COMS4040A Assignment 2 – Report

Tamlin Love (1438243) - BSc Hons

April 14, 2019

1 Introduction

Discrete Image Convolution is a widely used technique in digital image and signal processing. It can achieve a huge variety of effects, from blurring and sharpening to edge detection and noise removal.
Consider a discrete image $I \in \mathbb{R}^{M\times N}$ where each $Y(i,j) \in [0,1]$. Consider also a discrete filter $F \in \mathbb{R}^{L\times P}$. Then the discrete convolution of I by F is defined as:

$$(I*F)(i,j) = \sum_{s=0}^{L-1}\sum_{t=0}^{P-1} I(i-s, j-t)F(s,t), \tag{1}$$

where $i \in [0, M-1]$ and $j \in [0, N-1]$.
In serial, this can be expensive operation as it has an asymptotic time complexity of $\mathscr{O}(MNLP)$. However, because the calculation of each pixel $I(i,j)$ depends only on its local neighbourhood, we can parallelise the problem to improve efficiency. In this report, we consider several approaches to parallelising the problem on a GPU using CUDA. Before we can discuss these approaches, however, we must first discuss the different types of memory on a GPU.

1.1 GPU Memory

On the GPU there are 5 types of memory [2]:

- **Local Memory** - each thread has its own local memory which is not shared with any other threads, thus limiting its use to temporary variables in the convolution calculation.
- **Shared Memory** - this type of memory is shared among threads in a block. It is typically very fast, but is also quite small (48 KB on a Nvidia GTX 1060[1])
- **Global Memory** - global memory resides on the device and is accessible by all grids. As a consequence, it is fairly slow.
- **Constant Memory** - constant memory is a fast read-only memory type that is accessible by all grids. It is optimised for being read by multiple threads, but is fairly small (64 KB on a Nvidia GTX 1060[1]).
- **Texture Memory** - texture memory is a fairly large read-only memory type that is optimised for 2D spatial locality (i.e. nearby threads will access nearby points in texture memory). As a consequence, it is quite fast under these conditions.

2 Parallel Approaches

We now discuss four separate parallel approaches to the convolution problem.
The first approach is to naïvely assign each pixel to a thread and access its neighbours from global memory. While this certainly does the job of parallelising the problem, it is inefficient, as it makes roughly $2MNLP$ reads from global memory (as both the image and the filter are stored there) and then MN writes to global memory (as the output image is stored there). This approach is referred to as the **Naïve** approach.

A better approach is to make use of the faster shared memory for image reading. Due to the shared memory's small size, we can tile the image into overlapping blocks and only load one tile to the shared memory of the corresponding block. Thus each block deals with only a fraction of the total image. For a tile width and height of T, this reduces the number of reads from global memory to $\frac{MN}{T^2}(T+2\left\lfloor\frac{L}{2}\right\rfloor)(T+2\left\lfloor\frac{P}{2}\right\rfloor)$ (as the filter is still in global memory). This approach is referred to as the **Shared Memory** approach.
However, we can do better by making use of the read-only memory types. In one approach, we can load the filter into constant memory, thus making it much faster to access and reducing the number of reads from global memory to $MNLP$ if the image is in global memory or MN if the image is shared. For experimental purposes, we consider the first case. This approach is referred to as the **Constant Memory** approach.
In the final approach, we load the image into texture memory as a 2D texture, making use of the 2D spatial locality inherent to the convolution algorithm. This reduces global memory reads to $MNLP$ in the case of a global memory filter. This approach is referred to as the **Texture Memory** approach.
In each case, because MN threads are used, the overall asymptotic time complexity of each approach when using global memory accesses as a basic operation is $\mathcal{O}(LP)$.

