Problem 1

(a)

We'll start with

$$P(D=k)=\binom{n-1}{k}p^k(1-p)^{n-1-k}$$

Let's take the limit now

$$\lim_{n\to\infty}P(D=k)=\lim_{n\to\infty}\binom{n-1}{k}p^k(1-p)^{n-1-k}$$

Using $\lambda := (n-1)p$, we have

$$\lim_{n\to\infty}P(D=k)=\lim_{n\to\infty}\frac{(n-1)!}{k!(n-1-k)!}(\frac{\lambda}{n-1})^k(1-\frac{\lambda}{n-1})^{n-1-k}$$

Let's analyze each of these parts separately, starting with the factorial expression.

$$\frac{(n-1)!}{k!(n-1-k)!}=\frac{(n-1)((n-1)-1)\cdots((n-1)-k+1)}{k!}$$

The numerator can be approximated as $(n-1)^k+E$, where E represents the terms with powers $k-1$ or less. As $n\to\infty$, E effectively goes to zero, as the $(n-1)^k$ term dominates.
Now let's analyze the second part, $(\frac{\lambda}{n-1})^k$. We can see that this doesn't need much simplifying. But notice that the denominator here will cancel with the numerator we calculated in the first part.
Finally, let's analyze the third part. We can get the following using the approximation $(1+\frac{x}{n})^n=e^x$

$$\begin{aligned}&\lim_{n\to\infty}(1-\frac{\lambda}{n-1})^{n-1-k}\\ &=(1-\frac{\lambda}{n-1})^{n-1}(1-\frac{\lambda}{n-1})^{-k}\\ &=e^{-\lambda}1^{-k}\\ &=e^{-\lambda}\end{aligned}$$

Now we can put everything together.

$$\lim_{n\to\infty}P(D=k)=\frac{(n-1)^k}{k!}(\frac{\lambda}{n-1})^k e^{-\lambda}$$

$$\lim_{n\to\infty}P(D=k)=\frac{e^{-\lambda}\lambda^k}{k!}$$