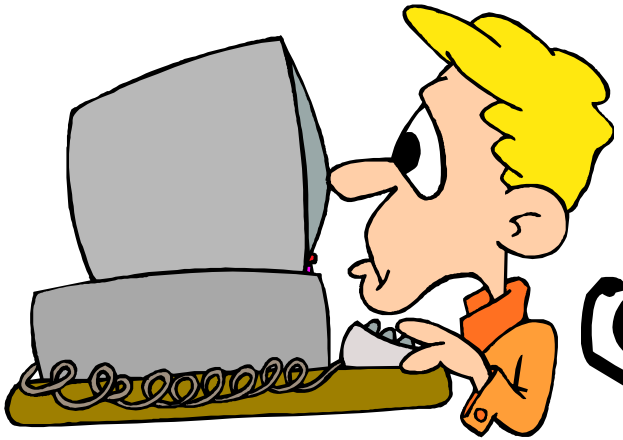
CS 1
Introduction to Computer Programming

Lecture 23: December 3, 2012

Advanced topics, part 1



Last time

- `Tkinter` widgets and GUI programming

Today

- Advanced topics, lecture 1
  - recursion
  - first-class functions
  - `lambda` expressions
  - higher-order functions
    - `map`, `filter`, `reduce`

Admin notes

- This is the last week of classes!
- Assignment 7 is due Friday at 2 AM
  - last assignment!
- Make sure you get caught up on all old assignments/redos!

Recursion

- A function is known as *recursive* when it calls itself
- Python allows you to define recursive functions
- Recursion has a reputation for difficulty, but it's really no big deal
- Let's look at a simple example

Sums

- How would you compute the sum of all the numbers from 0 to N (where N >= 0)?
- In Python, could just use the built-in **`sum`** function or a **`for`** or **`while`** loop
- Mathematically, we can define a solution like this:
  - **`sum_to_N(0) = 0`**
  - **`sum_to_N(n) = n + sum_to_N(n-1)`**

Sums

- `sum_to_N` definition:
  - `sum_to_N(0) = 0`
  - `sum_to_N(n) = n + sum_to_N(n-1)`
- We state that `sum_to_N(0)` is `0` by definition
- This is called a *base case*
  - it can be solved *immediately* (no recursion needed)
- To compute `sum_to_N(n)` for `n > 0`:
  - first compute `sum_to_N(n-1)`
  - then add `n` to it
  - This *must* be the correct answer!

Sums

- `sum_to_N` definition:
  - `sum_to_N(0) = 0`
  - `sum_to_N(n) = n + sum_to_N(n-1)`
- Another way to look at this:
- *If* `sum_to_N(n-1)` gives the correct sum from `0` to `n-1`
- *then* `sum_to_N(n)` will be the correct sum from `0` to `n`, because we've just added `n` to `sum_to_N(n-1)`

Sums

- `sum_to_N` definition:
  - `sum_to_N(0) = 0`
  - `sum_to_N(n) = n + sum_to_N(n-1)`
- The hard part:
- We *assume* that `sum_to_N` works correctly for all numbers `< n` when we define `sum_to_N` for the argument `n`
- How can we do this?

Sums

- `sum_to_N` definition:
  - `sum_to_N(0) = 0`
  - `sum_to_N(n) = n + sum_to_N(n-1)`
- This is an example of *mathematical induction*:
  - if `sum_to_N` works for some *base case* (`0`)
  - and we know that *if* `sum_to_N` works for some value `n-1`, *then* it will also work for the value `n`
  - then we conclude that `sum_to_N` works for all `n` from `0` to infinity!
    - works for 0, 1, 2, 3, ... as far up as we want to go

sum_to_N in Python

```
def sum_to_N(n):
    if n == 0:
        return 0     # base case
    else:
        return (n + sum_to_N(n - 1))
```

recursive call to `sum_to_N`

`sum_to_N` in Python

- Question 1: How is it possible that you can call a function while defining it?
- Answer: In fact, you *don't* call the function while defining it
  - You *never* call any functions while defining a function!
  - Instead, you define it with an internal call to what the function *will be* once it has been completely defined
  - The language implementation can handle the bookkeeping for you

`sum_to_N` in Python

- Question 2: What happens to the computation in progress in `sum_to_N` when a new recursive call to the same function starts?
- Answer: Every call to `sum_to_N` gets its own frame on the runtime stack, so when a recursive call to `sum_to_N` starts, a new frame is put on the stack, and the original call to `sum_to_N` still has all its own data in its own frame
  - recursion wouldn't work without a runtime stack!

