined by a neighbourhood of points of the old image.
- Global operations – each point of the new image is determined by the whole of the old image.

Linear spatial filtering

For an image f of size $M \times N$ with a filter of $m \times n$, linear spatial filtering is given by

$$g(x, y) = \sum_{s=-a}^{a} \sum_{t=-b}^{b} w(s, t) f(x + s, y + t), \quad (1)$$

where $a = (m - 1)/2$ and $b = (n - 1)/2$.

- To generate a complete filtered image the operation must be applied for $x = 0, 1, \ldots, M - 1$ and $y = 0, 1, \ldots, N - 1$.
- What happens at the border of an image?
  - Exclude the border pixels – the resulting image will be smaller than the original
  - Filter all pixels only with the mask coefficients which are contained in the image
  - Padding – by adding rows and columns of 0's (or other constant gray level) or padding by replicating rows or columns
  - At the end of the operation, the padding is stripped off – to keep the same size.

Linear spatial filtering cont.

A similar expression to (1) is

$$g(x, y) = \sum_{s=-a}^{a} \sum_{t=-b}^{b} w(s, t) f(x - s, y - t). \tag{2}$$

- The expression (1) is called correlation, and (2) convolution.
- Mechanically, correlation and convolution are the same process, except that the mask in convolution is rotated by 180 degrees prior to passing it around.

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Outline

1. Spatial Filtering
   - Smoothing Linear Spatial Filters
     - Mean filters
     - Gaussian Smoothing Filter
     - Kuwahara Smoothing Filter
   - Order-statistics filters
   - Sharpening Spatial Filters
     - Use of Second Derivative for Enhancement - the Laplacian
     - Use of First Derivatives for Enhancement – the Gradient

Smoothing linear spatial filters

- The idea is to replace the value of every pixel in an image by the average of the gray levels in the neighbourhood defined by the filter mask.
- The process results in an image with reduced “sharp” transitions in gray levels.
- Smoothing filters are used for blurring and for noise reduction.
- Edges also have sharp transitions in gray levels, so averaging filters have side effect of blurring edges.

Arithmetic mean filter

- This is the simplest of the mean filters.

$$\hat{f}(x, y) = \frac{1}{mn} \sum_{(s,t) \in S_{x,y}} g(s, t), \tag{3}$$

where $S_{x,y}$ represents the set of coordinates in a rectangular subimage window size of $m \times n$, centered at point (x, y). For example the following smoothing filter:

$$\frac{1}{9} \times \begin{pmatrix} 1 & 1 & 1 \\ 1 & 1 & 1 \\ 1 & 1 & 1 \end{pmatrix} \tag{4}$$

A spatial averaging filter in which all coefficients are equal is sometimes called a box filter or a uniform filter.

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Smoothing filter example

For example, consider a rectangular unweighted smoothing filter applied to an image. The bold region in the original image leads to the bold number in the output image. The same process is applied to all the other entries except the edge rows and columns.

0	**1**	**2**	**1**	0
1	**2**	**3**	**1**	0
2	**3**	**1**	**0**	2
3	1	0	2	3
1	0	2	3	3

0	1	2	1	0
1	1.67	**1.56**	1.11	0
2	1.78	1.44	1.33	2
3	1.44	1.33	1.78	3
1	0	2	3	3

Smoothing filter example cont.

0	**1**	**2**	**1**	0
1	**2**	**3**	**1**	0
2	**3**	**1**	**0**	2
3	1	0	2	3
1	0	2	3	3

0	1	2	1	0
1	2	**2**	1	0
2	2	1	1	2
3	1	1	2	3
1	0	2	3	3

Smoothing filter example cont.

