APPM4058A&COMS7238A: Digital Image Processing

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Contents

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Outline

Some basic gray level transformations

- Image negatives: for an image with gray levels in the range $[0, L-1]$,

$$s = L - 1 - r \tag{1}$$

- Log transformations: general form

$$s = c \log(r+1), \tag{2}$$

  where $r \geq 0$ and c is a constant. It maps a narrow range of low gray levels to a wider range of output levels.
- What does an inverse log transformations do?

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Some basic gray level transformations cont.

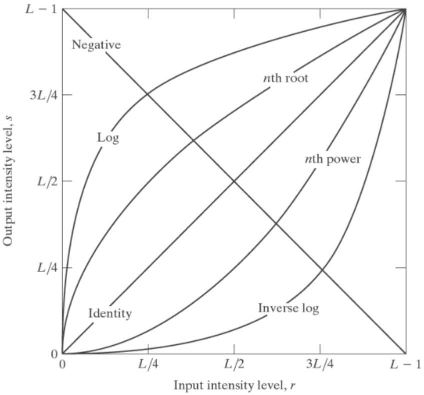

Figure: Some basic intensity transformations functions.

Some basic gray level transformations cont.

- Power-law transformations:

$$s = cr^{\gamma}, \tag{3}$$

where c and γ are positive constants.

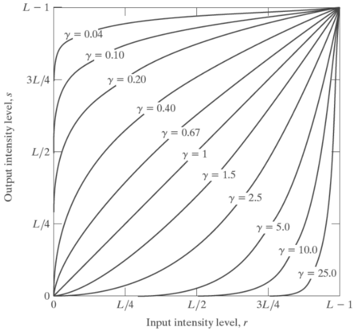

Figure: Plots of the equation $s = cr^{\gamma}$ for various values of γ ($c = 1$).

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Some basic gray level transformations cont.

(a) (b)

Some basic gray level transformations cont.

(a) (b)

Outline

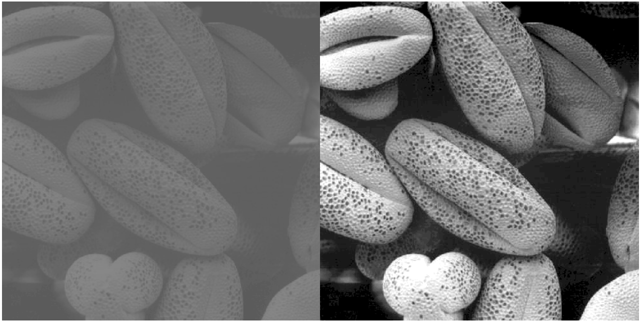
Histogram processing (1)

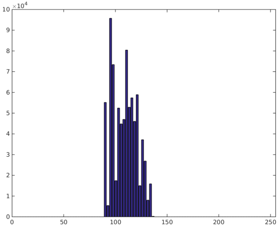
- The histogram of a digital image with gray levels in $[0, L-1]$ is a discrete function
$$h(r_k) = n_k, \tag{4}$$
where r_k is the kth gray level and n_k is the number of pixels in the image having gray level r_k.
- A normalized histogram is
$$p(r_k) = n_k/n, \tag{5}$$
where n is the total number of pixels.
- Loosely speaking, $p(r_k)$ is an estimate of the probability of occurrence of r_k;

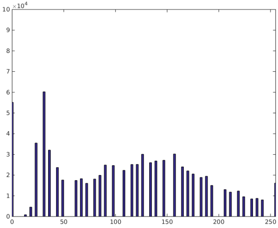
Histogram Processing (2)

- Histograms are the basis of numerous spatial domain processing techniques.
- Can be effectively used for image enhancement.
- Provides useful statistics.
- Useful in other image processing tasks such as segmentation and compression.
- Matlab function for dealing with histogram is `h=imhist(f,b)`, where `f` is the input image, `h` is its histogram, and `b` is the number of bins used in forming the histogram. the default of b is 256.

Histogram Processing (3)







Histogram Equalization (1)

- Consider the example below which has 16 elements and dynamic range $[0, 7]$ ($2^3 - 1 = 7$).
- Note the number of appearance of each distinctive value.

7	7	0	1
0	6	1	0
7	1	0	1
0	7	2	0

Histogram Equalization (2)

- We wish to equalise the histogram so that all 8 values are used equally often i.e. TWICE.
- It is important to preserve the ORDER from brightest to darkest.
- One possible solution to this is shown below.

7	7	0	1
0	6	1	0
7	1	0	1
0	7	2	0

7	7	2	4
2	5	4	1
6	3	1	3
0	6	5	0

Histogram Equalization (3)

- The concept is to modify the picture so that all brightness values are equally likely.
- The point is to allow all brightness values similar contrast.
- In principle this requires that the histogram of all the pixel values occurring is a straight line.
- This can only be approximately achieved in a real picture.