A better example: sorting

- Recursion may just seem like a cute trick with no practical value
- But recursion can sometimes allow us to solve problems that would be difficult to solve without it
- Case study: sorting a list of numbers
  - Could use the Python `sort` method, but we want to do it ourselves
  - Recursion will be an essential component of our solution

Quicksort

- We will write a function which is a variation on a sorting algorithm called "quicksort"
- The idea:
  - Take the first number of a list of numbers, called the *pivot*
  - Divide the rest of the numbers into (a) a list of numbers *smaller than* the pivot, (b) a list of numbers *greater than or equal to* the pivot
  - Sort these sublists (recursively) using quicksort again!
  - Combine the sorted lists to get the final sorted list

Quicksort

```
def quicksort(lst):
    if lst == []:     # base case (no items in list)
        return []     # nothing to sort, return as is
    pivot   = lst[0]
    less    = []
    greater = []
    for item in lst[1:]: # examine rest of list
        if item < pivot:
            less.append(item)
        else:
            greater.append(item)
    all = quicksort(less) + [pivot] + quicksort(greater)
    return all
```

Quicksort

- While writing the `quicksort` function, we assume that the `quicksort` function itself will work on all smaller lists than the list given as an argument
- We make sure that all the recursive calls to `quicksort` are on lists that are strictly smaller than the list that was given as the argument
- We make sure that we return the correct value for the base case (empty list, no recursion needed)
- And then our function will work!

Recursion

- Recursion is also very useful when using tree-like data structures
  - *e.g.* made out of nodes which have a value, a left subtree, and a right subtree
- In this case, functions that act on the tree-like data must first deal with smaller trees
- Whenever a function has to deal with data which is a "smaller version of the original data" (lists → sublists, trees → subtrees), then recursion is useful

Recursion

- You'll learn more about recursion if you take CS 2
  - and *lots* more about recursion if you take CS 4!
- We have to move on to other topics

First-class functions

- We have seen that Python functions can be treated as data
  - *e.g.* callback functions for event handlers in Tkinter
- This is a general phenomenon
- Python functions are also Python objects, so they can be treated like any other Python object
  - assigned to a different name
  - stored in data structures
  - returned from functions
  - *etc.*

First-class functions

```
def double(x):
    return 2 * x
>>> foo = double
>>> foo(10)
20
>>> lst = [foo, foo, foo]
>>> lst[0](42)
84
```

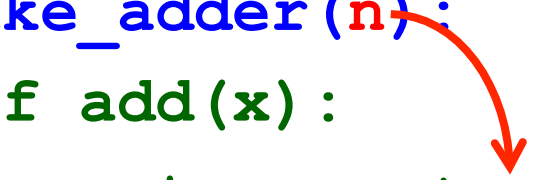
First-class functions

- You can return functions from functions:

```
def make_adder(n):
    def add(x):
        return x + n
    return add
>>> add5 = make_adder(5)
>>> add5(10)
15
>>> make_adder(15)(45)
60
```

First-class functions

```
def make_adder(n):
    def add(x):
        return x + n
    return add
```



- The `n` in `add` is the `n` which was the argument to `make_adder`
- We say that the `n` from `make_adder` is in *scope* inside the `add` function
- Even when `add` is returned, the `n` that was the argument to `make_adder` is retained

First-class functions

```
def make_adder(n):
    def add(x):
        return x + n
    return add
```

- Notice that the **`add`** function is a very trivial function that is only used in a single place
- The name **`add`** is not really important
- It would be nice if there was a more compact way to create a "function object" without having to give it a name

lambda

- In Python, an "anonymous function" can be created using a **lambda** expression
- The syntax:

```
lambda x: <expression involving x>
```

- is the same as writing

```
def func(x):
    return <expression involving x>
```

- except that the **lambda** expression has no name

lambda

- Examples:

```
lambda x : x * 2
```

- is the same as:

```
def double(x):
    return (x * 2)
```

lambda

- Examples:

```
lambda x, y: x * x + y * y
```

- is the same as:

```
def sum_squares(x, y):
    return (x * x + y * y)
```

lambda

- `make_adder` again:

```
def make_adder(n):
    return (lambda x: x + n)
```

- and that's it!
- We use `lambda` expressions primarily when we are defining a function that will only be used once
- `lambda` really only works for one-line functions
  - Python's indentation-based syntax gets in the way for longer functions

`lambda`

- As we will see shortly, **`lambda`** is often very handy when used with "higher-order" functions that take functions as arguments
- But first...

Interlude

- Another look at recursion!