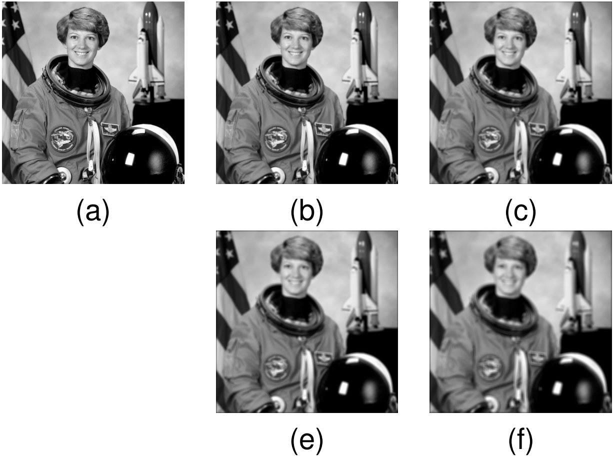

Figure: (a) Original image; (b) result using 3×3 averaging filter; (c) using 5×5; (e) using 7×7; (f) using 9×9.

Smoothing filter example - circular

$$A = \frac{1}{5}\begin{array}{|c|c|c|}\hline 0 & 1 & 0 \\ \hline 1 & 1 & 1 \\ \hline 0 & 1 & 0 \\ \hline\end{array}$$

$$A = \frac{1}{8}\begin{array}{|c|c|c|}\hline 0 & 1 & 0 \\ \hline 1 & 4 & 1 \\ \hline 0 & 1 & 0 \\ \hline\end{array}$$

Note that here the central pixel is multiplied by 4, the ones to left, to right, above, below by 1, and the corner pixels neglected (i.e. multiplied by 0).

Arithmetic mean filter cont.

- An important application of spatial averaging is to blur an image.
- What is the purpose of blurring an image? To get a gross representation of objects of interest, such that the intensity of smaller objects blends with the background and larger objects become 'bloblike' and easy to detect. The size of the mask decides the relative size of the small objects that will be blended with the background.

Gaussian Smoothing Filter

- A case of weighted averaging;
- Gives more weight to the central pixels and less weight to the neighbours.
- The farther away the neighbours, the smaller the weight.

Gaussian Smoothing Filter cont.

We want to construct a filter which mimics the shape of the 2D Gaussian bell function given by:

$$G(x, y; \sigma) = \frac{1}{2\pi\sigma^2} \exp\left(-(x^2 + y^2)/2\sigma^2\right), \tag{5}$$

where σ (standard deviation) is a measure of the spread. The 2D Gaussian function is the product of two 1D Gaussian (1 for each direction)

$$G(x; \sigma) = \frac{1}{\sqrt{2\pi\sigma^2}} \exp\left(-\frac{x}{2\sigma^2}\right). \tag{6}$$

- Need to produce a discrete approximation of the Gaussian function.

Some properties of Gaussian filter

- The Gaussian filter is a non-uniform averaging filter
- The filter coefficients diminish with increasing distance from the kernel's centre
- Central pixel have a higher weighting than those on periphery
- Larger values for σ produce a wider peak (greater blurring).
- Filter size must increase with increasing σ to maintain the Gaussian nature of the filter
- Gaussian kernel coefficients depend on the value of σ.
- At the edge of the mask, coefficients must be close to 0.
- The kernel is rotationally symmetric with no directional bias
- Gaussian kernel is separable, which allows fast computation
- Gaussian filters might not preserve image brightness.

Gaussian filter example

A 5×5 Gaussian filter using (5)

$$G(x, y; \sigma) = \frac{1}{2\pi\sigma^2} \exp\left(-(x^2 + y^2)/2\sigma^2\right),$$

where $\sigma = 1$.

0.0030	0.0133	0.0219	0.0133	0.0030
0.0133	0.0596	0.0983	0.0596	0.0133
0.0219	0.0983	0.1621	0.0983	0.0219
0.0133	0.0596	0.0983	0.0596	0.0133
0.0030	0.0133	0.0219	0.0133	0.0030