Histogram Equalization (4)

- The probablity of occurence of gray level r_k in an image is approximated by

$$p_r(r_k) = \frac{n_k}{n}, \quad k = 0, 1, \cdots, L-1. \tag{6}$$

- A transformation function

$$s_k = T(r_k) = \sum_{j=0}^{k} p_r(r_k) = \sum_{j=0}^{k} \frac{n_j}{n}, \quad k = 0, 1, \cdots, L-1. \tag{7}$$

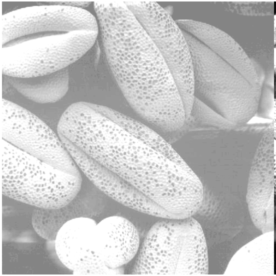
- Thus, a processed image is obtained by mapping each pixel with level r_k in the input image to a corresponding pixel with level s_k in the output image.

Histogram Equalization (5)

- (7) is called histogram equalization.
- (7) satisfies the following two conditions.
  1. $T(r)$ is single-valued and monotonically increasing;
  2. $0 \leq T(r) \leq 1$.
- (7) has the tendency of spreading the histogram of the input image so that the levels of the histogram-equalized image will span a fuller range of the gray scale.

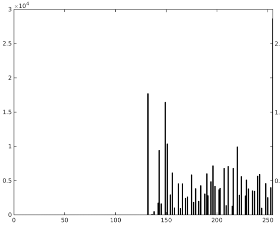
Matlab function for histogram equalization: `g=histeq(f,nlev)`, where `f` is the input image and `nlev` is the number of intensity levels specified for the output image. The default is 64.

Histogram Equalization Example (1)

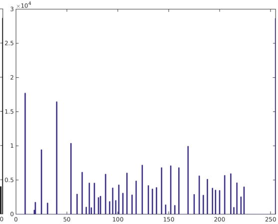


(a)

(b)

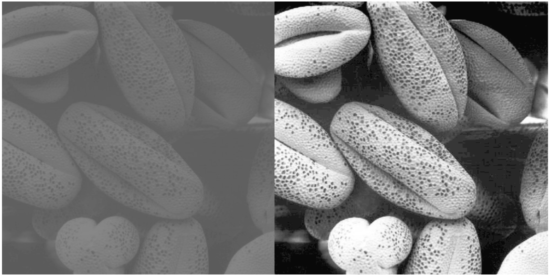


(c)

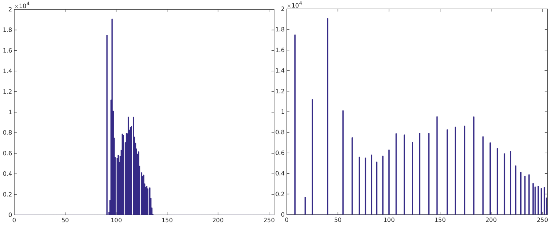


(d)

Histogram Equalization Example (2)



(a) (b)



(c) (d)

Histogram specification

- Histogram matching (or histogram specification): A method used to generate an image that has a specified histogram.
- Given an input image, we perform a histogram equalization as

$$s_k = T(r_k) = \sum_{j=0}^{k} p_r(r_j) = \sum_{j=0}^{k} \frac{n_j}{n} \quad k = 0, 1, 2, \ldots, L-1. \tag{8}$$

- Suppose we desire an ouput image with histogram with property

$$v_k = G(z_k) = \sum_{i=0}^{k} p_z(z_i) = s_k \quad k = 0, 1, 2, \ldots, L-1. \tag{9}$$

- The problem is to seek z_k that satisfies

$$z_k = G^{-1}(T(r_k)) = G^{-1}(s_k) \quad k = 0, 1, 2, \ldots, L-1. \tag{10}$$

Histogram specification cont.

- Since $v = s$, from (9) we need to find z's that satisfy $G(z) = s$ or $G(z) - s = 0$.
- Thus, we can find the value of z that corresponds to s by iterating on the values of z such that $G(z) - s = 0$ for $k = 0, 1, 2, \ldots, L - 1$
- The closest we can get to satisfying $G(z) - s = 0$ is to let $z = \hat{z}$ where $\hat{z}$ is the smallest integer in $[0, L - 1]$ such that

$$G(\hat{z}) - s_k \geq 0 \quad k = 0, 1, \ldots, L - 1 \tag{11}$$

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Histogram specification example cont.

Table: Intensity distribution and histogram for a 3-bit ($L = 2^3 - 1$) 64×64 digital image.

r_k	Original ($p_r(r_k)$)	CDF ($T(r_k)$)	CDF * (L-1)
0	0.19	0.19	1
1	0.25	0.44	3
2	0.21	0.65	5
3	0.16	0.81	6
4	0.08	0.89	6
5	0.06	0.95	7
6	0.03	0.98	7
7	0.02	1	7

Histogram specification example cont.

Table: The specified histogram

z_k	Specified ($p_z(z_k)$)	CDF ($G(z_k)$)	CDF * (L-1)
0	0	0	0
1	0	0	0
2	0	0	0
3	0.15	0.15	1
4	0.20	0.35	2
5	0.30	0.65	5
6	0.20	0.85	6
7	0.15	1	7

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Histogram specification example cont.