3 Empirical Analysis

The above approaches, along with the CPU serial approach, were run on a Nvidia GTX 1060 6GB GPU with the following specs

Memory Clock Rate	4.004 GHz
Memory Bus Width	192 bits
Peak Memory Bandwidth	192.192 GB/s
Theoretical Peak FLOPS	4.375 TFLOPS
Compute Capability	6.1

On the host side was an Intel i7-7700 CPU @ 3.60GHz.
An averaging filter, sharpening filter and Sobel vertical edge detection filter were implemented, with the former two being varied for sizes 3×3, 5×5 and 9×9. The 3×3 versions of each filter are shown below:

Averaging

1/9	1/9	1/9
1/9	1/9	1/9
1/9	1/9	1/9

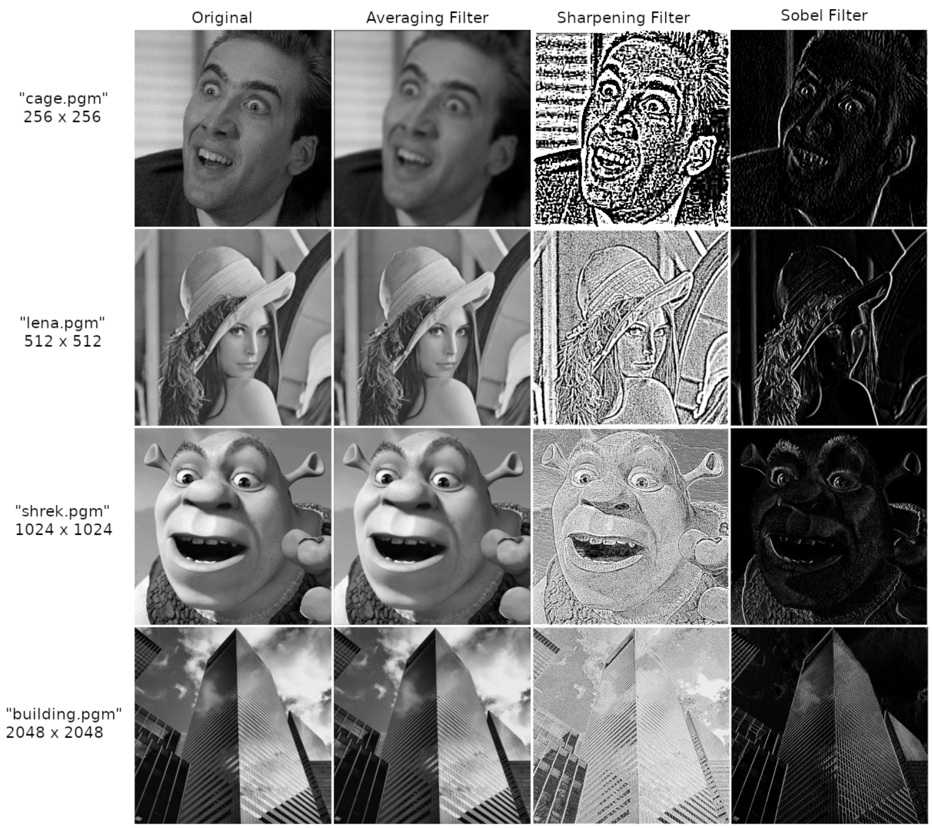
Sharpening

-1	-1	-1
-1	9	-1
-1	-1	-1

Sobel

-1	0	1
-2	0	2
-1	0	1

The experiments were run on four images of sizes ranging from 256×256 to 2048×2048. The images, along with some examples of the program output on these images, is shown below.



3.1 Kernel Time and Speedup

This first set of results shows the execution time for the kernel for various image sizes and filter sizes using both the averaging and the sharpening filters. All times are in seconds.