Gaussian smoothing filter example

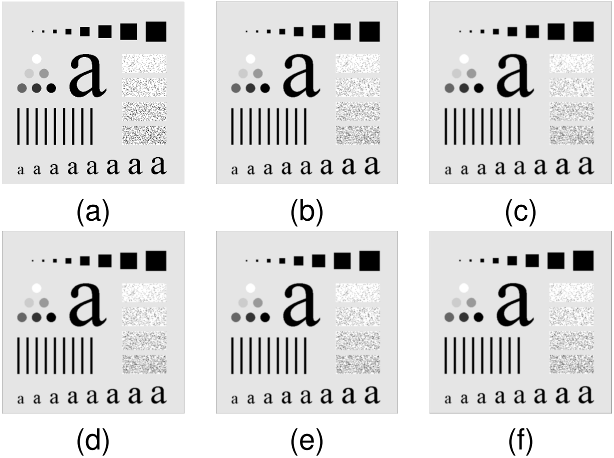

Figure: Example of Gaussian blurring using different sizes of masks ($\sigma = 1$). (a) original, (b) 3×3, (c) 5×5, (d) 9×9, (e) 15×15, (f) 30×30

Smoothing using averaging filter

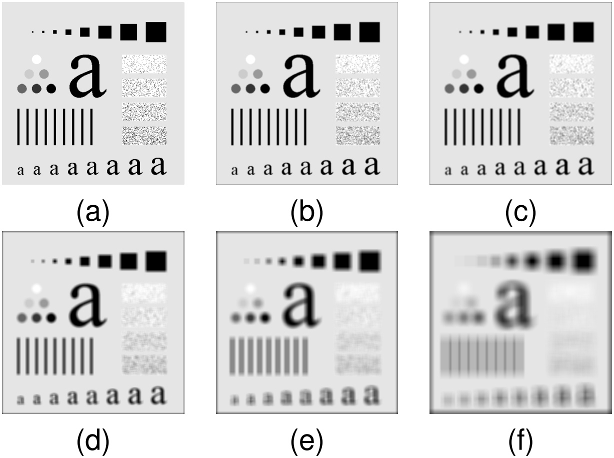

Figure: (a) original, (b) 3×3, (c) 5×5, (d) 9×9, (e) 15×15, (f) 30×30.

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Gaussian smoothing filter example

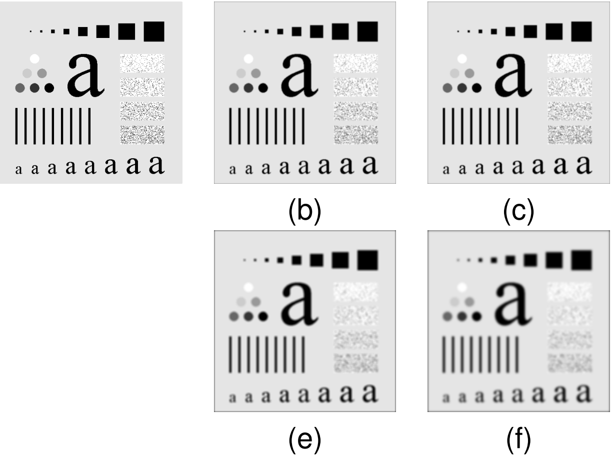

Figure: Gaussian blurring using different sizes of masks and σs. (b) 9×9, $\sigma = 1.0$, (c) 15×15, $\sigma = 1.0$,(b) 9×9, $\sigma = 3.0$, (c) 15×15, $\sigma = 3.0$

Kuwahara smoothing filter

- A further problem with smoothing for noise removal is that we smooth edges.
- This makes the picture look blurred. Several filters attempt to preserve or even enhance edges as they smooth.
- Kuwahara smoothing filter: The center pixel is determined by taking the mean and variance of each of the four squares that overlap at the pixel and assigning the value of the mean of the region with lowest variance.
- Kuwahara filter is an edge preserving filter.

Kuwahara smoothing filter cont.

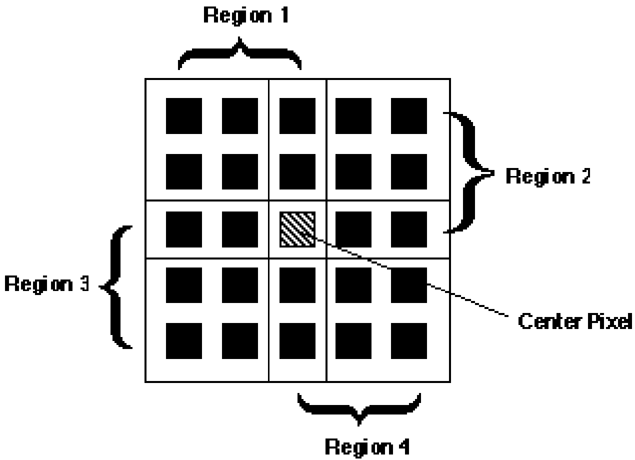

Figure: Kuwahara smoothing filter window

Kuwahara smoothing filter cont.