Higher-order functions

- Python functions can be used as data by just referring to them by name (no arguments) or by creating a `lambda` expression
- What's the use of this?
- In Python, can define functions which take other functions as their arguments
- These are known as "higher-order" functions
- Several useful ones are built-in

map

- The `map` function takes two arguments:
  - a function requiring one argument
  - a list
- and applies the function argument to every element of the list, collecting all the results into a new list
- Let's see some examples

map

```
def double(x):
    return x * 2
>>> map(double, [1, 3, 5, 7, 9])
[2, 6, 10, 14, 18]
>>> map(lambda x: x * 2, [1, 2, 3, 4, 5])
[2, 4, 6, 8, 10]
```

- Notice: we can use a `lambda` expression anywhere we can use a function

`map`

- **`map`** allows us to convert a list into another (related) list of the same size where the elements of the second list are functions of the elements of the first list
  - *n.b.* **`map`** doesn't change the original list
- **`map`** takes a function as its first argument, so it's a higher-order function
  - sounds scary, but it's no big deal

`filter`

- Sometimes, we want to pick out certain elements of a list satisfying particular properties
- The properties can be represented by a *predicate* (a function returning a boolean (`True`/`False`) value)

filter

- For instance, if we wanted to pick out all positive elements of a list, a predicate could be:

```
def positive(x):
    '''Return True if x is positive.'''
    return (x > 0)
```

- This is so simple, we can just write it as:

```
lambda x: x > 0
```

`filter`

- There is a higher-order function called **`filter`** that takes two arguments:
  - a function (a predicate *i.e.* returning a boolean)
  - a list
- and returns a new list consisting of all of the elements of the original list that satisfied the predicate (for which the predicate returned **`True`**)
- The original list is "filtered" to give the new list

filter

- Examples:

```
>>> filter(positive, [-3, 1, -4, 1, -5, 9, -2, 6])
[1, 1, 9, 6]
>>> filter(lambda x: x > 0, [5, -3, -8, 9, 7, -9])
[5, 9, 7]
>>> filter(lambda x: x != 0, [1, 0, 0, 2, 0, 0, 0])
[1, 2]
>>> filter(lambda x: x > 5, [4, 1, -2, 0, 3])
[]
>>> filter(lambda x: x > 10, [])
[]
```

reduce

- Another common thing to want to do is to take a list and collapse it into a single value which is a function of all the elements of the list
- For instance:
  - sum all the elements of a list together
  - multiply all the elements of a list together
  - find the largest/smallest elements of a list
- In all cases, we are "reducing" a list into a single value

reduce

- Example: sum the elements of a list
- If list elements are `[i, j, k, l ...]` we want to compute
  - `(i + j)`
  - `((i + j) + k)`
  - `(((i + j) + k) + l)`
- *etc.* until all the elements of the list are added together
- In each case, we're adding the previous sum to the next element

reduce

- Python has a built-in function called **reduce** that does this:

```
>>> reduce(lambda x, y: x + y, [1, 2, 3, 4, 5])
15
>>> reduce(lambda x, y: x * y, [1, 2, 3, 4, 5])
120
>>> reduce(lambda x, y: max(x, y), [3, 1, 4, 1, 5])
5
```

reduce

- The first argument to `reduce` must be a function of two arguments
- Let's call it `f(x, y)`
- Then `reduce(f, [1, 2, 3, 4, 5])` is:
  - `f(f(f(f(1, 2), 3), 4), 5)`
- Notice the nested `f`s
  - second argument to `f` is the next element of the list
  - first argument is the result of reducing all previous elements of the list

reduce

- If we have:

```
def f(x, y):
    return x + y
```

- Then `reduce(f, [1, 2, 3, 4, 5])` becomes

```
((((1 + 2) + 3) + 4) + 5)
```

- which is just the sum of the list
- **`reduce`** makes it easy to do repetitive computations like this without having to write explicit loops

reduce

- If we use reduce on an empty list:

```
reduce(lambda x, y: x + y, [])
```

- we get:

```
TypeError: reduce() of empty sequence with no
  initial value
```

- `reduce` takes an extra (optional) argument that (if present) gives a value to use for the empty list

```
>>> reduce(lambda x, y: x + y, [], 0)
0
```

Functional programming

- There is a style of programming which uses higher-order functions and `lambda` expressions a lot
- It's called *functional programming*
  - in contrast to *e.g.* object-oriented programming
- Python supports functional programming, but not as well as some other languages
  - *e.g.* Scheme, Ocaml, Haskell
- CS 4 will cover functional programming in much greater depth (using Scheme and Ocaml)

Next time

- Last lecture in the class!
- Advanced topics, lecture 2
  - command-line arguments
  - list comprehensions
  - iterators
  - generators
- Where to go from here

Code summary

```
def sum_to_N(n):
    if n == 0:
        return 0     # base case
    else:
        return (n + sum_to_N(n - 1))
```

Code summary

```
def quicksort(lst):
    if lst == []:     # base case (no items in list)
        return []     # nothing to sort, return as is
    pivot   = lst[0]
    less    = []
    greater = []
    for item in lst[1:]: # examine rest of list
        if item < pivot:
            less.append(item)
        else:
            greater.append(item)
    all = quicksort(less) + [pivot] + quicksort(greater)
    return all
```

Code summary

```
def make_adder(n):
    def add(x):
        return x + n
    return add

# Shorter:
def make_adder(n):
    return (lambda x: x + n)
```