Averaging Filter - Kernel Time

Image Size	Filter Size	Serial Time	Naive Time	Shared Memory Time	Constant Memory Time	Texture Memory Time
256 x 256	3 x 3	0.014537	0.000041	0.000053	0.000029	0.00004
	5 x 5	0.03184	0.000086	0.000103	0.00005	0.000062
	9 x 9	0.058421	0.000208	0.000266	0.000133	0.000135
512 x 512	3 x 3	0.033477	0.000117	0.00016	0.000076	0.00009
	5 x 5	0.047504	0.000293	0.000366	0.000156	0.000182
	9 x 9	0.119374	0.000778	0.001178	0.000415	0.000488
1024 x 1024	3 x 3	0.058487	0.000433	0.000592	0.000268	0.000301
	5 x 5	0.12551	0.001128	0.001415	0.000651	0.00065
	9 x 9	0.365407	0.003277	0.004373	0.001771	0.001693
2048 x 2048	3 x 3	0.171482	0.001637	0.002689	0.001026	0.001283
	5 x 5	0.442127	0.004866	0.00601	0.002485	0.002896
	9 x 9	1.402518	0.012336	0.017509	0.007144	0.006652

Sharpening Filter - Kernel Time

Image Size	Filter Size	Serial Time	Naive Time	Shared Memory Time	Constant Memory Time	Texture Memory Time
256 x 256	3 x 3	0.006454	0.000041	0.000052	0.000029	0.00004
	5 x 5	0.010588	0.000086	0.000102	0.000051	0.000062
	9 x 9	0.02632	0.000207	0.000281	0.000115	0.000132
512 x 512	3 x 3	0.01153	0.000118	0.000162	0.000076	0.000092
	5 x 5	0.028605	0.000294	0.000363	0.000155	0.000179
	9 x 9	0.086674	0.000792	0.001	0.000416	0.00045
1024 x 1024	3 x 3	0.061758	0.000419	0.000591	0.000267	0.000295
	5 x 5	0.112552	0.001127	0.001417	0.000632	0.000654
	9 x 9	0.349421	0.003057	0.00395	0.001751	0.001675
2048 x 2048	3 x 3	0.173789	0.001638	0.002322	0.001027	0.00111
	5 x 5	0.452518	0.00447	0.005788	0.002485	0.002555
	9 x 9	1.403984	0.012163	0.015792	0.006965	0.00662

We can immediately see that the time taken for the kernel function to run increases both with M and N but also with L and P as they increase for all approaches. This is of course predicted by the complexity analysis of these approaches. Restricting our discussion to the results of the sharpening filter, we can measure the speed-up, $S = \frac{Time_{Serial}}{Time_{Parallel}}$, of the various approaches and tabulate the results below:

Sharpening Filter - Speedup

Image Size	Filter Size	Naive Speedup	Shared Memory Speedup	Constant Memory Speedup	Texture Memory Speedup
256 x 256	3 x 3	157.4146	124.1154	222.5517	161.3500
	5 x 5	123.1163	103.8039	207.6078	170.7742
	9 x 9	127.1498	93.6655	228.8696	199.3939
512 x 512	3 x 3	97.7119	71.1728	151.7105	125.3261
	5 x 5	97.2959	78.8017	184.5484	159.8045
	9 x 9	109.4369	86.6740	208.3510	192.6089
1024 x 1024	3 x 3	147.3938	104.4975	231.3034	209.3492
	5 x 5	99.8687	79.4298	178.0886	172.0979
	9 x 9	114.3019	88.4610	199.5551	208.6096
2048 x 2048	3 x 3	106.0983	74.8445	169.2201	156.5667
	5 x 5	101.2345	78.1821	182.0998	177.1108
	9 x 9	115.4307	88.9048	201.5770	212.0822

Here we see that the approach that consistently gives the highest speedup is the Constant Memory approach, followed by the Texture Memory approach, then the Naïve approach and finally the Shared Memory approach. The fact that the Constant Memory approach is fastest is unsurprising, as it makes use of some of the fastest memory on the device. An interesting result is that the Shared Memory approach is the slowest. This is likely due to the increased overhead within the kernel required to handle the initialisation of the overlapping block.

3.2 Global Memory Accesses

Measuring the number of global memory accesses (read and write) performed by each approach, we arrive at the following results.