Table: 5×5 Kuwahara filter window

0	1	2	1	0
1	2	3	1	0
2	**3**	**1**	**0**	2
3	**1**	**0**	**2**	3
1	**0**	**2**	**3**	3

Outline

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Order-statistics filters

- The problem with averaging to smooth is that the noise is averaged in.
- Order-statistics filter: Filtering response is based on the ordering the pixels in the image area that are covered by the filter.
- Median filter, max filter, min filter etc.

Median filter

- The median filter considers the values of the pixels in the window and gives the median as the output.
- Thus, generally, “bad” outliers are ignored.
- The median, ξ, of a set of values is such that half of the values in the set are less than or equal to ξ, and half are greater than or equal to ξ.
- For a given pixel in an image, to perform median filtering,
    - first sort the pixel and its neighbours
    - determine the median
    - assign the median to the pixel in question.

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Median filter cont.

Use a 3×3 median filter on the image below:

Table: Example of median filter

0	1	2	1	0
1	2	3	1	0
2	**3**	**1**	**0**	2
3	**1**	**0**	**2**	3
1	**0**	**2**	**3**	3

0	1	2	1	0
1	2	1	1	0
2	2	1	1	2
3	1	**1**	2	3
1	0	2	3	3

Outline

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Sharpening spatial filters

- Fundamentally, the response of a derivative operator is proportional to the degree of discontinuity of the image at the point at which the operator is applied.
- Thus, image differentiation enhances edges and other discontinuities (such as noise) and deemphasizes areas with slowly varying gray-level values.
- A basic definition of the first-order derivative of a one dimensional function is

$$\frac{\partial f}{\partial x} = f(x+1) - f(x), \tag{7}$$

and second-order derivative is

$$\frac{\partial^2 f}{\partial x^2} = f(x+1) + f(x-1) - 2f(x). \tag{8}$$

First- and second-order derivatives in the context of image processing

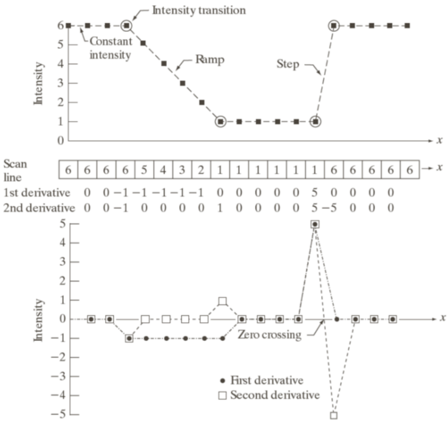

Figure: Illustration of 1st and 2nd derivatives using a line profile

Use of Second Derivative for Enhancement - the Laplacian

For a function $f(x, y)$, the Laplacian is defined as

$$\Delta^2 f = \frac{\partial^2 f}{\partial x^2} + \frac{\partial^2 f}{\partial y^2}. \tag{9}$$

The Laplacian is a linear operator.

The Laplacian cont.

A popular definition of digital second derivative in the x-direction

$$\frac{\partial^2 f}{\partial x^2} = f(x+1, y) + f(x-1, y) - 2f(x, y), \tag{10}$$

and in the y-direction

$$\frac{\partial^2 f}{\partial y^2} = f(x, y+1) + f(x, y-1) - 2f(x, y), \tag{11}$$

The digital implementation of (9) is

$$\Delta^2 f = (f(x+1, y) + f(x-1, y) + f(x, y+1) + f(x, y-1)) - 4f(x, y). \tag{12}$$

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The Laplacian cont.