Table: The mappings of all possible values of s_k to z_k

r_k	s_k	v_k	Map (z_k)
0	1	0	3
1	3	0	4
2	5	0	5
3	6	1	6
4	6	2	6
5	7	5	7
6	7	6	7
7	7	7	7

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Histogram specification example cont.

Table: The specified and actual histogram

r_k	Specified	Actual
0	0	0
1	0	0
2	0	0
3	0.15	0.19
4	0.20	0.25
5	0.30	0.21
6	0.20	0.24
7	0.15	0.11

Histogram specification cont.

The histogram matching method can be summarized as follows.

1. Obtain the histogram of the input image
2. Use (8) to precompute a mapped level s_k for each level r_k
3. Obtain the transformation function G from $p_z(z_k)$ using (9)
4. Precompute z_k for each value of s_k using the iterative scheme defined in connection with (11)
5. For each pixel in the original image, if the values of that pixel is r_k, map this value to its corresponding level s_k, then map level s_k to its final level z_k. Use the precomputed values in the previous steps 2 and 4.

Histogram specification example

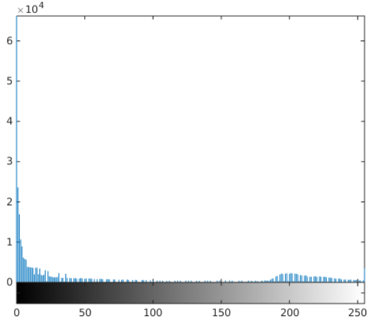
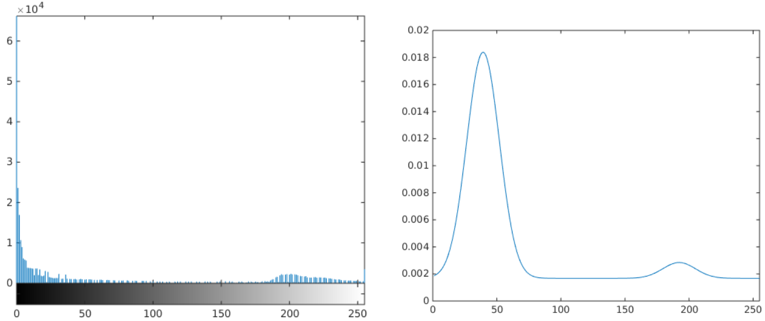


Figure: A Mars moon image and its histogram

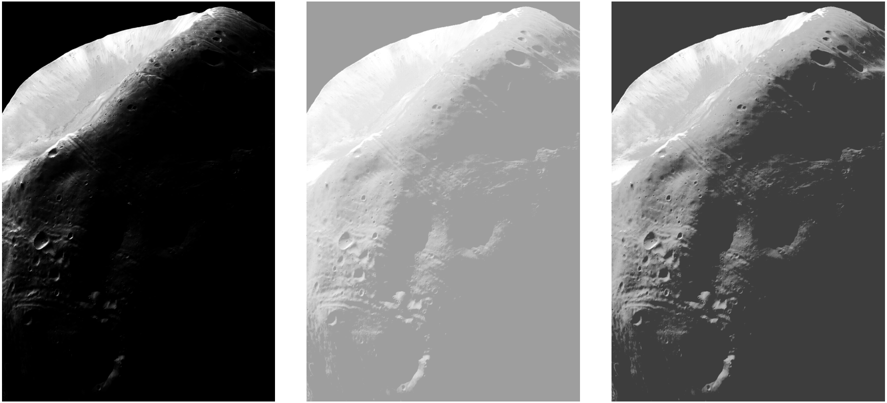
Histogram specification example cont.



(a) (b)

Figure: (a) The original histogram; (b) A specified histogram

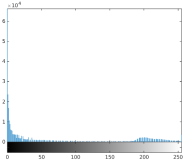
Histogram specification example cont.



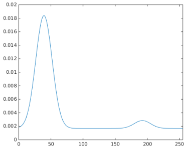
(a) (b) (c)

Figure: (a) The original image; (b) Histogram equalized image; (c) The resulting image from histogram matching

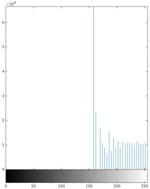
Histogram specification example cont.



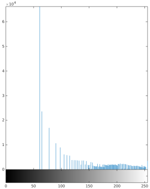
(a)



(b)



(c)



(d)

Figure: (a) The original histogram; (b) the histogram for histogram equalized image; (c) The specified histogram; (d) The histogram of the image from histogram matching

Adaptive histogram equalization

- Local enhancement: to devise transformation function based on the gray-level distribution in the neighbourhood of every pixel in the image.
- We can adapt the global techniques to local enhancement.
- The procedure is to define a square or rectangular neighbourhood and move the center of this area from pixel to pixel.
  1. At each location, the histogram of the points in the neighbourhood is computed and either a histogram equalization or histogram specification transformation function is obtained.
  2. The transformation function is applied to map the gray level of the pixel centered in the neighbourhood.
  3. The center of the neighbourhood is then moved to an adjacent pixel location
  4. Repeat steps 1 to 3.

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