Sharpening Filter - Global Memory Accesses

Image Size	Filter Size	Naive	Shared Memory	Constant Memory	Texture Memory
256 x 256	3 x 3	1245184	148480	655360	655360
	5 x 5	3342336	167936	1703936	1703936
	9 x 9	10682368	212992	5373952	5373952
512 x 512	3 x 3	4980736	593920	2621440	2621440
	5 x 5	13369344	671744	6815744	6815744
	9 x 9	42729472	851968	21495808	21495808
1024 x 1024	3 x 3	19922944	2375680	10485760	10485760
	5 x 5	53477376	2686976	27262976	27262976
	9 x 9	170917888	3407872	85983232	85983232
2048 x 2048	3 x 3	79691776	9502720	41943040	41943040
	5 x 5	213909504	10747904	109051904	109051904
	9 x 9	683671552	13631488	343932928	343932928

As predicted by the results in Section 2, we see that the number of global memory accesses in the Naïve approach is equal to twice that of the Constant and Texture Memory approaches, and that the number corresponding to the Shared Memory approach is further scaled by the size of the tile. The fact that the number of global memory accesses does not correspond to the kernel execution time points to the effects of the various memory speeds and the overhead associated with each approach.

3.3 Overhead Time

The overhead time (that is, the time taken for the relevant variables to be initalised and copied from the host to the device and vice-versa) was measured for all approaches using both filter types. The following results were recorded.

Averaging Filter - Overhead Time

Image Size	Filter Size	Naive	Shared Memory	Constant Memory	Texture Memory
256 x 256	3 x 3	0.084529	0.053155	0.0515	0.051605
	5 x 5	0.082068	0.053767	0.053193	0.052054
	9 x 9	0.073769	0.054243	0.054473	0.057575
512 x 512	3 x 3	0.074273	0.051962	0.052071	0.053216
	5 x 5	0.074455	0.054641	0.054124	0.054191
	9 x 9	0.071953	0.054384	0.05358	0.056806
1024 x 1024	3 x 3	0.075182	0.053532	0.05368	0.060639
	5 x 5	0.075999	0.0546	0.055844	0.05837
	9 x 9	0.07342	0.054135	0.053048	0.052906
2048 x 2048	3 x 3	0.080347	0.059241	0.054023	0.054079
	5 x 5	0.076506	0.058069	0.055145	0.055384
	9 x 9	0.089209	0.059943	0.054062	0.057342

Sharpening Filter - Overhead Time

Image Size	Filter Size	Naive	Shared Memory	Constant Memory	Texture Memory
256 x 256	3 x 3	0.066851	0.053856	0.05086	0.051455
	5 x 5	0.060539	0.053923	0.052218	0.052853
	9 x 9	0.063107	0.054783	0.055964	0.052518
512 x 512	3 x 3	0.063249	0.054889	0.052294	0.053067
	5 x 5	0.066288	0.05473	0.051798	0.050577
	9 x 9	0.064031	0.052632	0.051961	0.051078
1024 x 1024	3 x 3	0.0644	0.053654	0.05364	0.053326
	5 x 5	0.064512	0.053681	0.053515	0.053266
	9 x 9	0.070299	0.053248	0.051807	0.051805
2048 x 2048	3 x 3	0.069807	0.058458	0.055704	0.056625
	5 x 5	0.070972	0.059943	0.055607	0.063975
	9 x 9	0.065598	0.059443	0.062524	0.055691

We can immediately see that the overhead of the Naïve approach is substantially larger than that of the other approaches. This is to be expected, as the cost of loading an entire image and a filter into the device and loading the image back into the host is expensive. On average, the Texture Memory approach has the least overhead, indicating that loading an image into texture memory may be more optimal than the global memory. While the overhead time does increase with increases in M, N, L and P, it does so minimally, with changes in time occurring in the order of 0.01 seconds, indicating that the connection between device and host is not much of a bottleneck in this problem and on this hardware.

3.4 Measured Floating-point Computation Rate

The floating-point computation rate, $FLOPS = \frac{f}{Time}$, where f is the number of floating-point operations performed by the kernel, was calculated for each image size and filter size for each approach. The results are tabulated below, with all values in GFLOPS.