Table: Laplacian filters

0	1	0
1	-4	1
0	1	0

1	1	1
1	-8	1
1	1	1

0	-1	0
-1	4	-1
0	-1	0

-1	-1	-1
-1	8	-1
-1	-1	-1

The diagonal terms are incorporated in the second column masks.

The Laplacian cont.

The basic way in which we use the Laplacian for image enhancement is as follows:

$$g(x,y) = \begin{cases} f(x,y) - \Delta^2 f(x,y) & \text{if the center coefficient} \\ & \text{of Laplacian filter is negative} \\ f(x,y) + \Delta^2 f(x,y) & \text{if the center coefficient} \\ & \text{of Laplacian filter is positive} \end{cases} \tag{13}$$

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The Laplacian cont.

Note that (12) and (13) can be combined into one step as the following.

$$
\begin{aligned}
& g(x, y) \\
= & f(x, y) - (f(x+1, y) + f(x-1, y) + f(x, y+1) + f(x, y-1)) + 4f(x, y) \\
= & 5f(x, y) - (f(x+1, y) + f(x-1, y) + f(x, y+1) + f(x, y-1)).
\end{aligned}
$$

This equation can be implemented using the filter

Table: Composite Laplacian filter 1

0	-1	0
-1	5	-1
0	-1	0

The Laplacian cont.

If the diagonal neighbours are also included in the calculation of the Laplacian, then the filter shown in Table 5 can be used.

Table: Composite Laplacian filter 2

-1	-1	-1
-1	9	-1
-1	-1	-1

Use of First Derivatives for Enhancement – the Gradient

- First-order derivatives of a digital image are based on various approximations of the 2D gradient.
- First derivatives in image processing are implemented using the magnitude of the gradient.

$$\Delta f = \left[G_x \; G_y\right]^{\mathrm{T}} = \left[\frac{\partial f}{\partial x} \; \frac{\partial f}{\partial y}\right]^{\mathrm{T}} \tag{14}$$

The magnitude is

$$\Delta f = \mathrm{mag}(\Delta f) = \left[G_x^2 + G_y^2\right]^{1/2} = \left[(\frac{\partial f}{\partial x})^2 + (\frac{\partial f}{\partial y})^2\right]^{1/2}. \tag{15}$$

The gadient cont.

- Computation of the gradient of an image is based on obtaining the partial derivatives at every pixel location.
- One of the simplest ways to implement first-order partial derivative at point z_5 (see Table 6) is to use the following

$$G_x = z_8 - z_5, \text{ and } G_y = z_6 - z_5. \tag{16}$$

- Two cross-gradient operators by Roberts,

$$G_x = z_9 - z_5, \text{ and } G_y = z_8 - z_6. \tag{17}$$

These derivatives can be implemented for an entire image by using the 'Roberts' masks.

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The gradient cont.

An approach using masks of size 3×3 is given by

$$G_x = (z_7 + z_8 + z_9) - (z_1 + z_2 + z_3) \tag{18}$$

and

$$G_y = (z_3 + z_6 + z_9) - (z_1 + z_4 + z_7). \tag{19}$$

These two formulations can be implemented using Prewitt operators.

The gradient cont.

A slight variations of (18) and (19) use a weight of 2 in the center coefficient:

$$G_x = (z_7 + 2z_8 + z_9) - (z_1 + 2z_2 + z_3) \tag{20}$$

and

$$G_y = (z_3 + 2z_6 + z_9) - (z_1 + 2z_4 + z_7). \tag{21}$$

A weight of 2 is used to achieve some smoothing by giving more importance to the center point. The Sobel operators are used to implement these two equations.

The gradient cont.

- The Prewitt and Sobel operators are among the most used in practice for computing digital gradients.

Table: Various masks used to compute the gradient at point labelled z_5.

z_1	z_2	z_3
z_4	z_5	z_6
z_7	z_8	z_9

Roberts

-1	0
0	1

0	-1
1	0

Prewitt

-1	-1	-1
0	0	0
1	1	1

-1	0	1
-1	0	1
-1	0	1

Sobel

-1	-2	-1
0	0	0
1	2	1

-1	0	1
-2	0	2
-1	0	1

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