Averaging Filter - FLOPS

Image Size	Filter Size	Serial	Naive	Shared Memory	Constant Memory	Texture Memory
256 x 256	3 x 3	0.41476	155.04859	780.24936	259.88414	158.92480
	5 x 5	0.51869	195.84595	503.29103	402.39104	271.65729
	9 x 9	0.91089	257.41785	332.85389	482.40409	396.61416
512 x 512	3 x 3	0.72041	217.33306	1,033.83040	396.66526	282.53298
	5 x 5	1.39063	229.93518	566.54619	515.88595	370.17037
	9 x 9	1.78314	275.28489	300.64223	618.40717	438.87633
1024 x 1024	3 x 3	1.64941	234.90040	1,117.65449	449.94866	337.91320
	5 x 5	2.10534	238.90428	586.16510	494.48976	414.59082
	9 x 9	2.33012	261.42404	323.94836	579.64760	506.01689
2048 x 2048	3 x 3	2.25024	248.53237	984.23422	470.12179	317.10638
	5 x 5	2.39064	221.52407	552.02903	518.16955	372.21551
	9 x 9	2.42833	277.78424	323.63383	574.77934	515.14527

				Sharpening Filter - FLOPS		
Image Size	**Filter Size**	**Serial**	**Naive**	**Shared Memory**	**Constant Memory**	**Texture Memory**
256 x 256	3 x 3	0.93420	155.04859	795.25415	259.88414	158.92480
	5 x 5	1.55979	195.84595	508.22525	394.50102	271.65729
	9 x 9	2.02186	258.66141	315.08589	557.91082	405.62812
512 x 512	3 x 3	2.09170	215.49125	1,021.06706	396.66526	276.39096
	5 x 5	2.30940	229.15309	571.22839	519.21425	376.37435
	9 x 9	2.45588	270.41875	354.15654	616.92062	475.93700
1024 x 1024	3 x 3	1.56205	242.74910	1,119.54561	451.63386	344.78601
	5 x 5	2.34773	239.11627	585.33777	509.35575	412.05509
	9 x 9	2.43673	280.23768	358.63954	586.26836	511.45468
2048 x 2048	3 x 3	2.22037	248.38064	1,139.79579	469.66403	366.52927
	5 x 5	2.33574	241.14902	573.20222	518.16955	421.89281
	9 x 9	2.42579	281.73529	358.82122	589.55113	517.63540

Here the strengths of GPU parallelisation are evident, with FLOPS scaling vastly compared to the serial approach. As expected, the Naïve approach has a low floating-point computation rate when compared to the other approaches, although it is still substantially higher than the serial approach. We see the highest FLOPS in the Shared Memory approach, likely because the internal overhead is still large despite relatively few floating-point operations. Even then, the performance here is only a fraction of the theoretical maximum.
The general trend is that FLOPS increases with increases in M, N, L and P. This is true for all approached except the Shared Memory approach, whose FLOPS increases with increasing M and N but actually decreases with higher values of L and P. This is because the kernel time grows faster than the number of operations, as a smaller proportion of floating-point operations are dependent on L and P than in other kernels.

4 Conclusion

We conclude that the image convolution problem is a prime example of algorithms whose performances can be improved substantially by parallel approaches. After running the tests detailed in Section 3, we note that the best performance is achieved by the Constant Memory approach, and then by the Texture Memory approach. We hypothesize that the ideal parallel convolution algorithm would be a hybrid solution in which the image is loaded into texture memory while the filter is loaded into constant memory, as this approach maximises memory access speed while keeping overhead costs reasonable. Further work could include implementation of this approach, as well as other optimisation such as making use of the separability of certain filters.

References

[1] Nvidia, "CUDA C Programming Guide," March 2019.

[2] H. Wang, "COMS4040A & COMS7045A: High Performance Computing & Scientific Data Management - Introduction to CUDA C," 2019.